# THE BINAURAL LOCATION OF PURE TONES.

By R. V. L. Hartley and Thornton C. Fry.

Synopsis.

*Theory of the Binaural Location of Pure Tones.*—It is pointed out that to determine two coördinates of the sound source, angular position and distance, the simultaneous observation of both phase difference and intensity ratio for the two ears is necessary. Thus, contrary to the conclusions of certain observers, intensity ratio plays an important part in sound location. Using a method of analysis due to Stokes, the authors have prepared a series of *charts showing the intensity ratio and phase difference* for sources at various distances and angular positions emitting sound with a frequency from 310 to 1,860 cycles and for two angular separations of the ears, 165° and 180°, assumed to be placed on a spherical head. These charts show the theoretical position of the source for any combination of phase difference and relative intensity which might occur with actual sources. Experiments to test this theory are suggested. When unnatural combinations occur as a result of distortion of the sound in transmission or are produced artificially in the laboratory, the interpretation of the stimuli will depend on psychological factors and cannot be explained by any purely physical theory alone.

*Psychological Readjustment of Unnatural Combinations of Binaural Stimuli.*—Suggestions as to the nature of these adjustments are advanced and the results of laboratory experiments with such unnatural combinations are interpreted with the assistance of the charts and are shown to be in agreement with this theory.

## 1. Introduction.

In a paper on "The Function of Intensity and Phase in the Binaural Location of Pure Tones" Stewart [1] compares with theory the results of experiments which he performed to determine the effect of intensity when acting alone. He concludes that "clearly there exists neither quantitative nor qualitative agreement between the two curves (experimental and theoretical) for similar frequencies. In fact, the theory shows the existence of two values of $\theta$ for one value of relative intensity, whereas experiment shows only single values. The disagreement, quantitative and qualitative, is so very great that intensity cannot be an important factor in the location of a pure tone." The same general conclusion was also arrived at by Stewart and Hovda,[2] on the basis of less comprehensive data.

This conclusion seemed to the writers to be quite the opposite of what was to be expected in the light of their earlier work,[3] and further con-

[1] G. W. Stewart, Phys. Rev., May, 1920, p. 425.

[2] Stewart and Hovda, Psych. Rev., XXV., No. 3, May, 1918, p. 242.

[3] R. V. L. Hartley, Phys. Rev., June, 1919, p. 373.

sideration showed that the apparent disagreement could be attributed to the current practice of considering the effects of phase and intensity separately and that the results are not inconsistent with a theory based on the combined effect of the two.

It is the purpose of this paper to present such a theory, to apply it to published experiments, to suggest further lines of experiment and to furnish charts to facilitate such work.

## 2. General Theory.

We must note, first of all, that two coordinates are needed to define the position of a sound image in the horizontal plane; as, for instance, the polar coordinates angle and distance. To determine these coordinates two properties of the sound are needed. That is, the correspondence between location and stimulus is necessarily a *two to two* correspondence, and not a one to one correspondence, as most investigators in the binaural field have tacitly assumed. The assumption that any particular pair of properties is capable of determining the location of the image may be verified experimentally if it can be shown that when a stimulus is artificially set up in the laboratory, having values of these properties such as would result from a given source the observer obtains as sharp and accurate an image as if the tone actually came from that source.

A pair of properties of the stimulus which are apparently capable of determining the two coordinates of the image, are the difference in phase and the relative intensities at the two ears. To each position taken by a physical source of sound there corresponds a pair of values of these properties. No other pair can correspond to that position and only one or two positions at most can correspond to that pair.[1] There are, however, pairs of values of intensity and phase which cannot possibly arise by undistorted transmission from a single physical source. If, when such stimuli are set up by laboratory means, an image is obtained at all, it must be the result of an attempt upon the part of the observer to form a compromise between discordant data furnished by his sense organs. It follows, therefore, that experiments in which pure tones of arbitrary intensity and phase are applied to the two ears should be divided into two distinct classes according as the particular combination of intensity ratio and phase difference does or does not correspond to an actual source. The first of these, when compared with experiments on the location of actual sources, makes possible a direct test of the soundness of the above assumption, while the second can contribute to the final answer only indirectly.

[1] There are exceptions to this statement when the frequencies are high but they are unimportant for the purposes of this argument.

