Spectroscopy of 1f-2p Nuclei with Direct (d, α) and (d, p) Reactions. I. Cu⁶⁶ and Ni⁶¹[†]

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 $Zn^{68}(d, \alpha) Cu^{66}$ and $Cu^{65}(d, p) Cu^{66}$ angular distributions have been measured for deuteron energies of 12 MeV. The reaction products were magnetically analyzed, and the resolutions obtained were 11-12 keV for (d, α) and 8 keV for (d, p) reactions. Angular momentum transfers $L_{d,\alpha}$ and $l_{d,p}$ for the transitions were obtained by comparison with distorted-wave Born-approximation (DWBA) calculations. Level energies, J^{π} values or narrow limits for J^{π} , and (d, p) spectroscopic factors could be determined for about 65 Cu⁶⁶ levels below 3-MeV excitation. The reaction $Cu^{63}(d, \alpha)N^{161}$ was investigated as a function of deuteron energy, $10.5 \le E_d \le 12.5$ MeV, in order to support the assumption that (d, α) reactions in this mass and energy range proceed by a direct transfer mechanism. A high-resolution $Cu^{63}(d, \alpha) Ni^{61}$ spectrum and excitation functions for five well-resolved Ni⁶¹ levels are presented. It is found that (d, α) cross-section fluctuations do not exceed counting statistics (<15%). The reported DWBA calculations follow current DWBA theories, without arbitrary parameters or cutoffs; and include corrections for finite range and nonlocality effects. Microscopic form factors based on finite-well single-particle wave functions were used for the Zn⁶⁸ (d, α) Cu⁶⁶ calculations. Very good agreement with experimental (d, α) angular distributions was found if α potentials characterized by a real well with $r_0 \approx 1.4$ F, $V \approx 160$ MeV were used. Shallower and deeper α -potential families which fitted elastic α scattering equally well were found to lead to quantitative and qualitative disagreement with experiment.

I. INTRODUCTION

IRECT two-neutron transfer reactions of the type (p, t) or (t, p) have been widely and successfully used to investigate natural parity levels in eveneven nuclei, particularly those with collective enhancement. They exhibit distinctly structured angular distributions; strict selection rules generally allow unique spin and parity assignments.^{1,2} Total angular momentum selection rules for (neutron + proton) transfer reactions are somewhat less stringent because the transferred "deuteron" has spin 1. Until recently,³ few attempts had been made to measure and use (d, α) angular distributions for the determination of total angular momentum J and parity π of unknown final states. Instead, the emphasis in the spectroscopic use of deuteron transfer reactions has generally been placed on the interpretation of the observed transition strengths.⁴ Valuable quantitative structure information can indeed be gained from the observed magnitude of deuteron transfer cross sections,² however, not in the simple deductive way of single-nucleon transfer analyses. The data must be compared with theoretical predictions based on explicit wave functions for the nuclei involved, and since configuration mixing is extensive in all save a very few nuclei near magic numbers, sufficiently complete nuclear wave functions are extremely rare. The present study does provide absolute (d, α) cross sections, which can be used for a

test of forthcoming nuclear structure calculations, but its main purpose is to demonstrate the extent to which (d, α) angular distributions can be used in the otherwise difficult spectroscopy of odd-odd nuclei.

The scarcity of spectroscopic information for other than the lightest odd-odd nuclei is primarily a result of past experimental difficulties. Some odd-odd nuclei have been studied by thermal neutron capture (n, γ) reactions, a technique which greatly gained in precision with the introduction of high-resolution Ge(Li)counters.⁵ Generally, however, complicated decay schemes are encountered, and very time-consuming γ - γ coincidence measurements are needed before level schemes and J^{π} values can be suggested with some confidence. Studies with direct single-nucleon transfer reactions of sufficient resolution immediately determine the level scheme of the residual nucleus and generally succeed in determining the orbital angular momentum transfers l for the transitions. Hence, they provide the parity and J_f limits $(|J_i-j| \le J_f \le J_i+j,$ where $j=l\pm\frac{1}{2}$ for many final states. In special cases sum rules can be invoked that may suggest a few specific J^{π} assignments,^{6,7} but such assignments depend on the absence of fractionization of the expected J^{π} configurations and generally remain subject to considerable doubt. It is therefore necessary that experiments of the types mentioned above be supplemented by studies of reactions that show higher selectivity and more narrowly limit J^{π} .

We suggest that direct deuteron transfer reactions fill this need very well. Just as in single-nucleon transfers, the final level scheme is immediately clear, but,

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 ⁴ See, for instance, E. Rivet, R. H. Pehl, J. Cerny, and B. G. Harvey, Phys. Rev. 141, 1021 (1966); 137, 114 (1965).

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⁵ E.g., E. B. Shera and H. H. Bolotin, Phys. Rev. 169, 940

⁶ S. Yoshida, Nucl. Phys. 38, 380 (1962).
⁷ E.g., J. B. Moorhead, B. L. Cohen, and R. A. Moyer, Phys. Rev. 165, 1287 (1967).

furthermore, for 0^+ targets only pure J transfers (for the transfered neutron + proton pair) can populate a given state J_f ($J=J_f$). Our (d, α) data indicate that, as in other transfer reactions, the observed angular distributions are most characteristic of the transferred Lvalues. As parity is conserved and J = L + 1, recognition of two contributing L values determines J_f uniquely. A single dominant L contribution allows $J_f = L$ or $J_f = L \pm 1$ assignements, with a somewhat greater likelihood for the former value.⁸ Other well-known selection rules determine that the excitation of states of higher isospin is strongly inhibited and that 0⁺ states cannot be excited by direct (d, α) transitions; hence, $T_{\text{final}} =$ T_{initial} , and L=0 demands $J^{\pi}_{f}=1^{+}$. A further restriction applies to states with pure $(j^2)_{even}$ configurations, which are not excited by direct (d, α) or (α, d) reactions.² In practice, predominantly $(j^2)_{even}$ states in the Cu isotopes are still weakly excited owing to configuration mixing, so that very few known low-lying levels are actually missing in our high-resolution data.

Although the (d, α) selection rules by themselves allow a fair number of unique J^{π} assignments, it was deemed safer and more conclusive to also identify the Cu⁶⁶ levels and their parities from Cu⁶⁵(d, p)Cu⁶⁶ angular distributions. This additional information is often needed if (d, α) transitions with mixed L contributions are to be analyzed with confidence. For other levels the (d, p) limits for J^{π}_{f} help in uniquely fixing the J^{π} assignments.³ The Cu⁶⁵(d, p)Cu⁶⁶ reaction also yielded spectroscopic factors which are listed and may be helpful in future structure calculations.

II. TESTS OF THE DIRECT NATURE OF (d, α) TRANSITIONS

The validity of the selection rules cited above and the spectroscopic use of (d, α) reactions depends strongly on the condition that they be direct one-step reactions, and that compound contributions can be neglected. At this time it seems difficult to specify when two-step reactions (such as inelastic scattering plus pickup) contribute measurably to the observed cross section. Such effects are estimated to be at least an order of magnitude weaker than "typical" direct cross sections,⁸ but they could affect weak angular distributions. Typical $Zn^{68}(d, \alpha)Cu^{66}$ cross sections peak near 10 μ b/sr, whereas a few very strong transitions peak near or above 50 μ b/sr. Since we lack more quantitative guides, it seems reasonable to consider transitions with cross sections below or near 2 μ b/sr as uncertain with regard to the reaction mechanism. We shall have to allow the possibility that very weak excitation of a level may occur, although the direct one-step transition is forbidden by a selection rule. Assignments based on weak transitions are therefore bracketed in the tables. The question of compoundnuclear transitions is subject to more empirical tests.



FIG. 1. Energy dependence of the reaction $\operatorname{Cu}^{83}(d, \alpha)\operatorname{Ni}^{61}$. The measurements were performed with two standard Si-counter spectrometers and a 0.2-mg/cm² self-supporting Cu⁶³ target. The sum of all random errors is indicated by bars. The absolute cross-section scale is accurate to about 30%. The dotted lines are drawn as an aid to the eye.

An a priori consideration holds that for 12-MeV deuterons $Zn^{68}+d$ forms the compound nucleus Ga⁷⁰ at \approx 24-MeV excitation, more than 16 MeV above the neutron emission threshold, so that compound α emission to the low-lying Cu⁶⁶ states is strongly disfavored, and direct contributions are likely to dominate. As expected, all (d, α) distributions are strongly and consistently forward-peaked.3 In order to check for possible compound effects at larger angles, an "excitation" run with 10-keV (beam + target) resolution was made for $Cu^{68}(d, \alpha)Ni^{61}$ from 10.5 to 12.5 MeV. Cu rather than Zn targets were chosen for this purpose in order to be able to relax the requirements for detector resolution and because its low melting point makes Zn an unreliable target for charge monitoring. Excitation functions⁹ for 50° and 70° for several well-resolved states are shown in Fig. 1. Although data points are widely spaced, frequent and significant deviations from the smooth average curves would still be expected in a region of Erickson fluctuations. None were seen, and we conclude that in this mass and energy region (d, α) reactions are direct reactions to a degree that makes them fully useable for spectroscopic purposes. An ex-

⁸ P. J. Iano and N. Austern, Phys. Rev. 151, 853 (1966).

