Electromagnetic Decays of Baryon Resonances in the Harmonic-Oscillator Model*

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The harmonic-oscillator model for baryons is used to estimate the radiative widths of the observed N^* resonances. In the "second resonance region" of pion photoproduction, both the $S_{11}(1550)$ and $D_{13}(1525)$ resonances [and possibly the $P_{11}(1470)$, depending on what should be taken as its $N\pi$ width] are expected to be important, while in the "third resonance region" $F_{15}(1690)$, $S_{11}(1710)$, and $D_{13}(1690)$ are expected to have comparable contributions. The mixing angle between the two S_{11} resonances is determined to be about 35°. In inelastic electron scattering $ep \rightarrow eN^*$, the $S_{11}(1550)$ and $D_{13}(1525)$ resonances are expected to dominate over the $P_{11}(1470)$. The neutral $P_{11}(1470)$ radiative width is found to be smaller than the charged width, in contrast to an $SU(3)$ $\overline{\mathbf{10}}$ assignment for this resonance.

I. INTRODUCTION

IN a previous paper[1] (hereafter referred to as I), the authors began an investigation of a model that regards the baryon resonances as bound states of three paraquarks[2] interacting via harmonic-oscillator forces.[3] Particular attention was drawn to the fact that the experimental spectrum of N^* resonances[4] seems to occur as a series of distinct bands having alternating parity, and that with no exceptions the observed states fit very nicely into the spectrum predicted by the model,[1,2,5] completely filling up the two lowest-lying bands and quite heavily populating the third band. To wit: The nucleon and (3,3) resonance Δ fill the first band, the seven lowest-lying negative-parity N^* resonances fill the second band, and the next nine lowest-lying positive-parity N^* resonances occupy almost half of the 21 states predicted to be in the third band.

The second part of paper I was devoted primarily to calculating the rates with which the N^*'s decay into a nucleon (the ground state) by pion emission. The mechanism for this process was assumed to be the simple deexcitation of a single quark to its ground state. The calculations involved only two basic parameters: the "spring stiffness" α^2 (defined in I) for the harmonic oscillation and the quark-pion coupling constant $f_q^2/4\pi$. With many states predicted (and observed) to have the same quantum numbers, a certain amount of configuration mixing is to be expected in such states. However, in the first three bands there are six unique states which do not get mixed, and it was found possible to obtain values for the above parameters which yielded a reasonable simultaneous fit to the $N\pi$ widths of these unmixed resonances. Estimates were also given for some of the mixing angles, but this could be done for only a few cases due to the limited data available.

The values of the parameters were found to be $f_q^2/4\pi=0.055$ (to be compared to 0.081 for the pion-nucleon coupling constant) and $\alpha^2=0.10$ (GeV/c)^2, corresponding to a rms interaction radius of about 0.8 F. Apart from mixing angles, this knowledge of α^2 means that the wave functions of all the physical baryons are now completely determined. It is with this knowledge that we go on to examine in the present paper some further predictions of the harmonic-oscillator model for baryons, namely, the electromagnetic decay widths.

Here, the experimental situation is less clear, since only one radiative width, that for the process $\Delta^+(1236) \rightarrow p\gamma$, is known[6] with any certainty. Several others have been estimated,[7–10] but as yet no reliability can be placed on these numbers. Both the experiments and analyses of pion photoproduction are at present at a much cruder stage of development than the corresponding situation for elastic pion-nucleon scattering, where detailed experiments and high-powered phase-shift analyses have, after several years, begun to produce fairly consistent estimates of the $N\pi$ widths for quite a few of the N^* resonances. Nevertheless, interest[7–10] in pion photoproduction has been growing, particularly since various competing schemes for the baryons predict widely different electromagnetic couplings.

Therefore, in subsequent sections, stimulated by its reasonable degree of success for $N\pi$ widths, we use the

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[1] D. Faiman and A. W. Hendry, Phys. Rev. **173**, 1720 (1968).
[2] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).
[3] Essentially the same model has been used by N. S. Thornber [Phys. Rev. **169**, 1096 (1968)] to compute the inelastic electron scattering form factors of several resonances.
[4] A. Donnachie, R. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968); P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. **165**, 1730 (1968); B. H. Bransden, R. G. Moorhouse, and P. J. O'Donnell, *ibid.* **139**, B1566 (1965).
[5] R. H. Dalitz, in Proceedings of Topical Conference on πN Scattering, Irvine, Calif., 1967 (unpublished).
[6] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).
[7] Y. C. Chau, R. G. Moorhouse, and N. Dombey, Phys. Rev. **163**, 1632 (1967).
[8] R. K. Logan and F. Uchiyama-Campbell, Phys. Rev. **153**, 1634 (1967).
[9] J. Engels and W. Schmidt, Phys. Rev. **169**, 1296 (1968); C. Betourne, J. C. Bizot, J. Perez-Y-Jorba, D. Treille, and W. Schmidt, *ibid.* **172**, 1343 (1968).
[10] R. L. Walker (to be published).

harmonic-oscillator model to compute the radiative widths for the electromagnetic decays of several of the more prominent N^* resonances. The mechanism for the decay is again taken to be a one-quark deexcitation, this time by photon emission.

