Regularity of Multiplicity Distribution in NN and πN Collisions and the Structure of the Nucleon*

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The frequencies for the various numbers of charged secondaries emitted from pp, $\pi^{\pm}p$, pn, $\pi^{\pm}n$, and nn collisions from below production threshold to 27-BeV primary energy were found to define a single set of curves which can be expressed by a universal set of "multiplicity distribution functions" constructed from Poisson terms when conservation laws are suitably incorporated. The distribution functions so obtained contain no adjustable parameter and fit well the NN cosmic-ray jets at 100–250 BeV, the highest energy at which NN collision data are available. The ΛK° accompanying the pions produced by $\pi^{-}p$ were also found to follow the same distribution. The multiplicity regularity found points immediately to the existence of a number of nearly identical regions or "cells" inside the nucleon and the fireball.

THERE have been quite a large number of experiments[1] on nucleon-nucleon and pion-nucleon inelastic collisions. The rather extensive data acquired in the last few years on the number of charged secondaries emitted from such collisions make a systematic study possible. In this paper is reported a regularity found by the author for the distribution of such charged multiplicities in NN and πN collisions. This was obtained by analyzing about 50 different experiments[2] on pp, $\pi^{\pm}p$, pn, and $\pi^{-}n$ collisions, with KE extending from below production threshold to 27 BeV. Also included in the analysis are the data on $\pi^{+}n$ and nn collisions which were obtained from the corresponding $\pi^{-}p$ and pp experiments by isotopic invariance and charge conjugation:

$$p\leftrightarrow n\,,\quad \pi^{+}\leftrightarrow\pi^{-}.$$

Since charge is conserved in a collision, the number of charged secondaries, i.e., the number of prongs n_s resulting from pp, $\pi^{\pm}p$, and nn collisions will be even, and that from pn and $\pi^{\pm}n$, odd. Thus, in the following, the different types of collisions are treated under these two categories.

Distribution of charged multiplicities. In Figs. 1 and 2, the relative frequencies of occurrence of events for all the NN and πN collisions under study are plotted as a function of the mean charged multiplicity $\langle n_s\rangle$, for even and odd n_s, respectively.

For nn collisions, because of charge neutrality, the data points shown in Fig. 1 are displaced horizontally two units to the right, i.e., for the observed mean $\langle n_s\rangle$, the relative numbers of prong events are plotted at $\langle n_s\rangle+2$ instead $\langle n_s\rangle$, and (a)–(d) are for prong number $n_s=0$, 2, 4, and 6 instead of 2, 4, 6, and 8.

As will be seen in the discussion below, there is evidence that neutral pion production is independent of charged production. Thus for $\pi^{-}p$ collision, the charge-exchange reaction

$$\pi^{-}p\rightarrow\pi^{0}n \qquad (1)$$

as already distinct from non-charge-exchange reactions, was excluded from the analysis. Since the partial cross section for charge-exchange reaction (1) above 6 BeV is less than a few percent of the total inelastic $\pi^{-}p$ cross section, and even at 2.66 BeV it is only about 12%, it will be noted that the inclusion or exclusion of charge-exchange reaction (1) in the analysis will not affect the results to any significant extent.

It is seen that the data points for the different types of collisions in either Fig. 1 or 2 lie on a set of smooth curves, and when the two figures are displaced one unit along the $\langle n_s\rangle$ axis with respect to each other, the two sets of points are superposed and thus define one and the same set of curves within experimental errors. It is evident from the regularity shown that the production processes are similar in all the different types of inelastic collisions under investigation.

Distribution function for the regularity observed. In order to see the underlying processes involved in NN and πN inelastic collisions which show such extraordinary regularity, let us assume that the charged secondaries produced are only pions, the strange-particle and antinucleon production being much smaller in this energy range anyway. Then from charge conservation, the charged pions will be produced in pairs. If the production of neutral pions, for which the experimental data are scanty, is considered for the time being to be independent of charged-pion production, then the simplest case of production would be for these charged pions to be produced in pairs in small regions or "cells" inside the nucleon. If these cells of production are nearly independent, then we would expect the multiplicity distribution function to be given by some com-

