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Accuracy of Radiative Corrections to Electromagnetic Scattering from Protons

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Present calculations of radiative corrections for electron-proton and positron-proton scattering usually omit various parts independent of an infrared cutoff in two-photon-exchange contributions to the processes. These parts are examined for their effect on simple calculations which accompany the interpretation of experimental measurements. It is concluded that the simple calculations may not be adequate for large-angle scattering. More detailed analytic approximations are given for use at such angles if the asymptotic decrease of proton form factors is at least as fast as t^{-2}. A numerical method is outlined for slower decreases. The contribution of the $N^*(1236)$ resonance to radiative corrections is noted to depend most strongly on the choice of a form factor for N^* photoproduction.

I. INTRODUCTION

THE effective object of early experiments on electron-proton scattering was the measurement of the electromagnetic form factors of the proton. The form factors occur in the Rosenbluth[1] scattering cross section which is calculated on the lowest-order assumption of one-photon exchange between the electron and the proton. Higher-order terms in matrix elements and cross sections may be regarded as somewhat less significant because they are weighted by extra powers of the fine-structure constant α. Nevertheless, such questions as the effect of emission of soft photons undetected in the experiments require answers, and for that reason calculations have been extended[2,3] to the next order in perturbation theory. Finite corrections (radiative corrections) to the Rosenbluth cross section are found to come mostly from inelastic processes in which one photon is exchanged between the scattered particles and one soft photon is emitted, but the processes also supply terms dependent on a photon mass λ and divergent at $\lambda=0$. These terms are cancelled[2,3] by λ-dependent terms in matrix elements for two-photon elastic processes (e.g., the exchange of two virtual photons), whose contribution to the cross section is of the same order in α.

The simplest method of calculation,[4] which is almost universally used, expresses the radiative corrections as δ in

$$\sigma=(1+\delta)\sigma_0, \tag{1}$$

where σ is the corrected cross section and σ_0 is the Rosenbluth cross section. Here σ and σ_0 may also stand for differential cross sections. For (1) to hold as an identity, the matrix element for any part of the radiative correction must be proportional to the matrix element for one-photon exchange which determines σ_0. Relaxation of this requirement was first considered by Flamm and Kummer[5] in an approach based on dispersion theory. More directly, since the matrix element for scattering by exchange of two virtual photons contains an integral over the momentum of one of the photons, and this variable occurs in the arguments of the proton form factors in the calculations, it follows that the maintenance of (1) requires an approximation for the integral. The approximation, which in effect assumes[2,4] that form factors are significant only near zero momentum transfer (although its actual purpose is to simplify an integration), in a sense replaces the actual measurement of form factors for the purposes of a class of experiments[6] that compares electron-proton and positron-proton scattering under identical kinematical conditions to gain information about the real part of the amplitude for two-photon exchange. After presentation of the relevant matrix elements in Sec. II, the effect of this approximation and its relation to asymptotic behavior of form factors for large momentum transfer is discussed in Sec. III.

Two-photon-exchange scattering makes a finite contribution to δ, as well as a contribution dependent on λ, whose purpose has been mentioned above. The two parts are additive.[3,4] In simple calculations, some or all of the finite parts are neglected. Section IV examines the influence of that approximation on δ when the asymptotic behavior of form factors allows it to be used.

The last significant assumption of approximate calculations is that the proton remains a proton throughout the scattering event. However, in two-photon exchange, it is possible to excite a nucleon resonance [3,7,8] as an intermediate state. A summary of estimates, which are so far confined to the $N^*(1236)$ resonance, is given in Sec. V.