II. HARMONIC-OSCILLATOR MODEL

In the harmonic-oscillator model, the baryons are taken to be eigenstates of the Hamiltonian for three paraquarks interacting via harmonic-oscillator forces

$$H=\sum_j \frac{\mathbf{p}_j^2}{2M}+\tfrac{1}{2}M\omega^2\sum_{i<j}(\mathbf{r}_i-\mathbf{r}_j)^2.$$

For labeling of states however, it is customary (and slightly more convenient) to use the related shell-model Hamiltonian

$$H_{\mathrm{sm}}=\sum_j \frac{\mathbf{p}_j^2}{2M}+\tfrac{1}{2}M\omega^2\sum_j \mathbf{r}_j^2,$$

the eigenfunctions of which are products of three familiar three-dimensional harmonic-oscillator wave functions. The over-all spin-isospin-spatial wave function is taken to be symmetric,[2] and the resulting spectrum of eigenstates was described in I.

The ground state of the system is the $(1s)^3$ shell, corresponding[11] to the configuration **56**, $L^P=0^+$. To this, one normally allots the nucleon and $\Delta(1236)$.

The first excited level is the $(1s)^2(1p)$ shell, with the configuration **70**, $L^P=1^-$. Into this fit[12] the odd-parity N^* resonances $D_{15}(1680)$, $D_{13}(1690)$, $S_{11}(1710)$, $D_{13}(1525)$, $S_{11}(1550)$, $S_{31}(1640)$, and $D_{33}(1691)$. In principle, the observed S_{11}'s are mixtures of the two S_{11} eigenstates

$$S_{11}{}^a(1710)=\cos\theta_s\, S_{11}(8^{3/2})+\sin\theta_s\, S_{11}(8^{1/2}),$$
$$S_{11}{}^b(1550)=-\sin\theta_s\, S_{11}(8^{3/2})+\cos\theta_s\, S_{11}(8^{1/2}),$$

where θ_s is a mixing angle and $S_{11}(8^{3/2})$ and $S_{11}(8^{1/2})$ are eigenstates of H and the total quark spin. Unfortunately, a unique value of θ_s could not be deduced from the $N\pi$ widths alone, there being two allowed solutions: θ_s about 35° or 90°. A similar situation holds for the D_{13}'s, the corresponding mixing angle θ_d being about 35° or 127°. The present work was, in fact, partially initiated to see whether this ambiguity in the mixing angles could be resolved by consideration of the radiative widths.

The next excitation level originates from the $(1s)^2(2s)$, $(1s)^2(1d)$, and $(1s)(1p)^2$ shell-model states. These give rise to the even-parity configurations **56**, $L^P=0^+$ (P_{11},P_{33}); **56**, $L^P=2^+$ $(F_{15},P_{13},F_{37},F_{35},P_{33},P_{31})$; **70**, $L^P=0^+$ (P_{11},P_{13},P_{31}); **70**, $L^P=2^+$ $(P_{11},2P_{13},2F_{15},F_{17},P_{33},F_{35})$; and **20**, $L^P=1^+$ (P_{11},P_{13}). There are several known resonances which can fit into this band: $F_{37}(1950)$, $F_{17}(1983)$, $F_{15}(1690)$, $P_{11}(1470)$, $P_{11}(1750)$, $P_{33}(1690)$, $P_{13}(1860)$, $P_{31}(1930)$, and $F_{35}(1910)$. Some of these have been discovered only very recently.[4] Of these resonances, only the first two can be uniquely allocated in our model, all the others being, in principle, mixtures of configurations. An examination of the $N\pi$ widths showed, however, that each of these resonances belongs predominantly to one or two configurations, and these are listed in Table I.

The model can easily be extended to higher excited levels, but we have not done this because of the present dearth of knowledge about N^* resonances above 2 GeV.

