Fermion Quarks of Spin $\frac{3}{2}$

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The usual quark model of baryons requires quarks to obey effective Bose statistics if they are to be in space-symmetric states. We show that if quarks had spin $\frac{3}{2}$ they could obey the usual Fermi statistics, even with space-symmetric wave functions. Using spin-$\frac{3}{2}$ quarks, we construct a completely antisymmetric three-quark state of total spin $\frac{3}{2}$ corresponding to the baryon decuplet, and a mixed symmetry state of total spin $\frac{1}{2}$ corresponding to the baryon octet. A simple dynamical assumption about the spin dependence of quark-quark forces limits the s-wave baryon states to the octet and decuplet, and saturates the quark binding at three. We find very little difference in the experimentally tested quark-model predictions when spin-$\frac{3}{2}$ quarks are used. The main differences we do find are: the well-known prediction that $-\mu_p/\mu_n=\frac{3}{2}$ is lost with spin-$\frac{3}{2}$ quarks, and they must have anomalous magnetic moments. The predicted Σ^+ moment is essentially unchanged, but the Ξ^- and Ω^- moments change when spin-$\frac{3}{2}$ quarks are used, and their measurement would test the spin of the quark. With spin-$\frac{3}{2}$ quarks, mass relations within $SU(3)$ multiplets are kept while the less well satisfied relations connecting octet to decuplet mass differences get broken by a small amount.

I. INTRODUCTION

THE success of the quark model[1,2] has seemed to be linked with the requirement that spin-$\frac{1}{2}$ quarks obey Bose statistics. This is because the low-lying baryon octet and decuplet are obtained by requiring symmetric spin states for identical quarks. For quarks in s states this implies Bose statistics. Attempts to use antisymmetric p states[3] have not been successful because the many nodes of the antisymmetric spatial state make it difficult to explain why it would lie lowest, even among p states.[4] These nodes would also be expected to show up in the nucleon electromagnetic form factors,[5] but there is no evidence for them there. Models for quarks that would obey effective Bose statistics have been proposed using parastatistics[6] or unseen internal degrees of freedom.[7,8] We refer to this class of models as the "symmetric quark model."[9] While these models may be correct, it is of interest to ask whether it is indeed necessary to require effective Bose statistics for quarks.

We suggest here a model using fermion quarks that can couple in s waves to produce the low-lying baryon octet and decuplet. This can be done if the quarks have spin $\frac{3}{2}$.[10] In fact, most of the results obtained with spin-$\frac{1}{2}$ Bose quarks follow also for spin-$\frac{3}{2}$ Fermi quarks. In the following, we look at the multiplet structure and the static properties (masses and magnetic moments) of the low-lying baryon states. A reasonable dynamical assumption restricts the three quark s states to the spin-$\frac{1}{2}$ octet and spin-$\frac{3}{2}$ decuplet and also provides for quark saturation at three. The major difference in the predictions of the spin-$\frac{3}{2}$ fermion quark model is that the ratio $-\mu_p/\mu_n$ is predicted to be $\frac{5}{4}$ if we assume that the nucleon-quark magnetic moments are proportional to their charges. The same assumption with spin-$\frac{1}{2}$ Bose quarks leads to the prediction that $-\mu_p/\mu_n=\frac{3}{2}$, which is closer to the experimental value. This result means that spin-$\frac{3}{2}$ quarks would have to have anomalous moments. All other mass differences and magnetic-moment predictions are either the same as for spin-$\frac{1}{2}$ quarks or favor spin-$\frac{3}{2}$ quarks where they can be compared with experiment. The main experimental test of the spin of the quark would be experimental determinations of the magnetic moments of the Ξ^- and Ω^- hyperons, for which the two models give different predictions.

In the next section, we consider the baryon multiplet structure for spin-$\frac{3}{2}$ fermion quarks. In Sec. III, we consider baryon mass differences and in Sec. IV, magnetic moments. Mesons are treated in Sec. V. In Sec. VI we summarize the differences between spin-$\frac{3}{2}$ fermion quarks and spin-$\frac{1}{2}$ symmetric quarks.

