determine this singularity structure only because the models are based on a trivial exactly solvable canonical field theory. We have no idea what are reasonable hypotheses to make about the singularity structure in less trivial cases. We believe that this important unsolved problem must be faced if the program of defining dynamics exclusively in terms of currents is to be brought to a successful conclusion.

(Of course, exactly the same remarks could be made about conventional Lagrangian field theory; however, here, at least, perturbation theory can give us important clues.)

In particular, because we have found it necessary in all of our models to average over both timelike and spacelike directions, many results obtained by naive manipulation of the Sugawara equations are false in our models. The four false theorems of Sec. III are examples of this.

We find this extremely disquieting: The Sugawara model, which upon naive inspection appeared to be a set of logically connected propositions, each one necessarily following from its predecessors, has dissolved before our eyes into a collection of disconnected assertions, any one of which may or may not be true independently of the validity of the others, in any given theory.

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Interactions of Hadrons with Nuclei at High Energy. I*

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It is shown that under certain assumptions the Glauber multiple-scattering series can be summed explicitly to give a closed expression for the amplitude describing the interaction of a high-energy particle with a nucleus. The advantages of this description are discussed, and it is shown that available data are well described by the theory. Application of the theory to inelastic processes is given, and it is shown how quantities like σ_{A_1N} and $\sigma_{\Sigma N}$, total scattering cross sections which are not otherwise measurable, can be obtained.

I. INTRODUCTION

Recent high-precision work on proton-nucleus elastic scattering[1] has led to renewed interest in the problem of particle-nucleus interactions, both in the case of elastic[2] and inelastic[3] scattering. In this paper we wish to carry this work further along by making two points. First, we shall try to show why such interactions are potentially of great interest to particle physicists, and, second, we shall try to show that it is possible to formulate a theory of such interactions in a conceptually simple way, and that this theory agrees well with what data are available at the present time.

There are strong reasons why one could wish to understand particle-nuclear interactions. If one has a good theory for such processes, then one can use nuclear targets in high-energy experiments to obtain data which are not easily attainable by direct measurement, and in some cases data which are not directly attainable at all. In particular, there are three effects connected with nuclear targets which are potentially of great interest to particle physicists.

The first of these is the effect of *nuclear coherence*. If the cross section for a particular process to go on a hydrogen target is σ_H, then the cross section to go on a nucleus of atomic number A will be $\sigma_A = \sigma_H A^n$, where $n > 0$. Thus if we wished to examine rare production modes, which we frequently would like to do, we could look at production on a nucleus, and then use our theory to extract the production from a single nucleon.

The second useful effect is *nuclear rescattering*. One would frequently like to examine the interactions of very short-lived particles with nucleons. Unfortunately, most strongly interacting resonances have decay paths of the order of tens of Fermis, and thus cannot be made into beams for scattering experiments. However, if such a particle is produced inside of a nucleus, then it will have the chance to strike nucleons on its way out of the nucleus, and, once again, a good theory of particle-nucleus interactions would enable us to extract the

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resonance-nucleon scattering amplitude from the nuclear production amplitude.

Finally, one can look at *nuclear selection*. Hydrogen targets have spin and isotopic spin $\frac{1}{2}$, and this sometimes causes complications in the analysis of scattering data. There are many nuclei, however, with zero spin and isospin, so that if they were used as targets in scattering experiments, $T=0$ exchanges could be isolated. Such an ability to isolate particular quantum numbers in the exchange would be of great interest to Regge theorists, to give one example. Also, the intense collimation of production products which we will find should tend to make life easier from an experimental point of view.

It should also be mentioned that there is a complementary side to this approach—one could, in principle, use particle-nucleus scattering experiments to obtain information about nuclear wave functions, and particularly about short-range correlations.[4] This work is still in a very preliminary state, however, so that further comment at this time is probably premature.

From the above discussion, it is clear that a theory of particle-nucleus interactions could prove very useful in particle physics, and might also yield some information about nuclear physics.

In general, the theoretical work on this problem can be divided into two types. In those reactions where the final state of the nucleus can be defined, and where the nuclei are light (in practice, $A \lesssim 16$), one applies the multiple-scattering theory of Glauber[5] to fit the differential cross sections. In reactions where the energy resolution is not good enough to allow one to isolate the final nuclear state, present theories allow one to either treat the nucleus in some approximate way as an absorbing medium[6,7] or apply closure and sum the Glauber series in the limit $A \to \infty$.[8] In this work we also apply closure,[3,8] giving a single closed expression for particle-nucleus scattering. To do so, as we shall see, it will be necessary to assume an extremely simplified form for the nuclear ground-state densities. We shall argue, however, that to within the kinds of accuracy which are likely to be needed to extract new data from nuclear production experiments, the results are not terribly sensitive to the nuclear physics input, and that reasonable fits to the existing data can be obtained. The advantage of this technique is that the results should be applicable to light nuclei, where the limiting procedure mentioned above will not apply. In addition, we write down a single closed expression for the differential cross section which is valid in both the "coherent" and "incoherent" regions. Thus, in analyzing production data, no incoherent subtractions will have to be made.

In addition, we shall try to discuss the relation between the multiple-scattering approach and the approach via a model where the particles are pictured as traveling through a partially absorbing medium of some sort.[6,7] Throughout the rest of this paper, we shall refer to such models collectively as the "absorption model." The main difficulty with such models is that unless one inserts an "incoherent background" into the problem in a rather artificial way, one cannot match the data very well at large momentum transfers (we shall speak of momentum transfers which lie outside of the diffraction peak as large). We shall argue in Sec. III that this difficulty arises from the fact that the absorption models correspond to processes in which the nucleus remains largely unaffected by the scattering process, while large momentum transfer events are dominated by processes in which the nucleus is broken up. Experimental tests of this conjecture will be suggested.

We shall see that such "incoherent" events are well described by the mutliple-scattering model with closure. This will lead us, in turn, to consider production reactions in which we know that no coherence is possible, for example, the reactions

$$\pi^- + A \to K^* + A'$$

and

$$p + A \to \Sigma^+ A',$$

where A' is some hypernucleus. We shall see that it should be possible to determine the total scattering cross sections σ_{K^*N} and $\sigma_{\Sigma N}$ from such experiments, and shall discuss the significance of such results for the quark model.

In applying the absorption model, one usually assumes that the final state of the nucleus is the same as the initial state, so that the representation of the nucleus as an absorbing medium can be thought of as a sort of average over probability densities of a large number of nucleons. In reactions like those mentioned above, however, it is difficult to see how we could apply this idea. The main advantage of the multiple-scattering model, then, will be in the natural way in which results for completely incoherent processes like the above come out. This advantage is, of course, shared by formulations such as those of Ref. 8 where the multiple-scattering series is summed in an approximate way, rather than explicitly as in this paper.

This does not imply, of course, that the absorption model is in any sense wrong. The microscopic physical process which corresponds to absorption is, of course, just multiple scattering. In the multiple-scattering

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picture, a particle proceeds through a nucleus, scattering from individual nucleons as it goes. At each scattering, the probability that the particle will continue on without change of identity is something like $\sigma_{\mathrm{el}}/\sigma_{\mathrm{tot}}$ which, at high energies is $\sim\frac{1}{5}$. The probability that a particle will retain its initial identity after n scatterings should then be $\sim(\frac{1}{5})^n$. Thus a particle which comes into the nucleus with small impact parameter, traversing the nucleus in the region where the individual nucleons are most densely packed, will undergo many scatterings and its chance of emerging from the nucleus is small. This is the mechanism of the "removal of lower partial waves" in the absorption model.