II. THEORY

By standard techniques of quantum field theory applied to the single Feynman diagram for scattering

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[1] M. N. Rosenbluth, Phys. Rev. **79**, 615 (1950).
[2] Y. S. Tsai, Phys. Rev. **122**, 1898 (1961).
[3] J. A. Campbell, Nucl. Phys. **B1**, 283 (1967).
[4] N. Meister and D. R. Yennie, Phys. Rev. **130**, 1210 (1963).
[5] D. Flamm and W. Kummer, Nuovo Cimento **28**, 33 (1963).
[6] J. Mar. B. C. Barish, J. Pine, D. H. Coward, H. DeStaebler, J. Litt, A. Minten, R. E. Taylor, and M. Breidenbach, Phys. Rev. Letters **21**, 482 (1968), and references therein.
[7] S. D. Drell and S. Fubini, Phys. Rev. **113**, 741 (1959).
[8] G. K. Greenhut, Ph.D. thesis, Cornell University, 1968 (unpublished).

with the exchange of one photon, one may derive[3] the matrix element

$$M_1=\frac{i\alpha Z}{\pi}\frac{mM}{(E_1E_2E_3E_4)^{1/2}}\frac{1}{q^2}\times\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\Gamma_\mu(q)u(p_2)\,, \quad (2)$$

where m is the mass of the lepton which has initial four-momentum p_1 and final four-momentum p_3, and M is the mass of the proton which has initial four-momentum p_2 and final four-momentum p_4. The quantities E_i are the corresponding energies. Equation (2) describes the scattering of both electrons ($Z=1$) and positrons ($Z=-1$). The four-momentum q transferred by the photon is equal to p_1-p_3 or p_4-p_2. The proton vertex function is

$$\Gamma_\mu(k)=\gamma_\mu F_1(k^2)+(1.79/2M)\gamma\cdot k\gamma_\mu F_2(k^2)\,, \quad (3)$$

where F_1 and F_2 are the Dirac and Pauli electromagnetic form factors. From (2), the cross section for elastic scattering is

$$d\sigma=\frac{\pi^2E_1E_2}{[(p_1\cdot p_2)^2-m^2M^2]^{1/2}}\times\int d^3p_3d^3p_4Q\delta^{(4)}(p_3+p_4-p_1-p_2)\,, \quad (4)$$

where $Q=M_1{}^\dagger M_1$ gives the Rosenbluth cross section.

The calculation above is correct to the order of α^2. The first step in the derivation of the elastic contribution to (4) of the order α^3 is the construction of all the Feynman diagrams for scattering with two external lepton lines, two external proton lines, and four vertices. There are five such diagrams. If their matrix elements are subscripted 2–6, the substitution in (4) which gives an elastic scattering cross section to the order of α^3 is

$$Q=M_1{}^\dagger M_1+2\,\mathrm{Re}M_1{}^\dagger\sum_{i=2}^{6}M_i. \quad (5)$$

To follow the convention of Ref. 3, let M_2 be the matrix element for the virtual process in which the lepton absorbs a photon of four-momentum k before it emits a photon of four-momentum $q+k$ and M_3 be the matrix element for which the order of these two events is reversed. Then

$$M_2=\frac{e^4}{(2\pi)^6}\frac{mMZ^2}{(E_1E_2E_3E_4)^{1/2}}\int d^4k\,\bar{u}(p_3)\gamma_\nu\frac{\gamma\cdot(p_1+k)+m}{k^2+2p_1\cdot k}\times\gamma_\mu u(p_1)\bar{u}(p_4)\Gamma_\nu(q+k)\,\mathrm{Pr}(p_2-k,M)\Gamma_\mu(-k)u(p_2)\times\frac{1}{(k^2-\lambda^2)[(k+q)^2-\lambda^2]}, \quad (6)$$

where

$$\mathrm{Pr}(p_2-k,M)=[\gamma\cdot(p_2-k)+M]/[(p_2-k)^2-M^2] \quad (7)$$

is the propagator for the intermediate-state proton. Similarly, M_3 follows with the replacement of p_1+k by p_3-k in the intermediate lepton propagator

$$M_3=M_2\left(\frac{\gamma\cdot(p_1+k)+m}{k^2+2p_1\cdot k}\rightarrow\frac{\gamma\cdot(p_3-k)+m}{k^2-2p_3\cdot k}\right). \quad (8)$$

The γpp vertex function off the mass shell is assumed to be still defined by (3), because the insertion of an extra term with an axial-vector form factor proportional to F_2 has no significant effect on calculations.

