Chiral Dynamics without A_1

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The nonlinear realization method of Weinberg is used to construct a chiral-invariant Lagrangian consisting of π, N, $N^*(1236)$, and ρ, but not A_1. Chiral invariance can be, of course, maintained *without* invoking the A_1 meson. In view of the experimental status of A_1, it may be worthwhile studying such a model. The tree-diagram technique is used, with an off-mass-shell N^* propagator, to calculate pion-nucleon s-wave scattering lengths, isobar production parameters, and single-pion photoproduction differential cross sections. A comparison with the current-algebra method is made.

I. INTRODUCTION

CHIRAL Lagrangians have been generally very useful in understanding the dynamics of pions and nucleons.[1] This success has prompted attempts to widen the scope of the phenomenological Lagrangian to include other resonances such as the $N^*(1236)$ nucleon isobar,[2,3] the ρ meson, and the A_1 meson.[4,5] The motivation behind this is, of course, to seek an understanding of the production reactions involving these resonances.

Of the empirical resonances mentioned, the A_1 meson is the least established,[6] and it is perhaps questionable if chiral Lagrangians should contain the A_1 as a fundamental field. We make the simple observation that there can be chiral invariance of the Lagrangian *without* the A_1 field. (By chiral transformations we mean here the nonlinear transformations of Weinberg[7] and Schwinger.[8])

Therefore, we consider a chiral-invariant phenomenological Lagrangian model that would apply to reactions involving pions, nucleons, nucleon isobars, and ρ mesons, but no A_1 mesons. We use the model to determine the pion-nucleon s-wave scattering lengths, partly as a check, nucleon isobar s-wave production parameters, and single-pion photoproduction threshold cross sections, these parameters being most accessible to experimental analysis. The experimental fit is found to be quite good. (Even if we were to include the A_1 meson, its contribution is found to be negligible.)

In the case of the pion-nucleon scattering lengths, it is found that, if the usual N^* propagator is used to calculate, the experimental fit is not good unless large "contact" terms are postulated *ad hoc*. These contact terms correspond to the usual σ-meson terms of the current algebra. A bad feature of the matrix element so found is that its high-energy behavior is bad. By arranging the contact term, the isotopic even amplitude can have good behavior at infinity, but the isotopic odd amplitude would still be badly divergent. This prompts us to consider an alternative approach. A new "off-mass-shell" N^* propagator is proposed, and the results obtained using this propagator are in good agreement with experiment. This new propagator has singularities at $p^2=0$, but since the tree-diagram method is only an approximation scheme and does not include unitarity, these spurious singularities are not bothersome.

In constructing our Lagrangian, we have followed closely the nonlinear realization method of Weinberg.[7] We limit our discussion to the level of $SU_2 \times SU_2$.

An interesting aspect of our model in connection with our deleting A_1 from the Lagrangian is that the field-current identities no longer follow. The axial-vector current obtained by the Gell-Mann–Lévy construction[9] will not obviously ever be proportional to the A_1 field. This means that in the propagator for the axial-vector current there will be no dominance by a 1^+ state in the spectral functions. Nevertheless, the vector and axial-vector currents continue to obey the $SU_2 \times SU_2$ algebra.

II. PHENOMENOLOGICAL LAGRANGIAN

In this section, we write down the Lagrangian that we shall use in our applications. In writing down this Lagrangian, we have followed several over-all requirements: (i) We demand chiral invariance (and, of course, also isotopic invariance). The nonchiral-invariant part of the Lagrangian, following Weinberg,[7] is assumed to involve only the pion fields and will not affect the reactions considered in this paper. (ii) We construct the Lagrangian out of nucleons, nucleon isobars $N^*(1236)$, pions, and ρ mesons. Note first that our discussion is at the level of $SU_2 \times SU_2$ and not that of $SU_3 \times SU_3$. More importantly, we consider reactions involving π, N, N^*, and γ, and at the phenomenological level the Lagrangian should involve these fields (γ comes in through vector dominance). The most important fact about our requirements is that we do not consider the A_1 meson in our Lagrangian.

The experimental status of the A_1 meson[6] is not too clear and it is perhaps relevant theoretically to consider phenomenological theories where A_1 is not present.

[1] See, for example, W. A. Bardeen and B. W. Lee, in *Nuclear and Particle Physics*, edited by B. Margolis and C. S. Lam (W. A. Benjamin, Inc., New York, 1968). Also, for further references, see S. Weinberg, in *Proceedings of the 14th International Conference on High Energy Physics* (CERN, Geneva, 1968), p. 253.
[2] H. W. Huang, Phys. Rev. **174**, 1799 (1968).
[3] R. D. Peccei, Phys. Rev. (to be published).
[4] J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967).
[5] B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968).
[6] B. French, in *Proceedings of the 14th International Conference on High-Energy Physics* (CERN, Geneva, 1968), p. 91.
[7] S. Weinberg, Phys. Rev. **166**, 1568 (1968).
[8] J. Schwinger, Phys. Letters **24B**, 473 (1967).
[9] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).

We make the simple observation that theories without the A_1 meson can be, nevertheless, manifestly chiral-invariant. What does not follow in such theories is the field-current identity, where the axial current, for example, is directly proportional to the A_1 field. But, if the currents are defined in the manner of Gell-Mann and Lévy,[9] the currents obey the same $SU_2\times SU_2$ algebra as before.

In other words, in a chiral-invariant world without the A_1 meson, the currents still satisfy the $SU_2\times SU_2$ algebra. The axial-vector current propagator, in such a world, will not be saturated by a single 1^+ meson state.