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[1] For experiments performed up to 1965, see V. S. Barashenkov, V. M. Maltsev, I. Patera, and V. D. Toneev, Fortschr. Physik **14**, 357 (1966).
[2] For pp and nn, data from the following experiments have been used: *a, b, c, d, w, f, g, h, i, j, k, m, q, nn*, corresponding to Refs. 66, 63, 62, 58, 56, 50, 49, 48, 26, 22–24, 21, 20, (19, 18, 14, 16), 22–24 listed in Ref. 1. For πp: πa, πb, πc, πd, πe, πf, πq, πr, πs, πt, πn, πABC, corresponding to Refs. 108, 106, 105, 107, 99, 90, 117, 113, 112, 111, 110, in Ref. 1, and M. Deutschmann *et al.*, Aachen-Berlin-CERN Collaboration, Phys. Letters **12**, 356 (1964). For pn: *a, b, c, d, e, w*, corresponding to Refs. 66, 61, 55, 57, 8, in Ref. 1, and Wang Shu-Fen *et al.*, Zh. Eksperim. i Teor. Fiz. **39**, 957 (1960) [English transl.: Soviet Phys.—JETP **12**, 663 (1961)]. For πn: πa, πb, πc, πd, πe, πr, πs, πt, corresponding to 41, 99, 90, 86, 85, 70, 80, 30 in Ref. 1.

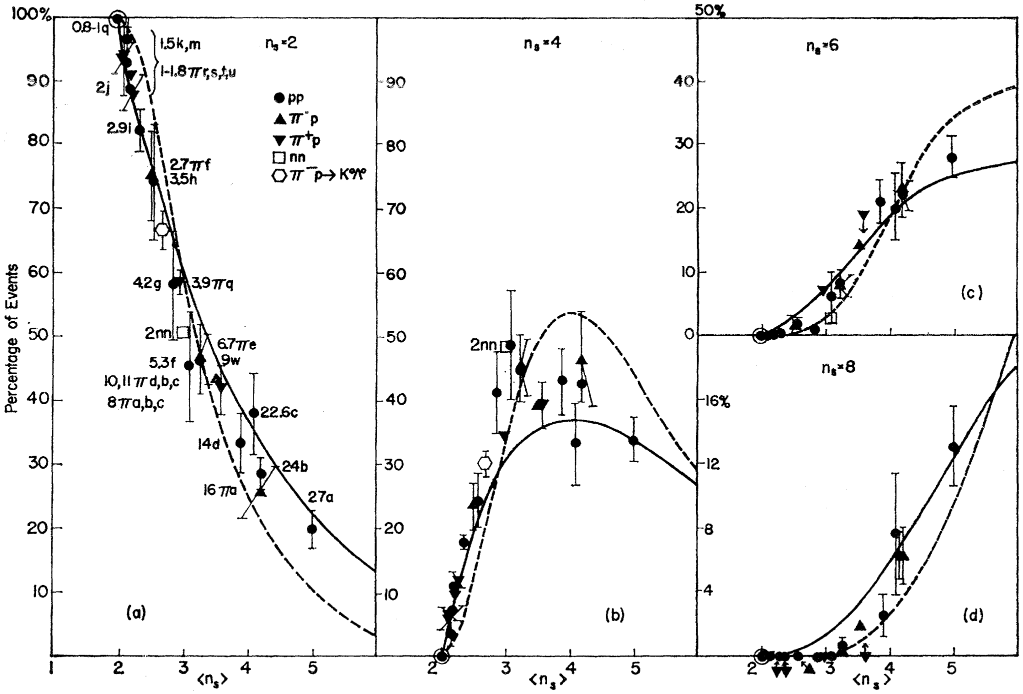

FIG. 1. Relative frequencies of events against mean multiplicity $\langle n_s \rangle$ of charged secondaries for pp, $\pi^{\pm}p$, and nn collisions at various energies. The numbers next to the data points are the laboratory kinetic energies of the primaries in BeV, and the letters are the references in Ref. 2. Solid and dashed curves are the parameterless distributions $W_{n_s(\mathrm{even})}{}^{\mathrm{I}}$ and $W_{n_s(\mathrm{even})}{}^{\mathrm{II}}$, respectively; see text. The neutral ΛK^0 points also fall on the curves within experimental errors. Because of charge neutrality, the nn data points are plotted against $\langle n_s \rangle + 2$ and (a)–(c) are $n_s = 0$, 2, and 4.

