Description of ^{16}O-^{16}O Elastic Scattering near the Coulomb Barrier*

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The excitation function at 90° in the c.m. system, just above the Coulomb barrier, and an angular distribution at 13-MeV c.m. energy, were analyzed with a conventional complex potential of Woods-Saxon shape. The excellent fit obtained led to almost the same parameters that were used to describe ^{16}O-^{16}O scattering data at higher energies by Siemssen *et al.* A real ^{16}O-^{16}O interaction with a repulsive core does not fit the excitation function at c.m. energies from 10–14 MeV, contrary to the conclusion of Brueckner *et al.* When absorption is added, the Coulomb potential masks the nuclear interior, and thus the elastic scattering data cannot be used as sensitive evidence for the existence of a repulsive core. The conclusion of Brueckner *et al.* seems to be affected by incorrect phase shifts.

I. INTRODUCTION

A STRAIGHTFORWARD method for calculating the potential between heavy nuclei has been presented recently by Brueckner, Buchler, and Kelly.[1] They find a pure real potential with a shallow attractive part and a strong repulsive core at short range. This potential was used subsequently to explain the elastic scattering of ^{16}O on ^{16}O for energies just above the Coulomb barrier. The high quality of the theoretical fits to the existing data of Bromley *et al.*[2] is very surprising, since for heavy nuclei channels other than the elastic one are open as soon as the Coulomb barrier is reached. The effect of these other channels would normally be accounted for by the inclusion of a complex term in the potential. In general, the elastic-scattering cross section of strongly absorbed charged particles at low energies does not depend very sensitively on the interior region of the potential. Therefore the presence or absence of a repulsive core in the potential should be hard to detect from the analysis of the elastic scattering data discussed above.

The use of a molecular type of potential such as that employed to explain α-α scattering phase shifts,[3] is very tempting for heavy nuclei scattering and in fact such an application exists already to ^{16}O-^{16}O scattering at high energies.[4]

Here an attempt was made to reproduce the calculated ^{16}O-^{16}O excitation function for energies up to 14 MeV in the c.m. system at a c.m. angle of 90° as well as the 13-MeV angular distribution using the real potential with a repulsive core of Brueckner *et al.*[1] It was found (i) that the calculated phase shifts η_l of Ref. 1 are incorrect, and (ii) that the high quality of the theoretical fits to the data could not be reproduced. In Sec. II the points (i) and (ii) will be demonstrated explicitly. It is also shown that the addition of an absorptive term to the real repulsive potential gives a better fit to the experimental data. However, a conventional complex Woods-Saxon potential yielded a completely satisfactory fit to the ^{16}O-^{16}O data near the Coulomb barrier.

II. RESULTS AND DISCUSSION

It should first be remarked that the total potential displayed in Fig. 1, and the data points of Figs. 2 and 3 were taken from figures in the respective references.[1,2] In particular, the data of Bromley *et al.*[2] may be much more accurate than represented here.

The theoretical potential W_0 of Ref. 1, is described here by the function

$$\begin{aligned} W_0(r) &= A\exp[-(r/b)^2] \\ &\quad -U/[1+\exp[(r-R)/a]]+V_c(r), \\ V_c(r) &= Z^2e^2/r, \qquad r\geq c \\ &= Z^2e^2(3-r^2/c^2)/2c, \qquad r<c. \end{aligned} \tag{1}$$

The values of the parameters that give the same W_0 as Ref. 1 are

$$A=88\text{ MeV}, \quad b=2.2\text{ F}, \quad c=3.8\text{ F},$$
$$U=30\text{ MeV}, \quad R=5.8\text{ F}, \quad a=0.5\text{ F}.$$

The resulting molecular-type of potential is illustrated in Fig. 1 and is a reasonably accurate representation of the one given in Ref. 1. For the calculations of the phase shifts and cross sections using this real potential two independent methods were employed. In one method the Schrödinger equation is numerically integrated in essentially a conventional manner. In contrast to the conventional optical-model calculation programs[5]