Returning to the construction of our Lagrangian, we have, based upon the two principle requirements, the following[10]:

$$\begin{aligned}\mathcal{L}=&-\bar{N}(\gamma\cdot\partial+m_N)N-\bar{N}_\mu g_{\mu\nu}(\gamma\cdot\partial+m_{N^*})N_\nu\\&-\tfrac{1}{2}D_\mu\boldsymbol{\pi}\cdot D_\mu\boldsymbol{\pi}-\tfrac{1}{2}\mathbf{V}_{\mu\nu}\cdot\mathbf{V}_{\mu\nu}-\tfrac{1}{2}m_\rho^2\boldsymbol{\phi}_\mu\cdot\boldsymbol{\phi}_\mu\\&-(g/2m_N)\bar{N}i\gamma_\mu\gamma_5\boldsymbol{\tau}N\cdot D_\mu\boldsymbol{\pi}\\&-(g_{N^*N\pi}/m_\pi)i\bar{N}_{\lambda j}g_{\lambda\mu}ND_\mu\pi^j\\&+(g_{N^*N\pi}/m_\pi)i\bar{N}N_{\mu j}D_\mu\pi^j\\&-(g_{N^*N^*\pi}/2m_{N^*})\bar{N}_{\lambda j}g_{\lambda\mu}i\gamma_\nu\gamma_5\boldsymbol{\tau}N_{\mu j}\cdot D_\mu\boldsymbol{\pi}\\&+f_\rho\bar{N}i\gamma_\mu\tfrac{1}{2}\boldsymbol{\tau}N\cdot\boldsymbol{\rho}_\mu\\&-(f_{\rho N^*N}/m_\pi)\bar{N}_{\lambda j}g_{\lambda\mu}i\gamma_\nu\gamma_5NV_{\mu\nu}{}^j.\end{aligned}\quad(2.1)$$

In writing down the Lagrangian, we have followed closely Weinberg's method,[7] where N and N^* fields are taken to transform linearly under chiral transformations, ρ_μ and π fields transform nonlinearly. Also,

$$\begin{aligned}D_\mu\boldsymbol{\pi}&=[1+\boldsymbol{\pi}^2/f_\pi^2]^{-1}\partial_\mu\boldsymbol{\pi},\\\mathbf{V}_{\mu\nu}&=\partial_\mu\boldsymbol{\rho}_\nu-\partial_\nu\boldsymbol{\rho}_\mu+f_\rho\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu,\\\boldsymbol{\phi}_\mu&=\boldsymbol{\rho}_\mu+(f_\rho/m_\rho^2)[\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}/(1+\boldsymbol{\pi}^2/f_\pi^2)],\end{aligned}\quad(2.2)$$

transform linearly. $N_{\mu j}$ is the Rarita-Schwinger notation[11] for the positive-parity nucleon isobar with $IJ=\frac{3}{2}$, $\frac{3}{2}$, where μ and j are the space-time and isotopic-vector indices, respectively. There is an implied isospin index, i.e., $N_{\mu j}=(N_{\mu j})^\alpha$, $\alpha=1$, 2 (see Appendix). This object is restricted by the usual free-field subsidiary conditions

$$\begin{aligned}\gamma_\mu N_{\mu j}&=0,\\\partial_\mu N_{\mu j}&=0,\\(\gamma\cdot\partial+m_{N^*})N_{\mu j}&=0,\end{aligned}\quad(2.3)$$

and

$$\tau_j N_{\mu j}=0.\quad(2.4)$$

Having this Lagrangian at our disposal, we then utilize the tree-diagram method to perform the calculations for any given process. (We consider those graphs given by lowest-order perturbation theory where no integration over internal momenta is required, or retain only those graphs which have the structure of trees.)

[10] The following metric is used: $g_{11}=g_{22}=g_{33}=-g_{44}=1$.
[11] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

The following choice is made for the N^* spin-$\frac{3}{2}$ projection operator:

$$\begin{aligned}\Lambda_{\mu\nu}(p)=&\sum_{s=-3/2}^{3/2}N_\mu(p,s)\bar{N}_\lambda(p,s)g_{\lambda\nu}\\=&\frac{(-i\gamma\cdot p+m_{N^*})}{p^2+m_{N^*}^2}\left\{\delta_{\mu\nu}+\frac{2}{3}\frac{p_\mu p_\nu}{-p^2}\right.\\&\left.-\tfrac{1}{3}i\frac{p_\mu\gamma_\nu}{(-p^2)^{1/2}}+\tfrac{1}{3}i\frac{\gamma_\mu p_\nu}{(-p^2)^{1/2}}-\tfrac{1}{3}\gamma_\mu\gamma_\nu\right\},\end{aligned}\quad(2.5)$$

which was constructed by taking the subsidiary condition $\partial_\mu N_{\mu j}=0$ as an operator identity. The usual spin-$\frac{3}{2}$ propagator is obtained by putting $p^2=-m_{N^*}^2$ or when $\Lambda_{\mu\nu}(p)$ is on the N^* mass shell. This freedom in picking a propagator is allowed because of the ambiguity in quantizing the N^* field, e.g., in quantum electrodynamics one has the freedom of taking $\partial_\mu A_\mu=0$ either as a matrix element or operator identity. Also, with this choice of N^* propagator a pole and branch cut are introduced at $p^2=0$, but since the tree-diagram method is an approximation scheme and we are working in lowest-order perturbation theory, it is possible that these singularities are cancelled by higher-order terms.

It is assumed that the coupling constants given in the Lagrangian are determined experimentally. The renormalized pion-nucleon coupling is g, where $g^2/4\pi=15$, and the universal vector-meson coupling constant is f_ρ, where $f_\rho^2/4\pi=2.4$. Also, $g_{N^*N\pi}=2.13$, which is based on an N^* width $\Gamma_{N^*}=120$ MeV, and $g_{N^*N^*\pi}=(9/5)g$, which can be determined from $U(6,6)$ or superconvergence arguments.[12,13] The ρNN^* coupling constant is determined from the dominant γNN^* coupling constant C_3 by employing vector dominance in the usual manner. Briefly, this is done as follows: Gourdin and Salin have given a phenomenological form for the γNN^* vertex[14,15] which involves three possible couplings, characterized by constants C_3, C_4, and C_5,

$$\begin{aligned}\mathcal{H}=&-(eC_3/m_\pi)\bar{N}_\lambda g_{\lambda\mu}i\gamma_\nu\gamma_5NF_{\mu\nu}\\&-(eC_4/m_\pi)\partial_\nu\bar{N}_\lambda g_{\lambda\mu}\gamma_5NF_{\mu\nu}\\&+(eC_5/m_\pi)\bar{N}_\lambda g_{\lambda\mu}\gamma_5\partial_\nu NF_{\mu\nu}.\end{aligned}\quad(2.6)$$