Our line of attack shall be as follows: First we shall consider how the multiple-scattering theory can be combined with closure to give differential cross sections for reactions in which the final state of the nucleon is not known. We shall consider two applications of this model: (1) elastic proton-nucleus scattering at 19–20 GeV/c, and (2) production of particle resonances by hadrons incident on nuclei. We would like to refer to this type of process as "hadroproduction" (in analogy with photoproduction). The main reason for looking at elastic scattering shall be to convince ourselves that the theory works, and also, in this particularly simple case, to try to obtain some physical understanding of the processes which occur in different regions of momentum transfer and for different nuclei.

Once the theory is checked against elastic scattering data, we shall turn to suggesting uses of the theory in extracting particle data through the nuclear rescattering and nuclear coherence effects described above. The treatment of photoproduction, because of some rather special problems which do not arise in hadroproduction, will be left to a later publication.

II. PRESENTATION OF FORMALISM

The Glauber theory has been presented in great detail and clarity elsewhere,[5] and we shall not go over the derivation here. The essential idea of the theory is that the phase shift suffered by a particle moving through a complex target can be written

$$\chi=\sum_{i=1}^{A}\chi_i\,, \tag{2.1}$$

where χ_i are the phase shifts due to the individual scattering centers, and the sum extends over all such centers. If we then note that the χ_i can be expressed in terms of scattering amplitudes as

$$e^{i\chi_i}=\frac{i}{2\pi ik}\int d\boldsymbol{\delta}\; e^{-i\delta\cdot\mathbf{r}}f_i(\delta)+1\,, \tag{2.2}$$

where f_i represents the amplitude for scattering on the ith nucleon, then for small-angle scattering, the elastic scattering of the particle from a complex target is given by

$$\langle\Psi_f|F|\Psi_i\rangle=\frac{ip}{2\pi}\langle\Psi_f|\int d^{(2)}b\; e^{i\Delta\cdot\mathbf{b}}\left\{1-\prod_{j=1}^{A}\left[1-\frac{i}{2\pi ik_j}\times\int d\boldsymbol{\delta}_j e^{-i\delta_j\cdot(\mathbf{b}-\mathbf{s}_j+\mathbf{s})}f_j(\delta_j)\right]\right\}|\Psi_i\rangle\,, \tag{2.3}$$

where Ψ_f and Ψ_i refer to the final and initial states of the target, $\boldsymbol{\delta}_j$ is the momentum transfer in the jth scattering, p and k are the incident momentum in the projectile-nucleus c.m. system and the projectile-nucleon c.m. system, respectively, $f_j(\delta_j)$ is the scattering amplitude of the jth scattering, $\boldsymbol{\Delta}$ is the total momentum transfer. The position vector of the ith nucleon is written as $\mathbf{r}_i=\mathbf{s}_i+\mathbf{z}_i$, where $\mathbf{s}_i$ is a vector in the plane perpendicular to the beam axis, and $\mathbf{b}$ is the impact parameter. $\mathbf{s}$ is defined by $\bar{s}=\sum_j\mathbf{s}_j/A$.

The essential kinematic condition in the derivation is that the momentum transfer is perpendicular to the incident momentum (i.e., that the scattering angle is small). Equation (2.3) has a very nice interpretation in terms of a multiple scattering progression, since if we expand the product, we find[5]

$$\langle\Psi_f|F|\Psi_i\rangle=\frac{ip}{2\pi}\langle\Psi_f|\int d^{(2)}b\; e^{i\Delta\cdot\mathbf{b}}\{\sum_i f_i(b-s_i+s)-\sum_i\sum_{j\neq i}f_i(b-s_i+s)f_j(b-s_j+s)+f^3-f^4\cdots\}\,, \tag{2.4}$$

where we have written

$$f_j(b-s_j+s)=\frac{1}{2\pi ik}\int d^{(2)}\delta e^{-i\delta\cdot(\mathbf{b}-\mathbf{s}_j+\mathbf{s})}f_j(\delta)\,. \tag{2.5}$$

Thus we can interpret the first term as the sum of contributions from single scattering, the second as those from double scattering, and so forth. This interpretation gives us a natural way of extending the Glauber approximation to inelastic processes.[3,8]

Another very important feature emerges from Eq. (2.3). The only expressions which enter are (1) the elastic projectile-nucleus scattering amplitude, and (2) the initial and final nuclear wave functions. In principle, both of these can be determined exactly from other experiments, so that there are *no free parameters* in this theory. Thus, the question of whether or not one can fit the elastic scattering data becomes rather crucial, since the theory will stand or fall on that test—there is no way to salvage it if it fails.

From Eq. (2.3), the differential scattering cross section for particle-nucleus scattering to a given final state Ψ_f is just

$$d\sigma/dt=(\pi/p^2)\langle\Psi_i|F^\dagger|\Psi_f\rangle\langle\Psi_f|F|\Psi_i\rangle\,. \tag{2.6}$$

Now in the type of experiment we want to consider, the final state of the nucleus cannot be determined.

Thus the cross section will be given by the above expression summed over all final states. If the particle energies are large compared to nuclear energies, we can use the closure statement that

$$\sum_f |\Psi_f\rangle\langle\Psi_f| = 1$$

to give us the differential cross section in the form

$$d\sigma/dt = (\pi/p^2)\langle\Psi_i|F^\dagger F|\Psi_i\rangle. \tag{2.7}$$

Thus, we see that we need know only the elastic particle-nucleon amplitude and the ground-state wave function of the nucleus in order to calculate the particle-nucleus cross section.

In Ref. 3, it was shown that for elastic proton-^{16}O scattering, the ground-state density

$$|\Psi_0|^2 \sim \prod_{j=1}^{A} e^{-r_j^2/R^2} \tag{2.8}$$

did very well in predicting the main features of the observed differential cross section, and that one had to go to more complicated harmonic-oscillator wave functions only to reproduce details of structure at high t. Thus, for a first approximation, we shall write the ground-state density for all nuclei as in Eq. (2.8).

We shall justify this assumption later in more detail, but for the moment, let us simply accept this as the simplest wave function which we can write. The single parameter R is determined by the relation

$$R^2 = \tfrac{2}{3}\langle r^2\rangle, \tag{2.9}$$

where $\langle r^2\rangle^{1/2}$ is the measured rms radius of the nucleus. If we now write the elastic particle-nucleon amplitude as

$$f(\delta) = (p/4\pi)(i+\alpha)\sigma\ e^{-B\delta^2/2}, \tag{2.10}$$

where σ_T is the total particle-nucleon cross section, we can calculate the cross section in Eq. (2.7) explicitly. We can write Eq. (2.7) symbolically as

$$\frac{d\sigma}{dt} = \frac{\pi}{p^2}\langle\Psi_0|\int e^{-i\Delta\cdot\mathbf{b}'}d\mathbf{b}'\{\sum_N \Phi^{(N)\dagger}(b')\}$$
$$\times\int e^{i\Delta\cdot\mathbf{b}}d\mathbf{b}\{\sum_M \Phi^{(M)}(b)\}, \tag{2.11}$$

where $\Phi^{(M)}(b)$ represents the term in Eq. (2.4) in which the scattering amplitude appears M times (we shall call this the Mth-order scattering term). Carrying out all of the integrals over momentum and nuclear coordinates, we find

$$\frac{d\sigma}{dt} = \tfrac{1}{4}\sum_{M=1}^{A}\sum_{N=1}^{A}\sum_{L=\min(0,A-M-N)}^{\min(M,N)}\left[-\frac{\sigma_T(1+i\alpha)}{2\pi}\right]^M\left[-\frac{\sigma_T(1-i\alpha)}{2\pi}\right]^N$$
$$\times\frac{A!}{L!(M-L)!(N-L)!(A-N-M+L)!}\left(\frac{1}{R^2+2B}\right)^{M+N-2L}\left(\frac{1}{4B(R^2+B)}\right)^L\frac{e^{-\Delta^2/4S^2}}{S^2T^2}, \tag{2.12}$$

where

$$T^2 = \chi + 2\lambda, \tag{2.13}$$

$$S^2 = \frac{-2\lambda^3 - 3\lambda^2\chi + 4\chi p}{4(\lambda+\lambda)^2}, \tag{2.14}$$

and where

$$\chi = \tfrac{1}{2}L\left(\frac{1}{R^2+B}+\frac{1}{B}\right)+\frac{M+N-2L}{R^2+2B}, \tag{2.15}$$

$$\lambda = \tfrac{1}{4}L\left(\frac{1}{R^2+B}-\frac{1}{B}\right), \tag{2.16}$$

$$P = \left[\frac{M-L}{R^2+2B}+\tfrac{1}{4}L\left(\frac{1}{R^2+B}+\frac{1}{B}\right)\right]$$
$$\times\left[\frac{N-L}{R^2+2B}+\tfrac{1}{4}L\left(\frac{1}{R^2+B}+\frac{1}{B}\right)\right]. \tag{2.17}$$

