Regge Poles, Argand Diagrams, and Resonances*

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Loops in the Argand diagram of s-channel partial-wave projections of Regge poles are investigated. In attempting to identify these with resonances, strong constraints on both Regge poles and resonances are deduced.

THE partial-wave projections of t-channel Regge poles give rise to circles in the Argand diagram. Ever since this fact was pointed out by Schmid,[1] two main attitudes have been taken. (a) Circles in a partial wave amplitude are identified with resonances, and the projection procedure is a tool in investigating them.[1,2] (b) Circles do not necessarily imply resonances, and the identification of resonances has to be reconsidered.[3,4] In this paper we will assume (a) to hold, and, by making standard assumptions about the properties of resonances, deduce strong restrictions about both the Regge poles and the resonances. Many of the restrictions agree with experiment. Some have not been checked. If the restrictions fail to agree with future experiment, (a) is incorrect and attitude (b) will have to be adopted.

The Regge representation, in terms of a few poles, is an approximate representation of the scattering amplitude. As such it can be used for an approximate phase-shift analysis. This approximate analysis may differ slightly from the experimental one. Very small features appearing in the approximate result, may not correspond to any experimental feature. It is thus important to consider carefully errors of the Regge representation.[5]

In treating the results of this approximate phase-shift analysis the definition of a resonance as a pole in the unphysical sheet is not of great help. We will consider as resonances only structures which can be resolved in an Argand diagram. The possibility of many, overlapping, nonresolvable resonances exists, but we cannot identify them.[6]

Lacking absolute criteria for the definition of resonances, we will demand a few necessary but not sufficient conditions for circles to be identified as resonances[7]:

(1) Resonances have well-defined quantum numbers in the s channel—i.e., spin, isospin, etc.

(2) Resonances factorize in the s channel.

(3) Resonances can be observed in "production experiments" (i.e., $\pi N\rightarrow\pi N^*$ in addition to $\pi N\rightarrow N^*\rightarrow\pi N$).

(4) There are no "exotic resonances." (By this we mean: $Y=2$, $B=2$, $I=\frac{5}{2}$, 2, etc.)

The first three conditions are essential. The fourth is a statement based on present experience. We will discuss all conditions but condition (3). It is connected to the dynamics of production processes, possibly to multiple Regge exchange.[8]

It has been noted[9] that resonance circles are generated by the oscillatory behavior of $\beta(t)$ (in which we have included the signature factor). Processes which contain resonances will therefore show a dip structure in the angular distribution. Processes without resonances will have a smooth angular distribution. This fact seems to hold in $\bar{p}p$ versus pp scattering.[10] Dips also appear in π^-p and K^-p [10] scattering. It would be interesting to compare these to K^+p data which should have no dips, even at large t.

A single Regge pole in the t channel contains all isospin values in the s channel, including exotic values whenever possible. To satisfy condition (4) two possibilities exist: (i) The partial wave projection of the trajectory contains no loops in the Argand diagram.

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[1] C. Schmid, Phys. Rev. Letters **20**, 698 (1968).

[2] H. R. Rubinstein, A. Schwimmer, G. Veneziano, and M. A. Virasoro, Phys. Rev. Letters **21**, 491 (1968); M. Kugler, *ibid.* **21**, 570 (1968).

[3] P. D. B. Collins, R. C. Johnson, and E. J. Squires, Phys. Letters **27B**, 23 (1968); V. A. Alessandrini and E. J. Squires, *ibid.* **27B**, 300 (1968); V. A. Alessandrini, D. Amati, and E. J. Squires, *ibid.* **27B**, 463 (1968).

[4] V. A. Alessandrini, P. G. O. Freund, O. Oehme, and E. J. Squires, Phys. Letters **27B**, 456 (1968).

[5] The error in a Regge representation with only a few singularities is of the order s^α; anything vanishing like e^{-s}, or faster, for large s is probably a feature of the approximation. This holds for fixed l, $s\rightarrow\infty$ and $l\gg s$ resonances in Ref. 4. Establishing the error at each energy is a more difficult task.

[6] This may be true in a world made mostly of resonances. G. Veneziano, Nuovo Cimento **57A**, 190 (1968); N. N. Khuri Phys. Rev. **176**, 2026 (1968); M. A. Virasoro (to be published).

[7] H. Harari, [in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 195] gives a similar discussion.

[8] G. F. Chew and A. Pignotti, Phys. Rev. Letters **20**, 1078 (1968).

