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Study of the ^{90}Zr(d, p)^{91}Zr Reaction*

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Measurements have been made of the differential cross section and polarization of protons from the ground state ($l_n=2$) and the 1.2-MeV state ($l_n=0$) in the ^{90}Zr(d, p)^{91}Zr reaction using 11-MeV deuterons. Measurements were also made of the elastic scattering cross section for 11-MeV deuterons and of the elastic scattering cross section and polarization for 14.8- and 16.0-MeV protons. An optical-model analysis was made of the elastic scattering results, and the potentials obtained were used in a distorted-wave Born approximation (DWBA) analysis of the (d, p) experiment. Except at forward angles, the DWBA calculation does not follow the trend of the polarization data for the protons from the 1.2-MeV state, where only the spin-orbit interactions contribute to the polarization. The polarization of the protons from the ground state was small at forward angles and could not be described by the DWBA analysis using the same geometrical parameters for the deuteron spin-orbit and real-central potentials.

I. INTRODUCTION

STUDIES of (d, p) reactions have been used in the past to obtain information about the spins, parities, and widths of the nuclear states involved. The most detailed description of these reactions has been the distorted-wave Born approximation (DWBA) treatment with the inclusion of spin-orbit terms in the proton and deuteron optical potentials. In many instances, this approach has been highly successful in describing the experimental cross-section data for (d, p) reactions. This approach has not been so successful in reproducing polarization results.

A treatment in which the incoming deuterons and outgoing protons are approximated by plane waves leads to zero polarization. Polarization of the protons in a (d, p) reaction may arise from the difference in the absorption of the deuterons and protons in nuclear matter. This effect was first suggested by Newns,[1] who showed that the sign of the polarization of the protons emitted at angles corresponding to the main stripping peak should be given by $P=\pm$ for $j_n=l_n\pm\frac{1}{2}$ if deuteron absorption were predominant and by $P=\mp$ for $j_n=l_n\pm\frac{1}{2}$ if proton absorption were predominant. Here, j_n is the angular momentum transfer to the nucleus, and l_n is the orbital angular momentum transfer. The direction of positive polarization is taken to be $\mathbf{n}=\mathbf{k}_d\times\mathbf{k}_p$. This model also predicts[2] that the maximum value of the proton polarization will depend on j_n and l_n, but in every case $|P|\leq\frac{1}{3}$, and $P=0$ for $l_n=0$.

Several experiments[3] show polarization near the main stripping peak with some features characteristic of the simple absorption model in which the deuteron absorption predominates. However, the spin-orbit interactions in the deuteron and proton channels appear to be very important as evidenced by the observation of nonzero polarization[4] in cases where $l_n=0$ and by values[5] of proton polarization in excess of $\frac{1}{3}$.

Recently, DWBA calculations with optical spin-orbit terms have been made for the ^{40}Ca, ^{58}Ni, and ^{88}Sr stripping reactions,[6,7] but do not reproduce the data well. A DWBA analysis has, however, been able to reproduce the general features of the proton asymmetry in the region of the main stripping peak as measured by Yule and Haeberli[8] with polarized deuterons.

There have been several recent attempts made either to include certain previously neglected corrections in the stripping calculations or to recalculate the distributions using new approximations. Butler *et al.*[9] proposed a theory for deuteron stripping reactions in which only proton-nucleus and neutron-nucleus optical potentials are used. May and Truelove[10] have used this theory to calculate proton polarization. Their calculation gives better agreement with the ^{40}Ca(d, p)^{41}Ca reaction than the DWBA analysis. As with the DWBA analysis, this model predicts larger polarizations than are observed experimentally for the ^{88}Sr(d, p)^{89}Sr (1.05 MeV) reaction. Other calculations[11,12] using the weakly bound projectile model have predicted a relationship between the polarizations when unpolarized protons scatter elastically with an energy corresponding to the energy of the protons emitted from the stripping reaction and the polarization of the protons from the

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stripping process. A simple diffraction-model calculation performed by Walls[13] for $l_n=0$ reactions has been successful in describing the $^{88}Sr(d, p)^{89}Sr$ (1.05 MeV) reaction.

Johnson and Santos[14] have included the d state of the deuteron in their calculations of stripping and pickup reactions and found a substantial correction to the calculated angular distributions, especially for large l_n transfers. It is possible that even larger corrections will be made to the proton polarizations through the addition of this component. Other attempts[15] have been made to improve the stripping calculations by a treatment of the deuteron distortion in the neighborhood of the nucleus.

In the experiment reported here, 11-MeV deuterons have been used to study the $^{90}Zr(d, p)^{91}Zr$ reaction leading to the ground state ($2d_{5/2}$) with $l_n=2$ and the 1.2-MeV state ($3s_{1/2}$), with $l_n=0$. Differential cross-section and polarization measurements have been made for these two outgoing proton groups. For the

TABLE I. $^{90}Zr(d, d)^{91}Zr$ differential cross sections for $E_d=11.0$ MeV.

