High-Energy Backward Elastic Proton-Deuteron Scattering and Baryon Resonances*

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Elastic proton-deuteron scattering at 1 GeV shows a peak at 180° which seems larger than that predicted by a simple nucleon-transfer mechanism. A straightforward generalization of the nucleon-transfer mechanism is obtained through a Regge extension of the model, permitting a calculation of the (p,d) process using the parameters of a (π^+,p) Regge model. A dynamic interpretation is attempted via a static field theory. It is estimated that a deuteron contains the $N^*(1688)$ resonance with a $\frac{1}{2}$–1% probability and that this accounts for about half of the backward proton-deuteron scattering at 1 GeV.

I. INTRODUCTION

FOR a long time it has been known that high-energy projectiles incident upon nuclei give rise to copious production of deuterons.[1] Recently, good energy-resolution experiments have been carried out at energies of 1 GeV and greater,[2] making it possible to identify the elastic scattering, thereby permitting a more detailed theoretical discussion. The basic problem and the specific topic of this work is the understanding of the mechanism for $p+d \rightarrow p+d$ backward scattering, which is probably one of the important processes causing deuteron production in this regime. In Sec. II we point out that the mechanism almost entirely responsible for the scattering is a fermion-transfer (pickup) process.[3,4] On the other hand, the results of the present work show that the high-energy backward proton-deuteron scattering is rich in information about the structure of the two-particle system and indicates that much more is involved than the simple nucleon-exchange process. We make two rather distinct generalizations of this mechanism, both of which imply the importance of excited nucleon states.

First, recognizing that the simple Born approximation pickup mechanism corresponds to a pole in the antiproton-deuteron (u) channel, we treat the reaction in the u-channel Regge-pole approximation. This model, discussed in Sec. III, allows the proton-deuteron backscattering to be calculated with the parameters which have previously been determined in π-nucleon scattering.

The Regge extension is seen to imply the importance of the $N^*(1688$ MeV) nucleon excited state (we refer to this as the nucleon resonance) in the process being created. Regardless of the validity of the Regge model, since this particular state has the same quantum numbers as the nucleon except for spin, it is not unexpected that it can play an important role in fermion-transfer reactions. Noting that the nucleon-nucleon potential at distances between 0.5–1.0 F reaches a depth of hundreds of MeV, it is to be expected that the proton and neutron in a deuteron can spend an important fraction of their existence in an excited state. This implies that the one-channel two-body interaction must be replaced by a many-channel interaction. In Sec. IV we use a static-field-theory model to derive the off-diagonal potential which couples the proton-neutron system to a nucleon plus nucleon resonance.

The effect on the deuteron is estimated in perturbation theory and the results expressed as an admixed component in the deuteron. We then discuss the contributions of the various components of the deuteron wave function to the fermion-exchange reaction at high energy.

II. QUALITATIVE DESCRIPTION OF PROTON-DEUTERON BACKWARD SCATTERINGS

The basis for the mechanism leading to backward proton-deuteron scattering at high energies can most easily be understood from considerations of momentum transfer. In Fig. 1 the nucleon-transfer or pickup process

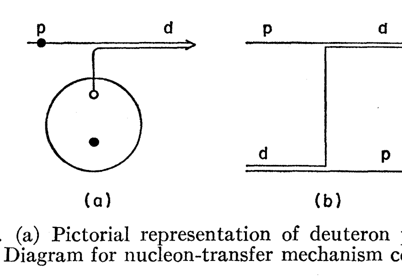

FIG. 1. (a) Pictorial representation of deuteron pickup reaction. (b) Diagram for nucleon-transfer mechanism corresponding to (a).

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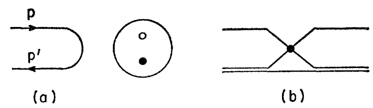

FIG. 2. (a) Pictorial representation for proton backward scattering by two-particle scattering (neglecting exchange). (b) Diagram corresponding to (a).