⁹ Previously reported by Y. S. Park and W. W. Daehnick, Bull. Am. Phys. Soc. 11, 366 (1966).











FIG. 4. Comparison of experimental and theoretical (see text) angular distributions for $Zn^{68}(d, \alpha)Cu^{66}$. Known and estimated experimental random errors were combined and are indicated by error bars. Measured level energies are listed at the right. The angular distributions are grouped according to the dominant L transfer. Where indicated, the theoretical DWBA curves (solid lines) are based on tentative mixtures of two L contributions; the curves contain no other adjustable parameters. Weak transitions, in particular oddparity transitions, are shown for completeness, although the suggested L assignments for weak transitions are at best tentative.

tension of this conclusion to include heavier targets and higher α energies does not run afoul of any theoretical expectations regarding reactions mechanisms. It has been found, however, that for lighter elements and low bombarding energies compound contributions may often be important.¹⁰

The data for Fig. 1 were taken with thin $300-\mu$ Si counters and detector resolutions of about 60 keV. When later work revealed extremely rich (d, α) spectra in all 1*f*-2*p* nuclei investigated, a Cu⁶³(d, α) spectrum was retaken with higher resolution in our split-pole spectrograph. The use of position-sensitive counters for particle discrimination¹¹ yielded spectra of about 11-keV resolution (for 20-MeV α 's) and negligible background. From Fig. 2, it can easily be seen that

the levels selected for Fig. 1 remained separated from their nearest neighbors even at 60-keV resolution, and that the lack of fluctuations in Fig. 1 is not due to averaging over several levels.

Figure 2 was compared to other recent high-resolution studies¹²⁻¹⁵ of Ni⁶¹. The excitation energy assignments up to 2.124 MeV are unambiguous, and the weighted literature average used should be accurate to 1 or 2 keV. Our own absolute energy scale calibration for the higher levels in Fig. 2 was uncertain by about 1%, but

¹⁰ D. A. Bromley, Argonne National Laboratory Report No. ANL-6848, 1964 (unpublished), and references therein. ¹¹ W. W. Daehnick, Phys. Rev. 177, 1763 (1969).

¹² R. H. Fulmer, A. L. McCarthy, B. L. Cohen, and R. Middle-ton, Phys. Rev. **133**, 955 (1964). ¹⁸ R. G. Tee and A. Aspinall, Nucl. Phys. **A98**, 417 (1967), and

references therein.

¹⁴ E. R. Cosman, D. N. Schramm, H. A. Enge, A. Sperduto, and

C. H. Paris, Phys. Rev. 163, 1134 (1968).
 ¹⁵ R. Bérand, I. Berkes, J. Daniére, M. Levy, G. Marest, and R. Rougny, Nucl. Phys. A99, 577 (1967); R. Schoeneberg and A. Flammersfeld, Z. Physik 200, 205 (1967); H. H. Bolotin and H. J. Fischbeck, Phys. Rev. 158, 1069 (1967).

by

l = 4

.922

* 2 5%

•4 •2

2.36

2.630

= 4

2.707

2.903

007 5

θ_{c.m}

• 4





Fig. 4 (Continued)

90

the relative spacing of neighboring levels could be ascertained to about 3 keV. The relative excitation energies measured by $Cu^{63}(d, \alpha)Ni^{61}$ agree better with the values of Ref. 13 than those of Ref. 14. The latter also are high, when compared with recent γ -ray data.¹⁵ Consequently, they have been given less weight in arriving at the average energies listed on Fig. 2.

III. EXPERIMENTAL PROCEDURE FOR THE MEASUREMENT OF ANGULAR DISTRIBUTIONS

A. $Zn^{68}(d, \alpha)Cu^{66}$ Experiment

Ninety-nine-percent-enriched Zn⁶⁸ targets on 20- $\mu g/cm^2$ carbon backings were used. The typical target thickness was about 30 μ g/cm². Since the targets slowly deteriorated during the long runs with 1–3 μ A of 12-MeV deuterons, elastic deuteron scattering from Zn⁶⁸ was monitored by a Si counter fixed at $\theta_L = 38^\circ$, when data were taken with the Enge split-pole spectrograph. Occasional discrepancies between charge and

counter monitor normalizations prompted us to remeasure the angular distributions for the more prominent groups with smaller beam currents, a five-Sidetector setup and two symmetrically mounted NaIcounter monitors in our standard 18-in. scattering chamber,¹⁶ in order to more accurately normalize some of the high-resolution runs.

In the spectrograph runs, resolution and counting rate were optimized¹¹ at some expense to beam current and solid angle (1.4 msr). The reaction α particles were detected for four position-sensitive counters in the spectrograph focal plane; the pulses were identified and processed in a manner described previously.¹¹ A typical (composite) $Zn^{66}(d, \alpha)Cu^{66}$ position counter spectrum is shown in Fig. 3. The resolution obtained was 11–12 keV for 16–19-MeV α 's. Particle discrimination was nearly perfect, and except at the smallest angle (10°) the peak-to-background ratio in the spectra was better than 10³. The (d, α) angular distributions obtained are shown in Fig. 4.

¹⁶ R. H. Fulmer and W. W. Daehnick, Phys. Rev. 139, B579 (1965).

Level	Farm	<i>H</i>	F_{min}		a(d, h)				a(d ∞)			J* (heet
No.	(MeV)	(MeV)	(MeV)	$l_{d,p}$	$(\mu b/sr)$	$(2I_{f}+1)S$	$J^{\pi_{d,p}}$	$L_{d, \alpha}$	$(\mu b/sr)$	$J^{\pi}{}_{d,lpha}$	$J^{\pi}_{n,\gamma}$	value)
				[1	190	0.16				-	-	
0	0	0	0	3	25	0.32	1+-3+	0+2	20	+-	+-	+
•				[1	110	0.09			Ę		:	
-	0.183	0.180	0.185	<u>_</u>	125	1.50	$2^{+3^{+}(1^{+})}$	7	6)	2+(1+3+)	(2+)	$2^{+}(3^{+})$
		(0.238)			•				:		(≪3+)	۸.
2	0.272	0.275	0.274	3	250	2.92	4+3+	4	13	4+(3+)	(3+)	$3^{+}(4^{+})$
3	0.383	0.386	0.383	1	1580	1.17	1+-3+	(4+2)	3	(3+)	(1+2+)	$2^{+}(3^{+})$
4	0.462	0.465	0.462	1	2350	1.72	2+3+	(2+?)	(11)	(2+)	(1+2+)	2+
ŝ	0.589	0.591	0.588	3	185	2.01	2+-4+	4	17	3+4+	(4+)	4+(3+)
6	0.724	:	0.727	1	88	0.06	0+-3+	4+2	10	3+	:	3+
7	0.819	0.822	0.819	1	1040	0.80	1+-3+	2	(8)	2+(1+3+)	(≤3+)	2+(1+3+
ø	1 015	[1.009]	1 012	3	40	0.40	1+-4+	2	10	(1+-3+)	(≤3)	$1^{+2^{+}(3^{+})}$
6	CTO.1	1.017	CT0.1	1	890	0.57	1+-3+	4	ŧ	3+4+	(3+)	$3^{+}(4^{+})$
10	1.051	1.053	1.048	1	270	0.18	0+-3+	0+(2)	21	1+	(1+2+)	1+
11	1.152	:	1.151	4	780	9.63	36-	5+(3)	10	6-5-4-	•	(.9)
12	1.209	1.213	1.208	1	570	0.35	0+-3+	2	(9)	$2^{+}(1^{+}3^{+})$	(2+)	2+(1+3-
13	1.247	:	1.247	2 4	600 (150)	$\left[0.55 \right]$	3-4-	ŝ	6	3-(2-4-)	:	3-(4-)
14	1.339	1.346	1.339	1	280	0.85	0+-3+	0	10	1+	(≤3+)	1+
15	1.433	1.440	1.434	1	660	0.39	0+3+	0+2	4	1+	(≤2⁺)	1+
16	1.544	1.549	1.543	1	0 <u>0</u>	0.04	0+-3+	ă	ot resolved		(3+)	3+(2+1+
17	1.557	1.562	1.555	1	98	0.05	0+-3+	ŭ	ot resolved		÷	1+2+3+
18	1.572	1.576	1.573	1	195	0.11	0+-3+	'n	ot resolved		:	1+2+3+
19	:	1.679	1.679	1	70	0.04	0+-3+	4+(2)	2.5	3+	:	3+
20	:	:	1.697	1	46	0.03	0+-3+	0+(2)	2	+	:	1 +
21	:	:	1.713	(1)	(140)	0.08	(0+-3+)	0+2	2	1+	÷	(1+)
22	1.730	:	1.730	4	270	3.10	36-	(5)	4	(2-)	:	5-4-(6-3-
23	:	1.746	:	:	not resolved	÷		D(ot seen		(<2+)	(2+0)