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[1] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN Reports Nos. TH 401 and TH 412. 1964 (unpublished).

[2] Reviews of the quark model have been given by R. H. Dalitz [in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, Cal., 1967), pp. 215–236, and in *Proceedings of the Second Hawaii Topical Conference in Particle Physics* (University of Hawaii Press, Honolulu, Hawaii, 1967)] and by G. Morpurgo [in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, Switzerland, 1968), p. 225]. These reviews have extensive quark bibliographies.

[3] See, for instance, the first work of Ref. 2.

[4] A. N. Mitra, Phys. Rev. **142**, 1119 (1966).

[5] A. N. Mitra and Rabi Majumdar, Phys. Rev. **150**, 1194 (1966). See, however, R. E. Kreps and J. J. de Swart, *ibid.* **162**, 1729 (1967), who indicate that antisymmetric wave functions can be found that give form factors in reasonable agreement with current experiments.

[6] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).

[7] M. Y. Han and Y. Nambu, Phys. Rev. **139**, B1006 (1965).

[8] J. Franklin, Phys. Rev. **172**, 1807 (1968). We refer to this work as I and generally follow its notation.

[9] O. W. Greenberg and M. Resnikoff, Phys. Rev. **163**, 1844 (1967).

[10] That quarks need not have spin $\frac{1}{2}$ was first suggested to the author by Don Lichtenberg (Temple University colloquium). R. Acharya and R. Narayanaswamy, Phys. Rev. **142**, 1085 (1966), have looked at the representations of a combination of spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ quarks satisfying the Bhabha equation, H. J. Bhabha, Phil. Mag. **43**, 33 (1952).

II. BARYON STATES

It is a straightforward matter to combine three spin-$\frac{3}{2}$ states to form states of definite $\mathbf{S}=\mathbf{s}_1+\mathbf{s}_2+\mathbf{s}_3$, using Clebsch-Gordan algebra. The result is that there is only one completely antisymmetric state under spin exchange. This state has $S=\frac{3}{2}$ and leads to the baryon decuplet with fermion quarks in a space-symmetric state. The other states have mixed spin symmetry leading to $SU(3)$ octets with $S=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$, or are completely spin-symmetric leading to $SU(3)$ singlets with $S=\frac{3}{2}, \frac{5}{2}$, or $\frac{9}{2}$, for fermion quarks in a space-symmetric state. Although it might be possible to fit all these states into the higher baryon spectroscopy, we prefer a model that does not contain them as s states. One dynamical assumption—that the $(\mathbf{s}_1+\mathbf{s}_2)=\mathbf{3}$ quark-quark force is repulsive (enough to prevent binding)—leaves only the $S=\frac{1}{2}$ octet as a possible bound state, along with the $S=\frac{3}{2}$ decuplet. We note that this dynamical assumption involves a strong breaking of $SU(6)$ symmetry. There is no loss of economy in requiring a dynamical assumption to limit the number of space-symmetric states, since, in models with unseen degrees of freedom, similar dynamical assumptions are required in order to achieve the effective Bose statistics. Here, at least, the dynamical assumption is made in the usual, seen space. As in the models with unseen degrees of freedom, this dynamical assumption also provides saturation at three quarks. That is, more than three quarks cannot be combined without introducing spin-3 combinations of quark pairs.

The multiplet structure of the higher orbital states of spin-$\frac{3}{2}$ quarks is just the same as in any theory that uses spin-$\frac{1}{2}$ quarks and assumes an $SU(6)\times O(3)$ structure.[11] This is true even though our dynamical assumptions violate $SU(6)$ strongly. For a state with mixed spatial symmetry, we find that the symmetries combine in just the same way as they did for spin-$\frac{1}{2}$ quarks to produce a $(70,1^-)$ state for spin-$\frac{3}{2}$ quarks. A similar situation holds for the other orbital states contained in $SU(6)\times O(3)$.