The origin of the sum over M and N in the above can be seen from Eq. (2.11). The sum over the index L also has a simple physical interpretation. In an expression like

$$\Phi^{(N)\dagger}(b)\Phi^{(M)}(b) \tag{2.18}$$

it is possible that one or more $f^\dagger(b)f(b)$ pairs act on the same nucleon, i.e., have the same nucleon index. If we call such an occurrence an overlap of order L when there are L pairs, then the index L in Eq. (2.12) simply sums over all such terms. For elastic scattering this is not too important, but it will be rather critical for processes in which quantum numbers are exchanged.

Turning our attention to hadroproduction process, we see that there are two types of complications which are not encountered in simple elastic scattering. Consider as an example the process

$$\pi A \rightarrow VA, \tag{2.19}$$

where V is a meson. Then instead of only one amplitude

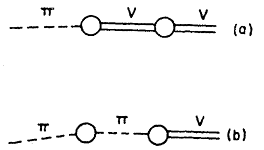

FIG. 1. Some multiple-scattering diagrams.

entering the problem, there will be three—those describing the processes

$$\pi N \to \pi N, \quad \pi N \to VN, \quad VN \to VN. \tag{2.20}$$

Thus, the Mth-order scattering term will no longer be just f^M, but will consist of the sum over all of the ways I can combine the above three amplitudes. For example, in second order, I will have both diagrams in Fig. 1.

The second complication concerns the fact that in the reaction (2.19) above, the condition in the Glauber approximation that the momentum transfer be perpendicular to the incident momentum cannot be satisfied, since even at zero production angle there is a minimum longitudinal momentum transfer given by

$$\Delta_m = (m_V^2 - m_\pi^2)/2P_{\text{lab}}. \tag{2.21}$$

The first of these difficulties is, of course, simply a matter of counting. To get around the second, one must extend the basic assumption in Eq. (2.1) to read

$$\chi_i(\pi N \to VN) = \Delta_m z_i + \chi_t, \tag{2.22}$$

where χ_t is the phase shift associated with the transverse momentum transfer, and $\Delta_m z_i$ is the extra phase added at the point of production due to the change of mass at that point. Such a term has been explained in terms of the absorption model,[9] and we need not go any further into it here.

Let us write

$$f_i(\delta) = [(i+\alpha)/4\pi] p\sigma_\pi e^{-(B/2)\delta^2} \tag{2.23}$$

as the amplitude for π-N elastic scattering on the ith nucleon, and

$$h_j(\delta) = [(i+\beta)/4\pi] p\sigma_V e^{-(D/2)\delta^2} \tag{2.24}$$

as the amplitude for V-N elastic scattering on the jth nucleon, and

$$g_l(\delta) = [G(i+\gamma)/4\pi] p e^{-(C/2)\delta^2} e^{i\Delta_m z l}, \tag{2.25}$$

as the amplitude to go from a π to a V on the lth nucleon. For the sake of simplicity, we assume that

$$D = B \tag{2.26}$$

(but not $B=C$). This is justified by the finding in Ref. 3 that the dependence of the cross section on these parameters is very weak indeed. If we redefine $\Phi^{(M)}$ as

$$\Phi^{(M)} = \sum_{k=0}^{M} g(b) f^{(M-k-1)}(b) h^{(k)}(b), \tag{2.27}$$

then Eq. (2.11) will describe the process (2.19). The meaning of the terms in the above definition is simple. The $k=0$ term corresponds to the case [as in Fig. 1(b)] where the V is made on the last nucleon from which scattering occurs, the $k=1$ term corresponds to the case [as in Fig. 1(a)] where the V is made on the penultimate scattering center, and so forth. This new definition of $\Phi^{(M)}$ then solves the counting problem outlined above.

We also note that now the problem of overlap is a bit harder, too. In fact, for the overlap term of order L, we can now distinguish five different types of terms. Let us call the case where two elastic amplitudes (either f or h) overlap "elastic overlap." Then these five cases are the following:

Case I. The $g_i^\dagger$ and g_i overlap, and there are $L-1$ elastic overlaps in addition.

Case II. The $g_i^\dagger$ overlaps with an h_i or f_i, and there are $L-1$ elastic overlaps in addition.

Case III. Same as *Case II* except that g_i overlaps with $h_i^\dagger$ or $f_i^\dagger$.

Case IV. Both $g_i^\dagger$ and g_i overlap with the appropriate elastic terms (but not with each other) and there are $L-2$ elastic overlaps in addition.

Case V. There are L elastic overlaps.

With this distinction in mind, we can proceed to put Eqs. (2.23)–(2.27) into Eq. (2.11) and evaluate the integrals. We find the result

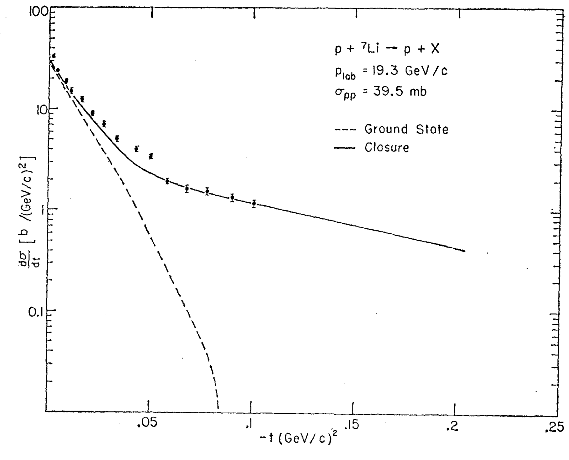

FIG. 2. p-^{7}Li scattering. Data from Ref. 10; $r=2.8$ F.

[9] M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966); S. D. Drell and J. S. Trefil, Phys. Rev. Letters **16**, 552 (1966); **16**, 832(E) (1966); J. G. Asbury, U. Becker, W. K. Bertram, P. Joos, M. Rhode, A. J. S. Smith, C. L. Jordan, and S. C. C. Ting, *ibid.* **19**, 832 (1966).