W. W. DAEHNICK AND Y. S. PARK

180)	SPEC	ΓR	o s	C O	РҮ	OF	1 <i>f</i>	- 2 p	Ň	U	CL	ΕI	w	ΙT	н	(d,	a) A	NI	D	(d,	p)	•	I		1069
	J≠ (best value)	i+	1-2-	(3+)	2-(1-)	(+) =	2+(3+1+)	~-	1-2-	36~	1-4-	۰.	14-	(1+)	1-4-	3+(4+5+)	1-4-	14-	4+(5+3+)	(2/)		1-4-	1+	(1+2+3+)	14-	(1+)	1-2-
	$J^{\pi}_{n,\gamma}$	(<3+)	÷	÷	:		•	(<3+)																			
	$\int \pi_{d,\alpha}$	1+		3+	(24-)	(+ 4)	3+2+(1+)			۰.	۰.			(1+)		3+(4+5+)			4+(3+5+)	, ,	7		1+	(1+2+3+)		+	
	$\sigma_{\max}(d, lpha) \ (\mu \mathrm{b/sr})$	19	2	3	2	1	(12)	not resolved		2.5	3	weak		2		6	weak		5.5	(7)	(1)		7	(2.5)	weak	3	not resolved
	$L_{d, \alpha}$	0+(2)	[2+(3)]	4+(2)	(3)		2	•		(3 or 4)	(3 or 2)	:		0+(2)	•	4+(2)		:	4	1 (2)	(2)(T	:	0	(2)		0	
()	$J^{\pi_{d,p}}$	1+-3 ⁺	1-2-	0+-3+	1-2- }	14-)	1+-3+		1-2-	36-	14-		14-	۰.	1-4-		14-	1-4-		(1+_3+)		14-		(0+-3+)	14-		1-2-
LE I (Continued	$(2I_f+1)S$	0.05 0.19	0.005	0.07	0.029	0.10	0.08		0.022	1.63	0.39	•	0.04	0.06 or 0.11)	0.35	:	0.46	0.28	•	0.08	(0.55)	0.41	•	0.04	0.12		0.008
TABI	$\sigma_{\max}(d, p) \ (\mu \mathrm{b}/\mathrm{sr})$	106 23	45	130	(320)	120	135 23		250	150	500	weak	55	(1)	450		009	360		(160)	(80)	550		80	15C	not seen	100
	l _{d,p}	1 3	0	1	0,	[2	3_1	:	0	4	2	•	J 2	(3 or 4)	[2		2	<u>]</u> 2	÷	(1)	(3)]2	÷	(1)	2		0
	$\frac{E_{\mathrm{Pitt}}}{\mathrm{(MeV)}}$	1.815	1.894	1.922	1.977	- - -	2.017	:	2 120		2.158	(2.173)	J 105	CCT - 7	1 JKK	2.200	2.329	1 261	100.2	2 301	1/0.7	2 440		2.501	2.525	2.537	2.557
	$\stackrel{E_{n,\gamma}}{(\mathrm{MeV})}$	1.821	:	1.927	:		2.018	2.026																			
	Emtr (MeV)	1.816	:	1.923	1.976		2.015	÷	2 122	1	2.159	2.173	107	161.2	7 767	107.7	2.328	7 360		2,301	1/0.7	7 440		:	2.519	:	:
	Level No.	24	25	26	27	i	28	29	30d	1000	31	32	33,1	100	24.4	neo	35	36a	36b	37	5	38a	38b	39	40	41	42

					Ĥ	ABLE I (Contin	rued)					
Level No.	Emtr (MeV)	$\stackrel{E_{n,\gamma}}{(\mathrm{MeV})}$	${E_{{f P}_{{f th}}} \over ({f MeV})}$	la,p	$\sigma_{\max}(d, p) \ (\mu \mathrm{b/sr})$	$(2I_f+1)S$	$J^{\pi_{d,p}}$	$L_{d, \boldsymbol{lpha}}$	$\sigma_{\max}(d,lpha) \ (\mu \mathrm{b/sr})$	$J^{\pi}_{d,lpha}$	$\int \pi_{n,\gamma}$	J≠ (best value)
43	:		2.567		not seen			6+(4)	50	5+(6+7+)		5+(6+7+)
44	2.581		2.581	0	200	0.057	1-2-		not resolved			1-2-
45a	007 0		102 0	Q	340	0.028	1-2-	:				1-2-
45b	2.002		2.004	:		•		0+(2)	3	(1+)		(1+)
46	:		2.630		weak	:		4	4	4+5+3+		4+5+3+
4				(0)	157	0.013						
47	2.645		2.644	2	115	0.08	(1-2-)		weak			(2-1-)
48	2.660		2.660	0	730	0.057	1-2-		weak			1-2-
49	:		2.682	1(or 2)	80	(0.04)	\$3	0+2	15			1+
50	:		2.707		not seen			4	17	4+5+(3+)		4+5+(3+)
51	2.740		2.743	2	480	0.34	1-4-	3	(10)	3-(2-4-)		3-(2-4-)
52	:		2.760		not seen	:		0+(2)	ŝ	1+		1+
53 a			004 0	0	75	0.006	1-2-					1-2-
53b	:		2.795	(2)	(13)	0.01		(0+2)	2	~		۰.
54	:		2.812	0	100	0.008	1-2-		weak			1-2-
55	:		2.837	0	110	0.008	1-2-		weak			1-2-
56	2.866		2.863	~	(22)	:		0	6	1 +		1+
57	:		2.903	(1)	62	0.03	(0+3+)	4+(2)	13	3+		(3+)
02	2 043			0	230	0.018	ţ		-			(
00	CH4.7		. .,	[2	70	0.05]	7_1		not resolved			(-7,-1)
59	÷		2.964	2+(0)	20	0.05	(14-)		not resolved			(14-)
8	:		2.986	(2)	45	0.03	(14-)	6(or 5)	2	(4-7)		(4-7)
61	3.009		3.007	(3)	135	(0.77)	(1+-4+)	4	11	4+(5+3+)		(4+)
69	3 030		2 037	0	620	0.053	ţ	:				(-6-1)
2			· · · ·	[2	280	0.19 ∫	7 1	:				(71)
62	2 001		2 000	0	850	0.068	- - -					ţ
3	100.0		000.0	[2	230	0.16 }	7_1	•				7 1
† References 5	5 and 18.	‡ Reference	5.									

180

337 w AND v S PARK JICK







FIG. 6. Comparison of experimental $\operatorname{Cu}^{66}(d, p)\operatorname{Cu}^{66}$ angular distributions with DWBA calculations. Experimental random errors are indicated by error bars where they exceed the size of the dots. Angular distributions are ordered according to level energy. The curves represent DWBA calculations with standard nonlocality and finite-range corrections. There is some Q dependence, and a sufficient number of calculations was made so that the Q values of data and curves do not differ by more than 0.5 MeV. Usually excellent agreement is found for pure l transfers. Where l mixing seemed necessary, it is so indicated, and the solid curve represents the (incoherent) sum of two l contributions. The solid l=0 curves used are empirical curves.

B. $Cu^{65}(d, p)Cu^{66}$ Experiment

Although 15-MeV data for this reaction have been published,¹⁷ it was apparent from our high-resolution Zn⁶⁸(d, α) Cu⁶⁶ data and previous level studies¹⁸ that many Cu⁶⁶ states had remained unresolved. Consequently, the Cu⁶⁵(d, p) reaction was re-examined at 12-MeV deuteron energy with our Enge split-pole spectrograph and standard nuclear emulsion techniques. 100- μ g/cm² Cu⁶⁶ targets (99% enriched) on 20- μ g/cm² carbon backings were used, and a total resolution of 8 keV was obtained. Figure 5 shows a typical plate spectrum for Cu⁶⁶ (d, p). The procedure for optimizing resolution and for beam and target monitoring was the

same as described earlier.¹¹ Experimental details regarding the calibration of the measured cross sections and excitation energies are discussed in part II of this paper. The good resolution and a peak-to-background ratio of better than 500 in (d, p) as well as in the (d, α) spectra, led to the identification of about 60 levels below 3 MeV, of which 17 levels had not been previously reported.^{5,18} There is good evidence (see Table I) that at least 7 of these 60 groups are doublets that remained unresolved, and one may estimate from the observed level density and experimental resolution that below 3 MeV more than 25% of all levels, in particular the weakly excited ones, still remain unresolved. By calibrating the (d, p) excitation energies at six angles, all strong impurity peaks could be traced and their accidental inclusion in neighboring peaks

¹⁷ S. A. Hjorth and L. H. Allen, Arkiv Fysik **33**, 207 (1966). ¹⁸ R. P. de Figueiredo, M. Mazari, and W. W. Buechner, Phys. Rev. **112**, 873 (1958).



avoided. The photographic plates generally were scanned with-0.2 mm resolution. Some rescans for poorly resolved levels were made with 0.1-mm resolution with the result of slightly improved energy resolution.

