A GENERALIZATION OF ELECTRODYNAMICS WITH APPLICATIONS TO THE STRUCTURE OF THE ELECTRON AND TO NON-RADIATING ORBITS.

BY LEIGH PAGE.

SYNOPSIS.

Generalization of Electrodynamics.—If every element of charge is assumed to be the center of equal electric and magnetic fields, each of which is capable of rotation, the electromagnetic equations assume a symmetrical form, containing Lorentz's formulation of Maxwell's equations as a special case.

Structure of the Electron.—If the electric field of each element of charge on the surface of the Lorentz electron is supposed to rotate about the normal to the surface, a magnetic field is produced which just annuls the intrinsic field due to the magnetic charge assumed to be distributed over the surface, and which exerts an inward stress equal to the outward stress of electrostatic repulsion. Not only is the surface in equilibrium, but the equilibrium is found to be stable for small volume changes.

Non-Radiating Electronic Orbits.—A suitable rotation of the fields of the elementary charges constituting the electron about an axis parallel to the acceleration would cause the radiation term to vanish. This suggests the possibility of radiationless orbits.

SEVEN years age the author of this paper proposed a kinematical interpretation of the electromagnetic equations[1] which both H. Bateman[2] and he[3] have since developed and generalized. In this interpretation electric charges are assumed to be the fundamental constituents of matter, magnetic poles existing only as secondary entities. Each element of electricity is supposed to emit uniformly in all directions with the velocity of light continuous streams of *moving elements*, or, as Bateman has termed them, *light particles*. Each moving element travels out from its source in a straight line uninfluenced by the subsequent motion of the source or by the presence or motion of neighboring moving elements. The nature of these moving elements is immaterial for the purposes of the theory other than that each must be susceptible of continuous identification and must move in a straight line with the velocity of light. A *line of electric force* is defined as the locus of a stream of moving elements emerging from a single source. To employ an analogy, a source may be likened to a machine gun firing bullets with the velocity of light. The bullets correspond to moving elements. If they are supposed to be

[1] Am. Jour. Sci., 38, p. 169, 1914.
[2] Phil. Mag., 41, p. 107, 1921.
[3] Proc. Nat. Acad. Sci., 6, p. 115, 1920.

strung along an endless perfectly elastic thread, such a thread constitutes a line of force. A charged particle, then, consists of a group of such guns pointing uniformly in all directions, each firing a continuous stream of bullets. If the charge is in motion, a line of force emerging from it will not in general have the same direction as the velocities of the moving elements which constitute the line. Thus arises a magnetic field, which has a direction at right angles to the electric lines of force and to the direction of motion of the moving elements constituting these lines, and an intensity proportional to the sine of the angle between these two vectors.

The number of lines of force emerging from an element of electric charge must be supposed to be very great. These lines may be grouped into bundles, each of which may be called a *tube of force*. The *electric intensity* **E** is defined as the number of tubes of force per unit cross section. If the light-velocity of the moving elements is denoted by the vector **c**, the magnetic intensity **H** of an elementary field is defined by

$$\mathbf{H} = \frac{1}{c}(\mathbf{c} \times \mathbf{E}).$$

The total electric or magnetic intensity at a point is the vector sum of the electric or magnetic intensities respectively of all the elementary fields which extend to that point.

Coulomb's and Ampère's laws follow at once as necessary consequences of this representation, and if the Lorentz-Einstein space and time transformations are used, Faraday's law and the law expressing the fact that the divergence of the magnetic intensity is zero, can be shown to hold exactly, provided the sources of moving elements do not rotate.[1] Thus the proposed representation contains the four field equations of electromagnetism. Furthermore, if it is assumed that the force on a charge e relative to an observer at rest with respect to the charge is given by the product $e\mathbf{E}$, the fifth equation of electrodynamics is easily deduced. In fact, the abundantly proven validity of this last relation, as interpreted on the basis of the representation under discussion, constitutes better experimental confirmation of the restricted relativity principle, perhaps, than any other evidence which has yet been adduced.

