Comments and Addenda

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Comment on Quantum Electrodynamics*

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It is shown that the supplementary condition in quantum electrodynamics with an indefinite metric is covariant as well as consistent.

INTRODUCTION

IN order to overcome well-known difficulties in the quantization of the electromagnetic field, the author[1] proposed a formulation of quantum electrodynamics in which an indefinite metric space was used for the photon states instead of the usual Hilbert space. Moreover, a supplementary condition was imposed on the physical states to ensure that the states of negative norm remain unobservable.

Recently, Haller and Landovitz[2] have drawn attention to some confusion regarding the supplementary condition and discussed its formulation in the presence of interaction. We shall, however, show that our original paper already gives the correct form of the supplementary condition, which fully satisfies the requirements of covariance and consistency both in the Heisenberg and in the interaction picture.

HEISENBERG PICTURE

The supplementary condition in the Heisenberg picture is given by[1]

$$\Omega^+\Psi=0\,, \tag{1}$$

with

$$\Omega\equiv\partial_\mu A_\mu\,, \tag{2}$$

where the superscript + denotes the positive-frequency part.[3] Since

$$\Box^2\Omega=\partial_\mu(\Box^2 A_\mu)=-c^{-1}\partial_\mu j_\mu=0\,,$$

it is clearly possible to split Ω into the positive- and negative-frequency parts in a Lorentz covariant manner, and thus

$$\Box^2\Omega^+=0\,,\quad \Box^2\Omega^-=0\,. \tag{3}$$

We can expand Ω^+ in powers of t for arbitrary values of the space coordinates as

$$\Omega^+=[\Omega^+]_{t=0}+t[\Omega_\mu{}^{+\prime}]_{t=0}+\frac{t^2}{2!}[\Omega_\mu{}^{+\prime\prime}]_{t=0}+\frac{t^3}{3!}[\Omega_\mu{}^{+\prime\prime\prime}]_{t=0}+\cdots\,, \tag{4}$$

where a prime denotes differentiation with respect to t. Since we can convert $[\Omega_\mu{}^{+\prime\prime}]_{t=0}$, $[\Omega_\mu{}^{+\prime\prime\prime}]_{t=0}$, $\cdots$ into space derivatives of $[\Omega^+]_{t=0}$ and $[\Omega^{+\prime}]_{t=0}$ by means of the relation $\Omega^{+\prime\prime}=c^2\partial_i{}^2\Omega^+$, it follows that if

$$\Omega^+\Psi=0 \quad\text{and}\quad \Omega^{+\prime}\Psi=0 \quad\text{at}\quad t=0\,, \tag{5}$$

then

$$\Omega^+\Psi=0$$

for all values of t. This proves the consistency of the supplementary condition.

INTERACTION PICTURE

The appearance of the positive-frequency part in the supplementary condition (1) complicates its transformation to the interaction picture. However, we observe that the supplementary condition in the interaction picture should satisfy the following requirements: (a) In the absence of interaction it should be equivalent to the supplementary condition in the Heisenberg picture. (b) It should be consistent with the equation of motion of the state vector in the interaction picture,

$$i\hbar\frac{d\Psi(t)}{dt}=\mathcal{H}(t)\Psi(t)\,,\quad \mathcal{H}(t)=-\frac{1}{c}\int A_\nu(x)j_\nu(x)d\tau\,. \tag{6}$$

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[1] S. N. Gupta, Proc. Phys. Soc. (London) **A63**, 681 (1950). For a simplified treatment of the indefinite metric space, see S. N. Gupta, Can. J. Phys. **35**, 961 (1957).

[2] K. Haller and L. F. Landovitz, Phys. Rev. **171**, 1749 (1968).

[3] Supplementary condition $(\partial_\mu A_\mu)^+\Psi=0$ should not be confused with $\partial_\mu A_\mu{}^+\Psi=0$, which holds only in the absence of interaction. This confusion is presumably responsible for the difficulties discussed in Ref. 2.