for proton-deuteron scattering is illustrated pictorially and schematically. For 180° proton scattering, it is seen that both the incident proton and the proton originally in the deuteron undergo a momentum transfer of approximately $\frac{1}{2}p$ in the c.m. system, where p is the (large) momentum of the proton. The "bounce" or scattering collision, the mechanism for which is backward nucleon-nucleon scattering, is depicted in Fig. 2. In this process the protons undergo much larger momentum transfers, approximately $2p$ and p for the incident and bound protons, respectively, for 180° scattering in the c.m. system. For example, at 1 GeV in the impulse approximation the deuteron wave function must provide a Fourier component corresponding to 2.5 F^{-1} for the transfer reaction compared to 10 F^{-1} for the scattering collision. Recently, estimates of the higher-order and exchange corrections have been made, with the result that the reaction is still underestimated by more than an order of magnitude.[5]

The cross section corresponding to the nucleon-exchange mechanism has been derived in Ref. 4 as the

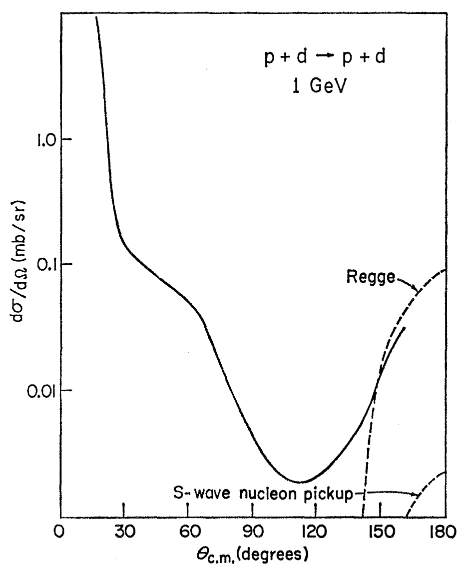

FIG. 3. The backward proton-deuteron elastic scattering cross section using u-channel nucleon Regge-pole exchange model. The parameters used were those of Ref. 13 for π^+-p scattering except for the residue function and the crossing matrix element. For comparison, the nucleon-transfer prediction with the S-wave Hulthén wave function (see text) is also shown (labeled S-wave nucleon pickup). The solid curve is the experimental cross section taken from Bennett *et al.* (Ref. 2).

[5] L. Bertocchi and A. Capella, CERN Report, 1967 (unpublished).

inverse process to the stripping mechanism of Ref. 3, and we introduce a simple relativistic generalization of that result. Using the Schrödinger equation for the deuteron wave function, one obtains for the Born approximation to the differential cross section in the c.m. system for the special case of proton-deuteron backscattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}}=(2\pi)^4\left(\frac{E_pE_d}{E_{\text{tot}}}\right)^2\frac{(\kappa_d^2+\Delta^2)^2}{M_N^2}|\Psi_{\text{deut}}(\Delta)|^4, \quad (1)$$

where $\Psi_{\text{deut}}(\Delta)$ is the Fourier component of the deuteron wave function corresponding to the momentum transfer Δ

$$\Delta=|\tfrac{1}{2}\mathbf{P}_d'-\mathbf{p}|\,,$$
$$\Psi_{\text{deut}}(\Delta)=\frac{1}{(2\pi)^{3/2}}\int d^3r\, e^{-i\Delta\cdot\mathbf{r}}\Psi_{\text{deut}}(\mathbf{r})\,. \quad (2)$$

In Eq. (1), the binding energy of the deuteron is $\epsilon_d=\kappa_d^2/M_N$, and E_p, E_d, and E_{tot} are the energies of the proton, deuteron, and the total energy, respectively, in the c.m. system. Relativistic kinematics has been used in obtaining Eq. (1). In order to illustrate the information that is contained in the 1-GeV reaction, let us try the Hulthén wave function, which is often used in low-energy-reaction calculations. The wave function in momentum space is

$$\Psi_d(\Delta)=N[1/(\Delta^2+\kappa_d^2)-1/(\Delta^2+\beta^2)], \quad (3)$$

where $\beta=1.44$ F^{-1}, $\kappa_d=0.232$ F^{-1}, and N is a normalization factor. Using the expression (3) in Eq. (1), one obtains the results shown in Fig. 3. The pickup peak is underestimated by more than an order of magnitude.

The most important fault in the calculation of Fig. 3 is the use of the wave function (3). At the large momentum transfers involved the behavior at small distance is important and one must use better wave functions. As will be shown in Sec. IV, one can account for about half of the p-d elastic backward scattering with the simple pickup mechanism using deuteron wave functions obtained with potentials that fit the low- and intermediate-energy two-body data. However, instead of proceeding with better conventional wave functions at this point, we shall discuss the relationship of this problem to backward π-nucleon scattering in Sec. III and argue that a new coupled-channel wave function should be utilized.