θ_{cm} (deg)	$d\sigma/d\Omega$ (mb/sr)	$d\sigma/d\sigma_R$
20.7	16 300	0.937
25.8	6 820	0.975
30.9	2 930	0.825
36.0	1 152	0.586
42.1	614	0.564
47.2	379	0.541
52.3	245	0.528
57.3	140	0.414
62.4	86.8	0.343
67.5	52.4	0.276
72.5	40.8	0.276
77.5	30.7	0.262
81.6	25.0	0.255
86.3	21.7	0.264
91.3	19.2	0.280
96.3	15.6	0.269
101.3	12.1	0.242
105.0	8.85	0.197
109.9	5.89	0.148
114.8	4.42	0.124
119.8	3.24	0.102
124.7	2.90	0.098
129.6	2.91	0.108
134.6	3.65	0.147
139.5	3.38	0.146
144.4	2.63	0.120
149.3	2.07	0.100
154.2	1.63	0.082
159.1	1.31	0.068

TABLE II. $^{91}Zr(p, p)^{91}Zr$ differential cross sections and polarizations for $E_p=16.0$ MeV.

θ_{cm} (deg)	$d\sigma/d\Omega$ (mb/sr)	P
20.5	7460	
25.6	3460	
30.6	1760	
35.7	858	
40.7	365	-0.005 ± 0.042
45.7	141	-0.061 ± 0.035
50.8	64.5	-0.052 ± 0.035
55.8	50.7	$+0.162\pm0.046$
60.8	57.1	$+0.238\pm0.052$
65.9	62.2	$+0.187\pm0.049$
70.9	58.5	$+0.159\pm0.045$
75.9	48.3	-0.035 ± 0.039
80.9	34.1	-0.198 ± 0.055
85.6	20.4	-0.320 ± 0.059
90.6	10.4	-0.634 ± 0.065
95.6	5.06	-0.506 ± 0.063
100.3	3.5	$+0.336\pm0.085$
105.3	4.66	$+0.631\pm0.096$
110.3	6.75	$+0.472\pm0.061$
115.3	8.4	$+0.282\pm0.059$
120.2	8.9	$+0.074\pm0.051$
125.2	7.91	-0.130 ± 0.050
130.2	6.16	-0.300 ± 0.058
135.1	4.46	-0.345 ± 0.067
140.1	2.55	
145.1	1.55	
150.0	1.34	
155.9	1.87	
159.9	2.90	
164.9	4.04	

[13] D. F. Walls, Nucl. Phys. **A90**, 353 (1967).

[14] R. C. Johnson and F. D. Santos, Phys. Rev. Letters **19**, 364 (1967).

[15] G. H. Rawitscher, Phys. Rev. **163**, 1223 (1967).

TABLE III. $^{91}Zr(p, p)^{91}Zr$ differential cross sections and polarizations for $E_p = 14.8$ MeV.

θ_{cm} (deg)	$d\sigma/d\Omega$ (mb/sr)	P
30.32	2071	
35.36	1009	
40.41	448	-0.020 ± 0.032
45.44	173	-0.034 ± 0.035
50.48	74.6	$+0.004\pm0.037$
55.52	51.9	$+0.105\pm0.038$
60.55	56.8	$+0.250\pm0.037$
65.57	62.2	$+0.205\pm0.036$
70.59	61.2	$+0.138\pm0.035$
75.61	52.7	$+0.027\pm0.036$
80.62	38.5	-0.045 ± 0.037
85.63	24.8	-0.255 ± 0.040
90.63	14.4	-0.332 ± 0.044
95.63	6.82	-0.452 ± 0.045
100.63	3.68	-0.107 ± 0.054
105.61	3.92	$+0.388\pm0.051$
110.59	5.47	$+0.427\pm0.046$
115.57	7.21	$+0.325\pm0.044$
120.55	8.74	$+0.139\pm0.042$
128.52	8.88	$+0.046\pm0.042$
130.48	7.46	-0.038 ± 0.044
135.45	6.11	-0.209 ± 0.047
140.41	4.25	-0.366 ± 0.051
145.36	2.83	
150.32	1.87	
155.27	1.56	
160.22	1.98	
165.16	2.65	
170.11	3.14	

1.2-MeV state, only the spin-orbit terms in the optical potential contribute to the proton polarization, while for the ground state both the spin-orbit and central potentials contribute to the proton polarization. Since neither ^{90}Zr nor ^{91}Zr are deformed or vibrational, a perturbation treatment is appropriate. Also, since these nuclei are moderately heavy, it was believed that the optical model would be able to reproduce the elastic scattering data and thus provide good parameters for the DWBA analysis. The DWBA analysis was made using optical-model potentials derived from our measurements of the deuteron elastic scattering cross section at 11 MeV and the proton elastic scattering cross section and polarization at 16.0 and 14.8 MeV. The analysis of this extensive set of data provides an appropriate test of the DWBA theory without relying on parameters interpolated from other analysis of data taken with neighboring nuclei or at different energies.

II. EXPERIMENTAL METHODS AND RESULTS

The Rutgers-Bell tandem Van de Graaff accelerator was used to produce the proton and deuteron beams

TABLE IV. $^{90}Zr(d, p)^{91}Zr$ (g.s.) differential cross sections and polarizations for $E_d = 11.0$ MeV.