Unlike M_2 and M_3, the remaining matrix elements M_4–M_6 do not give differing contributions to electron-proton and positron-proton scattering through their interference with M_1 in (2).

Subsequent sections of this paper deal with approximations made in calculations with (6) and (8), particularly the evaluation of the integrals over k.

III. EVALUATION OF INTEGRALS

The four-dimensional integrals in (6) and (8) have two noteworthy properties. Firstly, they are impossible to determine analytically in the absence of information about the functional forms of F_1 and F_2 in (3), and this limitation remains for quite a wide range of explicit substitutions of form factors. Secondly, they are functions of a photon mass λ.

The second property suggests a simple method of approximate evaluation. A gauge-invariant treatment of the complete calculation of order α^3 in perturbation theory involves the consideration of additional Feynman diagrams which illustrate the elastic process of one-photon exchange with the emission of one soft real photon. Taken separately, the matrix elements from these diagrams are also functions of λ, which cancel the λ-dependent parts of M_2 through M_6 when the entire order is examined. Largely independent of a prescription to distinguish between "soft" and "hard" protons, the inelastic calculation[2,3] produces only one function of λ:

$$K(p_i,p_j;\lambda^2)=p_i\cdot p_j\int_0^1\frac{dy}{p^2}\ln(p^2/\lambda^2)\,, \quad (9)$$

where $p=yp_i+(1-y)p_j$. Therefore, it is desirable to have an approximation for (6) and (8) which expresses λ dependence only through (9). The approximation is that the form factors [normalized so that $F_1(0)=1$] are negligible with respect to unity except when their arguments are near zero, so that

$$\int d^4k[F_1(k)g_1(k)+F_2(k)\gamma\cdot kg_2(k)] \approx F_1(0)\int d^4k\, g_1(k) \quad (10)$$

for any g_i.

With the approximation (10), whose consequence for M_2 is

$$M_2 \approx (-Z\alpha/2\pi)[K(p_2,-p_1;\lambda^2) + K(p_4,-p_3;\lambda^2)]M_1, \quad (11)$$

there is the additional bonus, through (4), that Eq. (1) is preserved. It is possible[3] to make a calculation a little more accurate than one which relies entirely on (10) but, at the time that this was first done, it seemed that the principal aim of experiments to which the theory provided a means of interpretation was the measurement of form factors. Hence (1) remained untouched, and no substitution was made for form factors in (6) or (8). However, since we now have a considerable (although not complete) understanding of form factors at low[9] and high[10] momentum transfers, it is meaningful to ignore (1), insert various possible form factors explicitly in (3), trace their effect through (6) and (8), calculate the appropriate cross sections for positron-proton and electron-proton scattering under the same kinematical conditions C, and determine the ratio $R(C)$ of these cross sections, for comparison with experiment. $R(C)$ may then take into account the influence of the proton's anomalous magnetic moment, which is discussed briefly in Sec. III B of Ref. 4 and neglected by (10).

Although the results reported may not be truly from the asymptotic region, present indications[10] are that the high-momentum-transfer behavior of form factors is according to a power law

$$F_i \propto (1-t/t_0)^{-n}, \quad (12)$$

where $n=2$, $q^2=t$ is measured in units of $(\mathrm{GeV}/c)^2$, and $t_0=0.71$ in those units. There are the further possibilities,[11] derived from the application of the Bethe-Salpeter equation to a model in the ladder approximation, that the exponent n in (12) may lie between 1 and 2, and that there may be an extra factor of $\ln(-t/M^2)$ multiplying that expression. Unless n is an integer, the conventional method of Feynman parametrization[12] offers little help even in the approximate evaluation of the integrals in (6) and (8). Therefore, they must be treated from first principles as four-dimensional integrals in Minkowski space.