III. REGGEIZATION OF THE NUCLEON TRANSFER REACTION

The proton-deuteron backscattering arising from the nucleon-transfer mechanism [Fig. 1(b)] is related to the scattering of antiprotons by deuterons (u channel) through considerations of the analytic continuation of the scattering amplitude. The nucleon-exchange mechanism in the (p,d) channel (s channel) corresponds to the nucleon pole in the u channel[6]; i.e., there is a pole in the scattering amplitude at $u=M_N^2$, where u is the square of the total energy in the $(\bar{p},d)$ barycentic system. Since the u variable in the s channel is zero near 180° and is negative for smaller angles, the u-channel pole at $u=M_N^2$ results in a back peak in the s channel—the pickup peak.

A 180° peak in π-nucleon scattering is also observed. This peak also should involve the u-channel pole, which, as in the (p,d) scattering, is a fermion exchange in the s channel. Here the situation is slightly more complicated because the π meson has isospin 1 in contrast to the zero isospin of the deuteron. For example, the u channel connected with π^--proton backscattering is π^+-proton, a pure isospin-$\frac{3}{2}$ channel. Thus the lowest-mass u-channel (fermion) pole is the 1236-MeV $\frac{3}{2},\frac{3}{2}$ resonance, and the nucleon exchange is not included. Therefore, the π^+-nucleon backscattering at high energy should be much more closely related to the (p,d) pickup process, for the u-channel π^--p process includes the nucleon pole. In fact, Heinz and Ross presented an argument that the nucleon pole dominates the backward π^+-p scattering (except for resonances),[7] and can fit the back peak.[8–10]

However, more recent experiments[11] have shown that the structure of the π^+-p cross section near 180° is more complicated than the prediction of the simple pole model. Among several attempts to explain the $\pi^\pm$-p backward scattering are Regge-pole models with and without resonances.[12,13] Chiu and Stack were able to obtain fairly accurate fits to the π^+-p data with the asymptotic expansion of a pure Regge-pole model including only the $I=\frac{1}{2}$ nucleon Regge trajectory. Briefly reviewing their work, the scattering cross section is given as

$$d\sigma/d\Omega=|f_N(\sqrt{s},\sqrt{u})|^2+\sin^2\theta|f_N(-\sqrt{s},\sqrt{u})|^2, \quad (4)$$

with

$$f_N(\sqrt{s},\sqrt{u})=C\frac{\beta_N(\sqrt{u})}{\cos[\pi\alpha_N(\sqrt{u})]}\left(\frac{s}{s_0}\right)^{\alpha_N(\sqrt{u})-1/2}$$
$$\times h_N(\sqrt{s},\sqrt{u})+(\sqrt{u}\leftrightarrow-\sqrt{s}). \quad (5)$$

Here $s=(p_1+p_2)^2$ and $u=(p_1-p_2')^2$ for $p_1+p_2\to p_1'+p_2'$, C is the appropriate crossing matrix element, $\alpha_N(\sqrt{u})$ is the nucleon Regge-pole position as a function of crossed-channel energy $\sqrt{u}$, $\beta_N(\sqrt{u})$ is the residue function, and s_0 is a scaling factor. The function $h(\sqrt{s},\sqrt{u})$ depends upon the masses of the projectiles, but is otherwise the same for all nucleon Regge-transfer processes.

We apply the same model to proton-deuteron scattering. At back angles an expansion in momentum transfer[14] has been used to justify the asymptotic Regge expansion,[15] since the argument of the Legendre polynomial $\cos\theta_u$ for the u-channel Regge pole at back angles does not become large with large s. Thus the use of the asymptotic form for the 1-BeV (p,d) scattering might be similarly justified.

Since the same Regge trajectory is involved in the deuteron-proton scattering as that used in Ref. 13 for π^+-proton scattering, we can make use of the parametrization of the trajectory from that paper. Except for the different crossing matrix element and possibly a different choice in the energy scaling factor s_0, the only difference in the expression[5] for (p,d) versus (π^+,p) is in the residue function. For both processes the function must vanish at $\sqrt{u}=850$ MeV as explained in Ref. 13.