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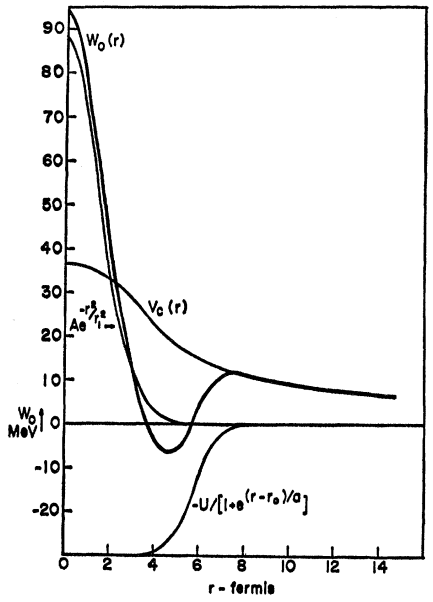

FIG. 1. The potential W_0 for the ^{16}O-^{16}O system.

the region of integration was divided into 2000 subintervals. For energies near the Coulomb barrier any rapid variation of the potential function causes appreciable changes in the calculated phase shifts. Therefore the small mesh size employed was necessary to ensure convergence.

The second method is based on numerical developments by Purcell[6] of a generalized R-matrix approach.[7] The scattering matrix is evaluated for each partial wave in a representation of radially truncated harmonic oscillator functions. There is no numerical integration of a differential equation involved. This latter approach was checked by reproducing the phase shifts for α-α scattering from the repulsive core potentials given by Ali and Bodmer[8] and by obtaining "perfect" agreement with the first method.

In each calculation all even l values up to and including 10 were incorporated. The corresponding phase shifts, calculated from W_0 shown in Fig. 1, are given in Fig. 2; two phase shifts of Ref. 1, divided by 2, are displayed for contrast. (The representation of the phase shifts in Ref. 1 is unusual, since they are zero at low energies and then at the top of the Coulomb barrier go upwards through very nearly two cycles of π.)

In Fig. 3, the 13-MeV angular distribution calculated with W_0 is compared with the data taken from Ref. 2. The fit is not satisfactory, in contrast to the one displayed in Fig. 5 of Ref. 1. An improvement of the fit is immediately seen when an imaginary term of magnitude $W=1.3$ MeV (as an example) is added to the real potential W_0 of Eq. (1). This complex potential, with a real repulsive core, will be referred to as W_0'.

In Fig. 4, it is seen that the ratio of the excitation function to Mott scattering at 90° in the c.m. system, calculated again with the real potential W_0, has a very large peak beginning at about 11 MeV. This is just the energy at which the low partial waves begin to reach the top of the Coulomb barrier. The corresponding phase shifts then go through $\frac{1}{2}\pi$, slowly enough to correspond to the large probability for transmission through the barrier (Fig. 2). There are also phase shifts that go sharply through $\frac{1}{2}\pi$. These correspond to a small probability of transmission through a thick part of the barrier. An example is the $l=8$ phase shift at 13.5 MeV, which causes a dip in the excitation function (Fig. 2). Such resonance behavior can be expected for any real potential of the type given in Eq. (1), with or without the core. Therefore one should not expect to fit the data, which show no large resonances at these energies, with a real potential; and curve 1 in Fig. 4 indeed is in contrast to the excellent fit claimed in Fig. 6 of Ref. 1. Again as soon as absorption is included, by using the potential W_0', the calculation (curve 2 of Fig. 4) comes closer to experiment. The inclusion of an imaginary absorptive term W in the real potential produces an averaging of the nuclear phase shifts over an energy range equal to twice the