The generalization of the classical equations of electromagnetism which is obtained by taking into account the possibility of rotation of the sources of moving elements[2] is particularly suggestive. The field equations assume the symmetrical form (in Heaviside-Lorentz units)

[1] See L. Page, "An Introduction to Electrodynamics," Chap. II., Ginn & Co., 1921.

[2] Proc. Nat. Acad. Sci., 6, p. 120, 1920.

$$\nabla \cdot \mathbf{E} = \rho, \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}(\dot{\mathbf{H}} + \epsilon \mathbf{u}), \tag{2}$$

$$\nabla \cdot \mathbf{H} = \epsilon, \tag{3}$$

$$\nabla \times \mathbf{H} = \frac{1}{c}(\dot{\mathbf{E}} + \rho \mathbf{v}), \tag{4}$$

where the velocity u of the free magnetism produced by a rotating field is in general equal to the velocity c of light. If the entire field rotates with the same angular velocity $\boldsymbol{\omega}$, then at a time r/c

$$\epsilon = \frac{e}{2\pi r^2 k^2 c^2} \frac{\boldsymbol{\omega} \cdot \mathbf{c} - \boldsymbol{\omega} \cdot \mathbf{v} k^2 \left(1 - \frac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)}{\left(1 - \frac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)^4}, \tag{5}$$

at a distance r from a charge e which is moving as a whole with velocity $\mathbf{v}$ at the time 0. The letter k stands for the reciprocal of $\sqrt{1 - \beta^2}$, β being the ratio of the velocity of the charge to that of light. If, now, the energy radiated for each complete rotation of the field is computed, it is found to have very closely the value

$$\frac{1}{32} h\nu,$$

where h is Planck's constant, and ν the frequency of rotation. This numerical coincidence suggests that quantum phenomena may be somehow connected with rotating electric fields.

The object of this paper is to propose a further generalization of the field equations, and to make applications to the structure of the electron and to non-radiating orbits.

Generalized Electrodynamics.

In the discussion immediately following, the terms *electric charge* and *magnetic charge* refer to an *element* of an electron or hydrion (positive electron), and not to charges so gross as even the entire charge on the electron. The postulates of the generalized theory are as follows:

1. With every electric charge is indissolubly associated an equal magnetic charge. By a charge is meant a center of uniformly diverging lines of force,—lines of electric force in the case of electric charges, and lines of magnetic force in the case of magnetic charges. Hence, from every element of the electron or hydrion radiate two sets of lines, one electric and the other magnetic. Since the Heaviside-Lorentz unit of charge is used in the following analysis, the magnitude e of the elementary charge is equal numerically both to the number of tubes of electric force and to the number of tubes of magnetic force radiating from it. The

electric intensity at any point is equal to the number of tubes of electric force per unit cross-section, and the magnetic intensity to the number of tubes of magnetic force per unit cross-section.

At first sight it may seem that this association of magnetism with electricity in the electron, for instance, is absurd in that it must give rise to a particle which has the properties of a magnetic pole as well as an electric charge. That such is not the case, however, will appear later on.

2. If $\mathbf{E}_1$ and $\mathbf{H}_1$ are the components of the electric and magnetic fields respectively at right angles to $\mathbf{c}$ due to an elementary charge e,

$$\mathbf{H}_1 = \frac{1}{c}(\mathbf{c} \times \mathbf{E}), \tag{6}$$

$$\mathbf{E}_1 = -\frac{1}{c}(\mathbf{c} \times \mathbf{H}), \tag{7}$$

just as in the classical theory. These are not independent relations, as either may be obtained from the other by forming the cross product with $\mathbf{c}$.

3. As in the classical theory, the force on an elementary charge e as measured by an observer relative to whom the charge is at rest at the instant considered, is

$$\mathbf{K} = e(\mathbf{E} + \mathbf{H}), \tag{8}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the total field strengths. It follows from this that the force as measured by an observer relative to whom e is moving with velocity $\mathbf{v}$ is

$$\mathbf{K} = e\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{H} + \mathbf{H} - \frac{1}{c}\mathbf{v} \times \mathbf{E}\right), \tag{9}$$

the method of proof being the same as that used in obtaining the corresponding expressions in the paper[1] already referred to.