$$\frac{d\sigma}{dt}=\frac{|G(1+i\gamma)|^2}{16\pi}\sum_{M=1}^{A}\sum_{N=1}^{A}\sum_{L=\min(0,A-M-N)}^{\min(M,N)}\left(-\frac{\sigma_\pi(1+i\alpha)}{2\pi}\right)^M\left(-\frac{\sigma_\pi(1-i\alpha)}{2\pi}\right)^N\left(\frac{1-q^M}{1-q}\right)\left(\frac{1-q^N}{1-q}\right)$$
$$\times C(N,M,L)\left(\frac{1}{R^2+2B}\right)^{M+N-2L+2}\left(\frac{1}{4B(R^2+B)}\right)^{L-1}\left\{\frac{L}{4C(R^2+C)}I_1(\Delta^2)\right.$$
$$+\frac{L(R^2+2B)e^{-(\Delta_mR)^2/2}}{2C(R^2+2C)(1+\beta)[R^2+2C/(1+\beta)]}[(M-L)I_2(\Delta^2)+(N-L)I_3(\Delta^2)]$$
$$\left.+\frac{L(L-1)B(R^2+B)e^{-(\Delta_mR)^2/2}I_4(\Delta^2)}{\{C(1+\beta)[R^2+2C/(1+\beta)]\}^2}+\frac{(N-L)(M-L)(R^2+2B)^2}{4B(R^2+2C)^2(R^2+B)}e^{-(\Delta_mR)^2/2}I_5(\Delta^2)\right\},\quad(2.28)$$

where

$$C(M,N,L)=\frac{A!}{MNL!(N-L)!(M-L)!(A-M-N+L)!},\quad(2.29)$$

$$\beta=B/C,\quad(2.30)$$

and

$$q=[\sigma_V(1+i\beta)/\sigma_\pi(1+i\alpha)],\quad(2.31)$$

and the quantities $I_j(\Delta^2)$ are defined in the Appendix. The main point of interest is that the terms containing $I_1(\Delta^2)$ through $I_5(\Delta^2)$ correspond to the five different overlap cases described above. We shall return to this point in Sec. IV.

We now identify the 4-momentum transfer t as

$$t=\Delta^2+\Delta_m{}^2.\quad(2.32)$$

We then have at our disposal expressions which depend only on the nuclear radius and particle scattering amplitudes. These expressions, then, are supposed to describe the elastic scattering and production of strongly interacting particles on nuclei. Our first concern, shall be to check that the theory is in agreement with what data exists at the moment, and then to see what new types of data could be extracted from hadroproduction processes. It is to these points that we now turn our attention.

III. ELASTIC PROTON-NUCLEUS SCATTERING

There are two pieces of very good data on elastic scattering of protons from nuclei. At 1.67 GeV/c lab momentum, there are data in which the energy resolution is good enough to identify unambiguously the final state of the target nucleus, at least for transitions to the ground state.[1] This data have been analyzed in great detail,[2,3] and one can say that the multiple-scattering theory seems to work very well, at least for scattering angles (c.m.) or less than $\sim 25°$.

Data also exist for scattering of protons from nuclei at momenta of about 20 GeV/c,[10] in which the final state of the nucleus was not determined. Typically, the energy resolution was $\Delta E \sim 50$ MeV, which is very large indeed on the nuclear scale. The approximations involved in closure should be very good for this type of data (which is the only type one can reasonably expect to gather at high energies). Thus we can use Eq. (2.12) to describe the scattering. In the graphs in Figs. 2–7, we show how the theory compares to experiment from $A=7$ to $A=208$. We also show, for comparison, the result one would obtain if one used Eq. (2.6) with the final state of the nucleus restricted to the ground state. The nuclear radii are taken from form-factor measurements except in the case of lead, where the large neutron excess makes the actual radius of the nucleus slightly larger than the electromagnetic radius. The nuclear radius of lead was picked to match the slope of $d\sigma/dt$. We have assumed that the pp and pn amplitudes are equal, which seems to be good to a few percent at momenta greater than 6 GeV/c.[11]

As expected, theory matches the gross features of the observed scattering data quite well. In addition, the quantitative fit is impressive when it is recalled that

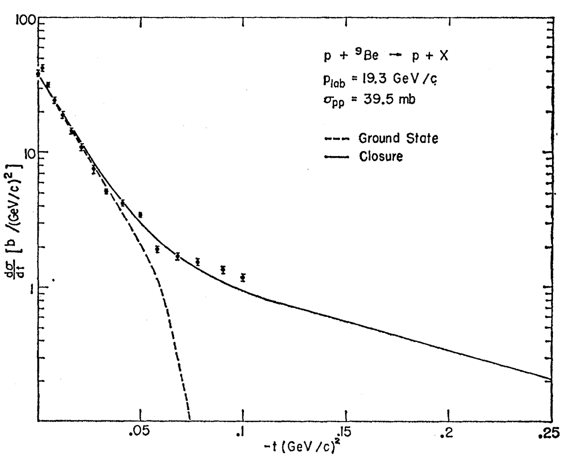

FIG. 3. p-^{9}Be scattering. Data from Ref. 10; $r=2.2$ F.

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[11] W. Rarita, R. J. Riddell, C. B. Chiu, and R. J. N. Phillips, Phys. Rev. **165**, 1615 (1968).

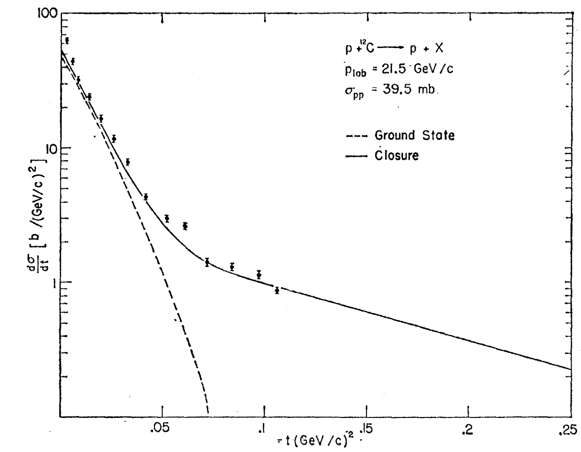

FIG. 4. p-^{12}C scattering. Data from Ref. 10; r=2.41 F.

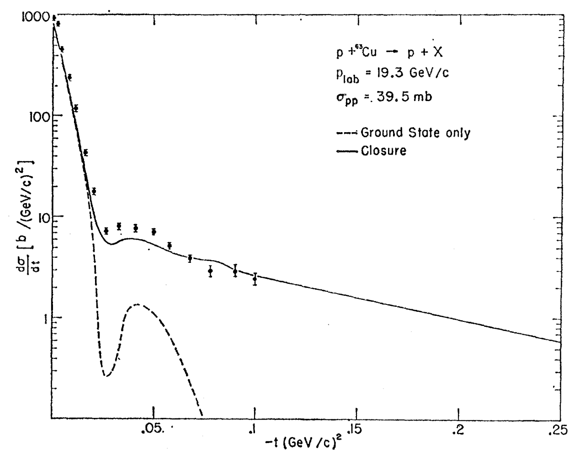

FIG. 6. p-^{63}Cu scattering. Data from Ref. 10; r=4.0 F.

there are *no* adjustable parameters in the theoretical curves. The points at very small t should not be taken seriously, since the Coulomb contributions have not been subtracted from the data. The deviations which still exist can probably be ascribed to deficiencies in the nuclear wave function of Eq. (2.8) (see discussion below). Before commenting on this point, however, let us see if it is possible to understand in a simple way what is going on in the various different regions of momentum transfer.

We shall find it convenient to introduce the following terms: elastic scattering—scattering in which the nucleus remains in its ground state; quasielastic scattering—scattering in which the nucleus retains its identity, but may be left in an excited state; and breakup scattering—scattering in which the nucleus does not retain its identity, but is fragmented in one way or another.

From the graphs, we see that at very small momentum transfer, the cross section is dominated almost completely by elastic scattering. This region is usually interpreted as the "coherent" region, in which the nucleus interacts with the particle as a whole. It also appears that almost any nuclear model which has the correct radius will match the data at small t. Thus the absorption model,[6,7] and the multiple-scattering model with and without closure all give about the same results in this region.