In order, then, to predict theoretically the result to be expected from a laboratory experiment, we must first of all be able to determine whether or not the combination of phase and intensity relations corresponds to an actual source, and, if so, where that source is located. This may be done if we know the relations which exist between the position of an actual source and the phase difference and intensity ratio which result from it. By assuming the head to be a rigid sphere, it is possible[1] to compute the relative intensities and phases of the pressure produced at any two points on its surface, by simple harmonic sound from a point source. Conversely it is possible to find the relative position of source and obstacle when the phase and intensity relations are known.

The location obtained from any pair of phase and intensity relations depends, of course, upon the frequency of the sound and the size of the sphere. This dependence reveals itself in the final formula of Stokes's analysis by the presence of the two quantities, $c$, the radius of the sphere, and $k$, which is $2\pi$ divided by the wave-length of the sound, so that $kc$ is the ratio of the circumference of a great circle on the sphere to the length of the sound wave. By virtue of dynamic similarity these occur in the combinations $kc$ and $r/c$ only, $r$ being the distance of the source from the center of the sphere. It is, therefore, convenient to construct charts in terms of these parameters.

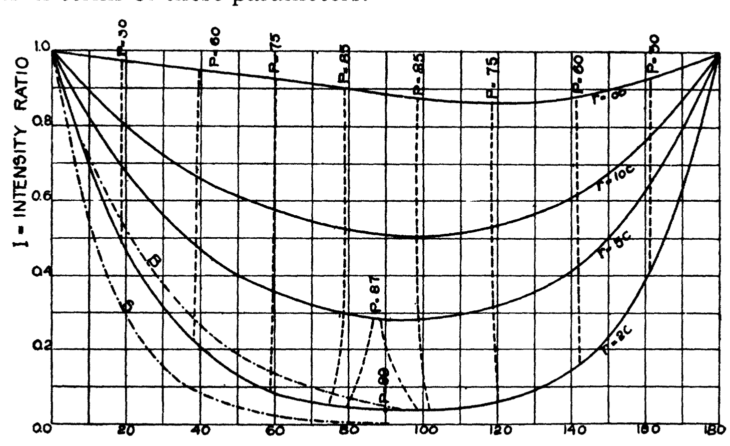


Fig. 1.

Ears 165° apart. $kc = 0.5$. Frequency 310 cycles.

Fig. 1 is such a chart drawn for the ratio $kc = 1/2$. If $c$ is taken as 8.75 cm., which is the average obtained by measuring the heads of a number of individuals, this corresponds to a frequency of 310 cycles.

[1] Rayleigh, Theory of Sound, Volume 2, Chapter 17. G. W. Stewart, PHYS. REV., 33, p. 467, 1911.

As measurements have also shown the ears of the average person to be about 165° apart, this separation has been used in making the computations for the chart.

The scale marked $\theta$ indicates the angular displacement of the sound source from the median plane, measured from front to back; the hammock-shaped curves correspond to various distances from source to observer; the series of approximately vertical lines indicate phase differences of various magitudes, and the scale at the left margin gives intensity ratios. Having any two of these quantities the other two may be determined. Thus it is possible to study theoretically the variation of the position of the image as the phase and intensity relations are altered. It will be observed that, as stated above, while there is an intensity ratio and phase difference corresponding to any source, there is not always a source corresponding to a pair of values of these variables.

For those experiments in which combinations are used which do correspond to actual sources the chart gives the theoretical location of the image directly.[1] Since, however, most experiments which have been

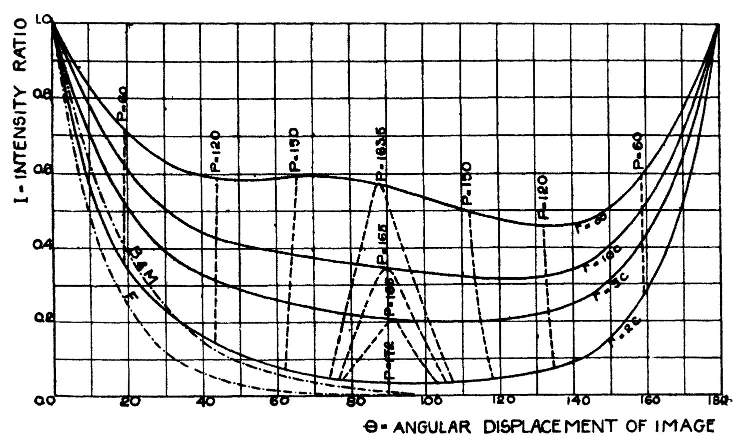


Fig. 2.