For the π^+-proton scattering the residue function is also known at the nucleon pole[16,7]

$$\beta_{\pi^+}(-m_n)=\tfrac{3}{4}\pi g^2\frac{d\alpha_N}{d\sqrt{u}}\bigg|_{-m_N}, \quad (6)$$

where g is the π-nucleon coupling constant. Recalling that the pion coupling constant is obtained from the vertex function $g=(1/i)(p_0 2p_0' 2q_0)^{1/2}\langle p|\phi_\pi|p\pi\rangle$, from Fig. 1(b) one can see that for the (p,d) case the residue at $\sqrt{u}=-m_n$, g^2, will be replaced by the deuteron wave function. One finds that (f is the nucleon field)

$$g^2\to 2p_0 2d_0|\langle p|f(0)|d\rangle|^2/16\pi \quad (7)$$

following the notation of Blankenbecler *et al.*,[17] who have evaluated this vertex function. The result is that

$$\beta_{dp}(-m_n)/\beta_{\pi^+p}(-m_n)\approx 1\times 10^{-3}. \quad (8)$$

The result of using (8) and otherwise keeping the parameters of Ref. 13 is shown in Fig. 3. Thus, the u-channel Regge model for the π^+p scattering is qualitatively able to predict the peak for (p,d) backscattering. Note that by increasing the magnitude of the scaling factor s_0 we are able to fit the magnitude of the back peak, for this parametrization overestimates the peak, as seen in Fig. 3. This is consistent with the increase of the scaling factor with the rest mass of the incident particles, as expected.[16]

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We should emphasize here that the (p,d) Regge model is quite crude, since the spin of the deuteron has been neglected and no best fit to the parameters within the freedom available to us has been used. Furthermore, recent changes in the π^+-p experimental data require a modification of the parameters.[9] It should also be noted that there have been numerous other calculations of π^+-proton 180° scattering using other techniques. However, just considered as a model, it is of interest that the π-nucleon and proton-deuteron can be related. We now turn to an attempt to gain some physical understanding from this result.

IV. RECOILLESS FIELD-THEORY MODEL

In applying a Regge-exchange model one adds to the nucleon-exchange-model dynamics associated with the baryon resonances on the nucleon trajectory. In this section we explicitly take such dynamics into account via a pickup model. The first excited state of the nucleon in the Regge model is the $N^*(1688)$, a spin-$\frac{5}{2}$ state, and the effect of including this resonance can be expected to be the most important contribution in addition to the nucleon. This suggests an extension of the nucleon-exchange process as depicted in Fig. 4.

The advantage of the proton-deuteron system as compared to the π-proton system is that there is a simple and accurate model of the fermionic structure of the deuteron, while the pion at best corresponds to a very strongly bound relativistic fermion-antifermion system, i.e., we know the deuteron wave function for sophisticated two-body forces that fit the two-body bound- and scattering-state information. This enables us to estimate the probability of virtual excitation of an $N^*(1688)$ in the deuteron and from this to carry out an explicit generalization of the nucleon-transfer reaction corresponding to Fig. 4.

We use a simple recoilless nonrelativistic model for the πNN^* coupling, with an interaction Lagrangian

$$\mathcal{L}_I = (g^*/M^3)\psi_{N^*}(x)\tau_\alpha\Sigma^3\cdot[\nabla\nabla\nabla]^3\phi_\alpha(x)\psi_N(x). \quad (9)$$

This Lagrangian is chosen by analogy to the static Lagrangian used in the Chew-Low theory[18] for low-energy π-nucleon phenomena. In Eq. (9), $\phi_\alpha(x)$ is the π field operator, τ_α is the isospin operator in the baryon space, $[\nabla\nabla\nabla]^3$ is a spherical tensor of rank 3 operating on the meson field, Σ^3 operates in the intrinsic baryon space and changes N to N^*, and ψ_N, ψ_{N^*} are field operators for recoilless nucleon and $N^*(1688)$ fields—the $N^*(1688)$ being treated as an elementary particle. The magnitude of the coupling constant $g_{NN^*\pi}\equiv g^*$ depends upon the definition of Σ^3, which is chosen simply to be the Clebsch-Gordan coefficient corresponding to a tensor of rank 3 acting between the nucleon and the N^*;

$$\langle \tfrac{5}{2}m|\Sigma_\mu{}^3|\tfrac{1}{2}m'\rangle = \mathcal{C}_{m'\mu m}{}^{\frac{1}{2}3\frac{5}{2}}. \quad (10)$$

The constant M is the nucleon mass.