θ_{cm} (deg)	$d\sigma/d\Omega$ (mb/sr)	P
9.9	1.90	
14.8	5.28	
19.8	7.41	
24.7	7.26	
29.6	5.71	
34.6	3.87	
39.5	3.04	-0.007 ± 0.080
44.5	3.50	-0.014 ± 0.056
49.4	4.21	-0.034 ± 0.056
54.4	4.26	$+0.032\pm0.046$
59.4	3.38	-0.011 ± 0.056
61.8		$+0.077\pm0.062$
64.3	2.20	$+0.027\pm0.060$
69.3	1.26	-0.018 ± 0.066
74.3	0.88	-0.007 ± 0.083
79.3	0.90	$+0.046\pm0.077$
84.3	1.07	$+0.055\pm0.083$
89.3	1.21	$+0.075\pm0.072$
94.3	1.22	$+0.030\pm0.066$
99.3	1.04	$+0.056\pm0.055$
104.3	0.795	
109.3	0.528	-0.152 ± 0.073
114.3	0.390	
119.4	0.283	-0.143 ± 0.085
124.4	0.296	
129.4	0.331	$+0.128\pm0.066$
134.5	0.374	
139.5	0.380	
144.6	0.366	
149.6	0.328	
154.7	0.265	
159.8	0.197	

used in this experiment. Detectors for the cross-section measurements and the polarimeter for the polarization measurements were attached to a rotating plate in a 15-in.-diam scattering chamber which is a modification of a Florida State University design.[16] The scattering angles could be set to an accuracy of $\pm0.10°$ and could be viewed remotely with a television camera.

TABLE V. $^{90}Zr(d, p)^{91}Zr^*$ (1.2 MeV) differential cross sections and polarizations for $E_d = 11.0$ MeV.

θ_{cm} (deg)	$d\sigma/d\Omega$ (mb/sr)	P
9.9	4.75	
14.8	2.57	
19.8	1.47	
24.7	4.07	
29.6	6.67	
34.6	6.85	
39.5	4.84	$+0.111\pm0.080$
44.5	2.70	-0.024 ± 0.082
49.4	1.29	-0.145 ± 0.090
54.4	1.01	-0.135 ± 0.084
59.4	1.31	-0.108 ± 0.094
61.8		$+0.068\pm0.088$
64.3	1.59	$+0.020\pm0.086$
69.3	1.65	-0.050 ± 0.083
74.3	1.45	-0.104 ± 0.080
79.2	1.06	-0.214 ± 0.090
84.2	0.691	-0.110 ± 0.100
89.2	0.449	-0.300 ± 0.115
94.2	0.348	
99.2	0.309	-0.014 ± 0.093
104.3	0.355	
109.3	0.363	$+0.068\pm0.101$
114.3	0.371	
119.4	0.334	-0.110 ± 0.099
124.4	0.283	
129.4	0.228	-0.130 ± 0.097
134.5	0.180	
139.5	0.143	
144.6	0.121	
149.6	1.25	
154.7	0.135	
159.8	0.138	

[16] E. J. Feldl, P. B. Weiss, and R. A. Davis, Nucl. Instr. Methods **28**, 309 (1964).

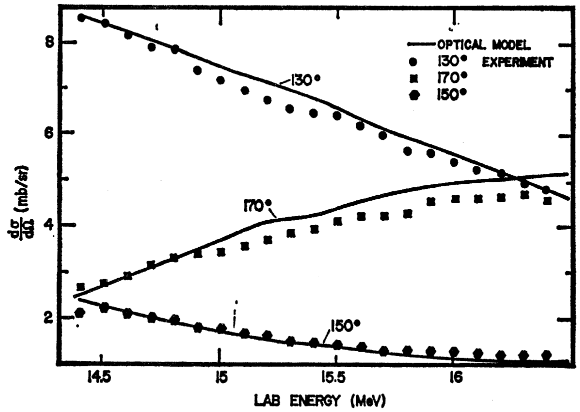

FIG. 1. $^{90}Zr(d, p)^{91}Zr$ experimental and theoretical cross sections at 130°, 150°, and 170° over the energy range of 14.4–16.4 MeV.

A. Cross-Section Measurements

For the proton cross-section measurements, 2-mm Li-drifted silicon detectors were used. For the deuteron cross-section measurements, 0.1-mm detectors were used in coincidence with the 2-mm detectors. The acceptance angle for the cross-section measurement was $\pm1.3°$ as defined by a collimator and antiscattering baffle before the detector. Measurements were made simultaneously with detectors at two or three angles. The pulses from the detectors were preamplified, and after further amplification and shaping were analyzed by an analog-to-digital converter[17] and accumulated in the memory of an SDS 910 [18] computer. After each run, the data were stored on magnetic tape. Data reduction was performed in an SDS 925 computer. The peaks were well resolved and were summed using light-pen routines.[19] Absolute normalization was obtained by measuring the elastic scattering yield at forward angles at 5 MeV and equating it with the known Rutherford cross sections.

The angular distribution of elastically scattered deuterons was obtained at 11 MeV using a 2.4-mg/cm^2 target of 99% enriched ^{90}Zr (see Table I). The angular distributions of elastically scattered protons at 16.0 and 14.8 MeV were obtained using a 6.2-mg/cm^2 target of 90% enriched ^{91}Zr (see Tables II and III). These energies correspond to the energies of the outgoing protons from the ground state and 1.2-MeV state, respectively, in the $^{90}Zr(d, p)^{91}Zr$ reaction carried out with 11-MeV deuterons. Angular distributions for the ground-state and 1.2-MeV state protons in the $^{90}Zr(d, p)^{91}Zr$ reaction were obtained at 11 MeV using the same 2.4-mg/cm^2 ^{90}Zr target used in the deuteron elastic scattering experiments (see Tables IV and V).