As an example of the calculation, consider the integral in (6) when (12) is substituted into (3). The denominator is

$$\begin{aligned} D &= (k^2-2p_2\cdot k)(k^2+2p_1\cdot k)(k^2-\lambda^2)(k^2+t_0^2)^n \\ &\qquad \times[(k+q)^2+t_0^2]^n[(k+q)^2-\lambda^2] \\ &= (E^2-r^2-2Mr)(E^2-r^2+2E_1E-2E_1rz) \\ &\quad \times(E^2-r^2-\lambda^2)(E^2-r^2+t_0^2)^n \\ &\quad\quad \times[t_0^2+E^2-r^2+a^2+2Ea-2raf(z,u)]^n \\ &\quad\quad\quad \times[-\lambda^2+E^2-r^2+a^2+2Ea-2raf(z,u)], \quad (13) \end{aligned}$$

where $k_0=E$, $\mathbf{k}^2=r^2$, $z=\hat{k}\cdot\hat{p}_1$, $u=\hat{p}_1\cdot\hat{p}_3$, $a^2=-t$, and $f(z,u)=zu+(1-z^2)^{1/2}(1-u^2)^{1/2}\cos(\phi_z-\phi_u)$, where ϕ_z and ϕ_u are the azimuthal angles associated with the vectors $\mathbf{k}$ and $\mathbf{p}_3$, respectively. It is assumed that the target proton is stationary, i.e., that $\mathbf{p}_2=0$.

The chief contributions to the integrals occur when one or more factors in (13) are small. If each term in (13) is considered separately, from left to right, the neighborhoods where this occurs are

(a) $r=E=0$ or $r=(E^2+M^2)^{1/2}+M$,

(b) $r=E=0$, or large $r=|E|$ near $z=1$,

or $r=(E^2+2E_1E)^{1/2}$ near $z=0$,

(c) $r=|E|$,

(d) $r=(E^2+t_0^2)^{1/2}$,

(e) $-a+[a^2+(E+a)^2+t_0^2]^{1/2}\leq r\leq a+[a^2+(E+a)^2+t_0^2]^{1/2}$,

(f) $-a+[a^2+(E+a)^2-\lambda^2]^{1/2}\leq r\leq a+[a+(E+a)^2-\lambda^2]^{1/2}$.

For integrals with k^2 in the numerator, the regions of large values of the integrand are still found in the union of neighborhoods under the six labels above. The analysis is made principally to check the possibilities for binomial expansion of the denominators in integrals of the form

$$\int_0^{2\pi} \frac{1}{[B+C\cos(\phi_z-\phi_u)]^n}\frac{d\phi_z}{A+B+C\cos(\phi_z-\phi_u)}, \quad (14)$$

where the azimuthal angle ϕ_z is the first variable to be integrated over in (6) or (8). For both equations (6) and (8) the result of the analysis is that $|C|<|B|$ and $|C|<|A+B|$, so that a single expansion can be used.

Terms proportional to $1-u^2$ appear in the integration. Experiments which record measurements in terms of q^2 have the choice of sampling small q^2 either by operating with beams of leptons of low energy or by setting detectors to measure large-angle scattering $(u\leq 0)$ at high energies. Since the Rosenbluth differential cross section falls off as $(1+u)(1-u)^{-2}$ with the cosine u of the scattering angle, the former choice is usually made. For analysis of such experiments, there is justification for the neglect of terms proportional to powers of $1-u^2$ in the present calculation, because these

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[9] V. Wataghin, Nuovo Cimento **54A**, 840 (1968).

[10] G. Weber, in *Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies* (Stanford Linear Accelerator Center, Stanford, Calif., 1967), pp. 59–75.