III. MAGNETIC MOMENTS

The baryon magnetic moments with spin-$\frac{3}{2}$ fermion quarks can be derived in the same way as for spin-$\frac{1}{2}$ boson quarks,[8,12] if we assume no orbital angular momentum and neglect relativistic and exchange effects. The results are

$$\mu_p=2\mu_{\mathcal{P}}-\mu_{\mathcal{N}}, \tag{1}$$

for the proton moment, and

$$\mu_\Lambda=\tfrac{1}{6}(10\mu_\lambda-2\mu_{\mathcal{P}}-2\mu_{\mathcal{N}}), \tag{2}$$

for the lambda moment. Here, $\mu_{\mathcal{P}}$, $\mu_{\mathcal{N}}$, and μ_λ are the magnetic moments of the $\mathcal{P}$, $\mathcal{N}$, and λ quarks, normalized so that $\boldsymbol{\mu}_i=2\mu_i\mathbf{s}_i$ to correspond to the usual normalization for spin-$\frac{1}{2}$ quarks. The other six moments of the spin-$\frac{1}{2}$ baryons follow from Eq. (1) by appropriate substitution of quark moments as in Eq. (3) of I.

If we make the assumption that the quark magnetic moments are proportional to the quark charges, Eq. (1) leads to $-\mu(p)/\mu(n)=1.25$. This is different from the corresponding symmetric-quark result[12] of 1.50 and not in as good agreement with the experimental ratio[13] 1.46. This result would be changed if quarks had anomalous moments, however, and need not be taken as a conclusive test of the model.[14] We take it as an indication that spin-$\frac{3}{2}$ quarks would have anomalous magnetic moments.

From Eqs. (1) and (2) and similar equations for the other baryons, we can derive linear relations for baryon moments of the type given in Eq. (5) of I. These will be independent of any assumptions about quark moments. The results for spin-$\frac{3}{2}$ fermion quarks are

$$\mu_{\Sigma^+}=(1/15)(17\mu_p+7\mu_n-9\mu_\Lambda)=2.70\pm0.10, \tag{3a}$$

$$\mu_{\Xi^0}=(1/15)(-4\mu_p+\mu_n+18\mu_\Lambda)=-1.74\pm0.20, \tag{3b}$$

$$\mu_{\Xi^-}=(1/15)(\mu_p-4\mu_n+18\mu_\Lambda)=-0.18\pm0.20, \tag{3c}$$

$$\mu_{\Omega^-}=(3/5)(\mu_p+\mu_n+3\mu_\Lambda)=-0.79\pm0.30, \tag{3d}$$

$$\mu(N^{*+},p)=(10/27)^{1/2}(\mu_p-\mu_n), \tag{3e}$$

where we have listed only those moments most likely to be measured. The numerical values in Eqs. (3) have been obtained using the experimental values $\mu_p=2.79$, $\mu_n=-1.91$, $\mu_\Lambda=-0.73\pm0.18$ (all in proton magnetons). The prediction for the Σ^+ moment is the same in either quark model and in agreement with experiment (2.4 ± 0.6). The only magnetic-moment predictions that have changed are the Ξ^- and Ω^- moments for which the corresponding symmetric-quark-model predictions are[8] $\mu_{\Xi^-}=-0.65\pm0.20$ and $\mu_{\Omega^-}=-2.19\pm0.48$. A good experimental determination of either of these moments would distinguish between spin-$\frac{1}{2}$ symmetric quarks and spin-$\frac{3}{2}$ fermion quarks if the nonstatic corrections are small. In an earlier paper[15] we have derived sum rules for the Ξ^- and Ξ^0 moments that are independent of the usual nonstatic corrections. Since those sum rules did not depend on the spin of the quarks, they would hold equally well here, given the same assumptions.

The prediction for the transition moment $\mu(N^{*+},p)$ given by Eq. (3e) is $(125/108)^{1/2}$ times that for spin-$\frac{1}{2}$ quarks. This transition moment has been used by Dalitz and Sutherland[16] to calculate the rate for $N^{*+}\rightarrow p+\gamma$

[11] K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters 14, 163 (1965).

