Force on an Electron near a Metal in a Gravitational Field

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The three principal theoretical treatments of the force on an electron near a metal in a gravitational field are briefly reviewed, and the central arguments restated. The essential conclusion is that the current theory of metal surfaces conflicts with the apparent experimental findings of Witteborn and Fairbank, but agrees with recent experiments by Beams. A simpler experiment, directed at this specific problem, is proposed.

EXPERIMENTS by Witteborn and Fairbank[1] have indicated that a vertical copper tube (standing, of course, in a gravitational field) generates an electric field just canceling the gravitational field on an electron in the space inside the cylinder; more precisely, they find a net force less than $0.09mg$ upon an electron in the vacuum within the cylinder. Prior work by Schiff and Barnhill[2] predicted this result; subsequent work by Dessler *et al.*[3] predicted a force larger than mg by a factor of 10^4–10^5. Herring[4] reconciled these theoretical studies, pointed out an error in one important estimate made by Schiff and Barnhill, and indicated that it would require a quite accidental cancellation of surface and body effects to yield theoretically the small observed force. A more recent measurement of fields near a spinning rotor by Beams[5] appears to support the large-field point of view. In spite of the controversy and the complexity of the exhaustive theoretical studies which have been made, the theoretical status seems quite clear.

The simpler problem is the force on an electron *within* a metal in a gravitational field. This problem was specifically addressed by Dessler *et al.*[3] The result may in fact be obtained almost trivially. A small dilatation $\Delta(\mathbf{r})$ in a simple metal causes a net potential $V=-\frac{2}{3}E_F\Delta(\mathbf{r})$, where E_F is the Fermi energy $\frac{1}{2}mv_F^2$. This follows from a linearized self-consistent Fermi-Thomas approximation, from the self-consistent electron-phonon interaction treated by Bardeen,[6] or from a pseudopotential treatment.[7] It is confirmed at least semiquantitatively by the comparison of theoretical and experimental ultrasonic attenuation and temperature-dependent resistivity. The pressure (measured in a horizontal plane) in a metal in a gravitational field varies with altitude z as ρgz, where ρ is the metallic density. Thus, the local dilatation is $-\rho gz/c$, where c is the elastic constant giving dilatation under uniaxial loading. Thus, the force downward on the electron, $\partial V/\partial z$, is

$$\partial V/\partial z=\tfrac{1}{3}mgv_F^2\rho/c$$

but c/ρ may be taken to be the velocity of sound v_s squared (though slight corrections are required to convert from c_{11}, which enters longitudinal sound velocity, to the modulus c). This force is greater than mg by a factor $v_F^2/3v_s^2$, which is equal to the atomic mass divided by the electron mass in free-electron-like metals and is of the order of 6×10^4 in copper, as suggested by Dessler *et al.*

One might naively assume that the tangential component of electric field is continuous across the surface, and that the field outside the metal is that given above. However, if the surface dipole varies with height (that is, with dilation), the tangential component will change across the surface. The usual conservation of tangential component is simply a restatement that the field is derivable from a potential; there is no reason why the potential in this case might not be shaped like a swimming pool. That is, the potential plotted in the z direction as a function of coordinates x and y would be flat outside of the metal, like the deck of the pool, while inside there is a gradient from the deep to the shallow end. This is suggested by the experiment.

It is not difficult to estimate the effect of the surface dipole. The electron wave functions extend beyond the positive ionic charge density at a metallic surface, giving rise to the electric dipole. This dipole must be sufficient to raise the rest energy of an electron outside the metal above the Fermi energy within the metal; otherwise the electron would not be bound to the metal. The difference in these two energies is called the work function and is of the order of a few electron volts. Herring points out that the Fermi energy is a constant of a system in equilibrium, and that the Witteborn-Fairbank finding of no force outside implies precisely that the work function measured at the top and at the bottom of the cylinder is the same; that the dilatation has not changed the work function. Any simple model of the surface, on the other hand, will yield a change in work function of the order of $E_F\Delta$, leading to a force outside the metal of the order of that found *within* the metal above. Such an estimate includes both surface and body effects and, as suggested by Dessler and by Herring *et al.*, there is no reason to expect cancellation of the two on the scale of one part in 10^5 or 10^6.