C. Experimental Errors

The error bars on the data points in Figs. 4 and 6 include all known random errors. For weak levels they are primarily due to statistics; however, for some highly excited levels large uncertainties are shown that are not of truly random nature, but result from the graphical analysis of imperfectly resolved peaks. Such errors are hard to compute, and at times may have been overestimated. The absolute scale error for the (d, α) cross sections is smaller than 20% and is primarily due to uncertainties in the target thickness. For the (d, p) data, the absolute scale calibration is uncertain to about $\pm 15\%$. The dominant contributions are uncertainties in the target thickness, monitor angle, and in the plate scanning efficiency. The latter is hard to assess for manual scanning, but rescanning generally reproduced spectra to within a few percent. The excitation energies could be more accurately obtained from the (d, p) spectra. The (scale) errors for all resolved levels is less than $\pm 0.4\%$ of excitation energy, while the separation of neighboring levels usually could be measured to better than 2 keV. The (d, α) spectra were calibrated to about ± 10 keV, and whenever the (d, α) excitation energy obtained was within 5 keV of the (d, p) energy, a weighted (d, p)- (d, α) average was used in Table I and in Figs. 3 and 5. Our agreement with the MIT¹⁸ (d, p) energies (within 2 keV) is unexpectedly good, probably due to a similarity in the spectrograph scale calibration. No such absolute accuracy is claimed here, but the precise level-by-level agreement indicates that the errors quoted in Ref. 18 may be overly conservative.

IV. DWBA ANALYSIS OF $Zn(d, \alpha)$ REACTIONS

A number of successful two-nucleon transfer calculations have been reported,¹⁹⁻²³ most of which employ two-nucleon transfer theories similar to those of Glendenning.² To date mainly (p, He^3) , (He^3, p) , (t, p), and (p, t) analyses have been published. It has been found¹⁹⁻²¹ that even if only one realistic configuration for the transferred nucleons is considered, the angular distributions (but not the absolute or relative cross sections) are, nevertheless, well predicted.

Experimental (d, α) angular distributions show a simple and systematic L-dependent structure, which hardly changes with target mass or Q value. Therefore, it would appear that-as in other two-nucleon transfers—details of the microscopic (d, α) form factors do not greatly affect predicted angular distributions for pure L. We verified this expectation by calculations with various test configurations for neutrons and protons in the $f_{7/2,5/2}$ and $p_{3/2,1/2}$ orbits.

Although the calculations cited above neglect nonlocality and finite range effects, we find that the latter approximations are not advisable for (d, α) calculations. Our initial zero-range DWBA calculations²⁴ produced fair agreement with L=0 and L=2 experimental angular distributions, but did poorly for higher Lvalues. A large sensitivity to radial integration cutoffs indicated that the poor momentum match in (d, α) led to significant contributions from the nuclear interior and that corrections had to be made for finite-range²⁵

- (1966).
 ²⁰ N. K. Glendenning, Phys. Rev. 156, 1344 (1967).
 ²¹ R. A. Broglia and E. Riedel, Nucl. Phys. A92, 145 (1966).
 ²² B. F. Bayman and A. Kallio, Phys. Rev. 156, 1121 (1967).
 ²³ B. F. Bayman and N. M. Hintz, Phys. Rev. 172, 1113 (1968).
 ²⁴ Devicement with DWBA code JULIE, written by R. M. Drisko (unpublished)

¹⁹ R. M. Drisko and F. Rybicki, Phys. Rev. Letters 16, 275

J. K. Dickens, R. M. Drisko, F. G. Perey, and G. R. Satchler, Phys. Letters 15, 337 (1965).

and nonlocality effects.²⁶ The systematic use of such corrections became practical after DWBA code DWUCK²⁷ was made available to us by Kunz, and led to significantly improved DWBA predictions.²⁸

The zero-range DWBA treatment of two-nucleon transfer reactions^{1,2,19,29} for a given L, S, J leads to the cross section

$$d\sigma/d\Omega \propto \sum_{m} \left| (-i)^{L} [4\pi (2L+1)]^{1/2} \right| \\ \times \int \psi^{(-)*} F_{LSJT}(R) Y_{L}^{m}(\hat{R}) \psi^{(+)} d\mathbf{R} \right|^{2}, \quad (1)$$

where the customary single-nucleon form factor has been replaced by

$$F_{LSJT}(R) = \sum_{\gamma} \beta_{\gamma LSJT} f_{L\gamma}(R),$$

with

$$f_{L\gamma}(R) = \sum_{N} g_{NL\gamma} U_{NL\gamma}(R). \qquad (2)$$

L, S, J and T are the quantum numbers of the transferred nucleon pair, while R and N refer to its c.m. motion, and γ to one of the $[j_1 j_2]_J$ nucleon-pair configurations that contribute to the reaction. $\beta_{\gamma LSJT}$ is the spectroscopic amplitude for this configuration, $U_{NL\gamma}(R)$ is a component of the two-nucleon radial wave function,² and the $g_{NL\gamma}$ are nuclear structure factors to be discussed below. The commonly used theories^{1,2,19,22} differ in the representation of the singlenucleon (bound) states u_{j1} , u_{j2} and in the representation of $U_{NL\gamma}(R)$. We have chosen to follow the procedure of Drisko and Rybicki,19 and have used their microscopic two-nucleon form-factor code MIFF³⁰ for the calculations of $f_{L\gamma}(R)$. As no detailed wave functions exist for Cu and Zn, the β_{γ} are unknown and the summation over γ could not be performed.

A. Microscopic Form Factor F_{LSJT}

The structure factors for pure configurations γ are given by

$$g_{NL\gamma} = g\Omega_n \langle n0, NL; L \mid n_1 l_1, n_2 l_2; L \rangle, \qquad (3)$$

and are the product of the symmetry factor g, the overlap integrals Ω_n , and a Moshinsky bracket. Ω_n measures the overlap of the relative motion of the transferred nucleons in the target with their relative motion in the (heavier) projectile.² (It is here assumed as pure S in the α particle.) The Moshinsky bracket results from the transformation of two single-nucleon wave functions $u_{n_1l_1}$ and $u_{n_2l_2}$ (expressed in harmonic-oscillator wave functions) into their c.m. and relative motion.²

It is clear from Eqs. (1) and (2) that the spectroscopic amplitudes β_{γ} cannot be factored out, unless we happen to have measurable contributions from only one configuration, i.e., only one term in $\sum_{\gamma} \gamma$; [in $Zn(d, \alpha)$ this might occur for $(1f_{7/2}{}^n 1f_{7/2}{}^p)_{J^{\pi}=7}^+$]. Generally, β_{γ} cannot be deduced from a comparison of measured and calculated cross sections; instead, experiments must be used to distinguish between predictions based on correct or incorrect theoretical nuclear wave functions. The scarcity of theoretical wave functions could thus limit meaningful experimentation to very few nuclei; fortunately, details of the form factor affect primarily the magnitude of the predicted cross sections,^{2,21} while the angular distributions are most sensitive to L. For our calculations for $Zn(d, \alpha)$, we have assumed various pure configurations $(n_1l_1; n_2l_2)$ in the 1*f*-2*p* shell in order to obtain the factors $g_{NL\gamma}$ and to calculate the corresponding coherent sums $f_{L\gamma}(R)$ [see Eq. (2)]. It was found that the radial dependence of the form factors $f_{L\gamma}(R)$ for different pure configurations differed little in this shell, especially near the nuclear surface, and the L-dependent structure of the predicted cross sections was not noticeably altered. This result may no longer hold if the terms in the sums for F_{LSJT} give much cancellation²⁰; however, such cancellations would lead to drastically reduced cross sections, and the corresponding transitions would not normally be analyzed. Therefore, it remains reasonable to analyze the stronger (d, α) transitions theoretically, even in the absence of detailed nuclear wave functions, with the goal of deducing L transfers from the direct comparison of theoretical and experimental angular distributions.

Calculations for two-nucleon transfers which use simple harmonic-oscillator single-nucleon wave functions for $u_{n_1l_1}$ and $u_{n_2l_2}$ without tail corrections give unsatisfactory agreement with experiment. This has been shown to be due to incorrect tails¹⁹ in $F_{LSJT}(R)$. The difficulty can be avoided by matching spherical Hankel functions of the correct asymptotic form to the oscillator-single-particle form factor.² Alternately, one

²⁶ F. G. Perey and D. Saxon, Phys. Letters 10, 107 (1964).

²⁷ Code DWUCK was written (in FORTRAN IV) by P. D. Kunz, University of Colorado. We are indebted to Dr. Kunz for making this program available to us, and for providing us with a complete set of instructions.

²⁸ The majority of DWBA curves shown in this paper were obtained with Code DWUCK at the University of Pittsburgh IBM 360-50 computer. The small (32-bit) word size of the IBM 360 necessitated that part of DWUCK be run in double precision, which led to a running time to about 8 min for a complete angular distribution. The inclusion of spin-orbit coupling in (d, α) increased the IBM 360 computing time (and cost) further. In order to reduce computing costs, code DWUCK was also run at the Carnegie-Mellon University Univac 1108 computer. This machine has a 36-bit word size (which eliminates the need for double precision) and is considerably faster. As a consequence (d, α) DWUCK runs including spin-orbit coupling now take only 20 sec each. The improvement factor in computing cost was better than 15.