Ears 165° apart. $kc = 1.0$. Frequency 620 cycles.

reported fall into one or the other of two classes, in each of which the stimuli are of a type which could never arise from an actual source, we shall consider these two classes in some detail.

Suppose that the intensity ratio is maintained at the constant value unity while the phase difference is adjusted to some value other than

[1] The chart shows two dotted lines for each value of $P$. One of these corresponds to an image in front of the observer, the other to an image behind him. Experience shows that while some observers localize the sound in one of these positions, and some in the other, it rarely, if ever, happens that both images occur simultaneously.

zero, say 60°. Since these do not correspond to any real source, it follows that purely physical considerations do not define the position of the image and that some assumption of a psychological nature is required. It seems reasonable in this connection to assume that the observer subconsciously judges either the intensity ratio or phase difference, or both, to be in error, and effects a readjustment of their values in such a manner as to obtain a combination which does correspond to a possible source. It seems logical to assume also that the observer will localize the image in that position for which the necessary readjustment is the least.

In the example under discussion, if the observer interprets the intensity ratio as 0.9 instead of 1.0 the two data become accordant and correspond to a *distant* image a little less than 40° from the median plane. In this case no readjustment whatever of the phase relation is necessary. On the other hand if the intensity ratio were not readjusted at all it would be

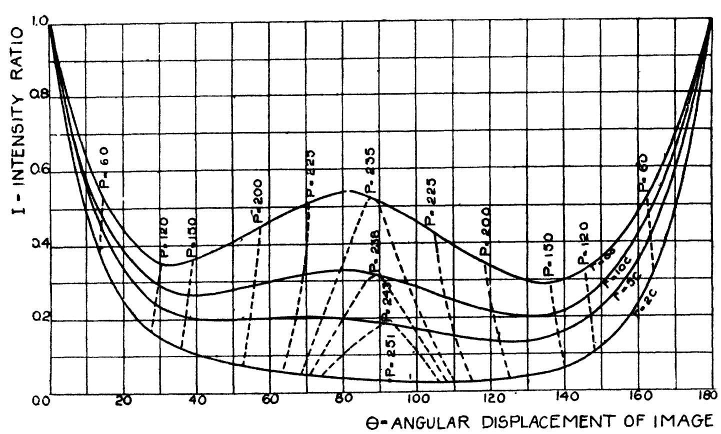


Fig. 3.

Ears 165° apart. $kc = 1.5$. Frequency 930 cycles.

necessary to conceive of the phase difference as zero instead of 60°. This is an enormous misinterpretation and it is scarcely conceivable that the observer would consent to it. Furthermore a moderate change in the phase difference does not greatly reduce the amount of readjustment of the intensity ratio which is necessary before physically possible conditions are obtained. We are, therefore, forced to the conclusion that the average observer will believe the sound to originate at a very great distance and at an angle from the median plane corresponding to the intersection of the phase curve with the curve of infinite distance. In our illustration this angle would be about 39°.

It may happen, of course, that for some other reason, such as the absolute intensity of the sound, the observer finds it difficult to believe that the source is more than a moderate distance away. He may then make a still further mental correction of the intensity ratio so as to bring the image closer to him. This will not have any appreciable effect on the direction of the image, since the line for $P = 60°$ is practically vertical.

Before discussing the results of such experiments, let us consider the other impossible condition which has been the subject of laboratory investigation, namely, the condition of equal phases but adjustable intensity ratio.

Equal phases can correspond only to the median plane—that is, to the vertical lines at the borders of the chart—and in this plane no intensity ratio other than unity can occur. As before, the observer may arrive at a combination of intensity ratio and phase difference that corresponds to an actual source in either of three ways. He may judge the phase difference to be correct and readjust the observed value of intensity ratio, or he may readjust the phase difference to conform to the intensity ratio, or he may readjust both. If he readjusts the intensity only, he must make the intensity ratio unity, obtaining thereby an image in the median plane. If he readjusts the phase difference only, the magnitude of the necessary readjustment depends upon how close to the head he is willing to place the image. The phase readjustment necessary to agree with an intensity ratio of 0.6 is obviously less if $r$ is taken as $2c$ than $5c$. Moreover, it may be made smaller yet by placing the image still closer to the head or even within the head.[1] However, as such sources are rather uncommon in everyday experience the observer may prefer to make a larger readjustment in phase and secure an image in a more usual position. Here as in the case of the experiment with equal intensities, the distance of the image depends upon the psychological prejudices of the individual, and we may expect rather wide variations between the results of different observers. Unlike the former case, however, the direction of the image is here dependent on the distance chosen, and so it too should be subject to similar variations.