[17] Designed by G. L. Miller, Bell Telephone Laboratories.
[18] Scientific Data Systems, Santa Monica, California.
[19] R. Van Bree, Rutgers University, New Brunswick, N.J.

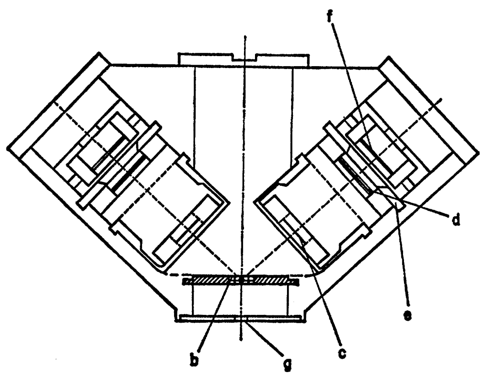

FIG. 2. The carbon-foil polarimeter: g, e collimators; b, carbon foil; c, d, and f, detectors.

Aluminum foils were used to stop elastically scattered deuterons at forward angles. A relative error of 3% and an absolute error of 8% is assigned for the cross-section measurements.

In order to investigate the possibility of resonances in this energy range, the $^{91}Zr(p, p)^{91}Zr$ cross sections at 130°, 150°, and 170° were measured in 100-keV steps from 14.4 to 16.4 MeV. These steps are small enough to detect any resonance phenomena which would affect this experiment, since the targets used were at least 100 keV thick. The results of this excitation function are shown in Fig. 1. The solid lines are theoretical calculations using the optical parameters obtained by fitting an angular distribution at 14.8 MeV. These parameters are discussed later.

B. Polarization

Polarization measurements were made with a carbon-foil polarimeter which had a mean second-scattering angle of 47° and a solid angle of 0.058 sr corresponding to particles scattered from 39° to 55°. The solid angle for the first scattering was 0.0052 sr with an angular spread of ±1.8°. The polarimeter is shown in Fig. 2.

A 14-mg/cm² carbon foil, with an energy loss of about 0.5 MeV for 15-MeV protons, was used in the polarimeter for the (d, p) polarization measurement. At angles larger than 100° a 28-mg/cm² carbon foil was used. The double-scattered protons were detected in two counter telescopes, each consisting of three silicon surface-barrier detectors, 400, 400, and 500 μ in thickness. The last detector had an active area of 300 mm².

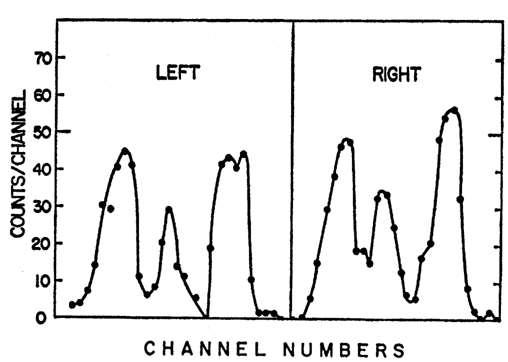

FIG. 3. (d, p) polarization spectrum at 50° with $E_d = 11$ MeV.

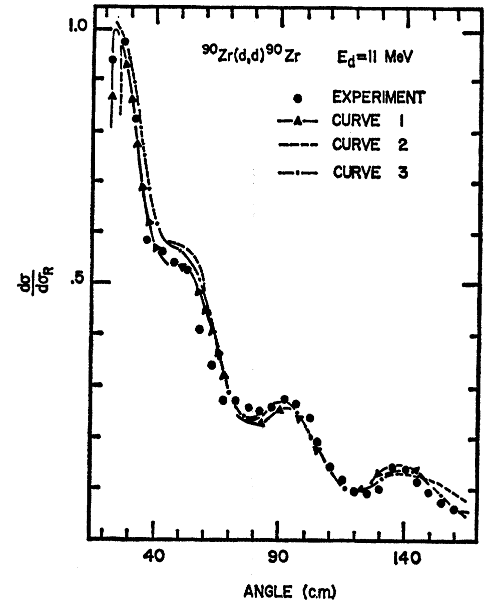

FIG. 4. $^{90}Zr(d, d)^{90}Zr$ experimental cross sections and optical-model fit at $E_d = 11$ MeV.

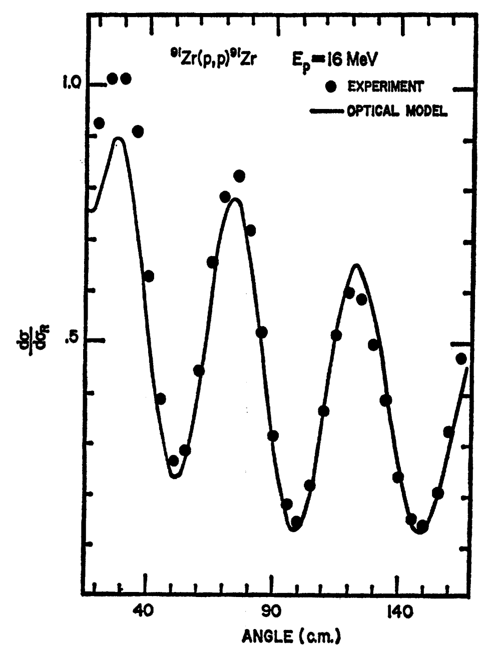

FIG. 5. $^{91}Zr(p, p)^{91}Zr$ experimental cross sections and optical-model fit at $E_p = 16$ MeV.