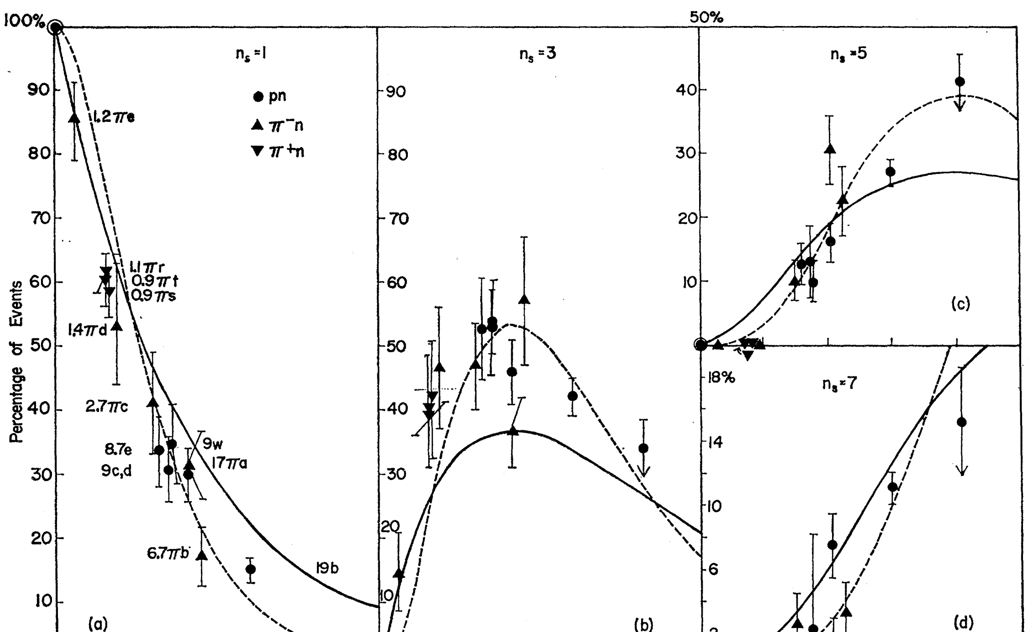

FIG. 2. Relative frequencies of events against mean multiplicity $\langle n_s \rangle$ of charged secondaries for pn and $\pi^{\pm}n$ collisions at various energies. The numbers next to the data points are the laboratory kinetic energies of the primaries in BeV, and the letters are the references in Ref. 2. Solid and dashed curves are the parameterless distributions $W_{n_s(\mathrm{odd})}{}^{\mathrm{I}}$ and $W_{n_s(\mathrm{odd})}{}^{\mathrm{II}}$, respectively. The data distribution and the theoretical curves are superposed on those of Fig. 1 when displaced one unit along the $\langle n_s \rangle$ axis; see text.

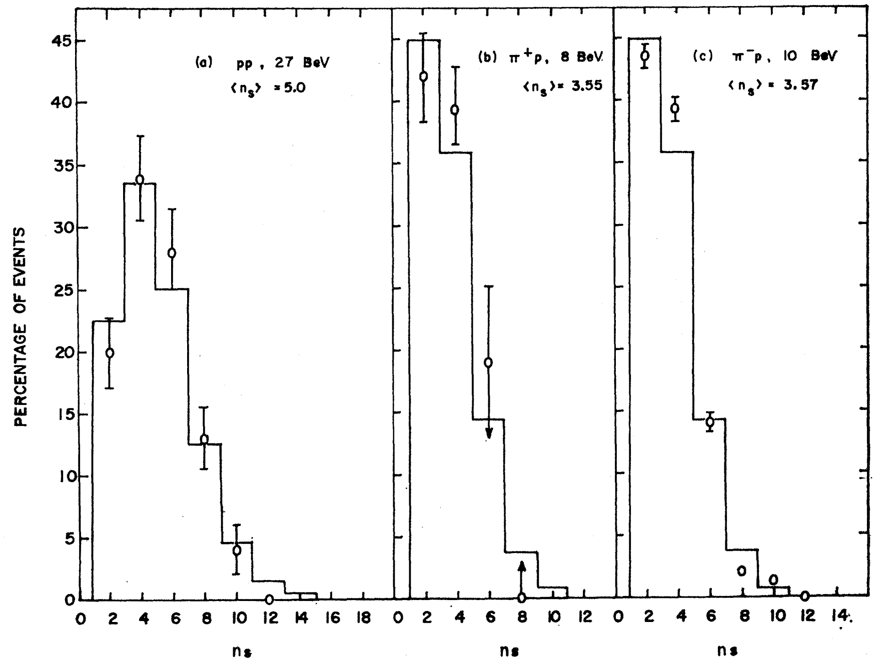

FIG. 3. Distribution of charged secondaries emitted from (a) pp, (b) π^+p, and (c) π^-p collisions. The experimental points follow closely the charge–baryon-number–conserved distribution $W_{n_s}{}^{\mathrm{I}}$ shown as solid lines. (a) Y. Baudinet-Robinet *et al.*, Nucl. Phys. **32**, 452 (1962); (b) M. Deutschmann *et al.*, ABC collaboration, Phys. Letters **12**, 356 (1964); (c) P. Fleury *et al.*, in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 597.