The expressions for the retarded electric and magnetic intensities due to an elementary charge e which has velocity $\mathbf{v}$ and acceleration $\mathbf{f}$ relative to the observer under consideration are obtained in the same manner as the corresponding expressions in the papers[1,3] referred to previously. They are

$$\mathbf{E} = \frac{e}{4\pi r^2 k^2 c\left(1 - \frac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)^3}\left\{\mathbf{c} - \mathbf{v} + \frac{1}{c}\mathbf{c} \times \mathbf{v} + \frac{rk^2}{c^3}\left[\left((\mathbf{f} - c\boldsymbol{\omega}_H) \times (\mathbf{c} - \mathbf{v})\right\} \times \mathbf{c} + \left(\left\{\left(\boldsymbol{\omega}_E + \frac{1}{c}\mathbf{f}\right) \times (\mathbf{c} - \mathbf{v})\right\} \times \mathbf{c}\right) \times \mathbf{c}\right]\right\}, \tag{10}$$

$$\mathbf{H} = \frac{e}{4\pi r^2 k^2 c\left(1 - \frac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)^3}\left\{\mathbf{c} - \mathbf{v} - \frac{1}{c}\mathbf{c} \times \mathbf{v} + \frac{rk^2}{c^3}\left[\left\{(\mathbf{f} + c\boldsymbol{\omega}_E)\right.\right.$$

$$\times (\mathbf{c} - \mathbf{v}) \Big\} \times \mathbf{c} + \left(\left\{ \left(\omega_H - \frac{1}{c} \mathbf{f} \right) \times (\mathbf{c} - \mathbf{v}) \right\} \times \mathbf{c} \right) \times \mathbf{c} \Big] \Big\}, \quad (11)$$

where $\boldsymbol{\omega}_E$ and $\boldsymbol{\omega}_H$ refer to the electric and magnetic fields respectively. If $\boldsymbol{\omega}_0$ is the angular velocity of rotation of a field relative to an observer with respect to whom the charge producing the field is momentarily at rest, then the components of $\boldsymbol{\omega}$ to be used in these equations by an observer relative to whom the charge has a velocity $\mathbf{v}$ in the x direction are related to the components of $\boldsymbol{\omega}_0$ by the following transformations,

$$\omega_x = \frac{1}{k^3} \omega_{0x},$$

$$\omega_y = \frac{1}{k^2} \omega_{0y},$$

$$\omega_z = \frac{1}{k^2} \omega_{0z},$$

which are the same as the transformations of the components of acceleration.

It is observed that (10) and (11) satisfy conditions (6) and (7). Furthermore, *the radiation field vanishes* provided

$$\boldsymbol{\omega}_E = -\frac{1}{c} \mathbf{f}, \quad (12)$$

$$\boldsymbol{\omega}_H = \frac{1}{c} \mathbf{f}. \quad (13)$$

Therefore radiationless orbits are possible. It is to be noticed that the rotation of the field of an element of an electron does not in any manner imply rotation of the electron as a whole, and that the electric and magnetic fields may rotate at different rates and about different axes.

The field equations of the generalized theory are the equations (1) to (4) already given. The divergence ρ of the electric vector may conveniently be split into two parts, the density ρ_0 of electric charge and the divergence ρ_H due to rotation of the magnetic lines of force. Similarly ϵ consists of the density of magnetic charge ϵ_0 and the divergence ϵ_E of the magnetic vector due to rotation of the electric lines of force. According to postulate (1) $\rho_0 = \epsilon_0$ always. The retarded expressions for ρ_H and ϵ_E due to a charge e moving with velocity $\mathbf{v}$, whose electric field has angular velocity ω_E and magnetic field angular velocity ω_H, are respectively

$$\rho_H = -\frac{e}{2\pi r^2 k^2 c^2} \frac{\boldsymbol{\omega}_H \cdot \mathbf{c} - \boldsymbol{\omega}_H \cdot \mathbf{v} k^2 \left(1 - \frac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)}{\left(1 - \frac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)^4}, \quad (14)$$

$$\epsilon_E = \frac{e}{2\pi r^2 k^2 c^2} \frac{\omega_E \cdot \mathbf{c} - \omega_E \cdot \mathbf{v} k^2 \left(1 - \dfrac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)}{\left(1 - \dfrac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)^4}. \quad (15)$$