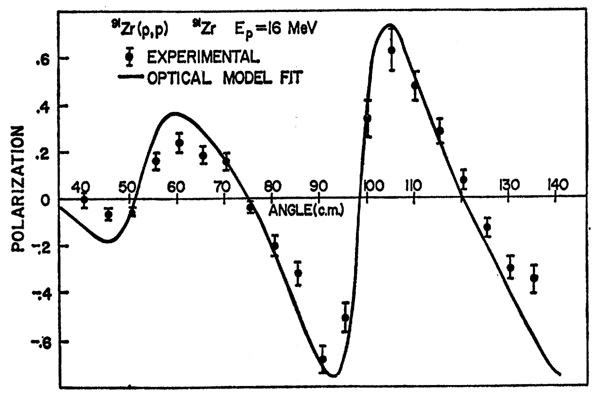

FIG. 6. ^{91}Zr(p, p)^{91}Zr experimental polarizations and optical-model fit at $E_p=16$ MeV.

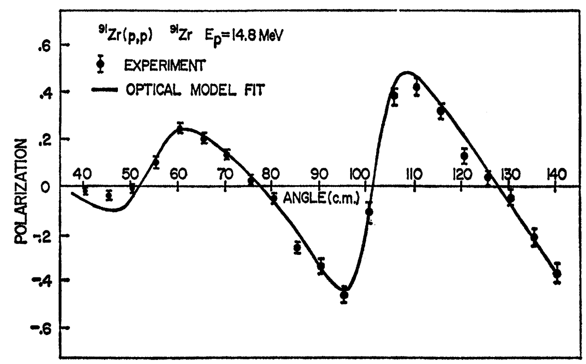

FIG. 8. ^{91}Zr(p, p)^{91}Zr experimental polarizations and optical-model fit at $E_p=14.8$ MeV.

For the proton elastic scattering polarization measurements, an 84-mg/cm^2 carbon foil was used. In some of the early elastic scattering measurements 2-mm lithium-drifted detectors were used.

Pulses from the three detectors were summed and analyzed by the analog-to-digital converter, which was gated by a fast-slow coincidence. The results were accumulated by the computer and stored on magnetic tape as in the cross-section measurements.

The proton polarization P was obtained from the asymmetry ϵ

$$\epsilon=(N_L-N_R)/(N_L+N_R)=P_1\bar{P}_2,$$

where $\bar{P}_2$ is the average analyzing power of the polarimeter, and N_L, N_R are the number of protons scattered to the left and right, respectively, in the polarimeter when the first scattering is to the left. Asymmetry measurements were made by placing the polarimeter to the left and right of the incident beam. Another set of two-asymmetry measurements was made with the target rotated through 180°. The average of the four-asymmetry values was taken in order to minimize false asymmetries due to the polarimeter, electronics, or target. The major asymmetry not corrected for with these sets of measurements was the effect of the angular distribution of protons nonuniformly illuminating the second scatterer. This effect was estimated to be less than 2% at all angles.

The analyzing power of the polarimeter $\bar{P}_2$ is the average polarization one measures when an unpolarized proton beam is scattered from the carbon second

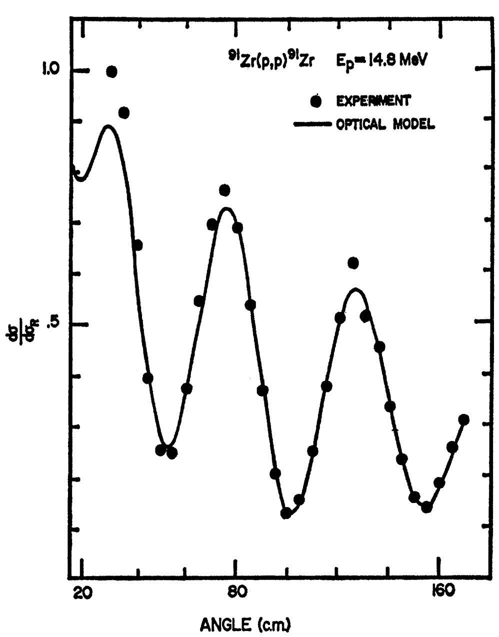

FIG. 7. ^{91}Zr(p, p)^{91}Zr experimental cross sections and optical-model fit at $E_p=14.8$ MeV.

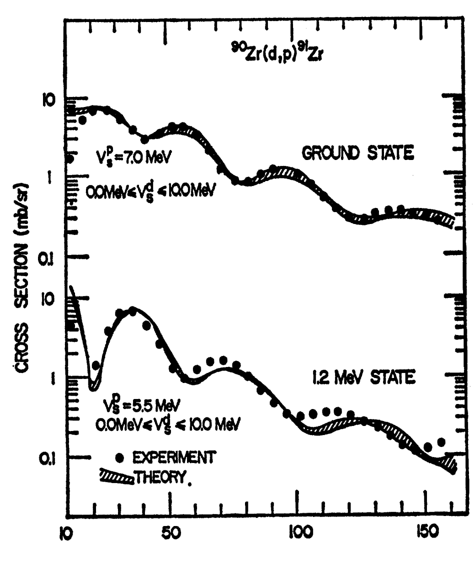

FIG. 9. ^{90}Zr(d, p)^{91}Zr (g.s. and 1.2-MeV state) experimental cross sections compared with the theoretical DWBA calculation upon varying the deuteron spin-orbit potential from 0 to 10 MeV.