III. CALCULATION OF ELECTROMAGNETIC WIDTHS

The mechanism for photon emission is taken as the deexcitation of a single quark. The nonrelativistic form of the interaction is

$$\sum_{j=1}^{3}\mathbf{J}_j\cdot\mathbf{A}(\mathbf{r}_j)=\sum_j q_j[-ig\boldsymbol{\sigma}_j\cdot(\mathbf{k}\times\mathbf{A})+(\mathbf{p}_j+\mathbf{p}_j')\cdot\mathbf{A}](e/2M).$$

Here eq_j is the charge of the jth quark; $\mathbf{p}_j$ and $\mathbf{p}_j'$ are the initial and final momenta, respectively, of the quark emitting a photon of momentum $\mathbf{k}$; $geq_j/2M$ represents the magnetic moment of a bound quark (g would equal 1 for a pure Dirac particle); and M is the effective mass of the bound quark. The electromagnetic field $\mathbf{A}$ is given by

$$\mathbf{A}(\mathbf{r}_j)=(4\pi)^{1/2}\sum\frac{\boldsymbol{\varepsilon}}{(2k_0)^{1/2}}(a_k^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}_j}+a_k e^{i\mathbf{k}\cdot\mathbf{r}_j}),$$

$\boldsymbol{\varepsilon}$ being a unit polarization vector.

The radiative widths may be determined by taking the matrix elements of this interaction between an N^* resonance state and the nucleon (the wave functions of the various states are given in I). All the algebra can be done exactly, without any simplifications such as the dipole approximation. The final expressions

TABLE I. Observed positive-parity resonances which can be mixtures of configurations. We list the predominant configurations for these resonances as deduced from their $N\pi$ widths. The brackets mean that the contribution from this configuration is small.

Resonance	Configurations
$P_{11}(1470)$	**56**, $L^P=0^+$; **70**, $L^P=0^+$
$P_{11}(1750)$	**56**, $L^P=0^+$; **70**, $L^P=0^+$
$P_{33}(1690)$	**56**, $L^P=0^+$; **56**, $L^P=2^+$
$F_{15}(1690)$	**56**, $L^P=2^+$; [**70**$(8^{1/2})$, $L^P=2^+$]
$P_{13}(1860)$	**56**, $L^P=2^+$; **70**$(8^{1/2})$, $L^P=2^+$
$P_{31}(1930)$	**56**, $L^P=2^+$; [**70**, $L^P=0^+$]
$F_{35}(1910)$	**56**, $L^P=2^+$; **70**, $L^P=2^+$

[11] We use the $SU(6)$ representations **56**, **70**, and **20** as a convenient notation for specifying the spin-isospin dimensionality and symmetry. L is the total quark orbital angular momentum.

[12] R. H. Dalitz, in *High-Energy Physics* (Gordon and Breach, Science Publishers, Inc., New York, 1966).

TABLE II. Decay rates for resonances in the first two bands. Here $B=(e/2M)^2\exp(-k^2/3\alpha^2)E/M^*$, where E and $\mathbf{k}$ are the total energy and momentum of the final-state nucleon, M^* is the mass of the decaying resonance, M is the effective mass of the bound quark, α^2 is the oscillator spring constant, and $e^2=1/137$.

Decay process	Radiative width
$\Delta\to N\gamma$	$(16/9)k^3g^2B$
$D_{15}{}^+\to p\gamma$	0
$D_{15}{}^0\to n\gamma$	$(2/45)(k^5/\alpha^2)g^2B$
$S_{11}{}^{a+}\to p\gamma$	$\dfrac{2}{9}\dfrac{k^5}{\alpha^2}\left(g+\dfrac{2\alpha^2}{k^2}\right)^2\sin^2\theta_s B$
$S_{11}{}^{b+}\to p\gamma$	$\dfrac{2}{9}\dfrac{k^5}{\alpha^2}\left(g+\dfrac{2\alpha^2}{k^2}\right)^2\cos^2\theta_s B$
$S_{11}{}^{a0}\to n\gamma$	$\dfrac{2}{81}\dfrac{k^5}{\alpha^2}\left(g(\sin\theta_s-\cos\theta_s)+\dfrac{6\alpha^2}{k^2}\sin\theta_s\right)^2 B$
$S_{11}{}^{b0}\to n\gamma$	$\dfrac{2}{81}\dfrac{k^5}{\alpha^2}\left(g(\sin\theta_s+\cos\theta_s)+\dfrac{6\alpha^2}{k^2}\cos\theta_s\right)^2 B$
$D_{13}{}^{a+}\to p\gamma$	$\dfrac{2}{9}\dfrac{k^5}{\alpha^2}\left(g^2-\dfrac{2\alpha^2}{k^2}g+\dfrac{4\alpha^2}{k^4}\right)\sin^2\theta_d B$
$D_{13}{}^{b+}\to p\gamma$	$\dfrac{2}{9}\dfrac{k^5}{\alpha^2}\left(g^2-\dfrac{2\alpha^2}{k^2}g+\dfrac{4\alpha^2}{k^4}\right)\cos^2\theta_d B$
$D_{13}{}^{a0}\to n\gamma$	$\dfrac{2}{27}\dfrac{k^5}{\alpha^2}\left[\left(\dfrac{14}{15}\cos^2\theta_d+\tfrac{1}{3}\sin^2\theta_d-\dfrac{\sqrt{10}}{15}\sin\theta_d\cos\theta_d\right)g^2+\dfrac{2\alpha^2}{k^2}\sin\theta_d[(\sqrt{10})\cos\theta_d-\sin\theta_d]g+12\dfrac{\alpha^4}{k^4}\sin^2\theta_d\right]B$
$D_{13}{}^{b0}\to n\gamma$	Same as $D_{13}{}^{a0}\to n\gamma$ with $\cos\theta_d\to-\sin\theta_d$, $\sin\theta_d\to\cos\theta_d$
$S_{31}\to N\gamma$	$\dfrac{2}{81}\dfrac{k^5}{\alpha^2}\left(g-\dfrac{6\alpha^2}{k^2}\right)^2 B$
$D_{33}\to N\gamma$	$\dfrac{2}{81}\dfrac{k^5}{\alpha^2}\left(g^2+\dfrac{6\alpha^2}{k^2}g+\dfrac{36\alpha^4}{k^4}\right)B$