It should be noticed that while ρ_H and ϵ_E are divergences in the electric and magnetic vectors respectively, they are due to the rotations of the lines of force and are not to be considered as true charges. A charge is detected by its field, and the divergences under discussion lack the characteristic uniformly diverging field of a charge. They are merely the end points of broken off lines of force which originate on a charge.

The energy equation is obtained by the usual methods from the field equations (1) to (4). It is

$$\frac{d}{dt}\{\tfrac{1}{2}(E^2 + H^2)\} + c\nabla \cdot (\mathbf{E} \times \mathbf{H}) + \mathbf{E} \cdot \mathbf{v}\rho + \mathbf{H} \cdot \mathbf{u}\epsilon = 0. \quad (16)$$

The last two terms represent the rate at which work is done on the divergences of **E** and **H**, whether these divergences are true charges or end points of broken off lines of force;

$$\tfrac{1}{2}(E^2 + H^2)$$

is the field energy per unit volume; and

$$(c\mathbf{E} \times \mathbf{H})$$

the Poynting flux.

Structure of the Electron.

The Lorentz electron is the simplest model which has yet been proposed. This electron is a uniformly chargek spherical shell relative to an observer with respect to whom it is at rest at the instant considered. It may easily be shown that all points of the electron are at rest relative to a given observer at the same time. If the electron is in motion, the relativity contraction causes it to appear shorter in the direction of motion than if it were at rest. Its dimensions at right angles to the direction of motion are unaltered. Each element of charge of the electron is assumed to act on every other element according to the same law of force as governs the actions of gross charges on one another. Hence the repulsive force expressed by Coulomb's law produces an outward stress at the surface of the electron which would cause it to explode in the absence of some inward force not explicable in terms of the classical electrodynamics. This outward stress is the same for an electron moving with constant velocity as for one at rest, and normal to the surface in both cases. Hence Poincaré has been led to suggest that it is counterbalanced by a uniform hydrostatic pressure of the ether. This hypothesis is open to several objections. In the first place, it requires the existence of forces of a character quite different from those contained in the electromagnetic scheme, as well as a second type of interaction between ether and electricity. Secondly, it implies the absence of ether from the interior of the electron, which is inconsistent with the assumption that electromagnetic forces are propagated across the inside of the electron from one element of charge to another. Thirdly, it is at variance with evidence showing that the ether (if one may be allowed to use the term) cannot have many, if any, of the properties of material media.

When an electron is moving with any other type of motion than constant velocity, its field exerts a drag on it. This retarding force is proportional, to a first approximation, to the acceleration, and accounts for the mass of the electron. The mass formula involves the radius of the electron, its charge, and the velocity of light. The last two being known, the first may be calculated, and comes out to be $1.88(10)^{-13}$ cm., which is of the order of magnitude to be expected. If the electron is assumed to have a volume distribution of charge expressible by any arbitrary function of the distance from its center, instead of the surface distribution of the Lorentz model, the value of the radius comes out somewhat different, though of the same order of magnitude, and the ether, if Poincaré's hypothesis be accepted, must be supposed to penetrate into the interior with a density which falls off as the center is approached. However, the objections to this hypothesis cited above still hold. The object of this section of the present paper is to show that the proposed generalized electrodynamics makes it possible to account for both the equilibrium and the stability of the electron without the necessity of introducing extra-electromagnetic forces.