The coupling constant g^* can be determined from the N^* lifetime and the branching ratio $N^*\to N+\pi$. We find a weak effective coupling constant

$$f^* = (\mu/M)^3 g^* \cong 0.12. \quad (11)$$

From the interaction Lagrangian, one can derive a coupling potential for the baryon-baryon system. The two-body interaction is considered to arise from diagrams like those of Fig. 5. The resulting two-channel interaction potential is written as

$$V = V_{NN} + V_{N^*N^*} + V_{NN^*}, \quad (12)$$

where V_{NN} and $V_{N^*N^*}$ are the nucleon-nucleon and nucleon-N^* potential, and V_{NN^*}, the potential arising from the last diagram in Fig. 5, is the only coupling interaction used in the present work. Through V_{NN^*} the two-nucleon system will virtually excite $N^*(1688)$ resonances. For low energies V_{NN^*} is treated as a perturbation, i.e., we assume that the fit to two-body data has determined $V_{NN'}$, from which we obtain the deuteron wave function Ψ_{deut} of Eqs. (1) and (2). Treating V_{NN^*} as a perturbation, we calculate the admixture into the physical deuteron wave function of (N,N^*) "configurations." Then we use the modified pickup mechanism of Fig. 4 to calculate the proton-deuteron backscattering.

The coupling potential is found to be

$$V_{NN^*} = (4\pi)^{-1}(gg^*/2M^4)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\Sigma_2{}^3\cdot[\nabla\nabla\nabla]^3 \times(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})(e^{-\mu r}/r)(1-P_{\text{ex}}). \quad (13)$$

The wave function of the deuteron including virtual N^* is taken as

$$\Psi_{\text{deut}} = (4\pi)^{-1/2}R_S(r)\chi^1 + R_D(r)(Y_2\chi^1)^1 + \sum_S a_S R_{D^*}(r)(Y_2\chi^S)^1 = \psi_S + \psi_D + \psi_{D^*}, \quad (14)$$

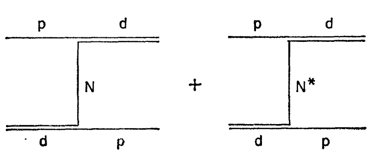

FIG. 4. Pickup mechanism extended to include nucleon resonances.

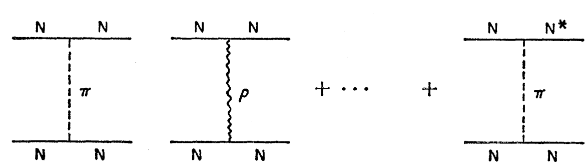

FIG. 5. Typical diagrams for two-channel, two-baryon potential. The last diagram corresponds to the longest-range coupling potential.

[18] G. F. Chew, Phys. Rev. **95**, 1669 (1954); G. F. Chew and F. Low, *ibid.* **101**, 1571 (1956).

in which $R_S(r)$ and $R_D(r)$ are the usual $L=0$ and $L=2$ components of the deuteron wave function, χ^S is the spin wave function corresponding to N^*N, so that $S=2$ or 3, and $R_{D^*}(r)$ is the radial wave function for the D^* component. Note that the main effect of the coupling interaction is to introduce a new $L=2$ component, Ψ_{D^*}. The constant a_S is determined from perturbation theory as

$$a_S=\langle R_{D^*}(Y_2\chi^S)^1|V_{NN^*}|\Psi_S+\Psi_D\rangle/(M_N-M_{N^*}). \quad (15)$$

In order to evaluate the matrix element (15) it is necessary to choose a model for $R_{D^*}(r)$. The model used is

$$\begin{aligned} R_{D^*}(r)&=N_1e^{-\kappa_0 r}, \quad r>c\\ &=N_2j_2(k_ir), \quad r<c. \end{aligned} \quad (16)$$