[12] G. Morpurgo, Physics 2, 95 (1965); H. R. Rubinstein, F. Scheck, and R. H. Socolow, Phys. Rev. 154, 1608 (1967).

[13] The experimental values in this paper are generally taken from the data compilation of A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. 40, 77 (1968), revised as UCRL-8030, August 1968 (unpublished).

[14] This point is discussed in I. See also footnote 3 of Ref. 26.

[15] J. Franklin, Phys. Rev. (to be published).

[16] R. Dalitz and D. G. Sutherland, Phys. Rev. 146, 1180 (1966).

decay, with the result[17] $\Gamma_{1/2}(N^{*+}\to p+\gamma)=0.40$ MeV for spin-$\frac{1}{2}$ quarks. For spin-$\frac{3}{2}$ quarks, this result would be (125/108) times as much, or $\Gamma_{3/2}(N^{*+}\to p+\gamma)=0.46$ MeV. The same factor (125/108) enters in calculating the rate for $N^{*++}\to p+\pi^+$ decay. This has been calculated for spin-$\frac{1}{2}$ quarks by Becchi and Morpurgo[18] with the result[19] $\Gamma_{1/2}(N^{*++}\to p+\pi^+)=80$ MeV, and thus we have $\Gamma_{3/2}(N^{*++}\to p+\pi^+)=93$ MeV for spin-$\frac{3}{2}$ quarks. The experimental estimates of the N^{*++} width range from[20] 100–120 MeV so that the spin-$\frac{3}{2}$ quarks seem to give a slightly better result, although still low. However, final-state interactions might be expected to increase the quark-model predictions for a strong decay width and this is not a conclusive test. The ratio of the γ decay to the pion decay of the N^* is independent of the quark spin and is given by[21] $\Gamma(N^{*+}\to p+\gamma)/\Gamma(N^{*++}\to p+\pi^+)=0.0050$ which agrees with the ratio (0.0054 ± 0.0002) estimated from pion photoproduction experiments.[16]

IV. BARYON MASSES

Mass formulas for spin-$\frac{3}{2}$ fermion quarks can be derived by the same method[12] as for symmetric quarks. For symmetric quarks, there are seven mass formulas that have been compared with experiment. These are given by Eqs. (21)–(24) of I. There is also a mass inequality given by the requirement that the left-hand side of Eq. (37) of I must be positive. Of the mass equalities, three relate masses within the same $SU(3)$ multiplet and four relate octet masses to decuplet masses. When spin-$\frac{3}{2}$ fermion quarks are used, the three mass formulas relating mass differences within $SU(3)$ multiplets remain. These are the "electromagnetic" mass relations

$$n-p\ (1.29)=\Sigma^--\Sigma^++\Xi^0-\Xi^-\ (1.6\pm0.7)\,, \quad (4)$$

$$\tfrac{1}{3}(N^{*-}-N^{*++})\ (2.1\pm0.9) = Y^{*-}-Y^{*+}+\Xi^{*0}-\Xi^{*-}\ (0.9\pm3.8)\,, \quad (5)$$

and the "medium-strong" relation

$$\tfrac{1}{3}(\Omega^--N^{*++})\ (145\pm1)=\Xi^{*0}-Y^{*+}\ (147\pm1)\,. \quad (6)$$

Here, we have used the particle symbol to denote its mass and have given experimental values (in MeV) in parenthesis. These relations hold independently of quark spin, since they just follow from the symmetry properties of three objects.

The symmetric-quark-model predictions relating octet masses to decuplet masses are lost in the following way for spin-$\frac{3}{2}$ quarks. Equation (23) of I is changed to

$$m_\lambda-m_{\mathcal{P}}+\tfrac{5}{6}(D_{\lambda\lambda}{}^2-D_{\mathcal{PP}}{}^2)+\tfrac{1}{6}(D_{\lambda\lambda}{}^0-D_{\mathcal{PP}}{}^0) =\tfrac{1}{3}(\Omega^--N^{*++})\ (145\pm1)\,, =\Xi^{*0}-Y^{*+}\ (147\pm1) \quad (7a)$$

$$m_\lambda-m_{\mathcal{P}}+D_{\lambda\lambda}{}^2-D_{\mathcal{PP}}{}^2=\Xi^0-\Sigma^+\ (122\pm1)\,. \quad (7b)$$