Lorentz's model will be adopted in that the electron will be supposed to be a uniformly charged spherical shell of charge e and radius a. Each element of charge is the origin of equal electric and magnetic fields. The magnetic fields will be supposed, normally, to be without rotation; the electric field of each element of charge, on the other hand, will be considered to have a constant angular velocity ω about the normal to the surface at the point considered. Consider, now, an electron permanently at rest relative to the observer. Expressions (10) and (11) for the electric and magnetic intensities respectively due to an element de of the electron's charge reduce to

$$d\mathbf{E} = \frac{de}{4\pi r^2 c}\left\{ \mathbf{c} - \frac{r}{c}\boldsymbol{\omega} \times \mathbf{c} \right\}, \tag{17}$$

$$d\mathbf{H} = \frac{de}{4\pi r^2 c}\left\{ \mathbf{c} + \frac{r}{c}\left[\frac{\boldsymbol{\omega} \cdot \mathbf{c}}{c^2}\mathbf{c} - \boldsymbol{\omega} \right] \right\}. \tag{18}$$

Symmetry shows that the resultant **E** and the resultant **H** are radial. Integrating over the surface of the electron, the intensities at a point P (Fig. 1) outside the electron and distant p from its center are found to be

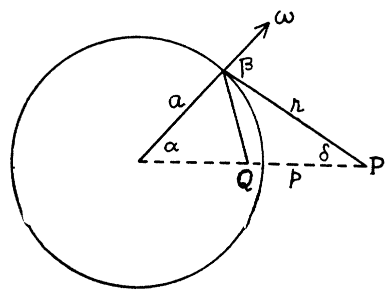

Fig. 1.

$$E = \frac{1}{4\pi}\int \frac{\cos\delta}{r^2}\,de$$
$$= \frac{e}{16\pi a p^2}\int_{p-a}^{p+a}\left[1 + \frac{p^2 - a^2}{r^2}\right]dr$$
$$= \frac{e}{4\pi p^2}, \tag{19}$$
$$\mathbf{H} = \frac{1}{4\pi}\int\left[\frac{\cos\delta}{r^2} + \frac{\omega}{cr}(\cos\beta\cos\delta - \cos\alpha)\right]de$$
$$= \frac{e}{16\pi a p^2}\int_{p-a}^{p+a}\left[1 + \frac{p^2 - a^2}{r^2}\right]dr$$
$$+ \frac{e\omega}{32\pi a^2 p^2 c}\int_{p-a}^{p+a}\left[r^2 - 2(p^2 + a^2) + \frac{(p^2 - a^2)^2}{r^2}\right]dr$$
$$= \frac{e}{4\pi p^2}\left[1 - \frac{2}{3}\frac{a}{c}\omega\right]. \tag{20}$$

Choose

$$\omega = \frac{3}{2}\frac{c}{a}$$
$$= 2.40\,(10)^{23} \text{ per sec.} \tag{21}$$

Then there is no magnetic field outside the electron, and the electric field follows Coulomb's familiar inverse square law.

At a point Q inside the electron, the same integral expressions for E and H hold, but the limits of integration are $a + p$ and $a - p$ instead of $p + a$ and $p - a$. Hence

$$E = 0, \tag{22}$$
$$H = -\frac{e\omega p}{6\pi a^2 c}$$
$$= -\frac{ep}{4\pi a^3}, \tag{23}$$

The electric field vanishes, and the magnetic field is directed toward the center and has a strength proportional to the distance p from the center.

The divergence of the electric vector is everywhere zeio (except at the surface). The divergence of the magnetic vector is zero outside the surface, but (23) shows that inside it must have the constant value

$$\nabla \cdot \mathbf{H} = -\frac{e\omega}{2\pi a^2 c}$$
$$= -\frac{3e}{4\pi a^3}. \qquad (24)$$

Thus the total volume distribution of divergence of the magnetic vector inside the electron is equal and opposite in sign to the surface magnetic charge, agreeing with the computed absence of magnetic field outside the electron. The divergences of **E** and **H** might have been obtained directly from (14) and (15).