We shall now show that the appropriate supplementary condition in the interaction picture is given by[4]

$$\Omega^+(x)\Psi(t)=0, \quad (7)$$

with

$$\Omega^+(x)\equiv\partial_\mu A_\mu{}^+(x)-\frac{1}{c}\int[D^+(x-x')j_0(x')]_{t'=t}d\tau. \quad (8)$$

In the absence of interaction, the condition (7) is, of course, equivalent to (1), and it only remains to be shown that (7) satisfies the consistency requirement. This can be established most conveniently by showing that (7) is compatible with the relations, which can be obtained by repeatedly differentiating it with respect to t and using the equation of motion (6).

By differentiating (7) with respect to x_0 and using (6), we obtain

$$\{\partial_0\Omega^+(x)-(i/c\hbar)\Omega^+(x)\mathcal{H}(t)\}\Psi(t)=0,$$

which can be expressed, with the help of (7), as

$$\{\partial_0\Omega^+(x)-(i/c\hbar)[\Omega^+(x),\mathcal{H}(t)]\}\Psi(t)=0. \quad (9)$$

But, according to (8),

$$\partial_0\Omega^+(x)=\partial_0\partial_\mu A_\mu{}^+(x)-\frac{1}{c}\int D^+(0)\partial_0 j_0(\mathbf{r}',t)d\tau'$$
$$=\partial_0\partial_\mu A_\mu{}^+(x)-\frac{1}{c}\int[D^+(x-x')\partial_0' j_0(x')]_{t'=t}d\tau',$$

which gives, on putting $\partial_0' j_0(x')=-\partial_i' j_i(x')$ and dropping the volume integral of a divergence term,

$$\partial_0\Omega^+(x)=\partial_0\partial_\mu A_\mu{}^+(x)-\frac{1}{c}\int[\{\partial_i' D^+(x-x')\}j_i(x')]_{t'=t}d\tau'$$
$$=\partial_0\partial_\mu A_\mu{}^+(x)+\frac{1}{c}\int[\{\partial_i D^+(x-x')\}j_i(x')]_{t'=t}d\tau'. \quad (10)$$

Moreover, in view of

$$[j_\nu(x),j_0(x')]_{t'=t}=ie^2c^2[\bar{\Psi}(x)\gamma_\nu\psi(x')-\bar{\Psi}(x')\gamma_\nu\psi(x)]_{t'=t}\delta(\mathbf{r}-\mathbf{r}'),$$

we have

$$[\Omega^+(x),\mathcal{H}(t)]=[\partial_\mu A_\mu{}^+(x),\mathcal{H}(t)]$$
$$=-i\hbar\int[\{\partial_\nu D^+(x-x')\}j_\nu(x')]_{t'=t}d\tau'. \quad (11)$$

Substitution of (10) and (11) into (9) yields

$$\left(\partial_0\partial_\mu A_\mu{}^+(x)-\frac{1}{c}\int[\{\partial_0 D^+(x-x')\}j_0(x')]_{t'=t}d\tau'\right)\times\Psi(t)=0. \quad (12)$$

By differentiating (12) with respect to x_0 and using (6), we obtain, by repeating the above procedure,

$$\left(\partial_0{}^2\partial_\mu A_\mu{}^+(x)-\frac{1}{c}\int[\{\partial_0{}^2 D^+(x-x')\}j_0(x')]_{t'=t}d\tau'\right)\times\Psi(t)=0, \quad (13)$$

and, in view of

$$\Box^2 A_\mu{}^+(x)=0, \quad \Box^2 D^+(x-x')=0, \quad (14)$$

we can express (13) as

$$\partial_i{}^2[\Omega^+(x)\Psi(t)]=0, \quad (15)$$

which is evidently compatible with (7).

It further follows that all the relations obtained by repeated applications of the above procedure are equivalent to those obtained from (7) or (12) by repeated applications of $\partial_i{}^2$.

[4] Transformation of the author's supplementary condition to the interaction picture was carried out by K. Bleuler, Helv. Phys. Acta **23**, 567 (1950). We shall, however, avoid his use of multiple time by treating Ω^+ as a function of a single time variable.