At the other end of the momentum transfer range, in the so-called "incoherent" region, however, the multiple-scattering model without closure and the

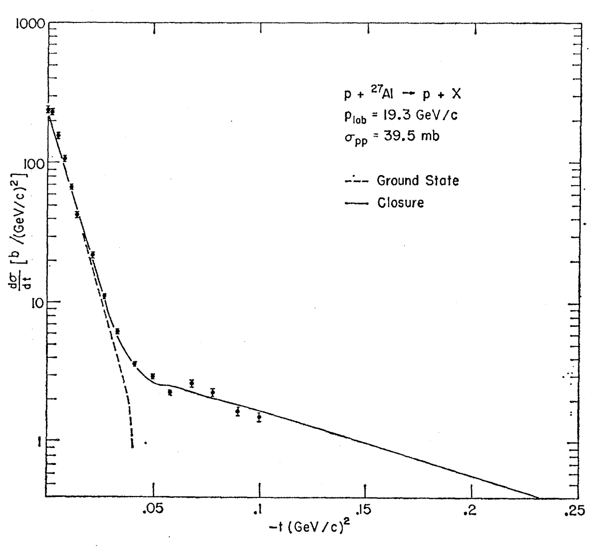

FIG. 5. p-^{27}Al scattering. Data from Ref. 10; r=3.0 F.

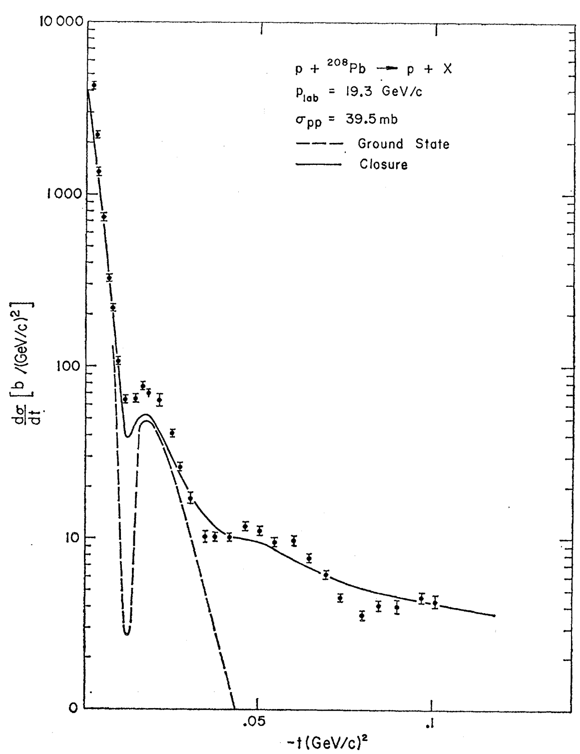

FIG. 7. p-^{208}Pb scattering. Data from Ref. 10; r=5.7 F.

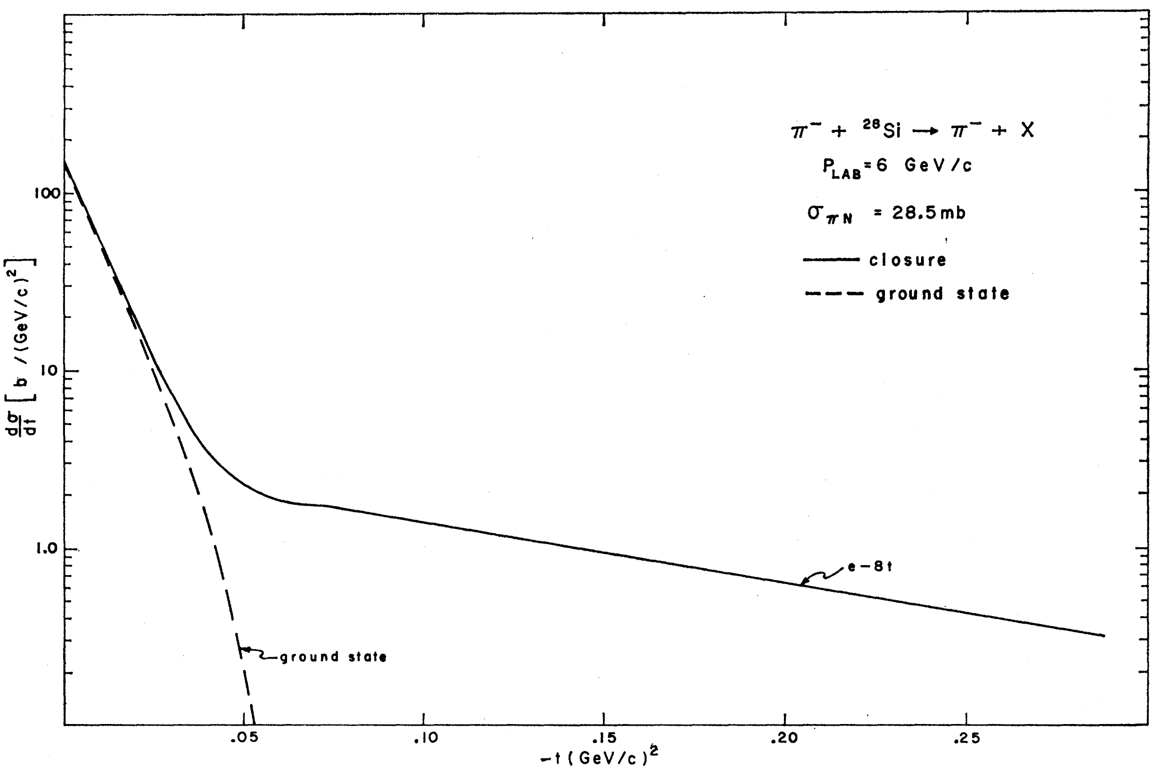

FIG. 8. Prediction for π-^{28}Si scattering. The results when breakup modes are removed should approximate the "ground-state" curve.

absorption model do not match the data at all, while the inclusion of closure results yields a very nice fit indeed. Also, we see that if the data were extended to higher t, we would expect to see a slope similar to that of elementary p-p scattering. In fact, in Table I, we give the slope in the incoherent region from our theory. These are to be compared with $B=10$ (GeV/c)$^{-2}$ which was used in the calculation.

For heavy nuclei, there is an additional t region. I shall call this the transition region, and it is here that structure is seen. Here the multiple-scattering model, while reproducing the gross structure, misses the data by 20–50%.

There is a very simple explanation of the shape of the differential cross section which shows how all of the above results come about, and which suggest a rather simple connection between the absorption model and the multiple-scattering picture. Suppose we write the sum over final states in Eq. (2.6) as

$$\frac{d\sigma}{dt}=\frac{\pi}{p^2}\Big(\sum_{n_Q}\langle\Psi_0|F^\dagger|\Psi_{n_Q}\rangle\langle\Psi_{n_Q}|F|\Psi_0\rangle + \sum_{n_B}\langle\Psi_0|F^\dagger|\Psi_{n_B}\rangle\langle\Psi_{n_B}|F|\Psi_0\rangle\Big), \quad (3.1)$$

where n_Q runs over elastic and quasielastic states, and n_B runs over breakup states. We would expect the latter type of process to become important around $\Delta\sim 200$ MeV/c or $t=0.04$ (GeV/c)^2. It is precisely at this value of momentum transfer that one enters the incoherent region. Thus one is very tempted to identify the sum over n_B in the above with the incoherent cross section.

This explanation also has the advantage that it now becomes obvious why the absorption models have difficulties in this region—they include only the ground state to ground-state transitions, so that when breakup modes become important, they will cease to be a good representation of the process.

This also explains why the closure result matches so well at large and small t, but not in the transition region. At small t, as we have pointed out, any model for the nucleus which has the right radius will match the data. At large t, one can, to a first approximation, think of the nucleus as an uncorrelated Fermi gas from which individual nucleons are ejected by the scattering. In this process, the details of nuclear shape are not important either, but only the radius.

TABLE I. Slopes of incoherent cross section for different nuclei. All results are determined to within ±10%.