derived for the widths are listed in Tables II and III for those resonances which are likely to be the most important ones in the analysis of pion photoproduction.

TABLE III. Decay rates of several states in the third band. Here $B=(e/2M)^2\exp(-k^2/3\alpha^2)E/M^*$, where E and $\mathbf{k}$ are the total energy and momentum of the final-state nucleon, M^* is the mass of the decaying resonance, M is the effective mass of the bound quark, α^2 is the oscillator spring constant, and $e^2=1/137$.

Decay process	Radiative width
$P_{11}{}^+(\mathbf{56},L^P=0^+)\to p\gamma$	$(1/27)(k^7/\alpha^4)g^2B$
$P_{11}{}^0(\mathbf{56},L^P=0^+)\to n\gamma$	$(4/243)(k^7/\alpha^4)g^2B$
$P_{11}{}^+(\mathbf{70},L^P=0^+)\to p\gamma$	$(1/54)(k^7/\alpha^4)g^2B$
$P_{11}{}^0(\mathbf{70},L^P=0^+)\to n\gamma$	$(1/486)(k^7/\alpha^4)g^2B$
$P_{11}{}^+(\mathbf{70},L^P=2^+)\to p\gamma$	0
$P_{11}{}^0(\mathbf{70},L^P=2^+)\to n\gamma$	$(1/1215)(k^7/\alpha^4)g^2B$
$P_{11}(\mathbf{20},L^P=1^+)\to N\gamma$	0
$F_{37}(\mathbf{56},L^P=2^+)\to N\gamma$	$(16/2835)(k^7/\alpha^4)g^2B$
$F_{15}{}^+(\mathbf{56},L^P=2^+)\to p\gamma$	$(2/135)(k^7/\alpha^4)[g^2-(4\alpha^2/k^2)g+12\alpha^4/k^4]B$
$F_{15}{}^0(\mathbf{56},L^P=2^+)\to n\gamma$	$(8/1215)(k^7/\alpha^4)g^2B$

IV. RESULTS

We comment first on the zeros which appear in Tables II and III. The result that in a quark model

$$\Gamma(D_{15}{}^+\to p\gamma)=0$$

was first discovered by Moorhouse,[13] and seems to be indicated by experiment.[14] It involves a transition from a three-quark state in the mixed $SU(6)$ representation **70** with quark spin $S=\frac{3}{2}$ and orbital angular momentum $L=1$ to a state with a symmetric $SU(6)$ representation **56**, with $S=\frac{1}{2}$, $L=0$. We also find

$$\Gamma[P_{11}{}^+(\mathbf{70},L^P=2^+)\to p\gamma]=0\,,$$
$$\Gamma[P_{11}(\mathbf{20},L^P=1^+)\to N\gamma]=0\,,$$

the first relation being true for essentially the same reason as the $D_{15}{}^+$ decay, and the second, which was first noted by Morpurgo,[15] follows because the transition requires a two-quark deexcitation.

The only radiative width known with any degree of reliability is for the process $\Delta^+(1236)\to p\gamma$. Dalitz and Sutherland[6] find

$$\Gamma(\Delta^+\to p\gamma)=0.65\text{ MeV}.$$

As pointed out by Becchi and Morpurgo,[16] a simple quark model predicts[17] that this decay is entirely through the magnetic dipole, with no contribution from the electric quadrupole. Experiment[18] seems to verify this. With the harmonic-oscillator wave functions and the knowledge of the spring constant α^2, we further obtain[19] from Table II

$$\mu=eg/2M=0.18\text{ GeV}^{-1},$$

and consequently

$$\Gamma[D_{15}{}^0(1680)\to n\gamma]=0.20\text{ MeV},$$
$$\Gamma[F_{37}(1950)\to N\gamma]=0.23\text{ MeV},$$

the charged and neutral decay widths for the F_{37} being equal. Both of these widths are quite appreciable, but neither has as yet been estimated experimentally.

