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THE DETECTING EFFICIENCY OF THE RESISTANCE-CAPACITY COUPLED ELECTRON TUBE AMPLIFIER.

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SYNOPSIS.

Detecting efficiency of the resistance-capacity coupled, two tube amplifier.—A *theoretical formula* is derived and compared with the results of *measurements* made by using a condenser potential divider to vary the high frequency input voltage A and a sensitive galvanometer to measure the rectified high frequency component of the output plate current b_0, the detecting efficiency being defined as b_0/A^2. Satisfactory agreement was found for the *various frequencies and coupling capacities* tried. The *effect of a third tube* was determined experimentally. For convenience of *comparison with transformer-coupled amplifiers*, curves are given showing the detecting efficiencies of the various amplifiers as a function of the frequency and hence their relative merits in detecting unmodulated signals.

Current amplification of the above amplifier as a function of the frequency was also determined.

1. INTRODUCTORY.

THE detecting efficiency of the electron tube amplifier means, in general words, the efficiency of the amplifier to make weak signals intelligible. From general considerations it can be seen that the detecting efficiency depends upon the relation between the input grid voltage change, and the resulting change in the output plate current. This has already been discussed in two papers,[1] one of which dealt with experimental measurements of the detecting efficiency of a three tube high-frequency transformer-coupled amplifier, and the other with the detecting efficiency of the single electron tube; this last considered the question from both a theoretical and an experimental standpoint. In the case of the high-frequency transformer-coupled amplifier the detecting efficiency was defined by the relation $\lim_{A \doteq 0} b_0/A^2$ where A and b_0 are the amplitudes, respectively, of the input grid potential and of the

[1] PHYS. REV., 16, 274, 1920; 16, 408, 1920.

rectified component of the output plate current. In the present paper is described an investigation, both theoretical and experimental, of the detecting efficiency of the high frequency resistance-capacity coupled amplifier. A theoretical analysis, in which certain simplifying assumptions were made, has been carried out, which showed how the detecting efficiency depended upon the constants of the coupling circuits. Experiments were then performed to obtain data on this type of amplifier, and to test the theoretical relations.

2. Theoretical.

Consider the high frequency amplifier of two tubes with resistance-capacity coupling as shown in Fig. 1. This amplifier may be used to receive modulated radio-frequency signals. There are other types of resistance-capacity coupling than the one shown in Fig. 1. For example,

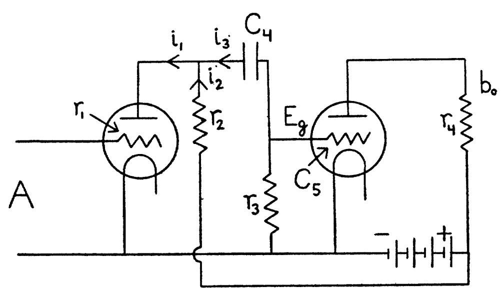

Fig. 1.

if separate plate batteries are used for the two tubes, the capacity C_4 and resistance r_3 may be omitted. Such an amplifier will, in general, possess different characteristics from those of the amplifier of Fig. 1. However, the connections shown in Fig. 1 are commonly used and it is this type of connection which was chosen for this investigation.

Let A be the amplitude of the radio frequency voltage impressed on the grid of the first tube, and let b_0 be the rectified component of the resulting radio frequency current in the plate circuit of the second tube. We now consider the action of the amplifier in the following way. Due to A, a high frequency component of the plate current of the first tube is produced. Call this i_1. This current divides into i_2 and i_3, as shown in Fig. 1. i_3 impresses a high frequency potential oscillation E_g on the grid of the second tube, which in turn causes the rectified high frequency component b_0 of the plate current of the second tube. Amplification is considered to take place in both tubes, but rectification to occur only in the second tube. If rectification occurs in the first tube, this may result in a shift of the operating point of the grid of the second tube, and hence a change in the characteristics of the amplifier. Experiment has shown, however, that for small input voltages the rectifying effect of the first tube is small, and so in the present treatment it is justifiable to ignore this effect.