It is then found by experimental fit to single-pion photoproduction in the region of the N^* that C_3 is the dominant coupling and C_4, $C_5\approx0$. Then, by using vector dominance,[16] we find $f_{\rho NN^*}=C_3f_\rho$, where $C_3=0.37$. We remark that the Gourdin-Salin analysis was done with the N^* propagator $\Lambda_{\mu\nu}(p)$ on the mass shell, i.e., $p^2=-m_{N^*}^2$. If the propagator (2.5), which is not on the mass shell, is used, we find that the result of Gourdin

[12] B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965).
[13] H. F. Jones and M. D. Scadron, Nuovo Cimento **48**, 545 (1967).
[14] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963).
[15] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 310 (1963).
[16] L. Stodolsky, Phys. Rev. **134**, B1099 (1964).

and Salin is reproduced because the data are fitted in the region of the N^*, where $p^2 \approx -m_{N^*}{}^2$. We have also investigated the effect on this analysis of the inclusion of the A_1 and found it to be negligible.

III. PION-NUCLEON s-WAVE SCATTERING LENGTHS

Let the amplitude for the process

$$N(p)+\pi^i(q) \to N(p')+\pi^j(k) \tag{3.1}$$

be given by

$$\langle \pi^j(k)N(p')|S|\pi^i(q)N(p)\rangle = \delta_{fi} + (2\pi)^4\delta(p'+k-p-q) \times [(m_N{}^2/p_0'p_0)(4q_0k_0)^{-1}]^{1/2}T^{ji},$$

and

$$T^{ji} = \bar{N}(p')[A^{ji}+\tfrac{1}{2}i\gamma\cdot(q+k)B^{ji}]N(p), \tag{3.2}$$

$$T^{ji} = T^+\delta_{ji} + T^-\tfrac{1}{2}[\tau_j,\tau_i], \tag{3.3}$$

where $T^\pm$, A, B are invariant functions of $s=-(p+q)^2$, $t=-(p-p')^2$, $u=-(p-k)^2$, and the s-wave scattering lengths are given by

$$a^\pm = (4\pi)^{-1}(1+m_\pi/m_N)^{-1}(A^\pm - \mu B^\pm)|_{\mathbf{p},\mathbf{q}=0}. \tag{3.4}$$

Using the Lagrangian (2.1), it can be seen that the processes that contribute to elastic πN scattering are N and N^*, s- and u-channel poles, and a ρ-meson t-channel pole, Fig. 1. The contributions of these processes to the amplitude T^{ji} are calculated using the tree-diagram method and are given as follows:

$$T_N{}^{ji} = \frac{g^2}{4\pi}\bar{N}(p')\left\{\frac{i\gamma\cdot k\gamma_5[-i\gamma\cdot(p+q)+m_N]i\gamma\cdot q\gamma_5 P_N{}^{ji}}{m_N{}^2-s} + \frac{i\gamma\cdot q\gamma_5[-i\gamma\cdot(p-k)+m_N]i\gamma\cdot k\gamma_5 P_N{}^{ij}}{m_N{}^2-u}\right\}N(p),$$

$$P_N{}^{ji} = \delta_{ji} + \tfrac{1}{2}[\tau_j,\tau_i],$$

$$T_{N^*}{}^{ji} = \frac{g_{N^*N\pi}{}^2}{m_\pi{}^2}\bar{N}(p')\left(\frac{k_\mu q_\nu \Lambda_{\mu\nu}(p+q)P_{N^*}{}^{ji}}{m_{N^*}{}^2-s} + \frac{q_\mu k_\nu \Lambda_{\mu\nu}(p-k)P_{N^*}{}^{ij}}{m_{N^*}{}^2-u}\right)N(p), \tag{3.5}$$

$$P_{N^*}{}^{ji} = \tfrac{1}{3}\{2\delta_{ji} - \tfrac{1}{2}[\tau_j,\tau_i]\},$$

$$T_\rho{}^{ji} = -\frac{f_\rho{}^2}{m_\rho{}^2-t}[\tfrac{1}{2}\bar{N}(p')i\gamma\cdot(q+k)]\tfrac{1}{2}[\tau_j,\tau_i]N(p),$$

where the subscripts indicate the appropriate pole diagram. Applying (3.3) and (3.4) to these results, we find that the s-wave scattering lengths are

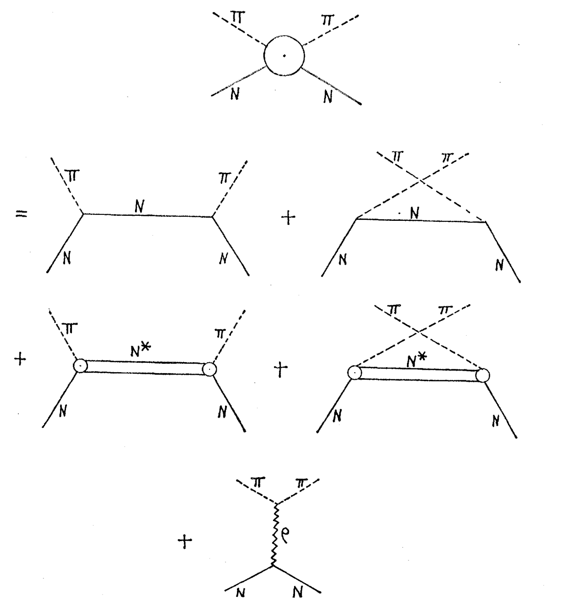

FIG. 1. Tree diagrams which contribute to elastic πN scattering.