The asymptotic form for (16) should actually be a Hankel function of imaginary argument, but in view of the crudeness of our calculation this makes little difference. The constants N_1, N_2, and k_i are determined by normalization and matching at c. This model corresponds to a potential uniform within c and zero outside. From considerations of the nucleon-nucleon potential c is expected to be approximately 1–1.5 F, so the D^* component will be maximum between 0.5 and 1 F where the potential is most attractive. We have used two values of κ_0 in the calculation: I, the model of an actually bound N^* corresponds to $\kappa_0=4.82$ F$^{-1}=\kappa^*$, and II, the tail corresponding to the coupled-channel Schrödinger equation with the potential of Eq. (12) gives $\kappa_0=\alpha+\mu$, where $R_s(r)_{r\to\infty}\to Ne^{-\alpha r}$ and μ is the inverse Compton wavelength of the π. From these calculations we find

$$\sum_{S=2,3} a_S^2\approx 0.01, \quad (17)$$

or that the probability of finding an $N^*(1688)$ in the deuteron is about 1%. We have neglected orbital states higher than two in the N^*-nucleon component.

Using form (14) for the deuteron wave function, the stripping mechanism of Fig. 4 gives in the c.m. system (E_p, E_d are the proton, deuteron total energies, respectively)

$$\begin{aligned}\frac{d\delta}{d\Omega}=&\frac{3}{16(2\pi)^4}\left[\frac{E_pE_d}{(E_p+E_d)M_N}\right]^2\\ &\times\{(\kappa^2+\Delta^2)^2[I_S^2(\Delta)+I_D^2(\Delta)]^2+50(\kappa^{*2}+\Delta^2)^2\\ &\times I_{D^*}^4(\Delta)\sum_t C_tP_t(\cos\theta_q)+120(\kappa^{*2}+\Delta^2)\\ &\times(\kappa^2+\Delta^2)I^2I_{D^*}^2(\Delta)P_2(\cos\theta_q)\}, \quad (18)\end{aligned}$$

where

$$\begin{aligned}C_t=[|\sum_{S_1,S_2}[(2S_1+1)(2S_2+1)]^{1/2}W(S_1S_2\tfrac{5}{2}\tfrac{5}{2};t\tfrac{1}{2})\\ \times W(2S_12S_2;1t|^2(C_{000}^{22t})^2] \quad (18')\end{aligned}$$

and

$$\begin{aligned}I^2=[|\sum_{l,S}a_S[(2l+1)(2S+1)]^{1/2}C_{000}^{l22}W(1S\tfrac{1}{2}\tfrac{5}{2};2\tfrac{1}{2})\\ \times W(l22S;12)I_l(\Delta)|^2]. \quad (18'')\end{aligned}$$

The quantity Δ, defined in Eq. (2), is in units of F^{-1}, and κ^* is defined in the preceding paragraph.[19] In Eq. (18) the C and W are standard Clebsch-Gordan and Racah coefficients, respectively, the $P_t(\cos\theta_q)$ are Legendre polynomials, $\cos\theta_q=(4+5\cos\theta)/(5+4\cos\theta)$, where θ is the c.m. scattering angle, $I_{l=0}\equiv I_S$, $I_{l=2}\equiv I_D$, where

$$\begin{aligned} I_S(\Delta)&=4\pi\int dr\, j_0(\Delta r)R_S(r)r^2,\\ I_D(\Delta)&=4\pi\int dr\, j_2(\Delta r)R_D(r)r^2, \quad (19)\\ I_{D^*}(\Delta)&=4\pi\int dr\, j_2(\Delta r)R_{D^*}(r)r^2. \end{aligned}$$

The first term in Eq. (18) corresponds to the usual neutron pickup process.

Calculations have been carried out using various nucleon-nucleon potentials. Typical results are shown in Fig. 6. The S state contributes very little to the 1-GeV backscattering, which explains the result with the Hulthén wave function (Fig. 3). The D state and the D^* state are seen to contribute roughly equally, although the D-state probability is approximately 6.5% compared to the 1.0–1.5% D^* state.

These results can easily be understood from Fig. 7, which shows separately the radial Fourier-transform functions for the three components of the deuteron considered in this work. The S-state component is completely dominant for momentum transfers less than 0.5 F^{-1}, but vanishes at about 2.0 F^{-1}. The Fourier transform of the D-state component vanishes at zero and about 4.0 F^{-1} and reaches its maximum for momentum transfers less than 0.5 F^{-1}. From this we can expect that the D-state component will dominate backward p-d scattering at energies somewhat less than those being considered in the present work.