bination of terms of the Poisson probability function. If the production is such that each cell can produce a pair at a time, then we could write down a distribution function for the multiplicity n_s of the charged secondaries for class-I collisions, the pp, $\pi^\pm p$, and nn collisions with even n_s, as

$$W_{n_s(\mathrm{even})}{}^{\mathrm{I}}=\frac{(\frac{1}{2}\langle n_s-2\rangle)^{(n_s-2)/2}}{[\frac{1}{2}(n_s-2)]!}e^{-\langle n_s-2\rangle/2},\quad n_s=\mathrm{even}. \tag{2a}$$

Here $\frac{1}{2}\langle n_s-2\rangle$ is the mean number of charged pion pairs produced. For nn collision, n_s-2 is to be replaced by n_s in the above expression because of charge neutrality.

Two conservation laws are involved here in Eq. (2a): Charge conservation is supposed to hold for each individual small creation region inside the nucleon, and baryon number conservation for the whole collision system.

Figure 3 illustrates how closely the experimental numbers of different types of collisions at three different energies follow the charge-baryon number conserved distribution $W_{n_s}{}^{\mathrm{I}}$.

For class-II collisions, the pn and $\pi^\pm n$ collisions with odd n_s, the corresponding distribution function would be

$$W_{n_s(\mathrm{odd})}{}^{\mathrm{I}}=\frac{(\frac{1}{2}\langle n_s-1\rangle)^{(n_s-1)/2}}{[\frac{1}{2}(n_s-1)]!}e^{-\langle n_s-1\rangle/2},\quad n_s=\mathrm{odd}. \tag{2b}$$

$W_{n_s(\mathrm{even})}{}^{\mathrm{I}}$ and $W_{n_s(\mathrm{odd})}{}^{\mathrm{I}}$ are one and the same set of functions of the mean number of pion pairs produced, $\frac{1}{2}\langle n_s-2\rangle$ and $\frac{1}{2}\langle n_s-1\rangle$ for collisions of classes I and II, respectively. Thus when plotted against this mean number, they give the same set of curves with no adjustable parameter. This universal set of "multiplicity distribution curves" is the set of solid curves shown in Figs. 1 and 2. The complete agreement between the parameterless distribution function $W_{n_s}{}^{\mathrm{I}}$ and the data points for low mean multiplicities is very striking and must reflect the interior cell structure of the nucleon as postulated above. The smooth continuation of both the inelastic data points and the curves to the double-circle points below production threshold relates the elastic and the inelastic processes in a very natural way.

For higher mean multiplicities $\langle n_s\rangle$ from about 3 to 4.5 for Figs. 1(a) and 1(b), and about 2–4 for Figs. 2(a) and 2(b), corresponding to about 1–3 average charged secondary particles produced, it seems that the agreement between the distribution $W_{n_s}{}^{\mathrm{I}}$ and the data points is not so complete although the experimental uncertainties there are becoming larger. On the other hand, one would expect the various resonance productions, with production cross sections up to a few mb as compared with the total inelastic cross section of about 20–30 mb, to show effects in this region.

In the above, we have applied charge conservation to each cell of production. If we relax the conservation law somewhat and demand charge conservation only for the whole collision system, then an alternative set of multiplicity distribution functions will result. For class-I collisions, since only an even number of particles are