$$a_N{}^+ = -\frac{g^2}{4\pi}\frac{1}{1+m_\pi/m_N}2\left(\frac{m_\pi}{2m_N}\right)^3\frac{1}{1-m_\pi{}^2/4m_N{}^2}\frac{1}{m_\pi},$$

$$a_N{}^- = \frac{g^2}{4\pi}\frac{1}{1+m_\pi/m_N}\left(\frac{m_\pi}{2m_N}\right)^3\frac{1}{1-m_\pi{}^2/4m_N{}^2}\frac{1}{m_\pi},$$

$$a_{N^*}{}^\pm = 0, \tag{3.6}$$

$$a_\rho{}^+ = 0,$$

$$a_\rho{}^- = \frac{f_\rho{}^2}{4\pi}\left(\frac{m_\pi}{m_\rho}\right)^2\frac{1}{1+m_\pi/m_N}\frac{1}{m_\pi}.$$

The scattering lengths $a^\pm$ are just the sum of the appropriate terms given here, and the result is

$$a^+ \approx -0.01 m_\pi{}^{-1}, \quad a^- \approx +0.08 m_\pi{}^{-1}, \tag{3.7}$$

which is in good agreement with experimentally determined values[17]

$$a^+ = -0.009 m_\pi{}^{-1}, \quad a^- = +0.093 m_\pi{}^{-1}. \tag{3.8}$$

Observe that the ρ-meson term $a_\rho{}^-$ is the dominant contribution (about ten times as large as the nucleon term) and is thus consistent with the usual ρ-dominance assumption. This situation would not be so if the usual on-mass-shell N^* propagator were used, since the N^* pole term would give a large contribution to a^+. It is then necessary to make assumptions about including

[17] See K. Raman, Phys. Rev. **164**, 1736 (1967), for references to experimental results.

other particles, whose existence is doubtful, in our Lagrangian, e.g., an $I=J=0$ meson, and to fix their couplings in such a way that their contribution cancels the large N^* value. We avoid this difficulty by our choice of N^* propagator and by including only known particles in the Lagrangian. It should be further noted that this selection of N^* propagator introduces a technical problem. If we calculate the amplitudes $A(s,t,u)$ and $B(s,t,u)$ using (3.5), we find that they have poles and branch cuts at s, $u=0$. This is in contradiction to what is known about the analytic properties of these amplitudes. These spurious poles and cuts do not cause much difficulty, since the tree-diagram method is just an approximation scheme used in lowest-order perturbation theory and it is possible that higher-order terms will cancel these singularities. Also, we are not too disturbed by these singularities since the poles are at s, $u=0$, which is far from the physical region, and the branch cuts, which are of the square-root type, can be picked away from the physical region in the s plane.

IV. COMPARISON WITH CURRENT ALGEBRA

In this section, we review the current-algebra calculation of pion-nucleon scattering lengths and compare the results with those in the previous section.[18] Consider the process

$$X(p)+\pi^i(q)\rightarrow Y(p')+\pi^j(k)\,, \tag{4.1}$$

where $X(p)$, $Y(p')$ are baryons of momentum p and p', respectively, and $\pi^i(q)$, $\pi^j(k)$ are pions of momentum q and k, respectively. The scattering amplitude for this process is given by

$$\langle\pi^j(k)Y(p')|S|\pi^i(q)X(p)\rangle = \delta_{fi}+(2\pi)^4 i\delta(p'+k-p-q) \times[(m_y m_x/p_0'p_0)(4q_0k_0)^{-1}]^{-1/2} \tag{4.2}$$

and

$$T^{ji}=(q^2+m_\pi^2)(k^2+m_\pi^2)i\int d^4x \times e^{-ik\cdot x}\langle Y(p')|T(\pi^j(x)\pi^i(0))|X(p)\rangle. \tag{4.3}$$

The partially conserved axial-vector current (PCAC) relationship $\partial_\mu A_\mu{}^i(x)=m_\pi^2 f_\pi \pi^i(x)$, where A_μ is the axial-vector current and f_π is the pion decay constant, is inserted in (4.3) and, applying the Ward-Takahashi technique, we obtain

$$T^{ji}=\frac{(q^2+m_\pi^2)(k^2+m_\pi^2)}{f_\pi^2 m_\pi^4}\Big\{i\int d^4x\, e^{-ik\cdot x}k_\mu q_\nu\langle Y(p')|T(A_\mu{}^j(x)A_\nu{}^i(0))|X(p)\rangle +iq_\nu\int d^4x\, e^{-ik\cdot x}\delta(x_0)\langle Y(p')|[A_4{}^j(x),A_\nu{}^i(0)]|X(p)\rangle -m_\pi^2 f_\pi\int d^4x\, e^{-ik\cdot x}\delta(x_0)\langle Y(p')|[A_4{}^i(0),\pi^j(x)]|X(p)\rangle\Big\}. \tag{4.4}$$

For the first commutator in (4.4), we insert the current-algebra relationship

$$\delta(x_0)[A_4{}^j(x),A_\nu{}^i(0)]=-i\epsilon_{jik}V_\nu{}^k(0)\delta(x)+\text{S.T.}, \tag{4.5}$$

where S.T. is the Schwinger term[19] that is symmetric in ij. It is customary to ignore this term since it only serves to make the T product covariant and does not affect the results. The last commutator in (4.4) is symmetric in ij and is assumed to be

$$\delta(x_0)[A_4{}^i(0),\pi^j(x)]=\delta_{ij}\sigma\delta(x)\,, \tag{4.6}$$

which is the usual σ-meson term. The resulting expression is

$$T^{ji}=\frac{(q^2+m_\pi^2)(k^2+m_\pi^2)}{f_\pi^2 m_\pi^4}\Big\{i\int d^4x\, e^{-ik\cdot x}k_\mu q_\nu\langle Y(p')|T(A_\mu{}^j(x)A_\nu{}^i(0))|X(p)\rangle +\epsilon_{jik}q_\nu\langle Y(p')|V_\nu{}^j(0)|X(p)\rangle-\delta_{ji}m_\pi^2 f_\pi\langle Y(p')|\sigma|X(p)\rangle\Big\}. \tag{4.7}$$

Having this relationship at our disposal, we let k, $q\rightarrow 0$ and evaluate it. This procedure gives off-mass-shell quantities which must be extrapolated to on-shell values by some suitable method.