²⁹ N. K. Glendenning, University of California Lawrence Radiation Laboratory Report No. UCRL-18268, 1968 (unpublished). ³⁰ Computer Code MIFF (unpublished) was written by R. Drisko and F. Rybicki, who kindly made it available to us.

can use the method of Drisko,¹⁹ who starts with more realistic finite-well single-particle wave functions φ_{i_1} and φ_{i_2} , which are expanded in a suitable orthogonal set of harmonic-oscillator wave functions $u_{nl}(r)$,

$$\varphi_j(r) = \sum_{n=1}^{10} \alpha_n u_{nl}(r).$$
(4)

The expansions always have one dominant term which corresponds to the oscillator-single-particle wave function $u_{nj}(r)$ used by Glendenning²; however, generally seven to ten terms are needed in order to obtain good overlaps (>99%) in the expansions.

In this study, the finite wells used to generate $\varphi_i(r)$ were real Saxon wells,³¹ with the radius parameters $r_0 = r_c = 1.25$ F, diffusivity a = 0.65 F, and $\lambda = 25$ for the spin-orbit interaction. The well depth is adjusted by code MIFF³⁰ to reproduce the specified separation energy E_S of the nucleon. Generally, the separation energies E_S for two transferred nucleons are not uniquely known. The energy balance merely determines $E_{S_1}+E_{S_2}$. The procedure adopted here was to consider the two possible sequential pickups and take the averages of E_8 .³²

Inasmuch as the product $\varphi_{j_1}(r)\varphi_{j_2}(r)$ becomes a sum of many harmonic-oscillator states involving different n_1 and n_2 , the Moshinsky transformation leads to a more complicated (c.m.) motion. For a relative s state of the transferred nucleons, the Moshinsky selection rule is

$$2(n+N)+L=2(n_1+n_2)+l_1+l_2.$$
 (5)

For oscillator-single-particle wave functions in Zn⁶⁸, for instance, $2(n_1+n_2)+l_1+l_2=10$; however, with finite-well wave functions the expansion of Eq. (4) leads to $2(n_1+n_2)+l_1+l_2 \leq 46$. The higher values of n_1 and n_2 lead to more and higher values for n and N, and more factors $g_{NL\gamma}$. Nevertheless, our dominant term $\beta_{\gamma NLSJT} g_{NL\gamma}$ is within about 1%³³ of Glendenning's dominant structure factor $G_{NLSJT\gamma}$.³⁴

B. Asymptotic Behavior of $f_{L\gamma}(R)$

If the nucleon separation energies E_{S1} and E_{S2} have been chosen such that $E_{s_1}+E_{s_2}=E_{n+p}$ (i.e., the correct separation energy of the neutron-proton pair), $f_{L\gamma}(R)$ will be realistic in the nuclear surface and interior, but regardless of the number of terms used in the expansions

it will show a somewhat too-rapid falloff at large radii, since $E_{n+p} = E_{S_d} + 2.235$ MeV. It is possible to search for a solution $f_{L\gamma'}(R)$ of a deuteron finite-well potential with adjustable parameters r_0' , and a', which accurately reproduces $f_{L\gamma}(R)$ for radii smaller than, say, 10 F, but in addition (by proper choice of V_0) possesses an asymptotic radial dependence which reflects the correct deuteron separation energy E_{S_d} . Code MIFF includes a search option of this kind, which had been used to study whether a prescription could be made for the geometry appropriate for a cluster-model form factor. It was found that for 2p-1f configurations the overlaps of $f_{L\gamma}'(R)$ with $f_{L\gamma}(R)$ are often surprisingly good ($\approx 99\%$). Typical best-overlap deuteron-well parameters were always close to $a' \approx 0.8$ F and $r_0' \approx 1.15$ F. When both nucleons have large l (as for $f_{7/2}^2$), overlaps are poorer, but even in the latter case the predictions for the angular distributions differ only insignificantly. Generally, it was therefore not necessary to read in external form factors for our DWUCK calculations in the 2p-1f shell. While the use of equivalent-cluster form factors $f_{L\gamma'}(R)$ is in no way essential for our calculations, the invariance of the results for $f_{L\gamma}$ and $f_{L\gamma}'$ indicates that the asymptotic "defect" of the neutron-proton form factors $f_{L\gamma}(r)$ does not significantly affect the predicted cross section.

C. Effects of Finite-Range and Nonlocality Corrections

The finite range²⁵ and nonlocality²⁶ corrections possible with code DWUCK did, indeed, produce significant improvements, as might be expected for reactions like (d, α) which have contributions from a large nuclear interior and exterior region. Finite-range and nonlocaity corrections separately and together had very similar effects in that they produced more correct slopes for the angular districutions. Figure 7 illustrates the effect for L=0 transitions for the commonly used nonlocality parameters $\beta_{deut} = 0.54$ and $\beta_{\alpha} = 0.2$, and a finite-range parameter $R_{d,\alpha} = 0.4$. Inasmuch as there was no previously suggested value for $R_{d,\alpha}$, the value 0.4 was found empirically. It is somewhat smaller than finite-range parameters used for other reactions²⁷ $(0.6 \le R \le 1.0)$. The agreement of data and dashed curve in Fig. 7 can be considered perfect, for it is expected that actual transitions to J^{π} = 1⁺ will generally have nonvanishing L=2 contributions which would fill in the predicted deep L=0minima. The effects of finite-range and nonlocality corrections were small for L=2, but very significant, and again quantitatively correct for higher L values (see Fig. 4).

D. Effect of Optical-Model Parameters on Calculated **Angular** Distributions

For the past years many investigators have used Wood-Saxon deuteron scattering potentials char-

^{at} R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240, 1962 (unpublished).

³² It was verified that for the $Zn(d, \alpha)$ reaction changes of several hundred keV in E_S would not noticeably affect the calculated angular distributions. However, it is not certain that the error remains small for more highly excited states. If the final configuration is, for instance, $[1/7n^2/2p_{3/2}^*]_{j}^{-1}$ the $f_{7/2}$ neutron is much more deeply bound than the $p_{3/2}$ proton, and average values E_{g} could be in error by several MeV. ⁸⁸ Y. S. Park, Ph.D. thesis, University of Pittsburgh, 1968 (unpublished)

⁽unpublished)

³⁴ N. K. Glendenning, University of California Radiation Laboratory Report No. UCRL-18270, 1968 (unpublished).



FIG. 7. Comparison of typical L=0 Zn⁶⁸ (d, α) data with DWBA calculations. The effect of finite-range and nonlocality corrections is demonstrated for L=0. The improved calculation shows nearly perfect agreement with the data. Note that the positions of maxima and minima are hardly effected. The effect of these corrections is smaller for L=2 and larger for L=4.

acterized by real-well depths of about 100 MeV and $r_0 \simeq 1.1$ F. The arguments in favor of this choice seem widely accepted and we have seen no reason to modify parameters that have been derived from fits to elastic deuteron scattering.³⁵ However, it was investigated (for L=4) whether the inclusion of spin-orbit coupling in the deuteron channel would modify the calculated (d, α) cross sections, and deuteron potential set 2 in Table II was used for this test. Small changes for $\sigma_{d,\alpha}$ were found at angles past the main stripping peak, but deviations from the potential set-1 predictions were not well correlated with the small experimental deviations often seen in Fig. 4. Hence, all DWBA curves shown in this paper were calculated with deuteron potential set 1.

There is, of course, no spin-orbit effect for α scattering, but it is for the α channel that we find a strong and most interesting sensitivity to optical-model families. A comprehensive optical-model analysis of elastic α scattering at 24.7 MeV was published by McFadden and Satchler³⁶ in 1966, which covered many targets from O¹⁶ to U. Three different potentials, which gave equally good fits $(X^2 \approx 1)$ for natural Cu over the measured range $10^{\circ} \le \theta \le 120^{\circ}$, are listed in Table II. $Zn(d, \alpha)$ calculations were made with all three potentials and are compared with typical data in Figs. 8(a) and 8(b). Figure 8(a) shows a comparison of zero-range calculations with L=0 data. Figure 8(b) shows a comparison of L=2 and L=4 data with theoretical curves which include all corrections discussed above. It is quite apparent that parameter set 2 ($V \approx$ 158 MeV, solid lines) produces good agreement with all data, while the shallower potential (92 MeV)-as well as the deeper potential (197 MeV)-fail for at least two of the three L values shown. Similar tests have been made for $N^{14}(d, \alpha)$, $Ti^{48}(d, \alpha)$, and $Pb^{208}(d, \alpha)$ angular distributions,³⁷ and it was consistently found that shallow α potentials failed to produce agreement with experiment, while potentials near $V \approx 160$ MeV were very successful. Plausibility arguments in favor of deep α potentials have been given before, although no definite preference could be established from elastic scattering analyses.³⁶ We find that the complete suppression of contributions from the nuclear interior does not give good DWBA predictions. In this respect, it is particularly interesting that the deepest (197-MeV) potential also failed badly in reproducing the data, particularly for L=0. Hence, this (d, α) study suggests that the most useful α potential lies near 160 MeV, and in this sense reduces the previous ambiguity for α potentials.