On the other hand, the D^* component is concentrated toward the origin in configuration space and thus the Fourier-transform function $I_{D^*}(\Delta)$ reaches its maximum for larger values of momentum transfer in the range $\Delta=1.5$–3 F^{-1} for the models we have used. Since the values of momentum transfer in the back peak go from 2.36 to about 4.0 F^{-1} for proton-deuteron scattering at 1 GeV, the very small D^* component can have a large or

[19] The factors $(\kappa^2+\Delta^2)$ and $(\kappa^{*2}+\Delta^2)$ in Eq. (18) are obtained from the coupled Schrödinger equation with the potentials of expression (12). Dropping the coupling potential V_{NN^*}, and using Hermiticity, one can replace the potentials V_{NN} and $V_{N^*N^*}$ by these kinematic factors in the Born-approximation scattering amplitude for the pickup process (Ref. 4). One finds $\kappa^2=ME_d$ and $\kappa^{*2}=1.286M_N(E_d+M_{N^*}-M_N)$, the factor of 1.286 here and in Eq. (18) coming from the reduced mass of the N^*-N system.

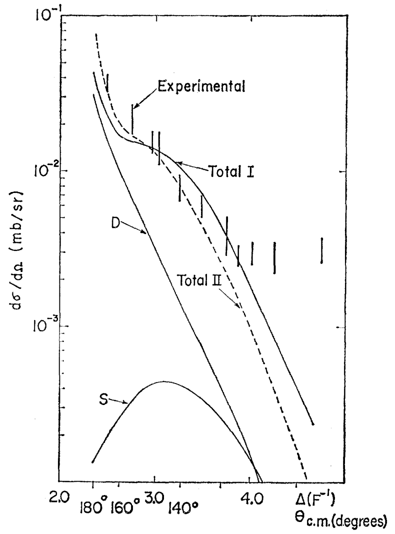

FIG. 6. Extended baryon-transfer mechanism for proton-deuteron scattering. The two curves labeled S and D show the differential cross section that would arise separately from the usual components [Eq. (14)]. The back peak at 1 GeV is obtained from the curves labeled "total," the 180° point corresponding to 2.36 F^{-1} and the angle in the c.m. system is given for scattering at 1 GeV. Total I corresponds to Model I with a 1.0% D^* state and $c=1.5$ F. Total II corresponds to Model II with a 1.5% D^* state and $c=1.5$ F. The Bressel wave function has been used for both calculations. C. M. Bressel, Ph.D. dissertation, M.I.T., 1965 (unpublished); C. N. Bressel, A. K. Kerman, and R. Rouben, Nucl. Phys. (to be published); C. Bressel, A. N. Kerman, and E. Lomon, Bull. Am. Phys. Soc. **10**, 584 (1965). The experimental points shown are taken from Bennett *et al.* (Ref. 2). The results of Coleman *et al.* (Ref. 2) are a little smaller. The energy dependence of the cross section (see text), which can be read from Figs. 6 and 7, is closer to the experimental values for Model II.

even dominant effect at some of these angles. This is the main reason for this unexpected importance of a virtual baryon resonance for this process.

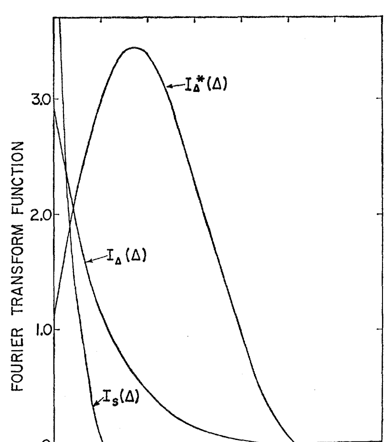

FIG. 7. Fourier-transform functions defined in Eq. (19) in the text. The function $I_{D^*}(\Delta)$ corresponds to Model I of Fig. 6. The units are fermi and fermi^{-1} for length and momentum, respectively.