The third possibility, that of readjusting both intensity and phase, is more likely to occur here than in the experiment with equal intensities, for, as the figure shows, a small readjustment in intensity reduces materially the necessary readjustment in phase and *vice versa*. As different observers may be expected to readjust the two by different relative amounts, there are obviously a wide variety of possible positions for the image in this case.

[1] Images within the head are not uncommon in the literature of binaural experiments. See Stewart, PHYS. REV., IX., No. 6, June, 1917, p. 502.

### 3. Comparison with Experiment.

While a large amount of experimental work has been done and Stewart in particular has published quantitative data taken under widely varying conditions, when this data is analyzed in the light of the above considerations it is found to be lacking in certain important features. In particular while the effect of intensity and phase on the *direction* of the image was studied their effect on its *distance* was not reported. In the absence of this data it is obviously impossible to check the theoretical deductions given above. The best that can be done is to determine whether or not such results as are available are consistent with the theory.

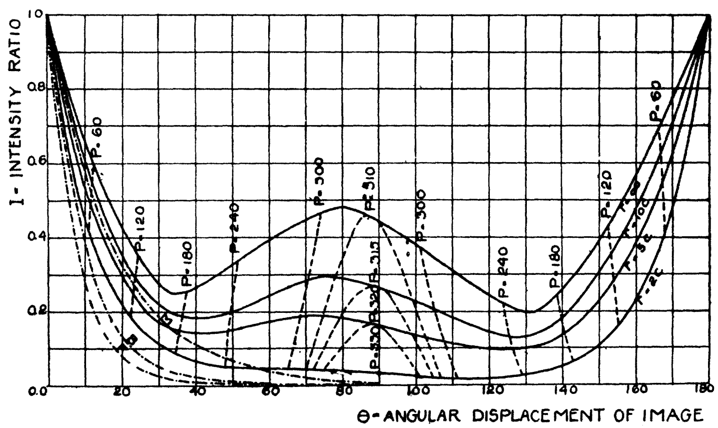


Fig. 4.

Ears 165° apart. $kc = 2.0$. Frequency 1,240 cycles.

As already indicated, almost all of the experiments were conducted with equal intensities or with equal phases, and so involve mental readjustments. For the experiments with equal intensities and varying phase differences a very good agreement has been established[1] between the direction of the image and that of a source which would produce the applied phase difference. The above theory indicates that although such an agreement is in a sense accidental, it is to be expected, since, as explained above, the observer will readjust the intensity ratio from unity to some value consistent with the phase difference observed. Also since the charts indicate that in most cases the direction of the image is practically independent of the readjustment made in the intensity ratio, the results of different observers should be in good agreement, as is found to be the case.

Of the experiments with equal phases and adjustable intensity ratios those of Stewart referred to at the beginning of the paper are by far the

[1] Hartley, *loc. cit.*, and Stewart, Phys. Rev., XV., No. 5, May, 1920, p. 432.

most comprehensive. An outstanding feature of these experiments is the wide variation in the results obtained by different observers both as regards the direction of the image and as regards its sharpness or "fusion." The variation in the direction of the image is to be expected from the theory. The observations in connection with the sharpness of the image are of interest for the light which they throw on the mental readjustments involved.

The observers were asked whether there was complete fusion and a rotation of the image about the head and, if there was partial fusion, where the other images were. It is stated[1] that "when there was only partial fusion, one image did not rotate but remained directly in front, or, by trial, in the position determined by the difference of phase. It could, therefore, be ascribed to the phase difference effect. Thus the terms 'partial' or 'incomplete' fusion really mean fusion in more than one image." This would indicate that in certain cases the observer

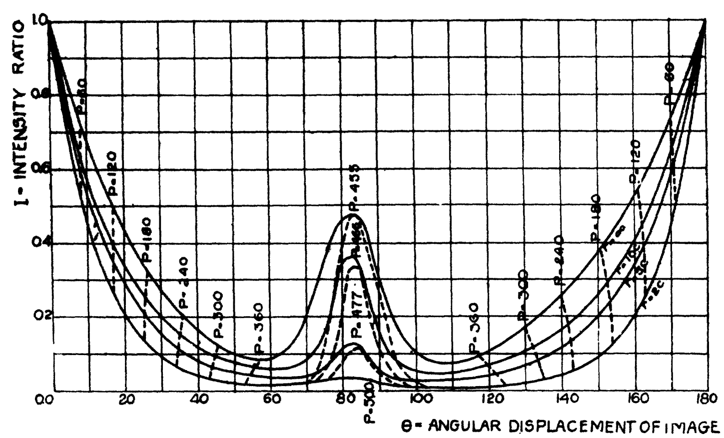


Fig. 5.