We proceed to derive the relation between A and b_0 in terms of the constants of the circuits and of the tubes. Let r_1 be the internal resistance between the filament and the plate of the first tube. Let C_5 be the filament-grid capacity of the second tube. Resistances r_2 and r_3, and capacity C_4 are as shown in Fig. 1. Let $\omega/2\pi$ be the frequency of the impressed voltage.

Let

$$\frac{1}{r_1} = g_1, \qquad \omega C_4 = x_4,$$
$$\frac{1}{r_2} = g_2, \qquad \omega C_5 = x_5, \tag{1}$$
$$\frac{1}{r_3} = g_3.$$

Consider the first tube. If the grid-plate capacity of this tube is neglected, it has been shown[1] that the relation between the impressed grid voltage A and the resulting high-frequency component i_1 of the plate current is given by the relation

$$i_1 = \frac{kA}{\dfrac{1}{g_1} + Z_p}, \tag{2}$$

k is the amplification constant of the tube, and Z_p the impedance of the plate circuit external to the tube. The amplification constant k and the internal resistance $1/g_1$ of tube are real quantities, and not complex, in the case of zero grid-filament capacity.

Considering the circuit shown in Fig. 1, the value of Z_p is found to be

$$Z_p = \frac{1}{g_2 + \dfrac{1}{\dfrac{1}{jx_4} + \dfrac{1}{g_3 + jx_5}}}, \tag{3}$$

where $j = \sqrt{-1}$. Substituting (3) in (2) gives

$$i_1 = kAg_1 \frac{g_2g_3 - x_4x_5 + j[x_4(g_2 + g_3) + x_5g_2]}{g_2g_3 - x_4x_5 + g_1g_3 + j[x_4(g_1 + g_2 + g_3) + x_5(g_1 + g_2)]}$$

The modulus of this is

$$i_1 = kAg_1 \sqrt{\frac{(g_2g_3 - x_4x_5)^2 + [x_4(g_2 + g_3) + x_5g_2]^2}{(g_2g_3 - x_4x_5 + g_1g_3)^2 + [x_4(g_1 + g_2 + g_3) + x_5(g_1 + g_2)]^2}}, \tag{4}$$

[1] G. Breit, Phys. Rev., 16, 388, 1920. Others have given similar formulas.

where i_1 now stands for the root-mean square value of the high frequency plate current of the first tube. It should be noticed that in the above treatment conductance through the coupling condenser C_4 has been neglected. The measured direct current value of this resistance was over 50 megohms. Terms expressing this condenser leakage were introduced into formula (4), but were found to be negligible.

Let us turn our attention to what takes place in the coupling circuit and find the relation between i_1 and E_g, the high frequency voltage impressed on the grid of the second tube. From Kirchhoff's laws

$$i_1 = i_2 + i_3,$$

$$\frac{i_2}{g_2} = i_3\left(\frac{1}{jx_4} + \frac{1}{g_3 + jx_5}\right),$$

$$E_g = \frac{i_3}{g_3 + jx_5}.$$

It is assumed that the grid-filament conduction current of the second tube is zero. Then

$$E_g = \frac{i_1}{(g_3 + jx_5)\left\{1 + g_2\left(\frac{1}{jx_4} + \frac{1}{g_3 + jx_5}\right)\right\}}.$$

The modulus of this is

$$E_g = \frac{i_1}{\left\{\left(g_2 + g_3 + \frac{x_5}{x_4}g_2\right)^2 + \left(x_5 - \frac{g_2g_3}{x_4}\right)^2\right\}^{1/2}}. \quad (5)$$

In this expression E_g stands for the root-mean-square value of the grid voltage of the second tube.

Consider now the second tube, in which amplification and rectification both occur. It has been shown experimentally in the previous papers[1] that the relation

$$\frac{b_0}{E_g^2} = n \quad (6)$$

is approximately true, where n is a quantity which is independent of the magnitude of E_g. It is assumed that in the present case n did not depend upon the frequency of the impressed voltage, because the impedance of the plate circuit of the second tube was independent of the frequency. This assumption was subsequently justified by experiment.