[9] Ref. 1 and C. B. Chiu and A. Kotanski, Nucl. Phys. **B7**, 615 (1968).

[10] Large-angle dips in π^-p, K^-p, and $\bar{p}p$ were observed by A. Ashmore *et al.*, Phys. Rev. Letters **21**, 387 (1968). For a review of small-t dips see V. Barger, in *Proceedings of the CERN Conference on High-Energy Collisions of Hadrons*; pp data: J. V. Allaby *et al.*, *ibid.*

(ii) A few trajectories cooperate in cancelling the loops in the unwanted s channels.

The partial-wave projection of a t-channel Regge pole, $R_t(I,J)$, is given by

$$R_t(I,J)=\int P_J(\cos\theta_s)\beta_I(t)\nu^{\alpha_I(t)}d\cos\theta_s, \qquad (1)$$

where J denotes the angular momentum, the subscript t the t channel, and I the isospin index (or later an SU_3 index). Equation (1) applies only for the spinless case. From $R_t(I,J)$ we separate the resonant part $\hat{R}_t(I,J)$. When dealing with an energy high enough so that the forward and backward peak are well separated, we can describe these by t- or u-channel Regge trajectories, respectively. The s-channel resonances $S(I,J)$ are given by

$$S(I,J)=X(I,I')\hat{R}_t(I',J)+X'(I,I'')\hat{R}_u(I'',J), \quad (2)$$

where $X(I,I')$ and $X'(I,I'')$ are crossing matrices. The condition for nonexistence of exotic states denoted by $I=E$ gives

$$S(E,J)=0. \qquad (3)$$

In our approximation we only deal with the leading Regge trajectories and we have an additional assumption:

(5) Regge trajectories having exotic quantum numbers can be neglected. [Assumption (5) is not independent of assumption (4).]

$$\hat{R}(E,J)=0. \qquad (4)$$

We have now to satisfy both (3) and (4). The solutions will be of type (i) or (ii) above. Type (i) is the trivial solution: $\hat{R}(I,J)\equiv 0$. Type (ii) will involve cooperation of Regge trajectories

$$\hat{R}(I,J)\propto \hat{R}(I',J) \qquad (5)$$

and a relation, following Eqs. (5) and (2), between $S(I,J)$ and $S(I',J)$, i.e., a cooperation between s-channel resonances.[11]

For a cooperation to occur, Eq. (5) has to be satisfied for all resonances. The cooperating trajectories must thus have similar $\alpha(t)$ and related residues. The Pomeranchuk trajectory cannot cooperate because of its unique intercept; it must correspond to a nonresonating part,[12] and must have no oscillations in β.

We will next give a brief list of results obtained from conditions (1)–(5):

$\pi\pi\to\pi\pi$: This process has the additional simplifying feature that the same trajectories are exchanged in the t and u channels; therefore

$$\hat{R}_t(I,J)=(-1)^J\hat{R}_u(I,J). \qquad (6)$$

The connecting factor $(-)^J$ stems from the properties of $P_J(\cos\theta)$ in Eq. (1). Condition (5) is expressed by the vanishing of $\hat{R}_t(2,J)$. The vanishing of $S(2,J)$ imposes one relation between $\hat{R}(1,J)$ and $\hat{R}(0,J)$, namely,

$$\hat{R}(1,J)=\tfrac{2}{3}\hat{R}(0,J). \qquad (7)$$

We thus have a single solution: Cooperating P' and ρ trajectories for $t<0$ and cooperating $I=0$ and $I=1$ resonances in two exchange-degenerate trajectories, for $s>0$.

$PP\to PP$ in SU_3: (P denotes a pseudoscalar meson). The demand that resonances or Regge poles behaving like **27**, **10**, and **10*** do not exist in any channel imposes three relations between the $\mathbf{8}_s$, $\mathbf{8}_a$, and **1** of SU_3. We thus have a unique cooperating solution demanding exchange-degenerate octet (odd l, odd signature) and nonet (even l, even signature), in both the t channel (for $t<0$) and s channel for large s.

$\bar{B}B\to\bar{B}B$ scattering in SU_3. Since no backward peak exists, we get exchange-degenerate trajectories of odd and even l.[1] The solution is nonunique but demands nonets of mesons in t and s channels. Octets only, are inconsistent with our conditions.