The situation for the P_{11} resonances is an interesting one. For the unmixed charged P_{11} states, we find that the radiative widths for $P_{11}{}^+(\mathbf{56},L^P=0^+)$, $P_{11}{}^+(\mathbf{70},L^P=0^+)$, $P_{11}{}^+(\mathbf{70},L^P=2^+)$, and $P_{11}{}^+(\mathbf{20},L^P=1^+)$ are in the ratio $1:\frac{1}{2}:0:0$. Although the physical P_{11}'s are mixtures of these states, it was suggested in I from a study of the $N\pi$ widths that the Roper resonance

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[13] R. G. Moorhouse, Phys. Rev. Letters **16**, 772 (1966). According to H. J. Lipkin (private communication to R. G. Moorhouse), this result also follows from $SU(6)_W$.

[14] S. D. Ecklund and R. L. Walker, Phys. Rev. **159**, 1195 (1967).

[15] G. Morpurgo, Phys. Letters **22**, 214 (1966).

[16] C. Becchi and G. Morpurgo, Phys. Letters **17**, 352 (1965).

[17] H. Harari and H. J. Lipkin, Phys. Rev. **140**, B1617 (1965).

[18] A. Donnachie and G. Shaw, Nucl. Phys. **87**, 556 (1967).

[19] Only a positive value of μ is considered here in analogy to the $SU(6)$ result that $\mu=\mu_p$ for the proton. Experimentally, $\mu_p=0.13\text{ GeV}^{-1}$.

$P_{11}(1470)$ might belong primarily to the configuration **56**, $L^P=0^+$. With this assumption and the above value of μ, we obtain

$$\Gamma[P_{11}^+(1470)\to p\gamma]=0.13 \text{ MeV},$$
$$\Gamma[P_{11}^0(1470)\to n\gamma]=0.05 \text{ MeV}.$$

Admixtures of other possible configurations will tend to reduce these numbers. Unfortunately, it is impossible to compare our numbers with any experimental determinations, even though several analyses[7–10] of pion photoproduction have been carried out in recent years over the appropriate energy region. The different analyses lead to contradictory results. On the one hand, Chau *et al.*[7] find $\Gamma[P_{11}^+(1470)\to p\gamma]$ quite large, being about 0.39 or 0.43 MeV for their two solutions. On the other, Schmidt *et al.*[9] find little or no evidence for the $P_{11}^+(1470)$ in their analysis. Our own value is somewhat smaller than the estimate by Chau *et al.*, but certainly not negligible.

Our prediction for the neutral P_{11} width is important, in that it differs radically from the prediction which follows from the popular assignment of the Roper resonance to an $SU(3)$ antidecuplet $\overline{\mathbf{10}}$ representation. In the latter case, it follows from U-spin arguments[20] that the decay amplitude $P_{11}^+\to p\gamma$ is zero, whereas $P_{11}^0\to n\gamma$ is nonzero (though no one knows how large or small this should be). In the present model, we find that the width of the neutral P_{11} is a factor of about 0.4 smaller than the P_{11}^+ width. It is obviously important to confront these opposite predictions with experimental test. This will no doubt take a considerable time and effort, through many experiments and detailed analyses.

The experimental situation with respect to the $S_{11}^b(1550)$ and $D_{13}^b(1525)$ resonances is also rather cloudy. Chau *et al.*[7] estimate $\Gamma(S_{11}^{b+}\to p\gamma)$ to be 0.10 or 0.04 MeV (depending on the solution) and $\Gamma(D_{13}^{b+}\to p\gamma)$ to be 0.22 or 0.21 MeV. The radiative width of the S_{11}^{b+} has also been calculated by Uchiyama-Campbell *et al.*[8] from a study of the processes $\gamma p\to\eta p$, $\pi p\to\eta n$, and they find it to be 0.34 or 0.13 MeV. These estimates range by almost an order of magnitude, and this uncertainty limits our investigation of these resonances.