Introducing (4) and (5) into (6) we have

$$\frac{b_0}{A^2} = nk^2g_1^2\frac{\dfrac{(g_2g_3 - x_4x_5)^2 + [x_4(g_2 + g_3) + x_5g_2]^2}{(g_2g_3 - x_4x_5 + g_1g_3)^2 + [x_4(g_1 + g_2 + g_3) + x_5(g_1 + g_2)]^2}}{\left(g_2 + g_3 + \frac{x_5}{x_4}g_2\right)^2 + \left(x_5 - \frac{g_2g_3}{x_4}\right)^2}. \quad (7)$$

[1] *Loc. cit.*

This expression gives the desired relation between the detecting efficiency b_0/A^2 and the constants of the tubes and of the circuits. In the former papers[1] the detecting efficiency has been defined by the relation $\lim_{A \to 0} b_0/A^2$. Although this is theoretically the proper definition (because it avoids any possibility of rectification in the first tube), we have been content in the present instance to measure the value of b_0/A^2 for finite values of A, and to call this the detecting efficiency.

3. Apparatus.

The apparatus consisted of a condenser potential divider, the amplifier, and a D'Arsonval galvanometer connected in the plate circuit of the last tube. These will be described in the order named. The arrangement is shown schematically in Fig. 2. The condenser potential divider

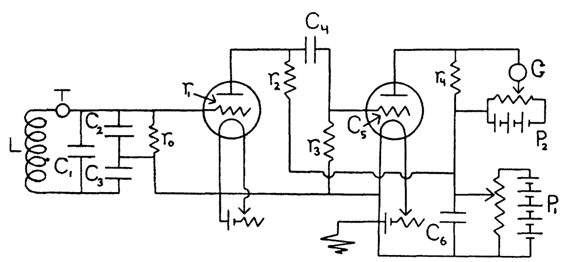

Fig. 2.

consisting of the three condensers c_1, c_2, c_3, the inductance coil L, and the thermogalvanometer T, were the same as described in a previous paper.[2] By coupling L to a suitable electron tube generating set, unmodulated high frequency voltage of small known amplitude and frequency was impressed on the grid of the first electron tube. The high resistance leak r_0 (1.2×10^6 ohms) was connected across c_2 to ensure a definite value of the grid potential during the experiment. The effect of r_0 upon the impedance of c_2 was negligible because c_2 was large (about 0.05 $\mu F\delta$) and the frequencies used were of the order of 3×10^5.

The amplifier was a two-tube one with resistance-capacity coupling. The tubes were Western Electric Company tubes, Type VTI; they were used with the filament current always 1.10 amperes and the plate voltage always 23.7 volts. Separate storage cells supplied each filament; the plate voltage supply was common to all the tubes. The plate battery was shunted by a 2 μF condenser c_6. The resistance r_2 was 27.6×10^3 ohms and r_3 was 393×10^3 ohms. The resistances r_0, r_2 and r_3, were

[1] *Loc. cit.*
[2] *Loc. cit.*

non-inductive. A resistance unit was made by painting both sides of a strip of cardboard about 4 mm. wide with black drawing ink. Two sturdy brass clips were pinched on to each end of the strip, so that the length of the strip between the jaws was about 0.5 mm. This was sealed in a small glass tube with sealing wax, the connecting wires being soldered to the brass clips and brought out through the wax seals. Such a resistance when well made, was found to change in value by not more than ten per cent. in two months. The resistance of 30,000 ohm units was found to be appreciably constant for direct currents varying from 10^{-4} to 10^{-5} amperes. The currents which they carried in the amplifier never exceeded these values. The high frequency resistance of a unit was assumed to be the same as the resistance found by a direct current measurement.

The change in the value of the rectified high frequency component of the plate current of the second tube of the amplifier (which we have designated by b_0) was measured by a D'Arsonval galvanometer, G, Fig. 2, connected across a resistance r_4, placed in the plate circuit. r_4 was 60,000 ohms. The galvanometer had a resistance of 16 ohms and a sensibility of 1.7×10^{-9} amperes per millimeter deflection at a scale distance of 120 cm. P_1 and P_2, Fig. 2, were potential dividers, P_1 serving to keep the plate voltage at a standard value, and P_2 to compensate for the potential drop in the resistance r_4, so that the galvanometer rested approximately at zero. When the input grid voltage was changed, a deflection of the galvanometer resulted which was proportional to the change in the rectified high frequency component of the output plate current.