A	B (GeV/c)$^{-2}$
7	9.5
9	9.9
12	10.5
27	9.8
64	11.0

In the transition region, however, we are in the range of momentum transfer in which the absorption model or the scattering model without closure start exhibiting "diffraction minima." It is well known that the details of such patterns are extremely sensitive to the details of nuclear structure, and to the influence of quasi-elastic final states. Thus in this region, we would expect our rather simple wave function [Eq. (2.8)] to begin to show serious deficiencies. Since the results in the coherent and incoherent region are not sensitive to such things, but only to the nuclear radius, we are not surprised by the success of our simplified wave function in those regions.

If one had chosen more realistic wave functions for the nuclear ground state, one could expect some improvement in this picture in the transition region. This point is being investigated, as is the question of large t dependence which could conceivably be somewhat more dependent on nuclear shapes than is the diffraction peak.[8] However, we shall see that the most useful application of this work is to the extraction of particle-nucleus cross sections from the dependence of the nuclear production cross sections on A. This sort of result is known to be relatively stable against variations of nuclear densities,[9] so that the degree of accuracy with which our simple wave function reproduces the data is good enough for our purposes at present.

This result is rather encouraging. It means that when we try to extract particle data from nuclear reactions, we can expect the results obtained from the coherent and incoherent regions to be more or less independent

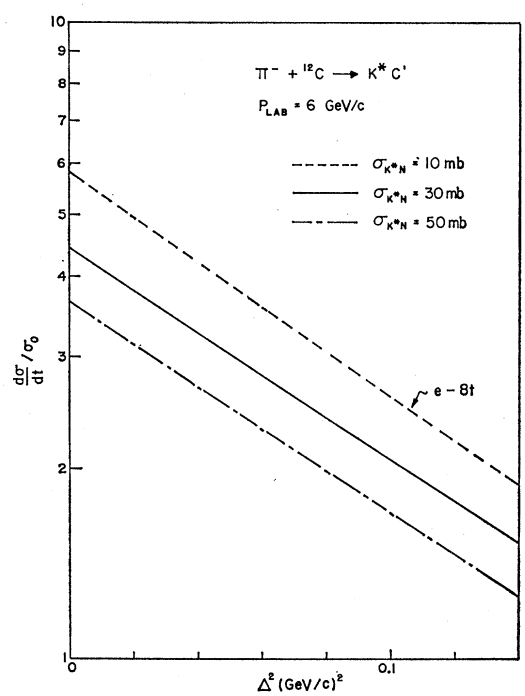

FIG. 9. Differential cross section for the incoherent reaction $\pi+{}^{12}C \rightarrow K^*+X$ for various values of σ_{K^*N}.

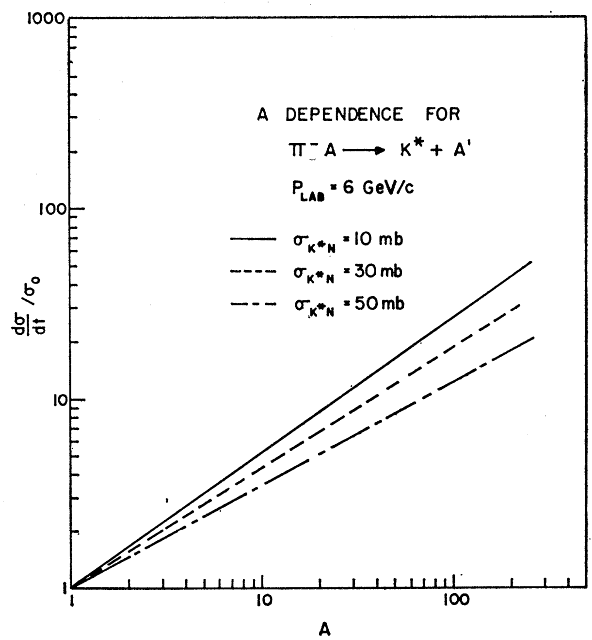

FIG. 10. The A dependence of the production of the K^* in the reaction $\pi^-+A \rightarrow K^*+X$ at $\Delta^2=0$ for various values of σ_{K^*N}.

of the nuclear physics input. On the other hand, the only hope of extracting useful nuclear physics information from low resolution experiments lies in a detailed examination of the transition region, which is the only place where the details of nuclear structure have any effect at all. In this transition region, the relative ease with which one can change the nuclear shape in the absorption model, and the difficulty of doing so in the multiple scattering theory means that for nuclear studies, one should probably use the former rather than the latter for the extraction of information.

It is particularly encouraging that the possibility of testing the above conjecture exists. Suppose we consider the reaction

$$\pi^-+{}^{28}Si \rightarrow \pi^-+{}^{28}Si.$$

This experiment has been done,[12] and it should be possible to separate experimentally those states in which the ^{28}Si retains its identity, and those in which it is broken up. This corresponds to experimentally separating the sums over n_Q and n_B in Eq. (3.1). Since this separation cannot be done theoretically when closure is applied, the above identification of large t process with breakup can only be verified by such an experimental procedure, and we await the experimental results with interest. The theoretical predictions are shown in Fig. 8.

We see, then, that particle-nucleus interactions in which the final state of the nucleus cannot be specified are well described by a multiple-scattering model with closure. We can now turn our attention to the question

[12] H. J. Lubatti (private communication).

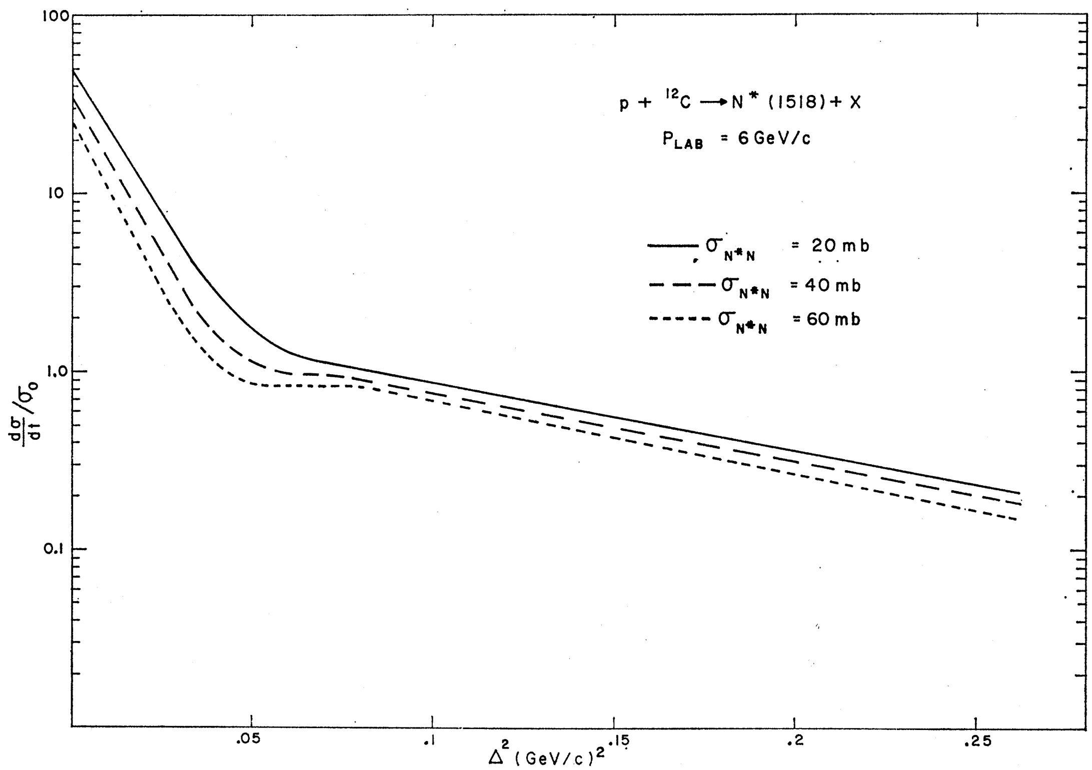

FIG. 11. Differential cross section for the coherent production reaction $p+{}^{12}C \rightarrow N^*(1518)+X$ for various values of σ_{N^*N}.

of how our possession of such a good description can aid us in obtaining new data about elementary particle interactions.