From Table II, our expressions for the S_{11} and D_{13} widths in MeV reduce to

$$\Gamma[S_{11}^{a+}(1710)\to p\gamma]=1.09(1+0.56/g)^2\sin^2\theta_s,$$
$$\Gamma[S_{11}^{b+}(1550)\to p\gamma]=0.63(1+0.83/g)^2\cos^2\theta_s,$$
$$\Gamma[D_{13}^{a+}(1690)\to p\gamma]=1.10(1-0.59/g+0.34/g^2)\sin^2\theta_d,$$
$$\Gamma[D_{13}^{b+}(1525)\to p\gamma]=0.56(1-0.89/g+0.80/g^2)\cos^2\theta_d,$$

where we have used the measured $\Delta^+\to p\gamma$ width. The

[20] H. J. Lipkin, Phys. Letters **12**, 154 (1964).

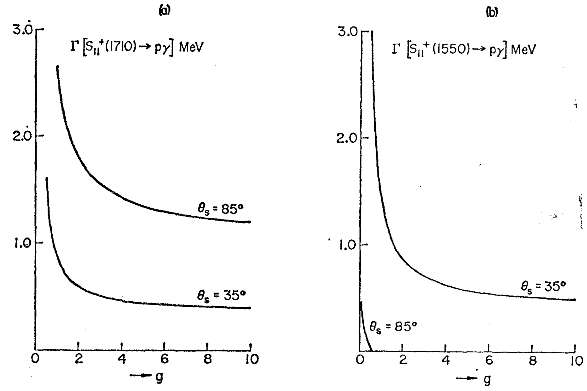

FIG. 1. Decay rates of $S_{11}^+(1710)$ and $S_{11}^+(1550)$ as functions of g, for the two values of the mixing angle $\theta_s=35°$ and $85°$.

mixing angles θ_s and θ_d were determined[21] from the $N\pi$ widths to be θ_s about 35° or 90° and θ_d about 35° or 127°. These widths are plotted as functions[22] of g in Figs. 1 and 2.

Examination of these graphs shows that the solution with θ_s near 90° is unlikely. With $\theta_s=85°$, say, g is required to be about 0.1 to yield an $S_{11}^{b+}(1550)$ radiative width in the range 0.05–0.4 MeV. This very small value of g, however, would imply radiative widths of hadronic proportions for the $S_{11}^{a+}(1710)$ as well as for the two D_{13} resonances.

Hence, $\theta_s=35°$ is the solution which is strongly favored. With this value of the mixing angle, Fig. 1(b) shows that the radiative width of $S_{11}^{b+}(1550)$ is then at least 0.42 MeV, this value being attained for g substantially greater than 1. Our estimate[23] is therefore larger than the one deduced by Chau *et al.*[7] but, depending on the size of g, could be close to one of the

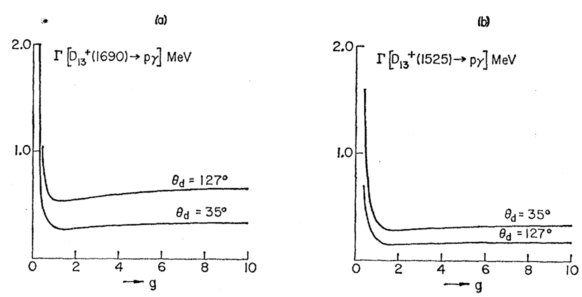

FIG. 2. Decay rates of $D_{13}^+(1690)$ and $D_{13}^+(1525)$ as functions of g, for the two values of the mixing angle $\theta_d=35°$ and $127°$.

[21] The value $\theta_s=90°$ would of course forbid the decay $S_{11}^+(1550)\to p\gamma$. Below, we shall use $\theta_s=85°$, which no longer yields the best fit for the $N\pi$ widths of both S_{11} resonances, but still yields acceptable values $\Gamma[S_{11}^a(1710)\to N\pi]=258$ MeV, $\Gamma[S_{11}^b(1550)\to N\pi]=22$ MeV compared to the experimental values 240 and 39 MeV, respectively.

[22] The g factor is as yet an undetermined quantity, since the mass M of the quark is unknown. Accurate evaluation of several radiative widths in principle immediately gives both g and M.

[23] Should the $S_{11}^{b+}(1550)$ radiative width indeed turn out to be as small as 0.04 MeV, the present model would have to be drastically revised.

solutions of Uchiyama-Campbell *et al.*[8] From Fig. 1(a), the width of $S_{11}{}^{a+}(1710)$ is likewise quite appreciable, also tending to about 0.4 MeV, and hence we expect this resonance to show up in photoproduction.

In the case of the $D_{13}{}^{+}$'s, it is difficult with the present data to decide which of the mixing angles $\theta_d=35°$ or 127° is the more appropriate one. From Fig. 2 with $g\gtrsim 1$, we find that $\Gamma[D_{13}{}^{b+}(1525)\rightarrow p\gamma]$ is either about 0.35 or 0.20 MeV, depending on the mixing angle, while $\Gamma[D_{13}{}^{a+}(1690)\rightarrow p\gamma]$ is either about 0.36 or 0.70 MeV. A mixing angle $\theta_d=127°$ would give agreement with the analysis of Chau *et al.*[7] for the $D_{13}{}^{b+}(1525)$.