Equation (4.7) can be used to evaluate the pion-nucleon scattering amplitude by letting X, $Y=N$. This analysis has been done by Raman[20] and the reader is referred to his paper for details. We will briefly describe his final result for the s-wave scattering lengths $a^\pm$ and compare it with the s-wave scattering lengths, given in

[18] For further details of current-algebra calculations, see S. L. Adler and R. F. Dashen, *Current Algebras* (W. A. Benjamin, Inc., New York, 1968).

[19] J. Schwinger, Phys. Rev. Letters **3**, 296 (1959).

[20] K. Raman, Phys. Rev. **159**, 1501 (1967).

(3.6), obtained using the phenomenological Lagrangian method.

Consider the T product in (4.7) which has contributions from N and N^* pole terms. The N pole gives the following values:

$$a_N^+ = -0.0105 m_\pi^{-1}, \quad a_N^- = +0.0008 m_\pi^{-1}, \tag{4.8}$$

and are consistent with those given in (3.6). The N^* pole yields the values

$$a_{N^*}^+ = -0.06\ m_\pi^{-1}, \quad a_{N^*}^- = 0.001 m_\pi^{-1}; \tag{4.9}$$

these are in contradiction with the values given in (3.6) which are $a_{N^*}^\pm = 0$. It is also noted that the value of $a_{N^*}^+$ is about ten times larger than the experimental value of a^+. This difficulty is resolved by assuming that the σ-meson term in (4.7) is adjusted in such a way that it gives the experimental result for a^+. This procedure fixes the value of the σ-meson term and the consistency can be checked by looking at the p-wave scattering lengths. Finally the vector-current term in (4.7) gives

$$a_V^- = \frac{1}{4\pi}\frac{m_\pi^2}{f_\pi^2}\frac{1}{1+m_\pi/m_N}\frac{1}{m_\pi}, \quad a_V^+ = 0, \tag{4.10}$$

and using the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSFR) relation,[21,22] we find this is identical with the ρ-meson contribution in (3.6). This is the dominant contribution to the s-wave scattering lengths in both methods.

V. $\pi N \to \pi N^*$ s-WAVE THRESHOLD PRODUCTION PARAMETERS

The tree-diagram method can be easily applied to the production process $\pi N \to \pi N^*$, where πN^* is produced in a relative s state. Of course, this method will give N^* in any l state, but the data are not sufficient to give information on states other than the s states accurately.

Let the matrix element for this process be given by (4.2), where X and Y are N and N^*, respectively, and T, the invariant amplitude, is given by

$$T = i\bar{N}_\nu(p')g_{\nu\mu}[Aq_\mu + Bk_\mu + i\gamma\cdot k(Cq_\mu + Dk_\mu)]\gamma_5 N(p), \tag{5.1}$$

where A, B, C, and D are invariant functions of s, t, and

[21] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 225 (1966).

[22] Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

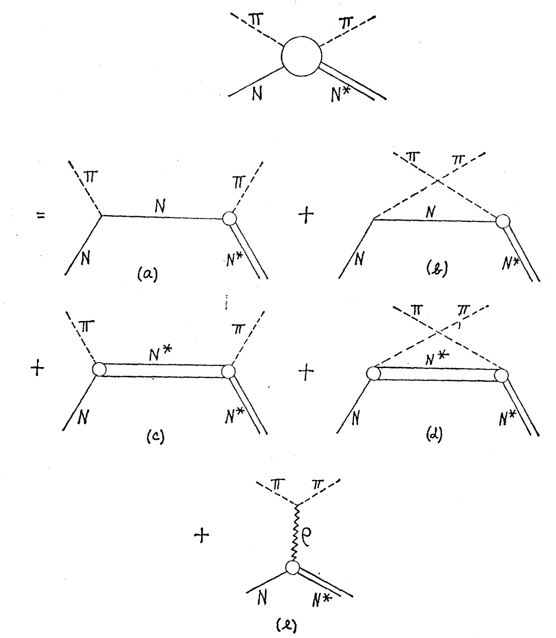

FIG. 2. Tree diagrams which contribute to isobar production.

u and multiplication by an over-all Clebsch-Gordan coefficient for specific charge states is understood. The value of T at threshold, which is what we are interested in, is given by the following simple two component forms:

$$T|_{\mathbf{p}',\mathbf{k}=0} = \frac{i\chi_j^\dagger(s')\boldsymbol{\sigma}\cdot\mathbf{p}q_j\chi(s)}{[2m_N(p_0+m_N)]^{1/2}} \times [A-(m_{N^*}+m_N+m_\pi)B]|_{\mathbf{p}',\mathbf{k}=0}$$

$$\equiv \frac{i\chi_j^\dagger(s')\boldsymbol{\sigma}\cdot\mathbf{p}q_j\chi(s)F}{[2m_N(p_0+m_N)]^{1/2}},$$

$$F = [A-(m_{N^*}+m_N+m_\pi)B]_{\mathbf{p}',\mathbf{k}=0}. \tag{5.2}$$

There are five processes that can contribute to $\pi N \to \pi N^*$, Fig. 2. These are the s- and u-channel N and N^* poles and the t-channel ρ pole. Both the N and N^* s-channel poles, Figs. 2(a) and 2(c), will not contribute to isobar production in a final-state s wave by the following arguments: (i) The s-channel N pole, Fig. 2(a), does not enter since conservation of angular momentum at the πNN^* vertex will permit only relative p waves or higher. (ii) The s-channel N^* pole, Fig. 2(c), is forbidden by parity conservation at the πN^*N^* vertex. If the final πN^* is in a relative s wave, then its parity is negative and the parity of the intermediate state is positive, thus, violating parity conservation.