IV. HADROPRODUCTION

The development of the conceptually simple but mathematically complicated multiple-scattering theory for particle-nucleus interactions would have something of the nature of an academic exercise if we looked only at elastic scattering experiments. The chief utility of the method is that we can now apply it to reactions in which resonances are produced inside of the target nucleus, scatter from nucleons inside of the nucleus, and then emerge. We shall consider two types of such reactions. First, reactions like

$$\pi^- + A \rightarrow K^* A' \tag{4.1}$$

and

$$p + A \rightarrow \Sigma + A' \tag{4.2}$$

which are completely incoherent—the final nucleus cannot be the same as the initial one. We shall see that by measuring such reactions, one should be able to deduce the total cross sections σ_{K^*N} and $\sigma_{\Sigma N}$ which would be very interesting to know, since most symmetry schemes make definite predictions about these quantities.

Second, we shall consider coherent production processes like

$$\pi^- + A \rightarrow A_1(1072) + A\,, \tag{4.3}$$

and

$$p + A \rightarrow N^*(1518) + A\,. \tag{4.4}$$

In these reactions, no quantum numbers need be exchanged with the nucleus, so the final nucleus can be the same as the initial.

The determination of σ_{A_1N} and σ_{N^*N} is a particularly interesting exercise because these quantities are of rather fundamental significance for the quark model[13] and other symmetry schemes in high-energy physics. The lowest-lying meson and baryon multiplets of mesons and baryons are generally taken to be made up of quark-antiquark and three quark states, respectively. There are, however, two ways one could build up higher spin multiplets. First, one could put the basic quarks into higher-angular-momentum states and then couple the spins and orbital angular momentum to the spin of the total particle, and, second, one could add a quark-antiquark pair for each unit of spin one wishes to add to the ground state. Call these two schemes L and Q excitation, respectively. In the L excitation scheme, the A_1 mseon would be made up of $(q\bar{q})$, while in the Q excitation scheme, it would be $(qq\bar{q}\bar{q})$. Now if one uses the additivity assumption[14] one easily

[13] R. H. Dalitz, in *High Energy Physics* (Gordon and Breach, Science Publishers, Inc., New York, 1966).

[14] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu, **2**, 105 (1965) [English transl.: Soviet Phys.—JETP Letters **2**, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966); H. J. Lipkin, *ibid.* **16**, 1015 (1966).

finds that for L excitation one expects

$$\sigma_{A_1N} \simeq \sigma_{\pi N} \sim 25 \text{ mb},$$

while for Q excitation one gets

$$\sigma_{A_1N} \simeq 2\sigma_{\pi N} \sim 50 \text{ mb}.$$

Thus, a determination of σ_{A_1N} would tell us rather easily which way one goes about constructing higher-spin resonance states. Of course, exactly similar arguments go through for σ_{N^*N}.

Consider the reactions (4.1) and (4.2). Recalling the discussion of overlap in Sec. II, we see immediately that the only nonzero contribution in Eq. (2.28) will come from the case of type-I overlap. This is the case where the inelastic operator g in F and $g^\dagger$ in $F^\dagger$ act upon the same nucleus. Any other case will give

$$\langle \Psi_0 | F^\dagger F | \Psi_0 \rangle = 0, \tag{4.5}$$

since it will be impossible to come back to the ground state. Thus the cross section for such reactions will be given by Eq. (2.28) with $I_2 \cdots I_5$ set equal to zero. In Fig. 9 we show the results for reaction (4.1) where the nucleus A is taken to be carbon. We note two important things: First, as expected, there is no coherent peak in these reactions, the slope being characteristic of scattering from single nucleons. Second, the magnitude of the cross section depends on the total cross section of the resonance on a nucleon. This is easily understood when we recall that a particle moving through a gas of density D has a mean free path given by

$$\lambda = 1/\sigma_{\text{tot}} D \tag{4.6}$$

so that variations in σ_{K^*N} or $\sigma_{\Sigma N}$ correspond to increasing or decreasing the transparency of the nucleus. In all that follows, we shall find it simplest to define

$$\sigma_0 = |G(1+i\gamma)|^2/16\pi \tag{4.7}$$

where $G(i+\gamma)$ is defined in Eq. (2.25) and (2.28), and deal only with the quantity

$$(d\sigma/dt)/\sigma_0 = H.$$

Since σ_0 is presumably determined from other experiments, it can be put in for any particular reaction to convert our results to cross sections.

Taking a cue from photoproduction, we can then ask what would happen if we did this experiment on a variety of nuclei and asked for the A dependence of H. Clearly, this will also depend on σ_{K^*N} and $\sigma_{\Sigma N}$. In Fig. 10 we show the curves for reaction (4.1) at zero-angle production. Clearly, since the shape of $d\sigma/dt$ does not depend on σ_{K^*N}, the A dependence of H will be independent of t.

Thus, one has two ways of determining σ_{K^*N}. One can either look at an individual nucleus and get the desired cross section from the magnitude of the differential cross section at several different t, or one can look at the A dependence. However, since the normalization of the differential cross section for production from a nucleus is hard to determine experimentally, and since it is not very sensitive to variations in σ_{K^*N}, it will clearly be better to do the experiment on a variety of nuclei and extract the desired cross section from the A dependence than to try to get it from the data for production on a single nucleus.

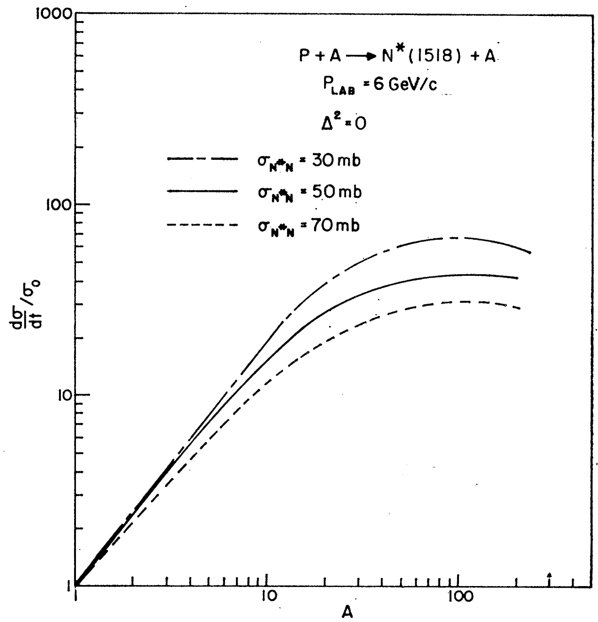

FIG. 12. A dependence for the production of $N^*(1518)$ in the reaction $p+A \to N^*+X$ at $\Delta^2=0$ for various choices of σ_{K^*N}.

We also note one other point from Fig. 10. The curves are normalized to the production cross section from hydrogen. Since this is normally a well-determined quantity, it is useful to be able to include it in the results. Also, including hydrogen increases by a decade the range of A over which the comparison between theory and experiment is to be made,[9] so that better determinations of the cross sections should be possible.

If we now turn our attention to reaction (4.4), we are confronted with a rather different situation. Here, there can be coherent production, so that one would expect that at small t both the multiple scattering picture and the absorption model would give similar results. We also see that Eq. (2.28) can be applied as it stands to these reactions.