The neutral S_{11} and D_{13} decays can be studied in a similar manner. Since the experimental determination of these widths seems unlikely for some time, we quote here only typical values of the widths (g somewhat greater than 1) to illustrate the sort of magnitudes which come out of this model. We obtain

$$\Gamma[S_{11}{}^{a0}(1710)\rightarrow n\gamma]\sim 0.11\text{ MeV},$$
$$\Gamma[S_{11}{}^{b0}(1550)\rightarrow n\gamma]\sim 0.13\text{ MeV},$$
$$\Gamma[D_{13}{}^{a0}(1690)\rightarrow n\gamma]\sim 0.25\text{ MeV},$$
$$\Gamma[D_{11}{}^{b0}(1525)\rightarrow n\gamma]\sim 0.14\text{ MeV},$$

these estimates being approximately the same for both values of each mixing angle θ_s and θ_d.

The remaining members of the first excited band, the **70**, $L^P=1^-$, are the S_{31} and D_{33} resonances. The corresponding widths for those resonances (again for large g) are

$$\Gamma[S_{31}(1640)\rightarrow N\gamma]\sim 0.10\text{ MeV},$$
$$\Gamma[D_{33}(1690)\rightarrow N\gamma]\sim 0.13\text{ MeV},$$

the charged and neutral decay rates being the same.

The other resonance which is important in this energy region is the $F_{15}(1690)$. Indeed, it has been known[24] for a long time that the F_{15} gives an appreciable contribution to the photoproduction cross section. Assuming that this resonance is primarily in the 56, $L^P=2^+$ configuration, we obtain

$$\Gamma[F_{15}{}^{+}(1690)\rightarrow p\gamma]\sim 0.26\text{ MeV},$$
$$\Gamma[F_{15}{}^{0}(1690)\rightarrow n\gamma]=0.11\text{ MeV}.$$

V. CONCLUSIONS

In the present paper, we have made estimates of the electromagnetic widths of most of the well-known baryon resonances, using the harmonic-oscillator model for baryons. There are as yet very little data with which we can compare our results. Nevertheless, a few general remarks can be made.

In the case of pion photoproduction from a proton or neutron, our investigation indicates that any future analysis should be carried out with a detail comparable to that of the present-day phase-shift analyses of pion-nucleon scattering. We anticipate that almost all the resonances will yield sizeable contributions, with no single resonance or two dominating at any one place above the (3,3) resonance energy.

In the "second resonance region," we expect from the above radiative widths and the known $N\pi$ widths that $S_{11}{}^{b+}(1550)$ and $D_{13}{}^{b+}(1525)$ will yield comparable contributions. The radiative width of $P_{11}{}^{+}(1470)$, taken as primarily 56, $L^P=0^+$, is smaller than for each of these two resonances. However, one cannot as yet state clearly what its effect will be, since there seems to be some doubt[25] as to its $N\pi$ width. The phase-shift-analyses[4] value of the latter is about 136 MeV, whereas direct observation[26] of mass enhancements in the 1400–1500-MeV region in πp and pp scattering experiments leads to estimates of the $N\pi$ width as low as 40 MeV. The harmonic-oscillator model of Ref. 1 gives 37 MeV. If, therefore, $P_{11}{}^{+}(1470)$ has an $N\pi$ width considerably smaller than that indicated by present phase-shift analyses, its effect in pion photoproduction will be less important than $S_{11}{}^{b+}(1550)$ and $D_{13}{}^{b+}(1525)$; however, if its $N\pi$ width is as large as 136 MeV, its contribution will be comparable to the other two resonances in this mass region.

In the "third resonance region," the striking feature is the expected absence[13] of a $D_{15}{}^{+}$ contribution, the observed cross section being due mainly to the $F_{15}{}^{+}(1690)$, $S_{11}{}^{a+}(1710)$, and $D_{13}{}^{a+}(1690)$ resonances.

As regards the mixing of the $S_{11}(1550)$ and $S_{11}(1710)$ states, our examination of their radiative widths shows that, of the two possibilities allowed by the $N\pi$ widths, viz., θ_s about 35° or 90°, the latter value is essentially ruled out. Because of the lack of data, the ambiguity of the D_{13} mixing angle θ_d could not be resolved.