Ears 165° apart. $kc = 3.0$. Frequency 1,860 cycles.

obtained one image, that in the median plane, by making the entire readjustment in the intensity ratio and another by readjusting the phase. There is, of course, no evidence to determine whether for the second image the entire readjustment was made in phase or whether both were readjusted.

It is stated further that "in the case where there was no fusion, and the tube at the left ear was being pinched the single image remained in front

[1] Stewart, PHYS. REV., XV., No. 5, May, 1920, p. 428. The exact meanings of the terms "fusion" and "partial fusion" as used by Stewart are not made entirely clear. At times the latter seems to mean that the observer had difficulty in forming any judgment at all, while in other cases such as that quoted above, it refers to the formation of multiple images.

until the ratio of intensities was very large, *i.e.*, of the order of 200 with observer *S*, and then there appeared directly at the right, or approximately so, a second image which increased in clearness with further pinching, the image in front simultaneously disappearing, giving in the limit, as would be anticipated, but the one image to the right." Here the observer very definitely appears to have readjusted the intensity ratio to unity to conform with equality of phase even when it involved quite large changes. When this readjustment became too great he changed over abruptly to a large readjustment of phase, giving the image at the side.

While the degree of fusion varied widely for different observers, the results of all four showed the same general variation of the completeness of fusion with frequency. As the frequency is increased the completeness of fusion falls off to a minimum around a thousand cycles per second and increases again up to 1,792 cycles, the highest frequency used. A very general qualitative explanation of this result may be obtained from the charts. Since one of the criteria given by Stewart for a "fused" image is that it move around the head as the intensity ratio is varied, its formation must involve a readjustment of phase difference; consequently the difficulty of forming a fused image should be greater the larger the readjustment of phase necessary to conform to the small intensity ratio. From Fig. 1 it will be seen that for an intensity ratio of 0.09, for example, an image may be obtained at $r = 2c$ by readjusting the phase difference from 0° to about 75°. In Fig. 3, which is drawn for a frequency of about 930 cycles, and therefore corresponds roughly to that of least fusion, the corresponding phase adjustment is nearer 200°. At the higher frequency, 1,860 cycles, represented by Fig. 5, a phase difference of 360°, which is indistinguishable from zero, is found to be consistent with an intensity ratio of 0.09 and no adjustment at all is required. Hence it should be easy to obtain a well fused image at the side for this frequency.

There remains the problem of analyzing the experimental relations between the direction of the image and the intensity ratio, the consideration of which led Stewart to the conclusion quoted in the first paragraph. Since the distance of the image was not recorded, we are not able to plot upon the chart of Fig. 1 curves which will accurately display the locations of the image as experimentally observed. However, if we arbitrarily assume no readjustment of intensity ratio, as was done implicitly by Stewart, and plot observed directions against applied intensity ratios, we get the curves *B* and *S* of Fig. 1. These curves correspond to his results, taken with observers *B* and *S*, using a frequency of 256 cycles, which is not widely different from the chart frequency of 310 cycles.

These curves indicate very considerable phase readjustments at the smaller values of intensity ratio. As drawn, they also indicate that the image should be quite close to the head in all cases. If, however, in the experiments this was not the case, it means only that intensity readjustment *did* occur and that each point on the curve should be moved upward far enough to give the observed value of $r$ keeping $\theta$ fixed at the observed direction. In Figs. 2 and 4 are shown similar curves plotted from results given by Stewart for frequencies of 512 and 1,024 cycles respectively.

Stewart's conclusion that theory and experiment are in hopeless disagreement was arrived at by comparing curves such as $B$ and $S$, Fig. 1, with the theoretical curve connecting intensity ratio and direction for a source at a distance of 477 cm., which was selected,[1] "as a common