Using the Lagrangian (2.1), we calculate these diagrams and obtain

$$T_N=\frac{g_{N^*N\pi}}{m_\pi}\frac{g}{2m_N}i\bar{N}_\lambda(p')g_{\lambda\sigma}q_\sigma\left[-1+\frac{2m_N(m_N+m_{N^*})}{m_N{}^2-u}+\frac{2m_N}{m_N{}^2-u}i\gamma\cdot q\right]\gamma_5N(p),$$

$$T_{N^*}=\frac{g_{N^*N^*\pi}}{2m_{N^*}}\frac{g_{N^*N\pi}}{m_\pi}i\bar{N}_\lambda(p')g_{\lambda\sigma}\times q_\sigma\left[-\frac{2}{3}\frac{(p-k)\cdot k}{u}+\frac{m_{N^*}-m_N}{3\sqrt{u}}+\frac{4}{3}\frac{m_{N^*}}{\sqrt{u}}\frac{(p-k)\cdot k}{m_{N^*}{}^2-u}-\frac{4}{3}\frac{m_{N^*}(m_{N^*}+m_N)}{m_{N^*}{}^2-u}\right]$$

$$+k_\sigma+i\gamma\cdot qq_\sigma\left\{-\frac{4}{3}\left[1+\frac{(p-k)\cdot k}{u}\right]\frac{m_{N^*}}{m_{N^*}{}^2-u}-\frac{1}{3\sqrt{u}}+\frac{2}{3}\frac{m_{N^*}}{\sqrt{u}}\frac{(m_{N^*}-m_N)}{m_{N^*}{}^2-u}\right\}+i\gamma\cdot qk_\sigma\frac{2m_{N^*}}{m_{N^*}{}^2-u}\times\gamma_5N(p),\quad(5.3)$$

$$T_\rho=\frac{2f_{\rho N^*N}f_\rho}{m_\pi}i\bar{N}_\lambda(p')g_{\lambda\sigma}\left(-\frac{m_{N^*}+m_N}{m_\rho{}^2-t}q_\sigma-\frac{1}{m_\rho{}^2-t}q_\sigma i\gamma\cdot q+\frac{1}{m_\rho{}^2-t}k_\sigma i\gamma\cdot q\right)\gamma_5N(p).$$

At threshold, these yield the following results:

$$F_N=-\frac{g}{2m_N}\frac{g_{N^*N\pi}}{m_\pi}\alpha_N\left(1+\frac{m_N}{p_0-\frac{1}{2}m_\pi}\right),$$

$$F_{N^*}=\frac{g_{N^*N\pi}g_{N^*N^*\pi}}{3m_\pi}\alpha_{N^*}\frac{1}{p_0-m_\pi}\left[1+\frac{3}{2}\frac{m_\pi}{m_{N^*}}-\frac{(m_{N^*}+m_N+m_\pi)(m_{N^*}-m_N+2m_\pi)}{m_{N^*}{}^2-(p_0-m_\pi)^2}\right],\quad(5.4)$$

$$F_\rho=2f_{\rho N^*N}f_\rho\alpha_\rho\frac{1}{m_\rho{}^2-(m_{N^*}-m_N)^2},$$

where the subscripts indicate the pole and the α's are the relevant Clebsch-Gordan coefficients. The full threshold amplitude F is given by

$$F=F_N+F_{N^*}+F_\rho.\quad(5.5)$$

The isotopic spin content of isobar production is fairly simple. Conservation of isospin allows production in only the $I=\frac{1}{2},\frac{3}{2}$ channels, or there will be only two independent amplitudes for this process. For convenience of calculation, we pick the processes $\pi^+p\to\pi^+N^{*+}$ and $\pi^-p\to\pi^-N^{*+}$ and list the pertinent Clebsch-Gordan coefficients in Table I. We can decompose the two amplitudes $F(\pi^+p\to\pi^+N^{*+})$ and $F(\pi^-p\to\pi^-N^{*-})$ into isotopic-spin invariant amplitudes for the $I=\frac{1}{2},\frac{3}{2}$ channels in the following manner:

$$F(\pi^+p\to\pi^+N^{*+})=-(\tfrac{2}{5})^{1/2}F_{3/2},$$
$$F(\pi^-p\to\pi^-N^{*+})=\tfrac{2}{3}(\tfrac{2}{5})^{1/2}F_{3/2}-\tfrac{1}{3}F_{1/2},\quad(5.6)$$

TABLE I. Clebsch-Gordan coefficients appropriate to each pole term for given initial and final charge states in isobar production.

Reaction \ Coefficient	α_N	α_{N^*}	α_ρ
$\pi^+p\to\pi^+N^{*+}$	$-\sqrt{\frac{2}{3}}$	$(8/27)^{1/2}$	$\sqrt{\frac{2}{3}}$
$\pi^-p\to\pi^-N^{*+}$	0	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}}$

where the subscripts represent the isotopic spin. In order to compare with experiment, we use the notation of Olsson and Yodh[23] and define production parameters in terms of the F's by multiplication with an appropriate scale factor. If we define the production parameters a_{2I} as

$$a_{2I}=\beta F_I,\quad(5.7)$$

where

$$\beta=m_\pi/500,$$

we obtain

$$a_3\approx0.016\text{ F},\quad a_1\approx0.023\text{ F},\quad(5.8)$$

and the results are in fermis.

Comparison of these values with the experimental results will require some discussion due to the ambiguities involved in extracting data on the final πN^* states from the observed $\pi\pi N$ states.[24] Isobar production has been analyzed by Olsson and Yodh[23] and they obtain the following values for a_1 and a_3:

$$a_3=0.0175\pm0.0008\text{ F},\quad a_1=0.059\pm0.005\text{ F},\quad(5.9)$$

where the errors are statistical. There are also errors which are caused by the specific model picked by these authors,[25] and they introduce further uncertainty in the production parameters. Also, Morgan[26] has analyzed isobar production in the $I=\frac{1}{2}$ channel and he finds a larger dipion resonance contribution to the process $\pi N\to\pi\pi N$ than do Olsson and Yodh. This can reduce a_1 by as much as 50%. The "adjusted values" for the production parameters are[24]

$$a_3=0.0175\text{ F},\quad a_1=0.023\text{ F},\quad(5.10)$$

which are consistent with the results we have obtained.