$\bar{B}\Delta\to\bar{B}\Delta$. Demanding no resonances in **35**, **27**, and **10**, and no Regge trajectory in **27** gives three homogeneous equations for $\hat{R}(8_s,J)$, $\hat{R}(8_a,J)$, and $\hat{R}(1,J)$. These equations have only null solutions. Two results emerge: (i) The coupling of vector and tensor trajectories produces no resonances; i.e., there are no dips in the angular distribution, or the coupling vanishes completely. (ii) High-energy resonances do not couple to $\bar{B}\Delta$, otherwise the assumption that no exotic states exist must be relaxed in the $\bar{B}\Delta$ channel. Similar properties have been investigated from a slightly different point of view.[13]

Next we turn to the results implied by factorization: condition (2). We will only consider factorization for s-channel resonances. The integration over a range of t makes the analysis of t-channel factorization much more difficult.

First, we discuss πN and ηN scattering. Both the elastic channels ($\pi N\to\pi N$ and $\eta N\to\eta N$) are dominated by P' and ρ exchange. If a resonance appears in both elastic channels when partial waves are projected, is should also appear in $\pi N\to\eta N$. This demands the existence of a cooperating trajectory having $I=1$ $G=-$, namely, the A_2. It should be emphasized that the existence of A_2 is necessary from factorization alone, regardless of SU_3.

A new situation appears in $\bar{B}B$ versus MM scattering. Both the elastic channels are dominated by vector and

[11] The relation between t-channel trajectories can be obtained from finite-energy sum rules, assuming resonance dominance of all trajectories but the Pomeranchuk one. H. Harari, Phys. Rev. Letters **20**, 1395 (1968). We obtain in addition relations between s-channel resonances.

[12] In agreement with the Harari conjecture (Ref. 11).

[13] J. L. Rosner, Phys. Rev. Letters **21**, 950 (1968); H. J. Lipkin Nucl. Phys. **B9**, 349 (1969).

tensor mesons. If the same resonance appears in both channels at high energy, then it should appear also in $\bar{B}B \rightarrow MM$. This last reaction is governed by baryon exchange. Since baryon exchange decreases faster with increasing energy than meson exchange does, the dominant circles appearing in this reaction at high energy will be much smaller than those in the elastic channels. Therefore, if dominant resonances appear in both $\bar{B}B$ and MM, we face apparent contradiction of factorization. Nondominant resonances in $\bar{B}B$ and MM elastic scattering may be much smaller than the dominant ones and may not lead to problems with factorization.

A few ways to resolve this contradiction appear. First: At high energy, the resonances appearing in $\bar{B}B$ do not appear strongly in MM, and vice versa. Second: There are overlapping resonances with identical quantum numbers which add coherently in the elastic channels but cancel in the inelastic ones. These resonances must have identical isospin, SU_3 properties, spin-parity, etc., but be distinguished only by their coupling to $\bar{B}B$ and MM. If the two possibilities are incorrect, the way out will lie in abandoning the identification between circles and resonances.

It may thus turn out that the $\bar{B}B$ system will display new "baryonic" resonances, which at high energy couple weakly to mesons, and "mesonic" resonances coupling weakly to $\bar{B}B$. This suggested possibility may open up a new field of research in meson spectroscopy. Only a few beginnings in this direction have been made.[14] Such "baryonic" resonances may have unexpected features. Conceivably, they may even have exotic quantum numbers.[13] In this case the results obtained in discussing $\bar{B}\Delta$ systems may have to be abandoned. If the division into "mesonic" versus "baryonic" resonances will hold in future experiments, a dynamical explanation for this division will be necessary.

We have deduced a large set of constraints on both Regge poles and resonances by identifying Argand circles with resonances. Most of the deductions were based on simple qualitative observation, and most of the constraints deduced are in rough agreement with experiments. In our opinion the crucial test of the assumption will arise from the factorization condition in $\bar{B}B$ versus MM scattering. Conceivably, new tests will arise in a more quantitative investigation of the point raised in this paper. We have also treated SU_3 without symmetry breaking; when breaking is included new results emerge.[15]

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[14] R. J. Abrams *et al.*, Phys. Rev. Letters **18**, 1209 (1967); J. Lys *et al.*, *ibid.* **21**, 1116 (1968); G. Alexander *et al.*, *ibid.* **20**, 755 (1968).

[15] C. B. Chiu and J. Finkelstein, Phys. Letters **27B**, 516 (1968).