One can further say what may be expected in an inelastic electron scattering type of experiment $ep\rightarrow eN^*$, which takes place via photon exchange. In the case of a real photon, the radiative width of $P_{11}{}^{+}(1470)$ is smaller by a factor of about $\frac{1}{3}$, compared to either the $S_{11}{}^{+}(1550)$ or the $D_{13}{}^{+}(1525)$, taking the mixing angles to be 35°. Thus, one would expect the 1450–1550-MeV region of the missing mass of the final state N^* to be dominated by the S_{11} and D_{13} resonances, at least for photons not too far off the mass shell. This indeed seems to be the case.[27]

For the $P_{11}(1470)$, we have another interesting prediction: that its charged state has a larger radiative

[24] J. Ashkin, in *Proceedings of the Tenth Annual International Conference on High-Energy Physics, Rochester, 1960*, edited by E. C. G. Sudarshan *et al.* (Wiley-Interscience Publishers, Inc., New York, 1961).

[25] This is further discussed in D. Faiman and A. W. Hendry, this issue, Phys. Rev. **180**, 1609 (1969), where it is suggested that there is possibly more than one resonance in the mass region 1400–1500 MeV.

[26] For a summary of the experimental situation, see J. G. Rushbrooke, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, Switzerland, 1968), p. 158.

[27] W. Panofsky, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, Switzerland, 1968), p. 23.

width than the neutral state. This is in direct contrast to an $SU(3)$ $\overline{\mathbf{10}}$ assignment for this resonance.

One can also take the matrix elements of the photon interaction operator between nucleon states. This gives the result[28] that the magnetic moments of the proton and neutron are in the ratio $-\frac{3}{2}$. Also, the quantity $\mu = eg/2M$ for the quark should be equal to the magnetic moment of the proton. The experimental value of the latter is 0.13 GeV^{-1}, which is somewhat less[29] than the value of μ deduced in Sec. IV from the physical radiative process $\Delta^+(1236) \to p\gamma$, viz., $\mu = 0.18$ GeV^{-1}. The situation is similar to one encountered in I, where the value of the pion-nucleon coupling constant $f_q/\sqrt{(4\pi)}$ as determined[30] by simply taking the matrix element of the pion emission operator between nucleon states is smaller than the value obtained by fitting the physical decay processes [$f_q/\sqrt{(4\pi)}$ equal to 0.17, compared to 0.24]. These two facts serve to remind us that the quark model is not yet quite numerically self-consistent.

However, our results for the $N\pi$ widths turned out to be reasonably good when compared with the experimental numbers. In the present absence of experimental data on the $N\gamma$ decay widths, we therefore believe that the results of the present investigation warrant consideration in any future analysis of pion photoproduction, proton Compton scattering, or inelastic electron scattering. The recent data[27] on the latter process certainly seem to be in general agreement with some of our expectations.

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[28] M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964); G. Morpurgo, Physics (N.Y.) **2**, 95 (1965).

[29] A similar discrepancy has been noted previously in the literature; see Refs. 12 and 16.

[30] C. Becchi and G. Morpurgo, Phys. Rev. **149**, 1284 (1966).

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Multiperipheral Dynamics at Zero Momentum Transfer*

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Following a suggestion by Goldberger and Low, the crude multi-Regge bootstrap model of Chew and Pignotti is reformulated through a generalization of the physical-region integral equation discovered in 1962 by Fubini and collaborators. When consideration is restricted to zero momentum transfer, Lorentz symmetry permits almost complete diagonalization of the kernel, Lorentz poles corresponding to eigenvalues thereof. Cuts also appear but in a manner dynamically and unambiguously related to the poles. Being an expression of unitarity, the equation encompasses "absorptive" effects.

1. INTRODUCTION

THEORETICAL study of strong-interaction dynamics heretofore has concentrated on reactions between two-particle channels, human capacities still not having mastered the combined requirements of Lorentz invariance, analyticity, and unitarity for this simplest reaction type. The time is nonetheless ripe for serious study of multihadron systems. It has long been recognized (a) that unitarity precludes dynamical isolation of two-particle from multiparticle channels, and (b) that indefinite proliferation of particle production characterizes any relativistic process. Theoretical attention to such questions has been inhibited not by belief in their unimportance but by the technical difficulties attendant on an indefinitely increasing number of spin-momentum variables. Recent experimental and theoretical developments, however, have suggested a general kinematical technique for decomposing arbitrarily large particle systems into finite subunits of manageable proportions; the approach may loosely be described as "multiperipheral." In this paper we propose a physically plausible and theoretically tractable dynamical equation suggested by multiperipheral kinematics.

The physical content of our equation is equivalent to that presented by Chew, Goldberger, and Low,[1] our work being stimulated by theirs. The difference between the two papers lies in the kinematical techniques employed. The principal advantage in the techniques of

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[1] G. F. Chew, M. L. Goldberger, and F. Low, Phys. Rev. Letters **22**, 208 (1969).