The notation here is that[12] $D_{ij}{}^S$ is the interaction energy of quarks of type i and j in a state of spin S. For spin-$\frac{1}{2}$ quarks the left-hand sides of Eqs. (7a) and (7b) were identical, implying a mass equality that was off by 24 MeV. That equality resulted because there was only one way to add up three spin-$\frac{1}{2}$ quarks to get total spin $\frac{3}{2}$ (the first two quarks had to be in a spin-1 state). For three spin-$\frac{3}{2}$ quarks, the completely antisymmetric state of total spin $\frac{3}{2}$ is formed from a linear combination of states with any two quarks in the spin-0 or spin-2 states. The numbers $\frac{5}{6}$ and $\frac{1}{6}$ in Eq. (7a) are just the squares of the appropriate combination coefficients. For the spin-$\frac{1}{2}$, three-quark state, any two identical quarks must be in a pure spin-2 state (spin 0 could not combine with spin $\frac{3}{2}$ to form total spin $\frac{1}{2}$) and this leads to the left-hand side of Eq. (7b). It is interesting to notice that the term responsible for the "breaking" of the equality of Eqs. (7a) and (7b) comes in with a factor of $\frac{1}{6}$. This could explain why the difference between Eqs. (7a) and (7b) is only 24 MeV ("weak-strong"), whereas the magnitude of other spin-dependent mass differences is of the order of[22] 100–200 MeV. A similar situation occurs with Eq. (24) of I, which is changed to

$$\tfrac{5}{6}(2\bar{D}_{\lambda\mathfrak{N}}{}^2-\bar{D}_{\mathfrak{NN}}{}^2-D_{\lambda\lambda}{}^2)+\tfrac{1}{6}(2\bar{D}_{\lambda\mathfrak{N}}{}^0-\bar{D}_{\mathfrak{NN}}{}^0-D_{\lambda\lambda}{}^0) =\bar{Y}^*+\bar{\Xi}^*-\bar{N}^*-\Omega^-\ (7\pm4)\,, \quad (8a)$$

$$2\bar{D}_{\lambda\mathfrak{N}}{}^2-\bar{D}_{\mathfrak{NN}}{}^2-D_{\lambda\lambda}{}^2 =3\Lambda+\bar{\Sigma}-2\bar{N}-2\bar{\Xi}\ (25.6\pm0.8)\,, \quad (8b)$$

and is no longer an equality between decuplet and octet mass differences. Again, the term breaking the equality enters with a factor of $\frac{1}{6}$, and this could account for the small magnitude of the difference between Eqs. (8a) and (8b).

The other two mass formulas that are lost when using spin-$\frac{3}{2}$ quarks relate electromagnetic mass differences of the decuplet to those of the octet. These equalities are satisfied, but with relatively large experimental errors caused by uncertainties in the decuplet masses. Therefore, they were not strict tests of the symmetric-quark model.

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[17] The subscript $\frac{1}{2}$ refers to the quark spin. The use of a form factor suggested in Ref. 16 for the N^* and N would reduce $\Gamma_{1/2}$ to 0.32 MeV and $\Gamma_{3/2}$ to 0.37 MeV for $N^{*+}\to p+\gamma$.

[18] C. Becchi and G. Morpurgo, Phys. Rev. **149**, 1284 (1966).

[19] This result is based on a pion-nucleon coupling constant of $f^2/4\pi=0.08$ as used in Ref. 18. The existence of form factors makes the result uncertain because the form factor for $N^{*++}\to p+\pi^+$ would be different than that for the unphysical process $N\to N+\pi$.

[20] M. G. Olsson, Phys. Rev. Letters **14**, 118 (1965) deduces $\Gamma=120\pm2$ MeV from an analysis of pion-nucleon total cross section data and this is the value listed in Ref. 13. However, the authors of Ref. 18 dispute this in their footnote 5 and suggest $\Gamma\simeq100$ MeV.