It was important that the filament voltage of the last tube and the voltages of P_1 and P_2 be constant. When a slow drift of the galvanometer occurred, the error was eliminated by averaging deflections. To insure that high frequency currents were not causing unknown disturbances in the galvanometer the following test was made. The deflection was observed with a certain voltage impressed on the amplifier. A 2 μF condenser was then connected across the galvanometer terminals, and it was found that the deflection remained unchanged. This showed that the high frequency currents did not appreciably affect the galvanometer.

4. The Two-Tube Amplifier.

(*a*) *The Variation of the Detecting efficiency with the Coupling Capacity.*— With the arrangement of apparatus as shown in Fig. 2, the resistances r_2 and r_3 were varied until the value of b_0 for a specified A at a frequency of 3×10^5 was approximately a maximum. This adjustment was not critical, and the values of r_2 and r_3 finally chosen were 27.6×10^3 and 393×10^3 ohms, respectively. After this was done the values of b^0 were observed for a series of values of A and for a series of values of the coupling condenser C_4. The values of b_0, shown in the family of

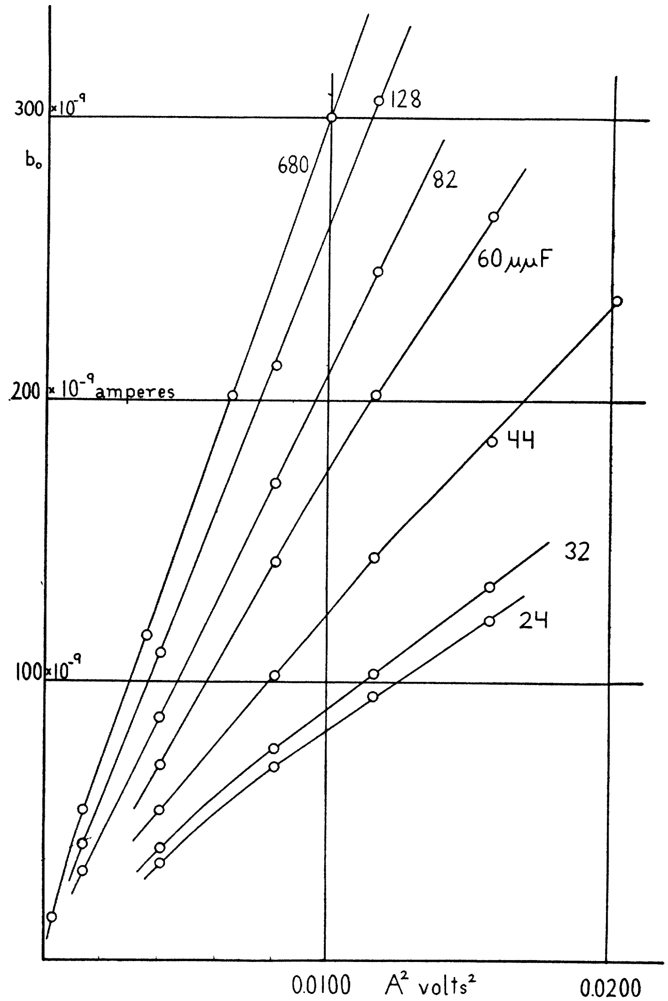

Fig. 3.

curves in Fig. 3, have been plotted as ordinates against the values of A^2 as abscissas. The frequency used was 3×10^5, corresponding to a wave-length of 1,000 m. It is seen that b_0 is approximately proportional to A^2, which is a first check on the correctness of formula (7). The fact that b_0 was approximately proportional to A^2 meant that the action of the last electron tube was represented approximately by the expansion of the plate-current grid-voltage relation, by Taylor's theorem, in which derivatives of higher order than the second were neglected. The slight bends in the lines of Fig. 3 may perhaps be ascribed to rectification occurring in the first tube, as discussed in section 2.