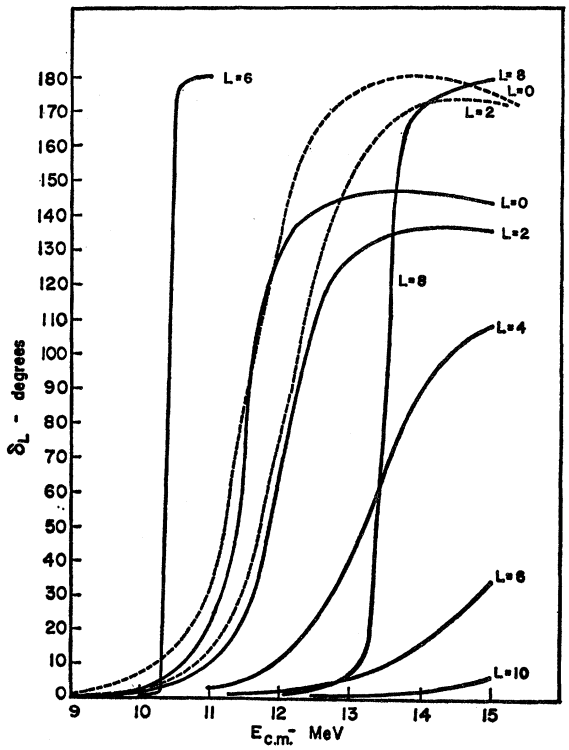

FIG. 2. Phase shifts calculated from the potential W_0 (solid line), and the first two phase shifts for W_0 from Ref. 1, divided by 2 (dashed line).

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imaginary well strength. This averaging destroys the strong resonant behavior and tends to smooth out the calculated cross section.

Before one continues to fit the data by varying the parameters of the complex potential W_0', one must ask what significance there would be in such a fit. A search was made in the parameters $U+iW$, R, and a of a complex Woods-Saxon potential [Eq. (1) with $A=0$], for a local minimum of the function D, which

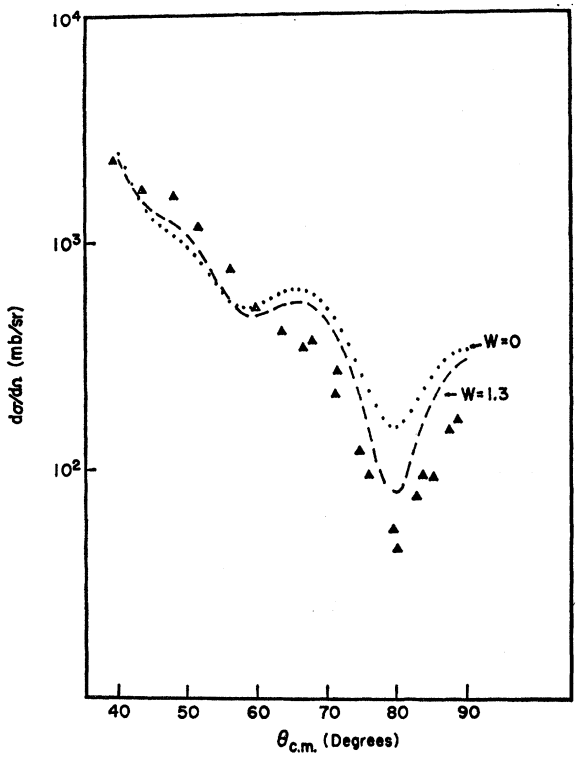

FIG. 3. The 13-MeV (c.m.) angular distribution calculated with repulsive core potentials W_0 (real), and W_0' (complex, $W=1.3$ MeV). The triangles denote data from Ref. 1.

represents the deviation of the calculation from experiment,

$$D=\sum_{i=1}^{N}[1-\sigma_i(\text{calc})/\sigma_i(\text{expt})]^2,$$

where N is the maximum number of data points. An excellent fit was obtained (Fig. 4) when $U=16.8$ MeV, $W=1.3$ MeV, $R=6.96$ F, and $a=0.49$ F. It appears that there would be no point in adding a repulsive core even to a complex potential to fit this data, because the conventional optical model works so well. It turns out that the parameters yielding the best fits to the data are almost identical to the ones found by Siemssen *et al.*[9] at higher energies. In particular, the shallowness of the real well depth is in agreement with

[9] R. H. Siemssen, J. V. Maher, A. Weidinger, and D. A. Bromley, Phys. Rev. Letters 19, 369 (1967).