[21] This ratio is the quantity that is directly measured by experiment. See Ref. 16.

[22] See, for instance, Eqs. (27) and (28) of I.

New electromagnetic mass inequalities are obtained when spin-$\frac{3}{2}$ quarks are used. These follow if the assumption is made that the mass splitting within an isotopic multiplet is caused by the electromagnetic energy due to the quark charges and magnetic moments, as well as an intrinsic nucleon quark mass difference. For spin-$\frac{1}{2}$ quarks, the magnetic (dipole-dipole) interaction[23] can have either the same sign (for $\mathbf{s}_1+\mathbf{s}_2=\mathbf{0}$) or the opposite sign (for $\mathbf{s}_1+\mathbf{s}_2=\mathbf{1}$) as the electric (Coulomb) interaction so that a definite sign is not, in general, predicted for electromagnetic energy differences. One inequality [that Eq. (37) of I is positive definite] can be derived by cancelling out the triplet interaction. For spin-$\frac{3}{2}$ quarks, the magnetic and electric interactions have the same sign for all the spin states ($\mathbf{s}_1+\mathbf{s}_2=\mathbf{0},\mathbf{1},\mathbf{2}$) for quark pairs in baryons. Then the following inequalities are satisfied:

$$\Sigma^+ + \Sigma^- - 2\Sigma^0 > 0 \ (1.75\pm0.18)\,, \tag{9a}$$

$$3(p-n)+\Sigma^- + \Sigma^0 - 2\Sigma^+ > 0 \ (7.25\pm0.25)\,, \tag{9b}$$

$$\Xi^{*-} - \Xi^{*0} + p - n - \tfrac{1}{3}(\Sigma^+ + \Sigma^- - 2\Sigma^0) > 0 \ (3.1\pm2.2)\,. \tag{9c}$$

These inequalities are all well satisfied and constitute tests of spin-$\frac{3}{2}$ quarks that do not apply for spin-$\frac{1}{2}$ quarks. There are other inequalities involving decuplet masses, but with experimental uncertainties too great to test the model.

V. MESONS

If we build the mesons from $q-\bar{q}$ states of spin-$\frac{3}{2}$ quarks, then the pseudoscalar and vector nonets arise, just as from spin-$\frac{1}{2}$ quarks, as the $S=0$ and 1 spin states. For spin-$\frac{3}{2}$ quarks, s states with J^P of 2^- and 3^- are also possible. These states are expected in any event as $L=2$ orbital states (or Regge recurrences) and at least one of them (the g meson,[24] which is likely to be 3^-) has been observed. It is probably neatest, however, to rule these states out as s states by making the dynamical assumption that the $S=2$ and $S=3$ $q-\bar{q}$ forces are repulsive (enough to prevent binding). This assumption then also provides saturation for the meson states, in that states like $qq\bar{q}$ would not occur since they would include repulsive spin states for either qq or $q\bar{q}$ pairs. Actually, given that the spin-3 qq force is repulsive (for baryon saturation), it is only necessary to require that the spin-2 $q\bar{q}$ force be repulsive to achieve saturation for the meson states.

There is not much that the quark model can say (without making additional symmetry assumptions) about the static properties of the mesons because restrictions to two-body interactions do not lead to any constraints for two-body states. There are some interesting inequalities, however, for the electromagnetic mass differences. For spin-$\frac{3}{2}$ quarks, these are