To see further to what extent the assumptions underlying (7) were in agreement with experiment, the values of b_0/A^2 for each value of C_4 in Fig. 3, for $A^2 = 0.010$ volts², have been plotted as ordinates against C_4 as abscissas in the full line curve of Fig. 4. In all cases the ratio b_0/A^2 has been expressed in amperes divided by volts². The numerical values of the circuit constants were as follows: $r_1 = 65 \times 10^3$ ohms, $r_2 = 27.6 \times 10^3$ ohms, $r_3 = 393 \times 10^3$ ohms, $\omega = 6\pi \times 10^5$, $C_4 = 680\ \mu\mu$F, and $C_5 = 38\ \mu\mu$F. The value for r_1 had been previously determined for this electron tube by measurements of its direct current characteristics for the filament and plate voltages used in these experiments. The value of C_5, *i.e.*, 38 $\mu\mu$F, was found by a measurement of the grid-to-filament capacity *in situ*, and included the capacity of the tube socket and connecting wires. Introducing the above values of the quantities into equation (7), and using the observed value of b_0/A^2, *i.e.*, 30×10^{-6}, for $C_4 = 680\ \mu\mu$F, the values of b_0/A^2 were computed for a series of values of C_4 from 680 to 10 $\mu\mu$F. The theoretical curve is shown in the dotted line of Fig. 4. The two curves agree closely for values of C_4 from 680 $\mu\mu$F to 200 $\mu\mu$F. For C_4 below 200 $\mu\mu$F the observed values of the detecting efficiency are seen to be less than the theoretical values.

If the more exact formula for the detecting efficiency had been used, *i.e.*, $\lim_{A \doteq 0} b_0/A^2$, instead of the formula b_0/A^2 where $A^2 = 0.010$ volts², the agreement between the observed and the theoretical detecting efficiencies would perhaps have been better. The data of Fig. 3 indicate this. At all events the agreement is sufficiently good to show that the simple theory embodied in formula (7) is an approximate statement of the facts.

(*b*) *The Variation of the Detecting Efficiency with the Frequency.*—The detecting efficiency of the amplifier was measured for a number of frequencies of the input voltage A. The coupling condenser C_4 was kept at its largest value, 680 $\mu\mu$F, throughout this series of measurements. The other capacities and resistances of the amplifier were the same as before. The values of A^2 were between 0.005 and 0.012 volts². The values of b_0/A^2, computed from the observed values of b_0 and A^2, have been plotted against wave-length in Fig. 5 for a range of wave-lengths from 400 to 1,600 meters. The points are shown as small circles in the figure; a smooth line, curve 1, has been drawn through them. The theoretical curve, computed from formula (7), is shown in the dotted curve 2, Fig. 5. This curve has been made to agree with the observed one at the wave-length 1,000 meters. The agreement between the theoretical and observed curves is good, and shows that formula (7) represents the behavior of the amplifier with fair accuracy.

(*c*) *Amplification.*—To determine the amplification due to the use of the first tube the detecting efficiency of the second tube alone was measured. This was done by disconnecting the first tube entirely and impressing the input voltage directly on the grid of the second tube. The values thus found for each wave-length are plotted in Curve 3,

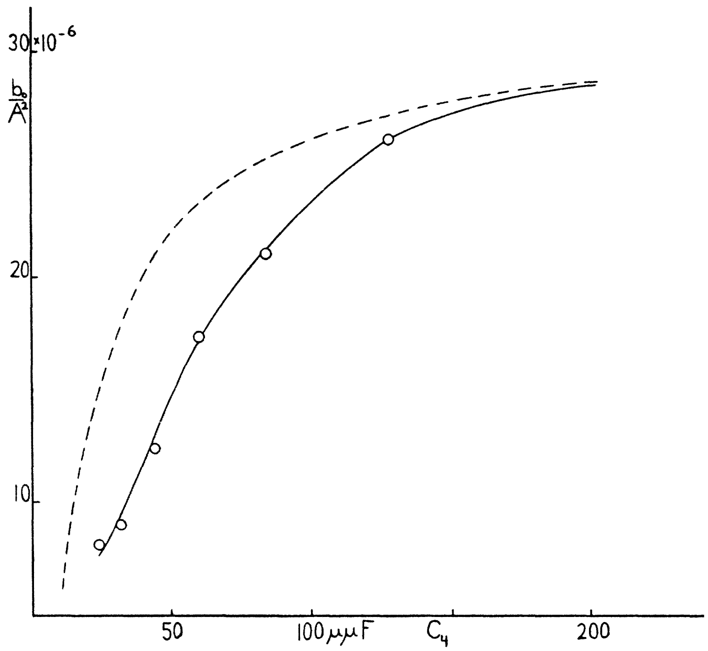

Fig. 4.