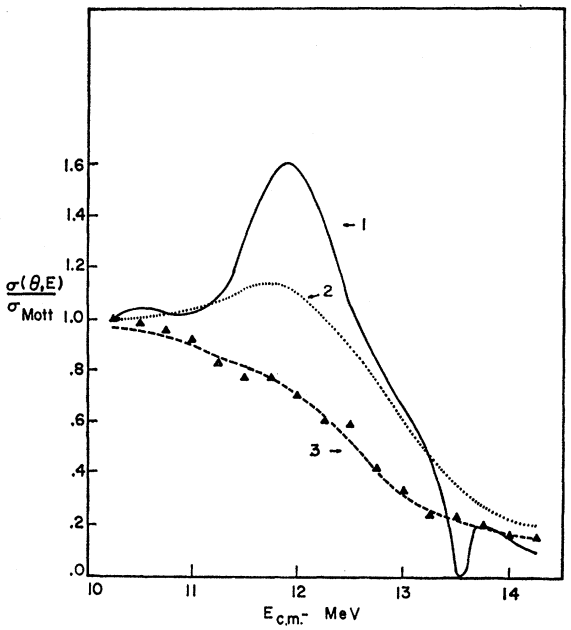

FIG. 4. Ratio of excitation function at 90° (c.m.) to Mott prediction. Calculated lines are for W_0 (solid line), W_0' (dotted line), and the best-fit optical potential (broken line) with $U=16.8$ MeV, $W=1.3$ MeV, $R=6.96$ F, $a=0.49$ F. The triangles denote data from Ref. 1.

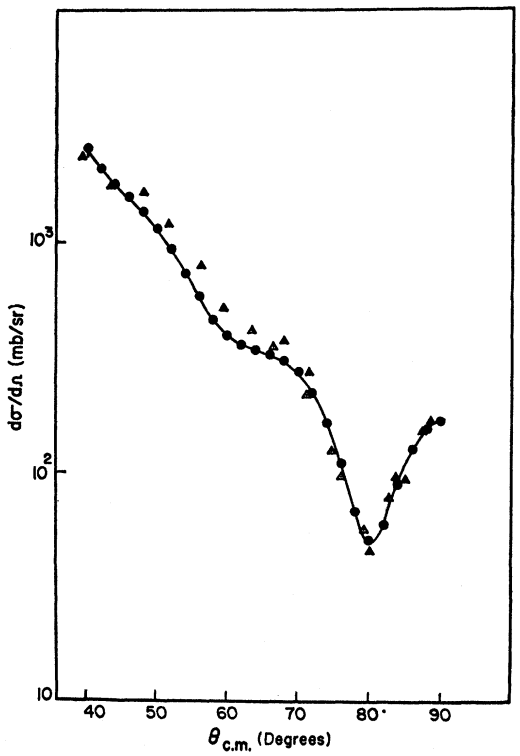

FIG. 5. The 13-MeV (c.m.) angular distribution calculated with the optical-model potential that gave the best fit of Fig. 4. The triangles are the data and the full dots are the calculated points.

recent analyses of ^{16}O-^{24}Mg and ^{16}O-^{40}Ca scattering data.[10,11] Finally, Fig. 5 shows that the angular distribution at 13 MeV is also predicted well by the Woods-Saxon parameters which fitted the excitation function, although no attempt was made to improve or alter this fit.

Where the figures contain a calculation using a complex potential, the cross sections are for coherent elastic scattering, i.e., no contribution to the elastic channel by way of the compound nucleus is included. This is justified since the compound nucleus ^{32}S is excited to more than 30 MeV for 13-MeV c.m. energy. At such a high excitation the large number of open reaction channels damps the compound elastic contribution considerably.