If we simply go ahead and do the calculation, we find that the results for production from carbon are those shown in Fig. 11. We see that, as expected, the shape of the curves closely approximates those of elastic scattering. The coherent and incoherent regions are easy to define. Again, one could hope to determine σ_{N^*N} from either the magnitude of the cross section from a single nucleus or from the A dependence for any given t (shown in Fig. 12).

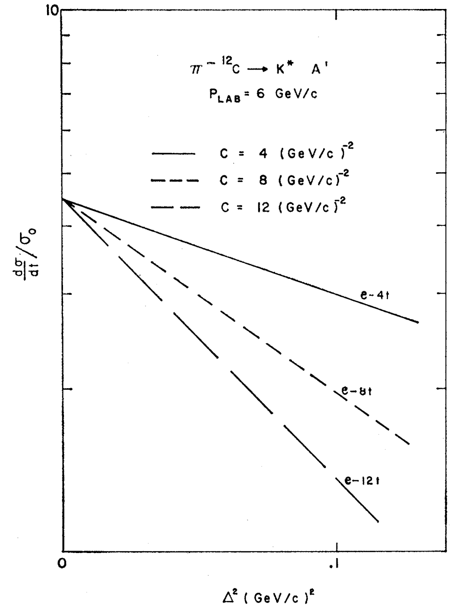

FIG. 13. The dependence of the differential cross section for the incoherent production reaction $\pi+{}^{12}C \rightarrow K^{*}+X$ on the parameter C in Eq. (2.25). The slope of the differential cross section from ${}^{12}C$ is seen to be equal to the slope for production of an individual nucleon. The dependence on the parameter B in Eq. (2.23) is insignificant.

Before leaving hadroproduction processes, we can say a few words about the use of "coherence" effects to determine production amplitudes from single nucleons. Consider reaction (4.1) as an example. Suppose that for some reason the reaction

$$\pi p \rightarrow K^{*}\Lambda \tag{4.8}$$

were too small to be measured directly, but that for some reason we wanted to have an estimate of its size. If we looked at production on several nuclei, we could determine $\sigma_{K^{*}N}$. Then from the value of $d\sigma/dt$ for a given nucleus, we could determine σ_0 and hence $|G(1+i\gamma)|^2$. In Fig. 13 we show the variation of the t dependence of the differential cross section for the above process with C. (The effect of variation of B is totally negligible.) Thus, since from Eq. (2.25) we have

$$(d\sigma/dt)(\pi p \rightarrow K^{*}\Lambda) = [|G(i+\gamma)|^2/16\pi]e^{-Ct},$$

we can determine all of the relevant parameters in reaction (4.8) from nuclear production.

This is not brought up to encourage experimenters to abandon hydrogen targets, but simply to point out that the possibility exists as a last resort. It might be, for example, that at very high energies, cross sections which fall rapidly with s may become too small to measure directly, so that such a technique might be needed.

V. CONCLUSION

It should be clear by now that the technique of nuclear rescattering is potentially a very useful one in particle physics. In order to use it properly, we have tried to show that the ordinary multiple-scattering theory gives a comprehensive and natural treatment of all particle-nucleus interactions, for all nuclei, and for all momentum transfer regions of interest. In addition, we have tried to show how the particle results which one obtains in this way are rather insensitive to the nuclear physics input except in what we have called the transition region. We have also tried to show how this theory "contains" the absorption-model results, and have tried to identify the absorption model with reactions in which it is reasonable to suppose that all or almost all of the cross section comes from processes in which the nucleus remains unexcited by the reaction. The success of even the simplest theoretical models[9] in deriving the total cross sections for ρ^0-N and ϕ^0-N scattering are examples of how nuclear rescattering can be used. The impact of these measurements on the quark model also illustrates how useful such information can be.

It has often been suggested that the model which has been presented is oversimplified—that many nuclear and particle effects have been left out. This is true. The successes of the multiple-scattering model in handling elastic scattering, however, lead one to suspect that terms which have been left out (spin and nuclear correlations, for example) cannot play all that great a role. In any case, we have presented here a host of theoretical predictions for various production reactions, and, indeed, there is hope that some of them may be checked soon.[12] If the theory does not match the new data well, then it will be time to examine our approximations in more detail. What is clear, however, is the fact that a theory of particle-nucleus reactions is needed in order to extract data from nuclear rescattering experiments, that such data are useful, and that there is no other way known at the present time to obtain such data.

ACKNOWLEDGMENTS

It is a pleasure for the author to thank Dr. A. Dar, Dr. A. Gal, Professor K. Gottfried, and Dr. J. Hufner for useful and illuminating discussions.

APPENDIX

In this section, we shall define the I_j, which appears in Eq. (2.28). Let us begin by defining

$$\xi_1(M,N) = \frac{L-1}{4}\left(\frac{1}{R^2+B}+\frac{1}{B}\right)+\frac{N-L}{R^2+2B} + \frac{1}{4}\left(\frac{1}{R^2+C}+\frac{1}{C}\right),$$

$$\xi_2(M,N)=\frac{L-1}{4}\left(\frac{1}{R^2+B}+\frac{1}{B}\right)+\frac{N-L-1}{R^2+2B}+\frac{1}{R^2+2C}+\frac{\beta^2}{[R^2+2C/(1+\beta)](1+\beta)^2},$$

$$\xi_3(M,N)=\xi_2(N,M),$$

$$\xi_4(M,N)=\frac{N-L}{R^2+2B}+\frac{L-2}{4}\left(\frac{1}{R^2+B}+\frac{1}{B}\right)+\frac{1}{C(1+\beta)}+\frac{1+\beta^2}{[R^2+2C/(1+\beta)](1-\beta)},$$

$$\xi_5(M,N)=\frac{N-L-1}{R^2+2B}+\frac{1}{R^2+2C}+\frac{L}{4}\left(\frac{1}{R^2+B}+\frac{1}{B}\right),$$

and then define

$$\lambda_1=\frac{L-1}{4}\left(\frac{1}{R^2+B}-\frac{1}{B}\right)+\frac{1}{4}\left(\frac{1}{R^2+C}-\frac{1}{C}\right),$$

$$\lambda_2=\lambda_3=\frac{L-1}{4}\left(\frac{1}{R^2+B}-\frac{1}{B}\right)+\frac{\beta}{[R^2+2C/(1+\beta)](1+\beta)^2}-\frac{1}{2C(1+\beta)},$$

$$\lambda_4=\frac{L-2}{4}\left(\frac{1}{R^2+B}-\frac{1}{B}\right)-\frac{1}{C(1+\beta)}+\frac{2\beta}{[R^2+2C/(1+\beta)](1+\beta)^2},$$

$$\lambda_5=\frac{L}{4}\left(\frac{1}{R^2+B}-\frac{1}{B}\right),$$

and also

$$\begin{aligned}\eta_1(M,N)&=\xi_1(N,M),\\ \eta_2(M,N)&=\xi_3(M,N),\\ \eta_3(M,N)&=\xi_2(M,N),\\ \eta_4(M,N)&=\xi_4(N,M),\\ \eta_5(M,N)&=\xi_5(N,M),\end{aligned}$$

where M, N, and L are the running indices in Eq. (2.28), and the subscripts $1\cdots 5$ refer to the quantities $I_1\cdots I_5$ which we are defining.

We can then define (for each i)

$$\begin{aligned}\gamma_i&=(2\eta_i+\lambda_i)/(2\xi_i+\lambda_i),\\ S_i&=\xi_i\gamma_i^2+\eta_i-2\lambda_i\gamma_i,\\ T_i&=\xi_i+\eta_i+2\lambda_i.\end{aligned}$$

In terms of these variables, we then find

$$I_j(\Delta^2)=(1+\gamma_j)^2e^{-\Delta^2(1+\gamma_j)^2/4S_j}/S_jT_j,$$

where Δ^2 is the transverse momentum transfer to the nucleus.