[11] M. Ciafaloni and P. Menotti, Phys. Rev. **173**, 1575 (1968).

[12] L. M. Brown and R. P. Feynman, Phys. Rev. **85**, 231 (1952).

TABLE I. Measurements of R as a function of kinematical conditions C, and associated theoretical predictions. The columns headed Δ and Δ' list radiative corrections in the sense of Eq. (1), and Δ' is the result of the more detailed calculation described in Sec. IV. Columns 7 and 8 contain theoretical values of R calculated without (1) for two different values of n, the assumed power of t^{-1} that reproduces the asymptotic behavior of proton electromagnetic form factors.

	C		Expt.	Radiative correction		Theor. values of R	
E_1 (GeV)	$\cos^{-1}u=\theta^0$	$-q^2$ $[10^6(\text{GeV}/c)^2]$	R	Δ	Δ'	$n=2$	$n=1$
10.0	2.60	0.204	1.010±0.020	0.001	0.001	1.001	1.001
4.0	12.5	0.689	0.986±0.016	0.006	0.005	1.005	1.007
10.0	5.0	0.731	0.965±0.045	0.002	0.002	1.002	1.003
4.0	20.0	1.54	1.003±0.022	0.015	0.010	1.010	1.013
4.0	27.5	2.44	1.040±0.043	0.028	0.019	1.019	1.028
4.0	35.0	3.27	1.111±0.123	0.044	0.025	1.026	1.041
10.0	12.5	3.79	1.024±0.034	0.014	0.008	1.008	1.011
10.0	15.0	5.00	1.038±0.059	0.020	0.012	1.012	1.015

have essentially no effect on measurements made near the forward direction.

Without further approximation, the integral over z is next performed. In the remaining respects, with the $-i\epsilon$ prescription and the transformation $E\rightarrow iE$ to Euclidean space, the method of integration complies with a standard pattern.[13] When integrals over E and r remain, the integrand contains logarithmic terms which cannot be treated in any simple way. Then, as a last step before numerical evaluation, the scaling transformations $w_1=(2/\pi)\tan^{-1}r$ and $2w_2-1=(2/\pi)\tan^{-1}E$ are made, so that w_1 and w_2 each range between 0 and 1.

The question of numerical calculation of these two-dimensional integrals is difficult, especially since the integrand is still a function of λ. The first difficulty is merely one of the time required for operation of numerical computer programs. The treatment of the infrared cutoff depends on the fact that the difference of the exact integral form for M_2 and the approximate expression in (11) must be independent of λ. If the two forms are evaluated separately by a Monte Carlo or other numerical program for a fixed small value of λ, and their difference remains stable for subsequent runs of the same programs with progressively smaller values of λ, then it is a crude but probably effective means of obtaining a credible result for the finite part of M_2. The same technique can be applied to M_3, whose approximate value under (10) is[2,3]

$$M_3=(Z\alpha/2\pi)[K(p_2,p_3;\lambda^2)+K(p_4,p_1;\lambda^2)]M_1. \quad (15)$$

This method has been adopted. The problem has been considered partly as a test of Monte Carlo schemes for eventual extensive application to the calculation of the part of the electron's magnetic moment that is proportional to α^3.

The abbreviation C in $R(C)$ covers the triplet (E_1,q^2,u) of kinematical conditions for a scattering experiment. In addition, the exponent n in (12) may be regarded as an unknown. The numerical programs must be run afresh for each new combination of the four parameters, but a simpler expedient is used here. The experimental results of Mar *et al.*[6] for measured values of $R(C)$ over eight different combinations C are reproduced in the first four columns of Table I. The radiative corrections[14] Δ obtained by the method of Meister and Yennie[4] are listed in column 5. Values of $R(C)$ calculated by the techniques outlined in this paper for the same sets of parameters C as in the experiment are given in columns 7 and 8. Although the application here is only to cases of integer n, the method can be applied for arbitrary n.