III. CONCLUSION

The theoretical prediction of a molecular type of potential to describe heavy-ion scattering is an interesting development in the study of such interactions. However, because of the masking of the nuclear interior by the Coulomb potential and the marked effect of other open reaction channels, the elastic scattering of ^{16}O-^{16}O near the Coulomb barrier cannot be used as reliable evidence for a repulsive core in the ^{16}O-^{16}O interaction. On the other hand, the present work does not preclude the possibility of a repulsive core in the ^{16}O-^{16}O potential. High precision experiments involving elastic scattering of heavy ions at higher energies and perhaps inelastic scattering experiments may provide justification for the theoretically predicted repulsive core. There again, great care must be taken to determine the significance of theoretical fits to experimental data using a repulsive core potential, since this potential involves introducing at least two new parameters into the potential function.

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Single-Particle and Core-Coupled States in ^{57}Co from the Decay of ^{57}Ni†

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The decay of 36.0-h ^{57}Ni produced by the ^{54}Fe(α, n)^{57}Ni reaction has been studied to obtain information on the level structure of ^{57}Co. γ-ray spectra were obtained with high-resolution Ge(Li) detectors, a Compton-suppressed Ge(Li) system, and Ge(Li)-NaI(Tl) γ-γ coincidence spectroscopy. The γ-ray energies in keV (and the relative photon intensities) observed in this investigation are as follows: 127.1±0.1 (200), 161.8±0.3 (0.22), 252.5±0.6 ($\leq$0.4), 380.0±0.2 (0.96), 673.4±0.2 (0.58), 906.8±0.3 (1.1), 1046.4±0.2 (1.6), 1223.5±0.4 (1.1), 1377.6±0.2 (1000), 1730.6±0.3 (0.72), 1757.6±0.2 (77), 1896.5±0.4 (0.28), 1919.5±0.2 (170), 2132.9±0.3 (0.47), 2730.6±0.2 (0.3), 2803.9±0.2 (1.7), and 3176.9±0.3 (0.24). On the basis of Ge(Li)-NaI(Tl) coincidence experiments, energy sums, and earlier nuclear reaction data, levels in ^{57}Co were assigned at 0 (7/2$^-$), 1223.5±0.4 (9/2$^-$), 1377.6±0.2 (3/2$^-$), 1504.7±0.2 (1/2$^-$), 1757.6±0.2 (3/2$^-$), 1896.5±0.4 (7/2$^-$), 1919.5±0.2 (5/2$^-$), 2132.9±0.3 (5/2$^-$), 2730.6±0.2 (5/2$^-$, 3/2$^-$), 2803.9±0.2 (3/2$^-$, 5/2$^-$), 3108.2 ($\leq$5/2$^-$), and 3176.9±0.3 (5/2$^-$) keV. The spin assignments are based on data from nuclear reaction studies, logft values, and relative photon intensities. An attempt was made to explain the experimental results and the structure of ^{57}Co in terms of single-particle states and particle-plus-core states based on coupling the 2$^+$ collective core vibration to the $f_{7/2}$ odd proton hole. Previous work on ^{57}Co and ^{59}Co indicates that levels in ^{57}Co at 1223.5 (9/2$^-$), 1683 (11/2$^-$), 1757.6 (3/2$^-$), and 2132.9 (5/2$^-$) keV belong to a core-coupled multiplet. Our γ-ray branching ratios are in agreement with this characterization of the states at 1223.5 and 1757.6 keV.

INTRODUCTION

THE properties of the low-lying states of ^{57}Co have been studied extensively by β- and γ-ray spectroscopy of the radioactive isotope ^{57}Ni and by studies of the nuclear reactions ^{56}Fe(p, $\gamma\gamma$), ^{58}Ni(t, α), ^{56}Fe(He^3, d), and ^{54}Fe(α, p).[1–11] Although several low-lying states have been observed in both the

† Work performed under the auspices of the U.S. Atomic Energy Commission.

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