TABLE VI. Optical-model parameters.

Parameter set	Nucleus	Energy (MeV)	V (MeV)	r_0 (F)	W (MeV)	r_0' (F)	V_s (MeV)	a (F)	χ^2/N
(a)	$^{90}Zr+d$	11	81.13	1.3273	19.91	1.1862	0	0.65	26
(b)	$^{90}Zr+d$	11	94.71	1.2352	21.17	1.0459	7.2	0.65	25
(c)	$^{90}Zr+d$	11	94.53	1.2350	23.10	1.0460	3.86	0.65	23
(d)	$^{91}Zr+p$	16	55.63	1.190	7.73	1.310	7.05	0.65	32
(e)	$^{91}Zr+p$	14.8	52.73	1.224	9.35	1.254	5.5	0.65	2

scatterer for the range of energies and angles over which protons are detected. The analyzing power is dependent on the incident proton energy. The polarimeter was calibrated between 5.7 and 8.0 MeV with protons scattered from a 1.6-mg/cm² carbon target at 50° in the first scattering and using measured polarization values of Moss and Haeberli.[20] Using aluminum foils to degrade the energy to this range, proton polarization from carbon at 50° was measured between 13.6 and 15.7 MeV, and these values were used to calibrate the polarimeter in the energy range used in this experiment. Within experimental error, the single value of $\bar{P}_2=-0.56\pm0.04$ was found to hold for the carbon foils and range of energies used in this experiment. This value agrees well with that obtained by numerical integration of known carbon polarization and cross-section data.

Polarization measurements were made for elastically scattered protons of 16.0 and 14.8 MeV using the 6.2-mg/cm² target of ^{91}Zr (see Tables II and III). The polarization measurements for the ground-state and 1.2-MeV-state protons in the $^{90}Zr(d, p)^{91}Zr$ reaction were made at 11 MeV using an 8.0-mg/cm² target. A set of left-right spectra at 50° for one run is shown in Fig. 3. Because of resolution and statistics, it is difficult to find the actual channels corresponding to the maxima and half-width of the peaks of interest. They were obtained by summing the four spectra and making a three-peak Gaussian fit to the data (see Tables IV and V).

III. ANALYSIS OF RESULTS

A. Optical Model

In order to determine the optical-model potentials to be used in the DWBA calculations for the (d, p) results, an analysis was made of the deuteron and proton elastic scattering data using optical-model Hunter Code,[21] in which the optical potential is given by

$$V(r)=-Vf(r)-iWg(r)-V_s h(r)\,\boldsymbol{\sigma}\cdot\mathbf{l}+V_c(r),$$

where

$$f(r)=\{1+\exp[(r-R)/a]\}^{-1},$$
$$g(r)=-4a(d/dr)\{1+\exp[(r-R')/a]\}^{-1},$$
$$h(r)=-(\hbar/m_\pi c)^2(1/r)(d/dr)\{1+\exp[(r-R)/a]\},$$
$$V_c=(Ze^2/2R)(3-r^2/R^2),\quad \text{for } r<R$$
$$=Ze^2/r,\quad \text{for } r>R$$

and

$$R=r_0A^{1/3},\quad\text{and}\quad R'=r_0'A^{1/3}.$$

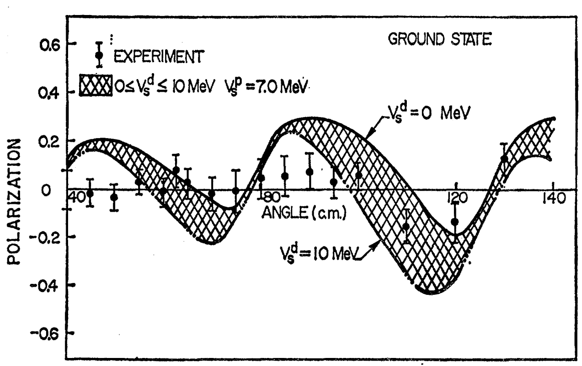

FIG. 10. $^{90}Zr(d, p)^{91}Zr$ (g.s.) experimental polarizations compared with the DWBA predictions allowing the deuteron spin-orbit potential to vary between 0 and 10 MeV.

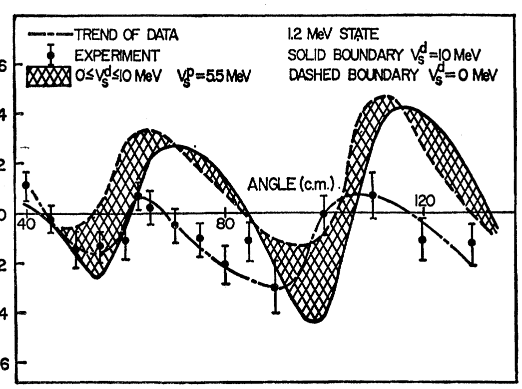

FIG. 11. $^{90}Zr(d, p)^{91}Zr^*$ (1.2 MeV) experimental polarizations compared with the DWBA predictions allowing the deuteron spin-orbit potential to vary between 0 and 10 MeV.