For the case $n=2$, the calculated values of R in the table do not differ at all from those obtainable[3] by various approximate treatments of the form factors involved in the integrals (6) and (8). The same holds, *a fortiori*, for form factors with an exponential dependence on t.

The case $n=1$ has been modelled in this calculation by a trivial adaptation of Wataghin's[9] three-pole fit for form factors, which takes account of contributions from the ρ^0, ω, and ϕ mesons. If the behavior of integrals (6) and (8) had been as insensitive here as for $n=2$, columns 7 and 8 of Table I would have contained identical entries. However, significant differences begin to appear as the scattering angle increases. The degree of "significance" must, of course, be assessed by reference to the experimental errors on measurements of R quoted in column 4. In principle, nevertheless, the chief source of error in beam monitoring[6] in that experiment need not recur in other experiments.

An extra factor of $\ln(-t/M^2)$ in (12) for large momentum transfers does not enter calculations unambiguously, for there is no evidence of such a term at small momentum transfers, and the question of how and where to introduce it for larger $-t$ has no single answer. But no plausible method of doing so seems to affect the results in columns 7 and 8 of Table I in any important way.

[13] S. S. Schweber, *Introduction to Relativistic Quantum Field Theory* (Harper and Row, Publishers, Inc., New York, 1961), pp. 519–522.

[14] Since this radiative correction applies to R, it is the difference of corrections calculated separately for positron-proton and electron-proton scattering.

Preliminary numerical estimates suggest, as Table I implies, that the best prospect for a measurement of R significantly greater than unity lies at large scattering angles,[15] above about 120°. The other conclusion of this work is that, while previous approximate methods of treatment of form factors may be adequate if $n=2$, the case $n=1$ requires a more detailed approach. One may either accept as highly likely Wataghin's recent proposal[16] that asymptotically $F_1(t)$ behaves like t^{-1}, and then be compelled to use the numerical methods of integration described here to calculate R accurately for large scattering angles, or reverse the chain of reasoning to use measurements of R as a test of the proposal. This may be an artificially simplified view, however, to the extent that indirect effects of strong interactions are not fully contained in the present calculation.

IV. APPROXIMATE EVALUATION OF INTEGRALS

A result of Sec. III is that numerical integration is probably unavoidable if the consequences of the substitution $n=1$ in (12) are to be explored fully. Is there a simpler way of proceeding in the case of large scattering angles if $n=2$, as seems to be most likely[10] from the experimental evidence?

This matter has been studied in a previous paper[3] which presents expressions for M_2 and M_3 that are more general than (11) and (15), and that contain parts independent of λ. Since the approximation (10) is justified if $n=2$, the expressions are proportional to M_1, and therefore allow the use of the simple relation (1) to calculate radiative corrections. With only the terms that have proved to be significant in numerical evaluation, and purged of typographical error, these extensions of (11) and (15) are

$$M_2=\frac{Z\alpha}{2\pi}\Big\{K(p_2,p_1;\lambda^2)+K(p_4,p_3;\lambda^2)+\Big[2\theta\ln\Big(\frac{s_1}{-s_2}\Big)+\tfrac{1}{2}\ln^2(A_2-1)+\tfrac{3}{2}\ln^2(-B_2)-\ln^2\Big(\frac{A_2-B_2}{-B_2}\Big)+2\,\mathrm{Li}_2\Big(\frac{1-B_2}{A_2-B_2}\Big)-\tfrac{1}{2}\ln^2(1-A_1)-\tfrac{1}{2}\ln^2(B_1)+2\,\mathrm{Li}_2\Big(\frac{A_1-B_1}{A_1}\Big)+\mathrm{Li}_2\Big(\frac{A_1-B_1}{1-B_1}\Big)-2\pi^2\Big]\times\coth\theta-K(p_2,p_1;-q^2)\Big\}M_1 \quad (16)$$