At the surface of the electron, E and H have the same values but opposite directions. Hence the charge on the surface is in equilibrium. The outward stress due to the electric field is

$$S = \frac{e^2}{32\pi^2 a^4}, \qquad (25)$$

per unit area, and the equal inward stress due to the magnetic field

$$S = \frac{e^2\omega^2}{72\pi^2 a^2 c^2}. \qquad (26)$$

Let the radius of the electron increase to $a + x$, the angular velocity ω retaining the value given above. Then the net outward stress is

$$S = \frac{e^2}{32\pi^2 a^4}\left[-2\frac{x}{a} + 7\frac{x^2}{a^2}\cdots\right]. \qquad (27)$$

The electron, therefore, is in stable equilibrium as regards changes in volume.

The generalized electrodynamics under discussion has been derived directly from the relativity principle, and hence all deductions to which it leads must be strictly in accord with this principle. Therefore the conclusions arrived at regarding equilibrium and stability of an electron permanently at rest apply equally well to an electron moving with constant velocity relative to the observer. In the latter case a magnetic field external to the electron is present, but it is the ordinary field due to motion of the electron as a whole, and not due to the rotations of the generalized theory. The divergences of both vectors are zero everywhere outside the electron.

These speculations as to the nature of the electron suggest that the positive electron or hydrion differs from the negative electron in the sense of the rotation ω. If, for instance, the rotation of the field of an element of charge on the surface of the negative electron is clockwise when viewed along the outward drawn normal, the rotation may be counterclockwise in the case of the positive electron. The fact that only two senses of rotation are possible is in accord with the existence of two and only two kinds of electricity.

Non-Radiating Orbits.

Integration over the entire surface of an unaccelerated electron has shown that **E** and **H** have the same values outside the surface as are predicted by Maxwell's classical equations. The rotations of the electric fields of the elements of charge constituting the electron annul one another in such a manner as to produce no resultant effect in so far as the electric intensity is concerned. An investigation of the external field and the dynamical equation of an accelerated electron is being undertaken in the hope that it may throw some light on quantum phenomena. For the results of Rutherford's experiments on the scattering of alpha rays are very excellent evidence, in the author's opinions that quantum phenomena cannot be accounted for by substituting for the inverse square electrostatic field of the nucleus amore complicatedfield involving repulsive as well as attractivef orces. Hence it would seem as though the quantum must be somehow or other connected with the electron.

The conditions for annulment of the radiation field have been already stated. The necessary small angular velocities in the direction of the acceleration must be supposed to be superimposed on the much larger angular velocity of the electric fields normal to the surface. In the case of an electron rotating in a circular orbit of quantum number n about a nucleus of atomic number Z,

$$\begin{aligned}\omega_H = -\omega_E &= \frac{1}{c}\mathbf{f} \\ &= \frac{16\pi^4 m e^6}{h^4 c}\frac{Z^3}{n^4} \\ &= 3.01\ (10)^{14}\frac{Z^3}{n^4}\text{ per sec.,} \qquad (28)\end{aligned}$$

which is quite negligible compared to (21).

Note Added in Course of Publication.—Further calculation shows that the mass and the cofficient of the rate ot change of acceleration are each one and a half times as great as for the simple Lorentz electron of the same radius. Hence, the mass being known, the radius of the proposed electron comes out one and a half times as gpeat as that of the Lorentz type, i. e., $2.82(10)^{-13}$ cm.

While the suggested superimposed rotations of the electric and magnetic fields of a revolving electron annul the radiation due to accelera. tion of the electron as a whole, there remains a radiation of the same order of magnitude due to the rotation about the normal to the electron's surface of the electric fields of each elemeut of charge. If the superimposed rotations are such os to acnul *entirely* the radiation field of eoch element of charge, the equilibrium of the electron's surface is interfered with, and the electromagnetic mass of the electron vanishes.

SLOANE PHYSICS LABOKATORY,
YALE UNIVERSITY,
April, 1921.