V. EXPERIMENTAL RESULTS

The resolution of 12 keV for the $Zn^{68}(d, \alpha)Cu^{66}$ data (Fig. 3) was sufficient to obtain differential cross sections for 45 levels below 3-MeV excitation. More highly excited levels were not analyzed, because the probability of misinterpreting unresolved multiplets increases rapidly above 2.5-MeV excitation in Cu⁶⁶. The extracted (d, α) angular distributions are displayed in Fig. 4, where they are compared with DWBA calculations (solid and dotted lines) obtained in the manner described above. Theoretical curves were calculated for excitation energies of 1.0 and 2.0 MeV, and were found to be nearly indistinguishable. The data seem to bear out this insensitivity to Q. (Note the equally good agreement for the strongly structured pure L=0 curves at 1.339, 2.449, and 2.863 MeV in Fig. 4.) In regions of strong configuration mixing, deviations from pure L transfers are expected for most J_{odd}^+ and J_{even}^- final states, i.e., for about half the transitions, and it is seen in Fig. 4 that the (incoherent) addition of two L values greatly improves the agreement of theory and data for many levels. In judging the quality of the agreement, it must be considered that apart from absolute scale normalizations and L mixing (where indicated) no adjustable parameters were used for the roughly 100 $Zn(d, \alpha)$ angular distributions shown in this and in the following article (part II).³⁸ It is felt that for all reasonably strong transitions which show agreement with the DWBA curves, dominant L

1076

G. M. Perey and F. G. Perey, Phys. Rev. 152, 923 (1966).
 L. McFadden and G. R. Satchler, Nucl. Phys. 84, 177 (1966).

 ³⁷ W. W. Daehnick, Y. S. Park, and M. B. Lewis, Bull. Am. Phys. Soc. 13, 1462 (1968).
 ³⁸ Y. S. Park and W. W. Daehnick, following paper, Phys. Rev. 180, 1082 (1969).

]	Real-well p	parameters			In	naginary-w	ell parame	ters
Channel	Set	Ref.	V (MeV)	(F)	7 0 (F)	<i>r</i> _c (F)	V_{so} (MeV)	W_{vol} (MeV)	W_{surf} (MeV)	a' (F)	r ₀ ' (F)
<i>d</i> -Zn	1	35	111.3	0.886	1.038	1.038	•••	•••	72.16	0.736	1.307
	2	35	114.5	0.84	1.028	1.028	7.14	•••	47.44	0.815	1.297
α-Cu	1	36	92.3	0.557	1.507	1.5	•••	16.7	•••	0.557	1.507
	2	36	157.8	0.544	1.443	1.5	•••	21.9	•••	0.544	1.443
	3	36	196.8	0.541	1.416	1.5	•••	24.6	•••	0.541	1.416
p-Cu	1	с	45.67	0.668	1.301	1.25		69.76	•••	0.343	1.305

TABLE II. Optical-model parameters which were considered in the present study. The calculations for Figs. 4 and 6 were made with d-Zn set^a 1, α -Cu set 2^b and p-Cu set 1^o.

^a Reference 35.

180

^b Reference 36.

^e F. G. Perey, Phys. Rev. 131, 745 (1963).

values can be extracted with confidence. The maximum cross sections $\sigma_{\max}(d, \alpha)$ listed in Table I refer to the largest values actually measured. Numbers in brackets are values at $\theta = 15^{\circ}$.

Typical Cu⁶⁵(d, p)Cu⁶⁶ spectra showed resolutions of about 8 keV (Fig. 5). The measured Cu⁶⁵(d, p)Cu⁶⁶ angular distributions are compared with DWBA calculations for single-nucleon transfers in Fig. 6. The calculations, which include finite-range and nonlocality corrections and a spin-orbit term (λ =25) in the neutron form factor, were performed with code DWUCK,²⁷ and without adjustable parameters. Details of these calculations are presented in the following paper.³⁸ The predicted (d, p) angular distributions change markedly with orbital angular momentum transfer l, and the agreement with experiment in the angular region investigated is sufficiently good to permit reliable extraction of dominant l values for almost all transitions. When mixing of l values occurs, the l assignments and, in particular, the extracted



FIG. 8(a). The data of Fig. 7 are compared with zero-range L=0 DWBA calculations based on different "equivalent" sets of optical-model parameters for α scattering. It is seen that only parameter set 2 (see Table II), i.e., the one used in Fig. 7, leads to acceptable predictions. (b) Comparison of typical L=2 and V=4 data with calculations based on different families of α parameters. All DWBA curves are corrected for finite-range and nonlocality effects. It is seen again that only set 2 leads to acceptable results.



FIG. 9. Distribution of spectroscopic strengths of various singleparticle states for $\operatorname{Cu}^{65}(d, p)\operatorname{Cu}^{66}$. The height of the vertical bars is proportional to S. Numbers above the bars refer to our best J^{τ} estimates. The scales at the right-hand side of the graphs indicate predicted spectroscopic sums for states of given J^{τ} in the absence of core excitation in Cu^{65} . All observed l=4, 3, and 2 transfers are compared to $g_{7/2}, f_{5/2}$, and $d_{5/2}$ spectroscopic sums, respectively.

spectroscopic factors, are less certain. Detectable admixtures are found for about 15 of 53 analyzed angular distributions.

The (d, p) DWBA predictions for l=0 did not agree well with the data for $\theta > 30^\circ$ (see curves for the 2.581-MeV level in Fig. 6). Hence, an empirical curve for l=0 was used in order to ascertain l=0 contributions. $\sigma_{\max}(l=0)$ quoted in Table I is an extrapolation for the l=0 main stripping peak. The cross section listed is the value at $\theta=0$ of the (l=0) DWBA curves fitted to the experimental points at 8°, 16°, and 20° (lab), and is about 60% larger than the largest actually measured cross section (at 8°). We feel that the extracted l=0spectroscopic factors are the least reliable ones and assign errors of $\pm 30\%$ to our procedure of extracting them, in addition to uncertain errors due to the DWBA treatment.

For all higher l values, the main stripping peak was observed, and the values $\sigma_{\max}(d, p)$ refer to directly measured cross sections. Random errors at the peak are generally small, but the absolute values of $S_{d,p}$ are subject to the $\pm 15\%$ experimental scale uncertainty. Since our data do not allow the distinction between $2p_{1/2}$ and $2p_{3/2}$ transfers, the l=1 spectroscopic factors given in Table I constitute an average; the actual $p_{1/2}$, and $p_{3/2}$ spectroscopic factors should be 4.5% larger and smaller, respectively. The l=2 curves were calculated for $d_{5/2}$ transfers. For eventual $d_{3/2}$ assignments the spectroscopic factors should be multiplied by 1.27. The l=3 spectroscopic factors are calculated for $1f_{5/2}$ transfers. The correction factor for $1f_{7/2}$ transitions would be 0.68. Finally, l=4 transitions were treated as $1g_{9/2}$. The spectroscopic factor correction for eventual $1g_{7/2}$ assignments would be 1.67. These correction factors show that the effect of the spin-orbit term in the form factor on the absolute cross sections can be quite considerable, i.e., $\pm 19\%$ for l=3 and $\pm 25\%$ for l=4. Its effect on the angular distributions is barely noticeable.

The (d, p) angular distributions and extracted l values can be compared to those obtained by Hjorth and Allen¹⁷ at 15-MeV bombarding energy. Up to 1-MeV excitation energy, very good agreement was found. Above 2-MeV excitation, where the level density is $\sim 40/\text{MeV}$, the groups observed in Ref. 17 do not generally correspond to resolved levels.

The (d, p) levels should also closely resemble those seen in Cu⁶⁵ (n, γ) by Shera and Bolotin.⁵ Up to about 2 MeV, the same positive parity states are seen in (n, γ) and (d, p), although the (n, γ) level energy assignments are systematically (up to 0.4%) higher than our measurements and the earlier MIT values¹⁸ (see Table I).

In Fig. 9, we show the distribution of the spectroscopic strengths and their sums for l=1, 2, 3, and 4transitions. We find practically all of the expected 2pand $1f_{5/2}$ strength²³ below 3 MeV. The 2p strength is widely fractionated, but there are only about five l=3 transitions of appreciable strength. This absence of pronounced fractionization led to the hope that some J^{π} assignments³ could be made on the basis of (d, p)spectroscopic strength.⁶ Unfortunately, the assignments of 4^+ and 3^+ to the two strongest $f_{5/2}$ transitions are in conflict with recent (n, γ) studies,⁵ which demand the reverse order $(3^+, 4^+)$, and it appears that the more general sum rules of Ref. 6 which allow for core excitation effects must be used. A similar difficulty seems to exist for l=4. Here only four levels are seen below 3 MeV, but as only about 50% of the $1g_{9/2}$ strength is accounted for, there must be several other, more highly excited $g_{9/2}$ components. Nevertheless, the temptation remains to accept the assignment¹⁷ $J^{\pi} = 6^{-1}$ for the very strongly populated level at 1.152 MeV. In the subsequent discussion of individual levels, we will continue to give some weight to the magnitude of spectroscopic factors, but it must be kept in mind that deviations of 20-40% from the simplified spectroscopic sum rules shown in Fig. 9 seem to occur in this nucleus.