The use of current algebra in the usual way, i.e., $q=k$, $q\to0$ for calculating isobar production, has certain technical ambiguities. Consider (4.7) with $X=N$, $Y=N^*$. There is an immediate difficulty if we

[23] M. G. Olsson and G. B. Yodh, Phys. Rev. **145**, 1309 (1966).
[24] For details see M. E. Arons, Phys. Rev. **175**, 1905 (1968).
[25] M. G. Olsson, Phys. Rev. Letters **15**, 710 (1965); **15**, 768(E) (1965).
[26] D. Morgan, Phys. Rev. **166**, 1731 (1968).

attempt to set $q=k$, $q\to 0$. If energy and momentum are to be conserved, this would require that $m_N=m_{N^*}$ or the baryons are extrapolated off the mass shell and we are faced with ambiguity of no choice of a definite mass. We might circumvent this by picking some mean mass. Next, note that the σ term cannot be ignored since it could be isotopic spin 2, thus connecting N^* and N, and this we would have to estimate somehow. The vector-current term does not contribute at all since it vanishes in the limit $k=q$, $q\to 0$, $m_N=m_{N^*}$. A parity and angular momentum analysis of isobar production in a final s state, tells us that the initial πN must be in a D wave and this presents a further difficulty if q, the initial pion momentum, is allowed to go to zero.

There are two alternative ways of using the expression (4.7). One is to let $q^2=-m_\pi^2$ and $k\to 0$; this yields

$$T=i(k_\mu/f_\pi)\langle N^*(p')|A_\mu(0)|\pi(q)N(p)\rangle|_{k\to 0,\,q^2=-m_\pi^2}, \tag{5.11}$$

and this expression has been used by Arons[24] to calculate isobar production with good results. The other way is to let $q^2=-m_\pi^2$, $k^2=-m_\pi^2$ in (4.7), and all this does is to give the usual Lehmann-Symanzik-Zimmermann formula for the process $\pi N\to\pi N^*$, which we are not interested in. We must keep in mind that with any of these procedures an additional assumption must be made as to how one extrapolates from the off-mass-shell to the on-mass-shell values.

VI. SINGLE-PION PHOTOPRODUCTION

The Kroll-Ruderman theorem[27] gives the threshold pion photoproduction amplitude to lowest order in m_π/m_N and to all orders in strong interactions in terms of the Born approximation alone. If this amplitude is used to calculate the threshold differential cross section, it is found to be in disagreement with experiment, indicating that higher-order corrections in m_π/m_N are not negligible. The tree diagrams can be evaluated without taking the m_π/m_N limit and can be expected to agree better with experiment than the Kroll-Ruderman theorem. Of course, in the limit of $m_\pi/m_N\to 0$, the tree diagrams reproduce the Kroll-Ruderman results.

Let the invariant amplitude for the process

$$\gamma(q)+N(p)\to\pi(k)+N(p') \tag{6.1}$$

be given by

$$\begin{aligned}T=(e/m_N)\bar{N}(p')[\tfrac{1}{2}Ai\gamma_\mu\gamma_\nu+BP_\mu\gamma_\nu\\+Ck_\mu\gamma_\nu+DiP_\mu k_\nu]\gamma_5N(p)F_{\mu\nu}(q),\\P_\mu=p_\mu+p_\mu',\\F_{\mu\nu}(q)=q_\mu\epsilon_\nu(q)-q_\nu\epsilon_\mu(q).\end{aligned} \tag{6.2}$$

At threshold, the differential cross section is

$$\frac{q}{k}\frac{d\sigma}{d\Omega}\bigg|_{k=0}=\frac{e^2}{4\pi}\frac{1}{(1+m_\pi/m_N)^3}\frac{(1+m_\pi/m_N)^2}{m_N^2}\times\frac{|m_\pi A-P\cdot qB-k\cdot qC|^2}{4\pi}. \tag{6.3}$$

Consider the process

$$\gamma p\to\pi^+n. \tag{6.4}$$

In the presence of electromagnetism, the interaction part of the Lagrangian for this process can be written as

$$\begin{aligned}\mathcal{L}_{\rm em}=e\bar{p}i\gamma_\mu pA_\mu-\frac{g}{\sqrt{2}m_N}\bar{n}i\gamma_\mu\gamma_5p\partial_\mu\pi^--i\frac{eg}{\sqrt{2}m_N}\bar{n}i\gamma_\mu\gamma_5pA_\mu\pi^-+ie(\pi^+\partial_\mu\pi^--\pi^-\partial_\mu\pi^+)A_\mu\\-\frac{eC_3}{m_\pi}\bar{N}_\lambda^{*+}g_{\lambda\mu}i\gamma_\nu\gamma_5p(\partial_\mu A_\nu-\partial_\nu A_\mu)-\frac{1}{\sqrt{3}}\frac{g_{N^*N\pi}}{m_\pi}i\bar{n}N_\mu^+\partial_\mu\pi^-+\frac{f_\rho}{\sqrt{2}}\bar{n}i\gamma_\mu p\rho_\mu^-+\frac{ef_{\rho\pi\gamma}}{m_\pi}i\epsilon_{\mu\nu\lambda\sigma}\partial_\mu\rho_\nu^+\partial_\lambda\pi^-A_\sigma,\end{aligned} \tag{6.5}$$

where we have added a $\rho\pi\gamma$ interaction term. The $\rho\pi\gamma$ coupling constant is determined from the ω^0 decay rate into $\pi^0\gamma$ by SU_3 arguments and based on a width

$$\Gamma(\omega^0\to\pi^0\gamma)=1.15\text{ MeV}; \tag{6.6}$$

we obtain

$$f_{\omega^0\pi\gamma}/3=f_{\rho\pi\gamma}=0.1375. \tag{6.7}$$

A similar Lagrangian can be written for the processes

$$\gamma n\to\pi^-p,\quad \gamma p\to\pi^0p. \tag{6.8}$$

The process $\gamma p\to\pi^0 p$ requires the addition of $\omega\pi\gamma$ and ωNN interaction terms,

$$\mathcal{L}_\omega=\tfrac{1}{2}f_\rho\bar{p}i\gamma_\mu p\omega_\mu^0+(e\times 3f_{\rho\pi\gamma}/m_\pi)i\epsilon_{\mu\nu\lambda\sigma}\partial_\mu\omega_\nu^0\partial_\lambda\pi^0A_\sigma. \tag{6.9}$$