[20] S. J. Moss and W. Haeberli, Nucl. Phys. **72**, 417 (1965).

[21] R. M. Drisco, Hunter Code (unpublished).

For protons, $\boldsymbol{\sigma}$ is the Pauli spin-$\frac{1}{2}$ operator and for deuterons $\boldsymbol{\sigma}$ is the spin-1 operator. A search routine was used to vary the parameters of this potential to fit the elastic scattering data. The best fit was defined by minimizing the quantity

$$\chi^2 = \sum_i \left(\frac{\sigma_T(\theta_i)-\sigma_e(\theta_i)}{\delta\sigma_e(\theta_i)}\right)^2 + \sum \left(\frac{P_T(\theta_i)-P_e(\theta_i)}{\delta P(\theta_i)}\right)^2,$$

where $\sigma_T(\theta_i)$ and $P_T(\theta_i)$ are the theoretical differential cross sections and polarizations at the angle θ_i, respectively, and $\sigma_e(\theta_i)$ and $P_e(\theta_i)$ the corresponding experimental values. The quantities $\delta\sigma_e(\theta_i)$ and $\delta P_e(\theta_i)$ are the experimental errors at the angle θ_i.

In order to fit the elastic deuteron data, we used an imaginary potential containing surface absorption and a real potential with a well depth of approximately 100 MeV, which has been shown[22] to be appropriate for elastic deuteron scattering. The diffuseness parameter was set at $a=0.65$ F. The parameter search was carried out, varying V, W, r_0, and r_0' to obtain the best fit to the deuteron cross-section data for three values of the spin-orbit parameter V_s. These fits to the data are shown in Fig. 4. Since deuteron-polarization measurements were not carried out, the deuteron spin-orbit potential was not determined, but the range of deuteron spin-orbit parameters spans the values of this parameter found in deuteron-polarization measurements on other nuclei.[23]

The fit to the proton elastic scattering cross-section and polarization data was carried out in a similar manner, varying V, W, V_s, r_0, and r_0' to obtain the best fits to the 16.0- and 14.8-MeV data. The best fits to these data are shown in Figs. 5–8. The proton parameters are similar to the set which was found by Rosen *et al.*[24] empirically to give the best over-all fit to proton scattering data for a large number of medium-weight nuclei. In Table VI is a list of the sets of parameters and the values of χ^2 obtained.

B. DWBA Analysis

The distorted-wave method has been described in detail by Satchler.[25] Calculations were performed with code JULIE[26] for the $^{90}Zr(d, p)^{91}Zr$ reaction. With this approximation, the transition amplitude is given by

$$A_{d,p} = \iint \chi_p^{(-)}(\mathbf{k}_p, \mathbf{r}_p)^* \langle F \mid V \mid I \rangle \chi_d^{(+)}(\mathbf{k}_d, \mathbf{r}_d)\, d\mathbf{r}_p d\mathbf{r}_d,$$

where $\chi(\mathbf{k}, \mathbf{r})$ are distorted waves for the scattering of a pair of particles with relative momentum $\mathbf{k}$ and separation $\mathbf{r}$. The other factor in the integrand is the matrix element of the interaction integrated over all the coordinates independent of $\mathbf{r}_p$ and $\mathbf{r}_d$. The wave function I includes the internal wave function of the deuteron, and the wave function F includes the wave function of the final nucleus assumed to be a shell-model wave function obtained by coupling the stripped neutron to the core. In the JULIE code, the interaction V is taken to be a zero-range proton-neutron interaction.

The deuteron parameter set c with values of the deuteron spin-orbit potential between 0 and 10 MeV, and proton parameter sets d and e for the ground state and 1.2-MeV states, respectively, were used in the DWBA calculations. The best fit to the (d, p) cross-section data was obtained with a cutoff for the radial integrals of 0 fm for the ground state and 5 fm for the 1.2-MeV state. These calculations and the data are compared in Fig. 9. The polarization calculations using the same parameters are compared with the data in Figs. 10 and 11. These calculations fail to reproduce the polarization data, although the fit to the cross-section data is quite good.

It was found that the calculated polarization was small when both the deuteron and proton spin-orbit potentials were small, indicating that the selective absorption is not as important as the optical spin-orbit potentials in determining the polarization. Varying the cutoff on the radial integrals did not affect the polarization significantly for values less than the nuclear radius.

Although the polarization predictions of the DWBA calculations do not adequately describe the measured polarizations for the protons corresponding to the ground state and 1.2-MeV state of ^{91}Zr, the polarization and cross-section distributions of protons corresponding to the 1.2-MeV state ($l_n=0$) appear to be related. The sign of the polarization for this state appears to follow the sign of the slope of the angular distribution at most angles, This behavior, along with the increase in polarization at the larger angles, indicates that the derivative relation between the angular distribution as described by Biedenharn and Satchler[27] may represent the behavior rather closely, i.e., $P(\theta) \propto (d\sigma/d\Omega)^{-1}(d/d\theta)(d\sigma/d\Omega)$. This was also suggested by the $^{88}Sr(d, p)^{89}Sr$ (1.05 MeV) polarization for an $l_n=0$ reaction.