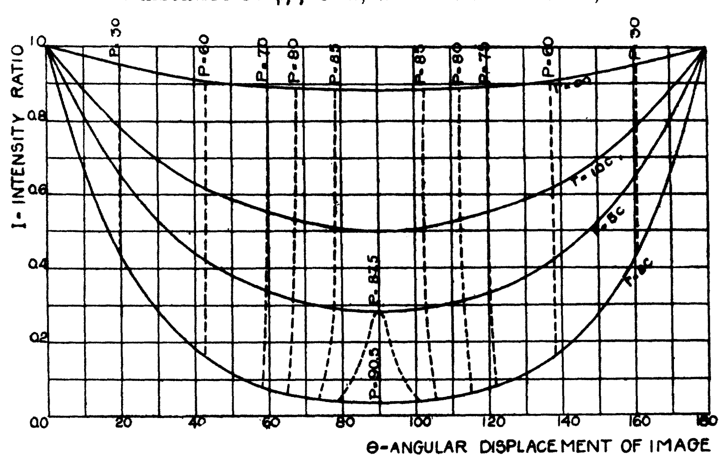


Fig. 6.

Ears 180° apart. $kc = 0.5$. Frequency 310 cycles.

one, though at distances of this magnitude or greater there is but little difference in relative intensities." Thus the curve he used is practically identical with that for a source at infinity and differs from the $r = \infty$ curve of Fig. 1 only in having been drawn for a different ear separation. The disagreement between his theoretical and experimental curves is, of course, obvious, but in view of the above considerations it is of no particular significance. For if the images actually were close to the head the experimental curves show excellent agreement with the theoretical ones *for sources in similar positions*. If they were not, all that is necessary to secure agreement is to assume that part of the inevitable readjustment was made in the intensity ratio instead of all being made in the phase difference. In general then the experimental results are entirely consistent with the theory outlined above.

[1] Stewart, PHYS. REV., XV., No. 5, May, 1920, p. 430.

## 4. Suggested Experiments.

In the light of these results it appears highly desirable that laboratory experiments be performed with both phase and intensity adjustable; and that the image be located not only with respect to angular displacement but with respect to distance as well, and that the ability to locate actual sources be investigated more fully. If it is found that with stimuli corresponding to an actual source the image is as accurate and sharp as for the source itself, it will show that phase and intensity are the controlling factors. If not other properties should be investigated. The results of experiments with other stimuli might also be of interest particularly to psychologists because of the light shed upon the prejudices of the observer and the dependence of these on his previous experience. The writers are not themselves in a position to carry out such experiments, but it is hoped that the charts here presented will prove useful to others who may be working in this field.

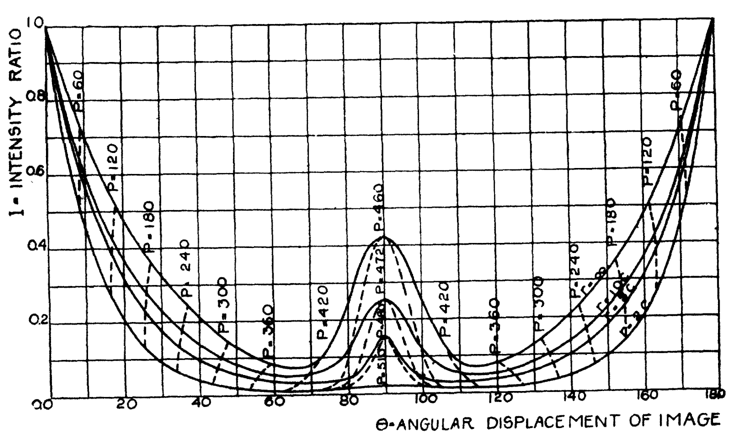


Fig. 7.

Ears 180° apart. $kc = 3.0$. Frequency 1,860 cycles.

Of these charts Figs. 1 to 5 correspond to an ear separation of 165°, while Figs. 6 and 7 correspond to a separation of 180°. Each chart is for a particular value of *kc*, which, as explained above, is equivalent to a particular frequency. The exact frequency depends, of course, on the values assumed for the radius of the head and the velocity of sound. The values given on the charts were computed for a sphere of 55 cm. circumference and a sound velocity of 340 meters per second.

With regard to the accuracy of these charts it is probably wise not to claim too much. The computations were carried out with considerable care and at the expense of a great deal of time, but the convergence of the series expansions used was so slow[1] for small values of $r$ that after the arithmetical results had been obtained a few obvious discrepancies remained which had to be removed by no more exact means than the judgment of the writers. Even this did not appear to suffice for values of $r$ less than $2c$, which accounts for the omission of such values from the chart. It is at least safe, however, to assert that the final accuracy is as great as that of the binaural sense itself, so that whatever residual errors remain are probably not of serious consequence.

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[1] In this connection see Rayleigh, Scientific Papers, Vol. 5, No. 292, p. 149.