$$\pi^\pm - \pi^0 > 0 \ (4.6)\,, \tag{10a}$$

$$\rho^\pm - \rho^0 > 0\,, \tag{10b}$$

$$K^\pm - K^0 > m_{\mathcal{P}} - m_{\mathfrak{N}}\,, \tag{10c}$$

$$K^{*\pm} - K^{*0} + \Xi^- - \Xi^0 > 0 \ (0.3\pm4.2)\,. \tag{10d}$$

Equation (10a) also follows using spin-$\frac{1}{2}$ quarks and is well satisfied. Equation (10b) cannot be derived for spin-$\frac{1}{2}$ quarks because the electric and magnetic mass shifts are then in opposite directions. It would be of interest to test the $\rho^\pm-\rho^0$ mass difference because a bootstrap calculation[25] predicts the opposite sign. Equation (10c) is quite important. It also follows using spin-$\frac{1}{2}$ quarks, but, for spin-$\frac{1}{2}$ quarks, the nucleon quark-quark magnetic interactions are proportional to their electric interactions and we can derive the equality[8]

$$m_{\mathfrak{N}} - m_{\mathcal{P}} = n - p + \tfrac{1}{3}(\Sigma^+ + \Sigma^- - 2\Sigma^0) = 1.9\pm0.2 \text{ MeV}\,, \tag{I.31}$$

and then Eq. (10c) is not satisfied (experimentally, $K^\pm - K^0 = -3.9\pm0.2$ MeV). With spin-$\frac{3}{2}$ quarks, the nucleon quark magnetic moments are not proportional to their charges and Eq. (I.31) no longer follows. In its place we have the lesser restriction

$$\Xi^- - \Xi^0(+6.6\pm0.7) > m_{\mathfrak{N}} - m_{\mathcal{P}} > 1.9\pm0.1 \text{ MeV}\,. \tag{11}$$

Then, Eq. (10c) leads to

$$K^\pm - K^0 + \Xi^- - \Xi^0 > 0 \ (2.7\pm0.7)\,, \tag{10c$'$}$$

which is satisfied. It is interesting to note that any quark model that uses the assumption that the nucleon quark magnetic moments are proportional to their charges to get a good ($-\frac{3}{2}$) proton-neutron magnetic moment ratio will predict a bad $K^\pm-K^0$ mass difference. Equation (10d) is similar to Eq. (10c′) but would not follow for spin-$\frac{1}{2}$ quarks.

The radiative decays of vector mesons, such as $\omega^0 \to \pi^0+\gamma$ have been calculated with spin-$\frac{1}{2}$ quarks by Becchi and Morpurgo.[26] They obtain $\Gamma_{1/2}(\omega^0\to\pi^0+\gamma) = 1.2$ MeV, in good agreement with experiment (1.1 ± 0.2 MeV). Using spin-$\frac{3}{2}$ quarks leads to a factor of (125/81) in this rate, so that $\Gamma_{3/2}(\omega^0\to\pi^0+\gamma)=1.8$ MeV. There are many uncertainties[27] in this calculation, however, so that this is not a strict test of the model. The same factor (125/81) enters into the rate for $\rho\to\pi+\pi$ which has been calculated by Becchi and Morpurgo[18] with spin-$\frac{1}{2}$ quarks. The uncertainties in

[23] See, for instance, Eq. (30) of I.

[24] D. J. Crennell *et al.*, Phys. Rev. Letters **18**, 323 (1967). See also Ref. 13.

[25] D. S. Beder, Nuovo Cimento **43**, 553 (1966).

[26] C. Becchi and G. Morpurgo, Phys. Rev. **140**, B687 (1965).

[27] This result follows if the radial overlap integral is set equal to unity, as is done in Ref. 26. Since the ω^0 radial wave function should be quite different from the π^0 wave function (because of the large difference in their masses), a realistic estimate of the overlap integral would reduce the rate given here. Other uncertainties in the predicted rate are discussed in Ref. 26.

this calculation are so great[18] that a strict comparison with experiment is not possible, but the order of magnitude remains reasonable with spin-$\frac{3}{2}$ quarks.

VI. SUMMARY

We have seen that most of the conclusions for spin-$\frac{1}{2}$ Bose quarks follow also for spin-$\frac{3}{2}$ Fermi quarks. We list here the main features of spin-$\frac{3}{2}$ Fermi quarks:

(1) The baryon multiplet structure follows from the dynamical assumption that the $(\mathbf{s}_1+\mathbf{s}_2)=\mathbf{3}$ quark-quark force is repulsive. The meson multiplet structure follows with repulsive $S=2$ and $S=3$ $q-\bar{q}$ forces. These same dynamical assumptions provide appropriate quark saturation for baryons and mesons.