IV. CONCLUSION

The $^{90}Zr(d, p)^{91}Zr$ polarization data measured here for the 1.2-MeV state ($l_n=0$) agree closely with the data obtained by Ludwig and Miller[7] earlier for the similar 1.05-MeV state ($l=0_n$) in the $^{88}Sr(d, p)^{89}Sr$

[22] C. M. Perey and R. G. Perey, Phys. Rev. **152**, 923 (1966).
[23] P. Schwandt and W. Haeberli, Nucl. Phys. **A110**, 585 (1968).
[24] L. Rosen, J. G. Beery, A. S. Goldhaber, and E. H. Auerbach, Ann. Phys. (N.Y.) **34**, 96 (1965).
[25] G. R. Satchler, Nucl. Phys. **55**, 1 (1964).
[26] R. H. Bassel, R. M. Drisco, and G. R. Satchler, Oak Ridge National Laboratory Progress Report No. ORNL-3085, 1961 (unpublished).

[27] L. C. Biedenharn and G. R. Satchler, Proceedings of the International Symposium on Polarization Phenomena of Nucleons, Basel, 1960 (unpublished); Helv. Phys. Acta. **6**, 372 (1961).

reaction. Those data also appear to follow the logarithmic derivative of the differential cross section, a result frequently noted in the comparison of elastic scattering cross-section and polarization distributions. This suggests that a simple method of calculation such as the diffraction method may be successful in describing these proton-polarization data. Reasonably good fits were obtained for the elastic scattering of protons and deuterons and the polarization of the scattered protons in order to supply the optical-model parameters at the appropriate nuclei for the stripping calculations. The deuteron stripping angular distribution was well described by the DWBA calculations using these parameters, but the proton polarization, in general, was not. Recent deuteron-polarization measurements[23] on other nuclei show that the deuteron distortion may be a sensitive function of the spin-orbit geometrical parameters which were not determined in this experiment. For the 1.05-MeV state, the DWBA calculations presented here fit the data well, out to about 60°, but not well at larger angles. The $^{90}Zr(d, p)^{91}Zr$ polarization for the ground state ($l_n=2$) was found to be small out to 100° and the DWBA calculations did not reproduce the data even at forward angles.

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High-Energy-Proton Fission Cross Sections of U, Bi, Au, and Ag Measured with Mica Track Detectors*

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The interaction of 0.6–29-GeV protons with U, Bi, Au, and Ag was studied with mica track detectors. Binary fission events were identified as correlated pairs of tracks. A fragment registered an acceptable track if its mass was ≥ 30 and its kinetic energy >6–8 MeV. The results show: The U fission cross section $\sigma_f(\text{U})$ is 1400 mb up to 1.0 GeV and decreases monotonically to 670 mb at 29 GeV; $\sigma_f(\text{Bi})$ increases from 125 mb at 150 MeV to a broad maximum of 215 mb at $\sim$600 MeV, and then decreases to 105 mb at 29 GeV; $\sigma_f(\text{Au})$ varies little, 59–76 mb, in the energy region 0.6–29 GeV (the highest values appear at 2 and 3 GeV); no binary events were observed from Ag targets: upper limits are 0.3 mb at 0.6 GeV, 2 mb at 1.0 GeV, 5 mb at 2–13 GeV, and 6 mb at 29 GeV. Effects of secondary particles were shown to be negligible; the experimental uncertainty varies from ± 10 to $\pm 20\%$. Ternary fission events (140) were seen for U starting at 1.0 GeV, and for Bi and Au starting at 2.0 GeV. The yields are about 1–2 per 1000 binaries. Many single unpaired tracks were also observed, and their yield increases with beam energy. From U, most of these seem to be from a small fraction of asymmetric fissions where one of the partners is below recording threshold. From Bi and Au, most of the single tracks are high-energy spallation residues.

INTRODUCTION

ALTHOUGH many measurements have been made of the proton fission cross sections of various elements at energies up to about 600 MeV, relatively few determinations have been carried out at higher incident proton energies.[1–4] One reason for the lack of such data is that in the GeV region, fission is no longer so clearly distinguishable from other types of processes. For example, at low and medium energies, radiochemical measurements of the yields of specific products give yield-versus-mass curves with well-defined peaks near half the target mass. At very high energies,[1] the peak may become very broad and its limits poorly defined (U^{238} at 28 GeV) or the peak may disappear altogether (Bi at 3 and 28 GeV). These observations indicate that other processes, such as spallation and fragmentation, may yield products in and near the fission-product mass region.

A technique in which two complementary heavy fragments are detected is superior to the radiochemical method for characterizing fission. In principle, multiparameter counter experiments[5] with Si detectors are well suited for the detailed study of high-energy fission. However, the complex nature of these experiments makes it impractical, at present, to use this method for measuring fission cross sections of a series of targets at a series of energies. The nuclear emulsion technique has been used[2,3] for measuring a few fission cross sections

* Research performed under the auspices of the U.S. Atomic Energy Commission.

[1] G. Friedlander, in *Physics and Chemistry of Fission* (International Atomic Energy Agency, Vienna, 1965), Vol. II, p. 265.

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[4] E. S. Matusevich and V. I. Regushevskii, Yadern. Fiz. **7**, 1187 (1968) [English transl.: Brookhaven National Laboratory Transl. No. BNL-TR-235 (unpublished)].

[5] L. P. Remsberg, Jr., J. B. Cumming, F. Plasil, and M. L. Perlman (unpublished).