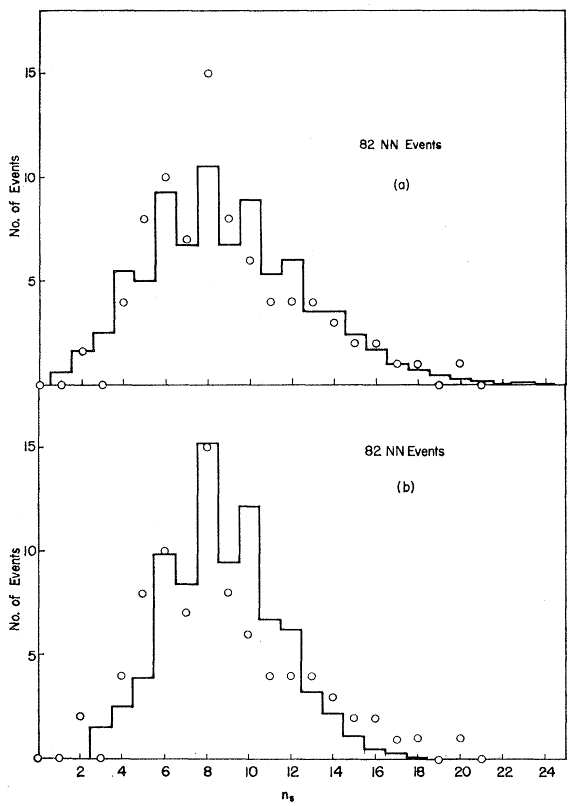

FIG. 4. Distribution of shower particles of 100–250-BeV *NN* jets. Circles in (a) and (b) are data points. Solid lines in (a) are the distributions calculated from $W_{n_s}{}^{\mathrm{I}}$, and in (b) from $W_{n_s}{}^{\mathrm{II}}$.

emitted, we have

$$W_{n_s(\mathrm{even})}{}^{\mathrm{II}}=\frac{\langle n_s-2\rangle^{n_s-2}}{(n_s-2)!}e^{-\langle n_s-2\rangle}\Bigg/\left(\sum_{n_s=2}^{\infty}\frac{\langle n_s-2\rangle^{n_s-2}}{(n_s-2)!}e^{-\langle n_s-2\rangle}\right). \quad (3a)$$

We note here that the odd Poisson terms are excluded or forbidden by charge conservation of the system. The summation in the denominator is the normalization for the distribution. For class-II collisions with n_s=odd, we have

$$W_{n_s(\mathrm{odd})}{}^{\mathrm{II}}=\frac{\langle n_s-1\rangle^{n_s-1}}{(n_s-1)!}e^{-\langle n_s-1\rangle}\Bigg/\left(\sum_{n_s=1}^{\infty}\frac{\langle n_s-1\rangle^{n_s-1}}{(n_s-1)!}e^{-\langle n_s-1\rangle}\right). \quad (3b)$$

The distribution functions $W_{n_s(\mathrm{even,odd})}{}^{\mathrm{II}}$ are again a single set of parameterless functions of the mean number of charged pions produced, $\langle n_s-2\rangle$ and $\langle n_s-1\rangle$ for class-I and class-II collisions, respectively. The set is plotted as dashed curves in Figs. 1 and 2. At low mean multiplicities, $W_{n_s}{}^{\mathrm{II}}$ does not agree with the experiments as completely as $W_{n_s}{}^{\mathrm{I}}$. For higher mean multiplicities, it seems that the agreement of $W_{n_s}{}^{\mathrm{II}}$ is better than $W_{n_s}{}^{\mathrm{I}}$. The larger experimental uncertainty and the resonance effects mentioned above for this region preclude one from saying whether $W_{n_s}{}^{\mathrm{I}}$ or $W_{n_s}{}^{\mathrm{II}}$ gives a better description. However, because of the complete agreement between $W_{n_s}{}^{\mathrm{I}}$ and the experimental points at low $\langle n_s\rangle$, one tends to believe that, at least at lower energies, the charge and other conservation laws hold for each creation cell.

Strange-particle production. The following strange-particle production[3] by π^-p at 7–8 BeV:

$$\pi^-+p\rightarrow\Lambda+K^0+n_\pi\pi$$

is consistent with this view. As can be seen from Fig. 1, where the relative numbers of ΛK^0 events are plotted on the appropriate charged-pion graphs, the distribution of the neutral K's and Λ's accompanying the charged pion produced follows exactly that of the charged π's produced. K^0 thus seems to emerge from the nucleon "core" and turns the remaining nucleon into a Λ while pions are emitted. Thus hypercharge is conserved in each cell.

π^0 production. So far, the π^0 production has not been considered. From the agreement between theory and experiment and the ΛK^0 distribution as seen above, it appears that π^0 production is indeed an independent Poisson process as postulated above, even though the π^0 may be produced in the same cells as the charged π's.