(2) If the assumption is made that the nucleon quark magnetic moments are proportional to their charges, then using spin-$\frac{3}{2}$ Fermi quarks leads to the result $-\mu_p/\mu_n=\frac{5}{4}$. This means that the quarks must have anomalous moments. Relations that are independent of quark moments can still be derived for combinations of baryon moments. The prediction for the Σ^+ moment (which is in agreement with experiment) does not depend on the quark spin, but the predictions for the Ξ^- and Ω^- moments change and measurements of these moments would test the quark spin.

(3) The predicted widths for $N^{*+}\to p+\gamma$ and $N^{*++}\to p+\pi^+$ are 15% larger with spin-$\frac{3}{2}$ quarks than with spin-$\frac{1}{2}$ quarks, in slightly better agreement with experiment.

(4) The mass relations between baryons within $SU(3)$ multiplets predicted with spin-$\frac{1}{2}$ quarks are retained with spin-$\frac{3}{2}$ quarks, but relations connecting decuplet masses to octet masses are dropped. The mass relations dropped were either poorly determined (experimentally) or were violated by $\sim$20 MeV. The small magnitude of this difference can be understood by a factor of $\frac{1}{6}$ in the term breaking the equality [Eqs. (7) and (8)].

(5) Electromagnetic mass inequalities, all of which are satisfied, can be derived using spin-$\frac{3}{2}$ quarks. An inequality for the $K^{\pm}-K^0$ mass difference, which was violated with spin-$\frac{1}{2}$ quarks, is modified with spin-$\frac{3}{2}$ quarks and is consistent with experiment [Eq. (10c′)].

On balance, spin-$\frac{3}{2}$ quarks are, if anything, in somewhat better agreement with experiment for the low-lying multiplets than spin-$\frac{1}{2}$ quarks. We have included here those tests we feel are the strictest tests of the quark model. Many other predictions of the quark model, such as most of the high-energy scattering relations,[28] do not depend on quark spin and would be the same for either model. It is an open philosophical question whether it is nicer to have quarks of the lowest possible spin and then have to contrive effective Bose statistics, or to settle for spin-$\frac{3}{2}$ quarks and normal Fermi statistics. This question could be answered experimentally, however, by measurement of the Ξ^- (or Ω^-) magnetic moments.

Neither model gives a good reason for the nonappearance, as yet, of quarks, and this remains an imposing difficulty. However, there is somewhat less compulsion to regard spin-$\frac{3}{2}$ quarks as necessarily light ($\sim\frac{1}{3}M_p$) because they could not be expected to have Dirac moments (the assumption of Dirac moments for spin-$\frac{1}{2}$ quarks "explains" the result $-\mu_p/\mu_n=\frac{3}{2}$, but also provides a measure of the quark mass). It is also true that there is less chance that spin-$\frac{3}{2}$ quarks with anomalous moments would be purely "elementary" rather than composites themselves.

Having broken the "spin barrier" for quarks, we might ask about quarks of higher spin. It does turn out, in fact, that any half-integral spin of $\frac{3}{2}$ or higher, would give results quite similar to those shown here for spin-$\frac{3}{2}$ quarks, although more complicated dynamical assumptions are required to restrict the multiplet structure and achieve saturation. We consider these models of higher quark spin in a subsequent paper.[29]

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[28] See, for instance, E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **2**, 105 (1965) [English transl.: JETP Letters **2**, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966); J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42A**, 711 (1966); H. J. Lipkin, Phys. Rev. Letters **16**, 952 (1966); J. J. J. Kokkedee, *ibid.* **22**, 88 (1966); C. A. Levinson, N. S. Wall, and H. J. Lipkin, *ibid.* **17**, 1122 (1966); J. J. J. Kokkedee and L. Van Hove, Nucl. Phys. **B1**, 169 (1967).

[29] J. Franklin, Phys. Rev. (to be published).