Tree diagrams, which contribute to the three photoproduction processes, are given in Figs. 3–5.[28] At threshold, the N^* intermediate states do not contribute to any of the three processes because of our choice of

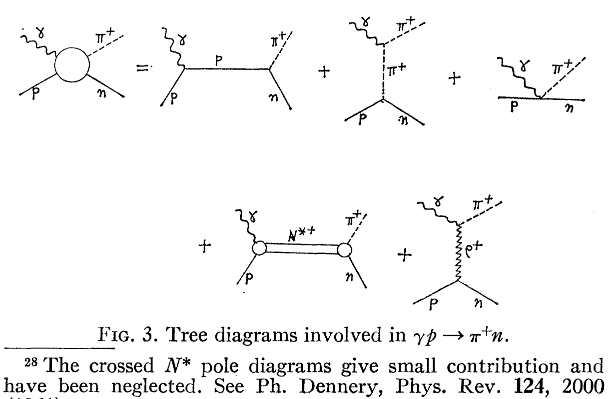

FIG. 3. Tree diagrams involved in $\gamma p\to\pi^+n$.

[27] N. M. Kroll and M. A. Ruderman, Phys. Rev. 93, 233 (1954).

[28] The crossed N^* pole diagrams give small contribution and have been neglected. See Ph. Dennery, Phys. Rev. 124, 2000 (1961).

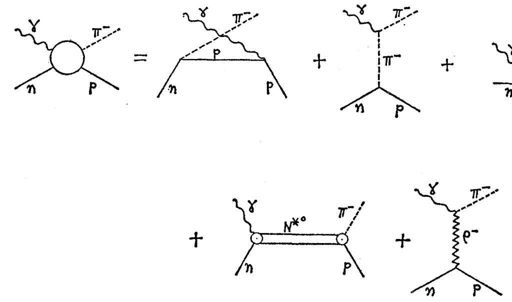

FIG. 4. Tree diagrams involved in $\gamma n \rightarrow \pi^- p$.

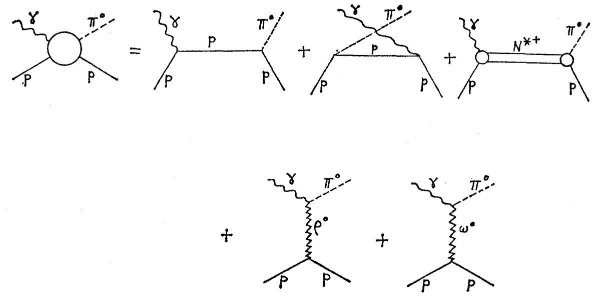

FIG. 5. Tree diagrams involved in $\gamma p \rightarrow \pi^0 p$.

N^* propagator. The ρ and ω t-channel diagrams give negligible contributions to all three processes, and thus, the nucleon and pion-pole diagrams and the contact term give the dominant contribution to all processes. The results for charged-pion photoproduction at threshold are

$$\frac{q}{k}\frac{d\sigma}{d\Omega}(\gamma p \rightarrow \pi^+ n) = \frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{1}{2m_N^2}\frac{1}{(1+m_\pi/m_N)^3} = 15\ \mu\text{b/sr}\,, \quad (6.10)$$

$$R = \frac{d\sigma}{d\Omega}(\gamma n \rightarrow p\pi^-) \bigg/ \frac{d\sigma}{d\Omega}(\gamma p \rightarrow n\pi^+) = 1 + \frac{m_\pi}{m_N} = 1.15\,, \quad (6.11)$$

to be compared with the experimental values[29]

$$\frac{q}{k}\frac{d\sigma}{d\Omega}(\gamma p \rightarrow \pi^+ n) = 15.6 \pm 0.5\ \mu\text{b/sr}\,, \qquad R = 1.265 \pm 0.075\,. \quad (6.12)$$

For neutral-pion photoproduction,

$$\frac{q}{k}\frac{d\sigma}{d\Omega}(\gamma p \rightarrow \pi^0 p) = \frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{1}{2m_N^2}\frac{1}{1+m_\pi/m_N}\frac{m_\pi^2}{2m_N^2} = 0.24\ \mu\text{b/sr}\,, \quad (6.13)$$

which is about half of the experimental value.

We note that a current-algebra calculation of single-pion photoproduction, using the PCAC condition with electromagnetic interaction, gives exactly the same results.[29] Furthermore, if the on-mass-shell N^* propagator is used, the charged-pion photoproduction results remain unchanged and the neutral-pion differential cross section is increased somewhat but not enough to explain the discrepancy with the experimental data.

[29] References to experimental results are given by G. W. Gaffney, Phys. Rev. **161**, 1599 (1967).

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APPENDIX

For convenience, we indicate here the method used to determine the isospin content of $N_{\mu j}$. In what follows, the space-time index μ is suppressed, and the isospinor index $\alpha = 1, 2$ is written explicitly. If I_l $(l = 1, 2, 3)$ are the isospin generators, then

$$[I_l, N_j{}^\alpha] = -(\tfrac{1}{2}\tau_l)_\beta{}^\alpha N_j{}^\beta + (T_l)_{mj} N_m{}^\alpha\,, \quad (\text{A1})$$

where $(T_l)_{mj} = -i\epsilon_{lmj}$. Using this relation and the subsidiary condition

$$\tau_j N_j{}^\alpha = 0\,, \quad (\text{A2})$$

we find

$$\begin{aligned}
N_1{}^1 &= N^{*0}/\sqrt{6} - N^{*++}/\sqrt{2}\,, \\
N_2{}^1 &= i[N^{*0}/\sqrt{6} + N^{*++}/\sqrt{2}]\,, \\
N_1{}^2 &= N^{*-}/\sqrt{2} - N^{*+}/\sqrt{6}\,, \\
N_2{}^2 &= i[N^{*-}/\sqrt{2} + N^{*+}/\sqrt{6}]\,, \\
N_3{}^1 &= (\sqrt{\tfrac{2}{3}})N^{*+}\,, \\
N_3{}^2 &= (\sqrt{\tfrac{2}{3}})N^{*0}\,.
\end{aligned} \quad (\text{A3})$$

The convention that all operators destroy when operating forward is used.