In total, the calculation was carried out for six different potentials: those of Bressel, Feshbach-Lomon, Hulthén with hard core, Hamada-Johnston, Reid soft core, and Reid hard core, all of which are determined from the two-body data. The purpose of this study was to determine to what extent the calculation depends upon the details of the "conventional part" of the deuteron wave function, the first two terms in Eq. (14). The results were all surprisingly similar to each other and thus to Fig. 6, although there are, of course, quantitative differences. The explanation seems to be that for potentials which fit the vanishing of the nucleon-nucleon S-wave phase shift and other prominent features of the medium energy (100–300 MeV) information, the resulting deuteron S-wave Fourier

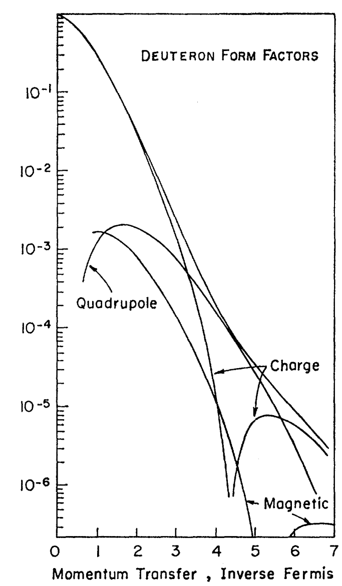

FIG. 8. Contributions to the deuteron form factor. The vanishing of the quadrupole form factor near 4 F^{-1} corresponds to the vanishing of the S-state component of the deuteron wave function shown in Fig. 6. (Taken from Ref. 20.)

component vanishes between about 2 and 4 F^{-1}, and the D-wave Fourier component vanishes at about 4 F^{-1}. Thus the S wave gives almost no contribution and the D wave is too small to fit the 1-BeV p-d backscattering. For this reason there seems to be some quantitive significance to even the crude estimate of the D^* contribution made in the present work.

It is interesting to compare this result with the electron-deuteron scattering,[20] shown in Fig. 8. Note that at momentum transfers of 4–5 F^{-1} the quadrupole form factor dominates. This is related to the vanishing of the Fourier transform of the S-wave part of the wave function at about 2 F^{-1}. The difference is that in dealing

[20] G. C. Hartman, Ph.D. dissertation, M.I.T., 1967 (unpublished).

with the electron scattering it is the form factor that appears—the Fourier transform of the charge and magnetic moment distribution (involving the square of the wave function)—rather than the Fourier transform of the wave function.

The energy dependence of the cross section at angles somewhat smaller than those at which the nucleon-transfer mechanism is entirely dominant (i.e., near the bottom of the back peak) can be found from the experiments of Coleman *et al.*[2] At $\cos(180°-\theta)=0.9$, the ratio of the differential cross section at laboratory kinetic energies 1 versus 1.3 versus 1.5 GeV is 2.6 and 2.4, respectively. The theoretical values can be obtained from Figs. 6 and 7 for $\Delta=2.79$, 3.18, and 3.43 for the three points, respectively, with the result that these ratios are 1.4 and 1.35 for Model I, and 1.93 and 1.64 for Model II. Obviously, this feature depends rather sensitively on the model of the D^* component. The increased effect of inelasticity, which has been neglected in the theoretical calculations, would tend to cut down the back peak and may account for our theoretical cross sections falling too slowly with energy.

V. CONCLUSIONS

Both from the Regge model and especially from the static-field-theory model based on the width for the process $N^*(1688)\rightarrow N+\pi$, it is suggested that $N^*(1688)$ resonances are important in the high-energy baryon-transfer (p,d) reaction. From the field-theory model we conclude that a deuteron has $\frac{1}{2}$–1% component consisting of an N^* and a nucleon. Although these are simple models, the suggestion that excited nucleon modes may be detectable and important in short-range nuclear correlations opens a new direction for nuclear-structure physics.

Aside from the considerations of the Regge model, one can see how the 1688 spin-$\frac{5}{2}^+$ particle would be particularly important for the problem being treated in this paper. Among the other resonances, the $I=\frac{3}{2}$ modes should not be important because both nucleons must be excited to this state to form an $I=0$ state. The $I=\frac{1}{2}$, $J=\frac{1}{2}$ 1480 resonance would occur mainly in an S-state configuration and would therefore not contribute much to the high-energy pickup process. Thus, it is the special value of $\frac{5}{2}$ for the spin leading to an $L=2$ configuration that makes the 1688 resonance so important for the transfer process in this domain.

However, we should remark that the other resonances will probably play a role in the nucleon-nucleon interaction to produce interesting nonlocalities via the coupled channels which are implied. Not only will they affect the elastic scattering but obviously they will be crucial in determining the rates for pion production in nucleon-nucleon scattering. Our analysis is to be considered just the first tentative step in the direction of unravelling what happens when two nucleons approach distances less than a fermi. This forms an interesting program for the future.

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