VI. J^{π} ASSIGNMENTS: DISCUSSION OF INDIVIDUAL LEVELS

For the lower-lying levels $(E_x < 1.5 \text{ MeV})$, it is virtually certain that the same states are seen in

 $(d, \alpha), (d, p)$, and (n, γ) experiments if the measured excitation energies agree to within a few keV. In some cases our J^{π} limits for (d, p) and (d, α) overlap only for a single J^{π} (usually 3^+); for others, a $J^{\pi}=1^+$ assignment from L=0 in (d, α) is rigorous, especially if (d, p) data confirm the parity. No further discussion seems required for such J^{π} assignments. For levels where the evidence for J^{π} is not as convincing and/or disagreement with assignments by other investigators^{5,17} is found, a brief explanation for the preferred J^{π} values shall be given. Alternative J^{π} values consistent with but not suggested by our data are listed in parenthese in Table I.

The Cu⁶⁶ ground state is the only state in Cu⁶⁶ for which definite spin and parity assignments $(J^{\pi}=1^+)$ are listed in the current *Nuclear Data Sheets*.³⁹ The 1⁺ assignment is confirmed by the L=0 component in Zn⁶⁸(d, α)Cu⁶⁶, and is in agreement with the J^{π} limits from our Cu⁶⁶(d, p)Cu⁶⁶ data.

The first excited state seen in all reactions lies at 185 keV. Its spin is limited by angular momentum selection rules to $J^{\pi} = 1^+$, 2^+ , or 3^+ . The 2^+ value is preferred from the (d, α) work on account of a completely pure L=2 angular distribution³ (see Fig. 4); it also is the suggested (n, γ) assignment.⁵ $J^{\pi} = 1^+$ is ruled out by the (d, p) spectroscopic factor, which exceeds the maximum $f_{5/2}$ strength for 1⁺, and by the fact that the state is fed by γ decay from several levels with $J^{\pi} \ge 3^+$. The value 3⁺ is not firmly excluded by selection rules; in fact, the 185-keV level is not fed by any of the higher 1⁺ states.⁵ An important argument against a 3⁺ assignment is based on the $f_{5/2}S(d, p)$ sum rule, which prohibits the $J^{\pi} = 3^+$ assignment for more than one of the three strong $f_{5/2}$ transitions (i.e., 0.185, 0.274, or 0.588 MeV) (see Fig. 9).

In the (n, γ) work,⁵ a level at 238 keV is suggested. There is no evidence for this state in either the (d, p) or the (d, α) spectra.

The level at 274 keV is limited by (d, α) and (d, p) selection rules to 3⁺ or 4⁺. It has the largest $f_{5/2}(d, p)$ spectroscopic factor and fills the entire spectroscopic strength for the $(p_{3/2}\pi f_{5/2}\gamma)_{3+}$ configuration (Fig. 9). This fact would favor the choice $J^{\pi}=4^+$, as long as the strong level at 1.013 MeV which contains additional $f_{5/2}$ strength is a single 3⁺ level. The latter has now been shown to be a closely spaced doublet,⁵ at least one level of which is likely to have $J^{\pi}\neq 3^+$. With this assumption our data permit $J^{\pi}=3^+$ for the 274-keV level as well as 4⁺, which is in better agreement with the γ -decay scheme.

The 383-keV level is very strongly excited in (d, p) which provides the limits $J^{\pi} = 1^+$, 2^+ , 3^+ . The (d, α) angular distribution, while in fair agreement with L = (4+2), is very weak and somewhat atypical. This fact might indicate a j_{even}^2 neutron-proton configuration

and favor $J^{\pi} = 2^+$, in agreement with the strong γ transition to the 1⁺ ground state.

The 462-keV level is limited by (d, p) to $J^{\pi} = 2^+$ or 3⁺. The (d, α) transition resembles L=2, but is not well fitted. The γ transitions demand $(1^{+}2^{+})$, so that $J^{\pi} = 2^+$ remains as the only acceptable choice.

The 588-keV level must have $J^{\pi} = 3^+$ or 4⁺. It has the second-largest $f_{5/2}$ spectroscopic factor and, hence, must be $J^{\pi} = 4^+$ if the 272-keV level is assigned 3⁺ or vice versa. The 4⁺ assignment is preferred by the (n, γ) data.

The level at 727 keV is limited to $J^{\pi}=3^+$ by selection rules. It was not observed in (n, γ) . The level at 819 keV is excited by a pure L=2 (d, α) transition, suggesting $J^{\pi}=2^+(1^+, 3^+)$. It γ decays only to levels preferentially assigned as 2^+ and is most strongly fed by the 1.815-MeV 1⁺ level.

In the 1.009–1.017-MeV doublet,⁵ one member must have $J^{\pi}=3^+$ or 4⁺, and the other one $J^{\pi}=1^+$, 2⁺, or 3⁺. There is no appreciable broadening in the (d, α) or (d, p) peaks, which leads us to believe that (d, α) populates a 3⁺ state—the level, at 1.017 MeV, according to (n, γ) . (d, p) may primarily populate the 1.009-MeV state and the exhausted $f_{5/2}$ spectroscopic sum for 3⁺ leads us to favor $J^{\pi}=1^+$ or 2⁺. The level at 1.048 MeV uniquely has $J^{\pi}=1^+$ on the basis of the (d, α) angular distribution, in agreement with the (d, p) and (n, γ) limits.

The level at 1.511 MeV has a (d, p) l=4 strength close to the $g_{9/2}$ 6⁻ limit. The relatively strong (d, α) transition is dominated by L=5. If this is a single level, $J^{\pi}=6^{-}$ is the most likely assignment. However, the possibility of a negative-parity doublet must be considered because of the large l=4 strength and some weak L=3 admixture in the (d, α) angular distribution.

Level 12 at 1.208 MeV is classified as $2^+(1^+, 3^+)$ by (d, α) and (d, p). The (n, γ) work strongly favors 2^+ . The level at 1.247 MeV has negative parity and is not seen in (n, γ) . It is limited to $3^-, 4^-$ by (d, p) and well fitted by pure L=3 in (d, α) ; hence, $J^{\pi}=3^-$ is preferred. Levels 14 and 15 are clearly 1^+ on the basis of our (d, α) and (d, p) data. The assignment is in agreement with the (n, γ) limits.

The triplet made up of levels 16-18 consists of positive-parity states with $J^{\pi} = 1^{+} - 3^{+}$. All three states are strongly excited in (d, α) , which excludes the 0⁺ value; however, the triplet is not resolved at all angles, hence no L assignments have been made. Level 16 (at 1.543 MeV) was suggested as (3^{+}) in (n, γ) . This is in agreement with the limits from this experiment.

Level 19 at 1.679 MeV, and level 20 at 1.697 MeV, have rather unique assignments of 3^+ and 1^+ , respectively. The small cross sections, however, may cast some doubt on the assumed reaction mechanism. The weak level at 1.713 MeV has $J^{\pi}=1^+$ on the basis of the (d, α) data; however, in (d, p) it is not resolved at larger angles from its strongly excited neighbor

³⁹ Nuclear Data Sheets, compiled by K. Way et al. (Academic Press Inc., New York, 1966).

(level 22), which makes the l=(1) assignment for (d, p) somewhat uncertain.

Level 22 has the second largest $g_{9/2}$ strength in (d, p), $J^{\pi}=3^{-}-6^{-}$, and, as expected, it only weakly excited in (d, α) . The L=5 calculation fits perfectly within the large statistical uncertainties, so that 5^{-} or 4^{-} are the most likely assignments.

Level 23 postulated for 1.746 MeV in the (n, γ) study⁵ is definitely not seen in (d, α) ; it is not indicated by the (d, p) data either, but a weak level at that energy would not be resolved from level 22. A 0⁺ assignment would be consistent with all experimental observations. Level 24, also seen in (n, γ) , has definitely $J^{\pi}=1^+$. Level 25 (at 1.894 MeV) has not been reported before, and is only weakly excited in (d, p)and (d, α) . However, the (d, p) data agree very well with l=0, so that the assignment of $J^{\pi}=1^-$ or 2^- for a level at this energy seems justified.

Level 26 has $J^{\pi}=3^+$, unless we excite an unresolved doublet. Similarly, the 2⁻ assignment for level 27 is unique, if (d, p) and (d, α) excite the same 1.977-MeV level. The weakness of the (d, α) transitions must be expected for most negative-parity levels; nevertheless, it reduces the significance of the (d, α) limits and 1⁻ is not firmly ruled out.

The 2.017-MeV state (level 28) must be $J^{\pi} = 1^+$, 2⁺, or (3⁺), with 3⁺ least likely because of the exhausted $3^+(f_{5/2})$ strength. 2⁺ is most likely because of the almost pure L=2 shape in (d, α) , which definitely shows no L=0 admixture.

A level at 2.026 MeV has been postulated in the (n, γ) study. There are no indications for its existence in (d, α) or (d, p).

Level 30 (near 2.120 MeV) appears to be a multiplet. A significant l=0 component in (d, p) calls for $J^{\pi}=1^-$ or 2^- , while the strong l=4 component in (d, p) calls for 3^--6^- . The weak (d, α) angular distribution is best fitted by L=4; however, it is also consistent with L=3 ($J^{\pi}=2^--4^-$). Hence, we have at least a negative-parity doublet.