and

$$M_3=(Z\alpha/2\pi)[K(p_2,p_3;\lambda^2)+K(p_4,p_1;\lambda^2)-K(p_2,p_3;-q^2)]M_1, \quad (17)$$

where

$$p_i\cdot p_j=mM\cosh\theta,$$
$$[yp_i+(1-y)p_j]^2=s_1(y-A_1)(y-B_1),$$
$$[yp_i-(1-y)p_j]^2=s_2(y-A_2)(y-B_2),$$
$$A_i>B_i,$$

and

$$\mathrm{Li}_2(x)\equiv-\int_0^x\frac{\ln(1-y)}{y}dy$$

in a Spence function, or dilogarithm.

The entries Δ' in column 6 of Table I are radiative corrections calculated with the assistance of (16) and (17). The table indicates that there is no difference of present importance between Δ' and Δ for small scattering angles. For larger angles, detailed inspection of (16) and (17) shows that the most significant terms involve the λ-independent functions $K(p_2,p_j;-q^2)$. Therefore, the approximate relation[14]

$$\Delta'-\Delta\approx-(2\alpha/\pi)[K(p_2,p_3;-q^2)-K(p_2,p_1;-q^2)] \quad (18)$$

can be used to take more accurate account of the effect of scattering with the exchange of two virtual photons on radiative corrections. For substitution into (18),

$$K(p_2,p_j;-q^2)=\frac{1}{4}\Big[\ln^2\Big(\frac{M(2E_j-M)}{\rho a}\Big)-\ln^2\Big(\frac{m^2}{\rho a}\Big)+\ln^2\Big(\frac{2ME_j}{\rho a}\Big)\Big]+\frac{1}{2}\Big[\ln\Big(\frac{2ME_j}{\rho a}\Big)\ln\Big(\frac{2E_j(2E_j-M)}{m^2}\Big)-\mathrm{Li}_2\Big(\frac{2E_j-M}{2E_j}\Big)+\mathrm{Li}_2\Big(\frac{M}{2E_j}\Big)-\frac{\pi^2}{6}\Big], \quad (19)$$

where again $a^2=-t=-q^2$, $\rho=(2ME_j-M^2)^{1/2}$, and terms of order m^2/M^2 with respect to unity have been neglected. Equation (19) holds if $2ME_j>M^2+m^2$, but a similar equation is easy to obtain by integration if $2ME_j<M^2+m^2$. The quantity E_3 does not appear in the headings of columns 1–3 of Table I, but is determined by these kinematical quantities through the relation $-q^2\approx 2E_1E_3(1-u)$.

V. EFFECTS OF NUCLEON RESONANCES

It has been assumed above that two-photon–exchange scattering occurs without the excitation of nucleon resonances in M_2 or M_3. The assumption is expressed in (7), which is just the proton propagator for substitution into (6) and (8). Drell and Fubini[7] first made an approximate nonrelativistic estimate of the effect of excitation of the $N^*(1236)$ resonance by the use of information about the amplitude for proton Compton

[15] For large angles, the approximation which neglects powers of $1-u^2$ in the numerical integration must be removed. However, the purpose of the approximation was to ensure a single binomial expansion in an integral form like (14). Other single binomial expansions are available in any suitably chosen range of u.

[16] V. Wataghin, Nucl. Phys. (to be published).

scattering. More exact calculations may be describeP readily, but are difficult to perform because of the many terms that are generated. To carry through a project of this type in a reasonable time requires, firstly, a computer program to handle the necessary algebraic operations and substitutions of integral forms, and, secondly [since it is not always possible to use (1)], a numerical program to produce values of radiative corrections from the simplified algebraic expressions.