Production at cosmic-ray energies. It would be interesting to see whether the charge–baryon-number–conserved distribution functions $W_{n_s}{}^{\mathrm{I}}$ and $W_{n_s}{}^{\mathrm{II}}$ also hold at cosmic-ray energies. Plotted in circles in Fig. 4 are 82 cosmic-ray jets produced by nucleon-nucleon collisions at 100–250-BeV primary energy, 55 of which were obtained by Guseva *et al.*,[4] and 27 by Lohrmann *et al.*[5] We consider jets with even numbers of shower particles n_s to be *pp* events, and jets with odd n_s to be *pn* events. Theoretical distributions may thus be calculated separately for even and odd n_s of the 82 jets, from either $W_{n_s}{}^{\mathrm{I}}$ or $W_{n_s}{}^{\mathrm{II}}$, and then combined to give the final distribution. The results are plotted in Fig. 4. It seems that $W_{n_s}{}^{\mathrm{I}}$ and $W_{n_s}{}^{\mathrm{II}}$ account equally well for the multiplicity distribution of the 82 jets. Thus if fireballs with no nucleon embedded are produced at such high energies, then such fireballs will also have the cell structure like the nucleon. From symmetry consideration, one also expects a (spherically) symmetrical distribution of

[3] M. I. Soloviev, in *Proceedings of the Tenth Annual International Conference on High-Energy Physics at Rochester, 1960*, edited by E. C. G. Sudarshan *et al.* (Wiley-Interscience Publishers, Inc., New York, 1961), p. 388; V. A. Belgakov *et al.*, in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 252.

[4] V. V. Guseva *et al.*, J. Phys. Soc. Japan **17**, Suppl. A-III (1962).

[5] E. Lohrmann, M. W. Teucher, and M. Schein, Phys. Rev. **122**, 672 (1961).

these cells about the nucleon core or the center of the fireball.

It will be noted that the corresponding charge–baryon-number–conserved binomial distribution with cell number $N=6$, 4, or as small as 3 will also fit the data in Figs. 1 and 2 equally well (cell number $N=6$ might correspond to the two nucleons, each with three quarks, in NN collisions), except Fig. 2(d) for $n_s=7$. The datum point at $\langle n_s\rangle=5.2$ (27 BeV) in Fig. 2(d) is $\leqslant(15.2\pm3.4)\%$, while for $N=3$, 4, and 6, the predicted values are 34.1, 25.4, and 23.6%, respectively. These values are obtained in the following way from the binomial probability function $B(N,p)$, where N in the present case is the cell number, and p is the elementary probability: The usual mean value Np for the binomial distribution is now set equal to the mean number of the pion pairs produced, i.e.,

$$Np=\tfrac{1}{2}\langle n_s-1\rangle$$

for pn collisions. For a given N, since $\langle n_s\rangle=5.2$ for this particular experiment, p may be computed and hence $B(N,p)$ is determined.[6]

Thus, if there are only a few cells, or a few "excited" cells at low energies inside the nucleon, then the number of these cells has to increase with energy, as already begins to show at 27 BeV in pn collisions. In any case, at high energies the number of cells has to be quite large in order to account for the high multiplicity cosmic-ray-jet events.

That high- and low-multiplicity states have much in common in their production mechanism was also indicated recently by the similarity among the "internal t distributions $F(t)$" for such states resulting from π^+p collisions by Bialkowski and Sosnowski.[7]

The cell structure of the nucleon and the fireball envisaged here also accounts for the long-perplexing situation that the transverse momentum of shower particles produced in NN and πN collisions has almost the same mean value of about 0.4 BeV/c and the same distribution[8]

$$dN/dp_\perp \propto p_\perp \exp(-p_\perp/\tfrac{1}{2}\langle p_\perp\rangle)$$

for an enormous energy range extending from a few BeV to 10^5 BeV. This is clearly a superposition effect and thus provides additional evidence for the cell structure of the nucleon.

The author wishes to thank Mrs. Tsing Tsing Lin for helping him with the numerical analyses and computation.

[6] See, e.g., Natl. Bur. Std. Appl. Math. Ser. **6** (1949).

[7] G. Bialkowski and R. Sosnowski, Phys. Letters **25B**, 519 (1967); also in rapporteur talk of L. Di Lella, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Wiley-Interscience Publishers, Inc., New York, 1968), pp. 191 and 192.

[8] G. Cocconi, J. Koester, and D. H. Perkins, University of California Radiation Laboratory Report No. UCRL-10022, 1961 (unpublished); P. H. Fowler and D. H. Perkins, Proc. Roy. Soc. (London) **278A**, 401 (1964); also for elastic collisions, J. Orear, Phys. Letters **13**, 191 (1964).