Level 31 is strongly excited by l=2 in (d, p). The (d, α) assignment is uncertain; hence, only the limit $J^{\pi}=1^{-}4^{-}$ can be given. Level 32 has been resolved at various angles, but no l assignments could be made.

Level 33 appears to be a mixed-parity doublet. The (d, p) transition demands $J^{\pi} = 1^{-} - 4^{-}$. The (d, α) transition, though very weak, agrees perfectly with $J^{\pi} = 1^+$. Near 2.266 MeV we again have a mixed-parity doublet. The strong (d, p) and (d, α) reactions obviously do not populate the same level.

The limits $J^{\pi}=1^{-}4^{-}$ claimed for levels 35, 36a, and 38a are based on the strong $d_{5/2}$ neutron transfer strengths and are self-evident. Level 36b and 38b are hole states strongly excited in (d, α) transitions, the latter uniquely showing $J^{\pi}=1^{+}$. The angular distributions for level 37 (at 2.391 MeV) are weak and hence not conclusive. Several possible l and L values are indicated in Table I.

Level 39 is weakly excited in (d, p) by l=(1), and

in (d, α) by L=(2). Tentatively assigned limits are $J^{\pi}=1^+$, 2^+ , or 3^+ . Level 40, stronger in (d, p) has J^{π} limits 1^- to 4^- . The negative parity is reflected in the very weak (d, α) transition.

Level 41, at 2.537 MeV, is only seen in (d, α) . The angular distribution is well represented by L=0. The 1⁺ assignment is called tentative because of the weakness of the transition.

Above 2.5 MeV many l=0 or l=0+2 transitions are seen in (d, p). These levels must have $J^{\pi}=1^{-}$ or 2⁻. As might be expected they are not, or only weakly, excited in (d, α) ; this fact supports the 1⁻, 2⁻ assignments for the levels 42, 44, 45a, 47, 48, 53a, 54, 55, 58, 62, and, 63.

The state at 2.567 MeV (level 43) is very strongly excited in (d, α) and not seen at all in (d, p). Hence, it seems reasonable to assume that we excite an $f_{1/2}$ hole configuration. The contributing L values are L=6and, very probably, L=4, and thus lead to a $J^{\pi}=5^+$ assignment. On the other hand, data and DWBA curves do not agree well above 60° [See Fig. 4(c)]. As there is negligible uncertainty in the data, the disagreement indicates the need for improvements in the L=6 calculation or a doublet. Hence, we must call the L=4 admixture tentative, and cannot rule out 7⁺ or 6^+J^{π} values with certainty.

Level 45b at 2.604 MeV has L=0+(2) and, consequently, $J^{\pi}=1^+$. The assignment is marked tentative because of the small (d, α) cross section. Level 46 again is only observed in (d, α) and well represented by L=4; hence, $J^{\pi}=4^+$, 5⁺, or 3⁺.

Level 49 at 2.682 MeV is weakly excited in (d, p), consistent with l=1 or 2, but shows a very strong L=0+2 transition in (d, α) , hence $J^{\pi}=1^+$. Level 50 represents another hole state with L=4, $J^{\pi}=4^+$, 5^+ , or 3^+ .

Level 51, at 2.743 MeV, is seen in both reactions with l=2 and L=3, respectively. The assignment is 3⁻, (2⁻), or (4⁻).

Level 52 is populated primarily by L=0, thus $J^{\pi}=1^+$. Level 56 at 2.863 MeV shows a strong, pure L=0 transition, which again determines $J^{\pi}=1^+$.

Level 57 has a strong L=4 component in (d, α) . The fragmentary (d, p) angular distribution agrees best with l=1; hence, the tentative assignment is (3^+) .

If level 60 is a singlet, $J^{\pi}=4^{-}$ is the only assignment consistent with the (d, p) and (d, α) data. However, the weak (d, α) cross sections agree best with L=6, hence a doublet at this energy is a possibility.

A level near 3.007 MeV is relatively strongly excited in both reactions by l = (3) and L=4, respectively, and judged to have $J^{\pi} = (4^+)$. The 4⁺ assignment helps augment the greatly deficient $f_{5/2}(4^+)$ strength, while 3⁺ would further aggravate the $f_{5/2}(3^+)$ surplus strength.

VII. SUMMARY AND CONCLUSIONS

The study of Ni⁶¹ and Cu⁶⁶ by high-resolution (d, α) reactions has yielded information on almost all known levels in these nuclei below 3-MeV excitation. In some

contrast to the situation in very light nuclei,4 few transitions were markedly enhanced. The strongest transitions observed in our early work with ≤ 60 -keV resolution⁹ for $Cu^{63}(d, \alpha)Ni^{61}$ invariably have been found to populate multiplets of two or more levels (see Fig. 2). Similarly, the strongest transition observed in the high-resolution $Zn^{68}(d, \alpha)Cu^{66}$ study $(\sigma_{max}=84)$ $\mu b/sr$) appears to lead to an unresolved doublet⁵ near 1.013 MeV. Many transitions are observed with peak cross sections between 9 and 20 μ b/sr. We tend to ascribe the apparent lack of selectivity in these (d, α) transitions to considerable configuration mixing in the 1f-2p shell. The same argument would explain the fact that evidence for appreciable contributions from two L values was seen in almost half of all $Zn^{68}(d, \alpha)$ angular distributions.

The investigation of the energy dependence near $E_d = 12$ MeV in the Cu⁶³(d, α) Ni⁶¹ reaction yielded good evidence that for this mass and bombarding energy region (d, α) reactions are direct, and helped justify our attempt at a detailed DWBA analysis of the angular distributions observed. The use of current twonucleon transfer theories^{2,19} and refined DWBA codes²⁷ led to very good agreement between measured and calculated angular distributions. The empirical $L_{d,\alpha}$ assignments of Ref. 3 were fully confirmed by the calculations, and as a consequence L and J^{π} assignments or narrow J^{π} limits were obtained for all states significantly excited by (d, α) . Where possible, a comparison of the (d, α) assignments with our $\operatorname{Cu}^{65}(d, p)\operatorname{Cu}^{66} l$ values as well as with previous (n, γ) studies⁵ was made. Good agreement was found, and the (d, p) and (n, γ) information helped to support J^{π} assignments or further narrow down J^{π} limits.

The (d, p) studies provided additional new J^{π} information, particularly on negative-parity states, which are weakly excited in (d, α) . Frequently, large spectroscopic factors were taken into consideration if the (d, α) , (d, p), and (n, γ) selection rules did not yield a unique choice for J^{π} . The total extracted spectroscopic strength for l=1 and l=3 transitions happened to agree to within about 10% with the expected sums.^{6,28} However, predictions for transitions to particular J^{π} levels by the simplified sum rules,⁷ shown in Fig. 9, at times led to much greater discrepancies. Hence J^{π} assignments based predominantly on spectroscopic strengths do not appear reliable in the Cu⁶⁶ region. Recent theoretical work has cast doubts on the customary³¹ DWBA method of extracting spectroscopic factors S_i for levels significantly removed in energy from the corresponding single-particle states.⁴⁰ This problem definitely arises here for the extracted l=2 and l=0 spectroscopic strengths, which are listed in Table I for completeness only. The larger l=3 and l=1 transitions are probably treated with adequate accuracy, and it is possible, but not certain, that the ordinary Saxon-well form factors here are still sufficiently reliable for the $g_{9/2}$ transfers. Nevertheless, a further study of this question is indicated. The failure to get good agreement between calculated and observed l=0 angular distributions might well be due to the crudeness of the (d, p) form factors used.

The DWBA studies for (d, α) have demonstrated the importance of finite-range and nonlocality corrections and revealed a very strong preference for the family of optical-model α parameters characterized by a real potential depth of $V_0 \approx 160$ MeV. Microscopic form factors based on pure (1f1f), (1f2p), or (2p2p)neutron-proton configurations for positive-parity states and on (1g1f) or (1g2p) configurations for negativeparity states produced very similar calculated L-dependent angular distributions. Configurations involving at least one low l value (2p) led to microscopic form factors which can be reproduced to high accuracy by (renormalized) deuteron-cluster form factors generated by real Woods-Saxon wells with $r_0 \approx 1.15$ F, $r_c = 1.3$ F, and a=0.8 F, and overlaps of 98% are easily obtained. For (1g1f) and (1f1f) configurations the best cluster well geometrics were similar, but overlaps dropped to $\sim 90\%$. It is concluded that for the prediction of typical (d, α) angular distributions in the 1f2p shell reasonable assumptions for the microscopic form factor or correctly chosen "cluster" form factors are adequate, and that $L_{d,\alpha}$ transfers usually can be deduced without explicit knowledge of the nuclear wave functions.

It appears, therefore, that the study of direct (d, α) transitions is a powerful spectroscopic tool for the investigation of odd-odd nuclei, even where detailed nuclear wave functions are not available. An analysis of the absolute cross sections can produce information on the spectroscopic amplitudes not otherwise obtainable,^{2,22} but it depends critically on the existence of explicit theoretical structure predictions. Hence, an analysis of the absolute $\operatorname{Zn}^{68}(d, \alpha) \operatorname{Cu}^{66}$ cross sections could not be performed at this time.

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⁴⁰ R. J. Philpott et al., Nucl. Phys. (to be published).