The assumptions governing the input to the algebraic program are the following:

(a) The propagator for a spin of $\frac{3}{2}$, to replace (7), is

$$\mathrm{Pr}_{\rho\sigma}(p,M^*)=(\gamma\cdot p-M^*)^{-1}[\delta_{\rho\sigma}-\tfrac{1}{3}\gamma_\rho\gamma_\sigma-(1/3M^*)(\gamma_\rho p_\sigma-\gamma_\sigma p_\rho)-(2/3(M^*)^2)p_\rho p_\sigma],$$

where M^* stands for the $N^*(1236)$ mass.

(b) Photoproduction of the $N^*(1236)$ resonance occurs principally in $M1$ mode,[17,18] so that the Lagrangian for the γpN^* vertex is

$$e(c/M)v_\nu\gamma_\mu\gamma_5 u\partial_\nu A_\mu+\text{H.c.}, \tag{20}$$

where v_ν is a Rarita-Schwinger spinor.

(c) The dimensionless constant c in (20) is 1.437, which can be derived from the work of Iddings.[19]

(d) The $M1$ form factor for the γpN^* vertex is given, in general, by

$$G(t)=0.64M^4/(t+0.8M^2)^2, \tag{21}$$

an expression which matches[17] early experimental data on photoproduction. Further details of the calculation have been presented in a previous paper.[3]

In that paper, the approximation (10) for form factors was used on $G(t)$, for the somewhat unphysical reason that the algebraic calculation would otherwise have overflowed the available storage of the computer. There are now stronger justifications for the approximation. Section III indicates that (10) produces no significant error if $G(t)$ falls off with t no more slowly than t^{-2}. Dufner and Tsai[18] have made a phenomenological analysis of recent data, which leads to a dependence

$$G\propto(1+9.0a)\exp(-6.3a). \tag{22}$$

The expression (22) falls off much more rapidly with $t=-a^2$ than does (21). Moreover, Caneschi[20] has concluded from a model based on the Bethe-Salpeter equation that G does not decrease asymptotically as slowly as t^{-2}.

No completely general conclusions can be drawn from the numerical computer programs produced in the present work, because of the time needed for them to generate results for a set of kinematical conditions which is large enough to be representative. For angles near 135° and momentum transfers between 1 and 5 $(\text{GeV}/c)^2$, the absolute value of their contributions to radiative corrections for lepton-proton electromagnetic scattering ranges between 0.007 and 0.011 as E_1 varies between 4 and 10 GeV, with the assumption of (21) for $G(q^2)$, where q is the total four-momentum transferred in the scattering process. Since $G(q^2)$ is factored out of integrals under the approximation (10), it is possible that numerical predictions should be multiplied by a ratio of (22) to (21) to be in accordance with the most recent results. Hence, they may be reduced for large momentum transfers. On the other hand, for some kinematical conditions Greenhut[8] has reported that contributions for the $N^*(1236)$ resonance under different means of approximation may be up to 50% larger than those calculated by the present method. Further study of these questions is required.

The effects of nucleon resonances of higher mass may be incorporated in the calculation when accurate experimental measurements of their photoproduction become available. Dufner and Tsai[18] have remarked that this situation has not yet been reached.

VI. CONCLUSIONS

For small scattering angles (below, say, 90°) it is sufficient to calculate radiative corrections for comparison with present experiments by the simplest available methods.[2,4] For larger scattering angles, more detailed analytic approximations of the type quoted in Sec. IV are likely to be both accurate and noticeably different from the results of the simple methods if the asymptotic decrease of proton electromagnetic form factors is at least as rapid as t^{-2}. If the form factors fall off only as t^{-1}, analytic approximations should be replaced by numerical procedures, as stated in Sec. III. The contribution of the $N^*(1236)$ resonance to radiative corrections at large angles is not negligible, but its precise importance depends most critically on the choice of the analytic expression for a γpN^* form factor.

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[17] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).
[18] A. J. Dufner and Y. S. Tsai, Phys. Rev. **168**, 1801 (1968).
[19] C. K. Iddings, Phys. Rev. **139**, B446 (1965).
[20] L. Caneschi, Nuovo Cimento **56A**, 551 (1968).