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A direct measurement of the change in work function with pressure has apparently not been made, but from related experiments it seems clear that the above estimate is essentially correct. Specifically, Beams[5] has measured fields near the surface of a high-speed aluminum rotor. Here again there are gradients in the local dilatation. The observed fields are roughly in accord with those expected within the metal. This would seem to rule out an explanation of the Witteborn-Fairbank result in terms of simple surface effects. However, there are many differences in the two experiments, one conspicuous one being the order of magnitudes of difference between the dilatations involved.

Both experiments are difficult ones. For that reason, it is not easy to make large changes in the gradient of dilatation, nor to look for relaxation effects over wide time scales. The question is sufficiently important to the understanding of metallic surfaces that a simpler and more direct experiment is much to be desired. For example, a metallic bar bent as a horseshoe could be compressed on one arm. A compression of 0.1% would be expected to yield a contact potential difference of the order of $10^{-3}E_F$ or 6 mV; that is, a field in the space between arms corresponding that potential drop. The Witteborn-Fairbank experiment would suggest a field smaller by a factor of 10^5.

Note added in proof. Recent experiments by P. P. Craig [Phys. Rev. Letters **22**, 700 (1969)] utilized uniform compression and do not, therefore, specifically address the question of dilatation gradients.

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Possible Zero at a Wrong-Signature Sense Point on the Δ Trajectory*

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We comment on the proposed vanishing of the Δ Regge trajectory residue at $\alpha_\Delta=\frac{1}{2}$, giving experimental tests of this hypothesis and an interpretation of such a result.

THE possibility that, in backward meson-baryon scattering involving exchange of the Δ Regge trajectory, the residue functions vanish at the wrong-signature sense point, where $\alpha_\Delta=\frac{1}{2}$, has been considered by Igi *et al.*[1] In this paper we would like to suggest some experimental tests of this hypothesis, and to comment on the interpretation of such a zero.

According to some conventional Regge-pole fits to backward π^-p elastic scattering,[2] the point where $\alpha_\Delta=\frac{1}{2}$ is at $u\approx0.35$ $(\text{GeV}/c)^2$. Other fits[3] give an even larger value of u at this point. It is difficult to detect a zero in $d\sigma/dt$ even at $u\approx0.35$ $(\text{GeV}/c)^2$ in elastic scattering, where $u<0.14$ $(\text{GeV}/c)^2$ for $E_L>2.5$ GeV. In addition, the Carnegie-BNL measurements of backward π^-p elastic scattering[3] show less of a flattening than the Cornell-BNL data[4] that Igi *et al.*[1] concentrated on, so one would like additional predictions to test the presence of the zero. These are available in experiments (which are currently in progress[5]) involving inelastic backward scattering. For example, as shown in Table I, in backward production of $N^*(1688)$ at $E_L=2.5$ GeV, we find $u=0.35$ in the physical region. The most favorable situation not involving resonances but with reasonable cross sections appears to be $K^-p\rightarrow\Sigma^+\pi^-$. If there is a turnover in backward π^-p elastic scattering due to a zero at a wrong-signature sense point on the Δ trajectory, more pronounced dips should appear in these inelastic reactions.

We would also like to note that the treatment of ghost-eliminating zeros in Regge-pole amplitudes should in principle be different at wrong-signature and at right-signature points. Because of the zero of the signature factor in the wrong-signature situation, two of the four possibilities in the right-signature situation become special cases of the other two. In the right-signature case each of the four mechanisms[6] has its own

TABLE I. The value of u in the backward direction for various reactions, in $(\text{GeV}/c)^2$.

E_L (GeV)	$\pi N\rightarrow N^*(1688)\pi$	$\pi N\rightarrow N^*(1240)\pi$	$K^-p\rightarrow\Sigma^+\pi^-$	$\pi^-p\rightarrow p\pi^-$
2.5	0.43	0.23	0.18	0.14
4.0	0.29	0.16	0.12	0.09
6.0	0.20	0.11	0.08	0.06

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