Fig. 5. It is seen that the detecting efficiency of the second tube in this case was appreciably constant with wave-length. This bears out the assumption underlying formula (6). By dividing the ordinates of curve 1 by those of curve 2, Fig. 5, for the same wave-length, the amplification of the rectified high frequency component b_0 of the plate current of the second tube due to the use of the first tube with the resistance-capacity coupling was determined. Since the detecting efficiency of the second tube, when alone, was 10×10^{-6}, the curve 1, Fig. 5, also shows the amplification. The marginal numbers on the right of Fig. 5 refer to the amplification. These numbers give the *current* amplification. The

power amplification or, if telephones are used, the *sound intensity* amplification is proportional to the square of these numbers.

5. COMPARISON WITH OTHER AMPLIFIERS.

A third tube was coupled to the two-step amplifier by means of resistance-capacity coupling. The plate resistance of this coupling was 19.4×10^3 ohms, the grid resistance was 134×10^3 ohms, and the coupling capacity was 680 $\mu\mu$ F. These correspond to r_2, r_3 and C_4 of Fig. 2, respectively. The detecting efficiency of this three-tube ampli-

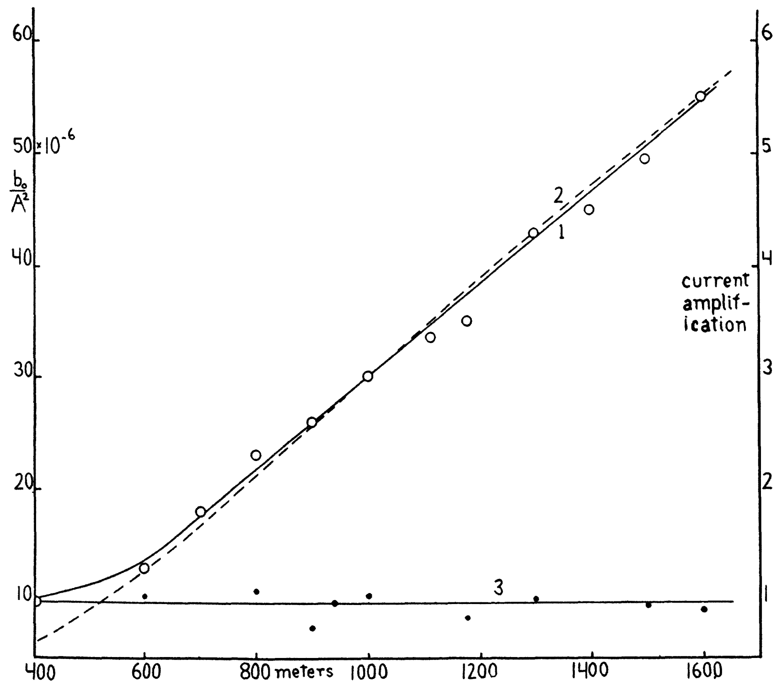

Fig. 5.

fier was measured for the wave-length range from 400 to 1600 meters. The curve is shown in curve 2, Fig. 6. Curves 1 and 3, Fig. 6, are reproductions of curves 1 and 3, Fig. 5, and refer to the two and one tube amplifiers respectively; they are placed in Fig. 6 for the sake of comparison. It is seen that for wave-lengths below 500 meters the addition of one and two tubes with resistance capacity coupling to the single detector tube produced no increase in amplification.

To show the comparison with a transformer-coupled amplifier data from a former paper[1] have been used and have been plotted in curves

[1] PHYS. REV., 16, 274, 1920. It should be noted that in Fig. 3 of this paper the number 15 should be 1.5.

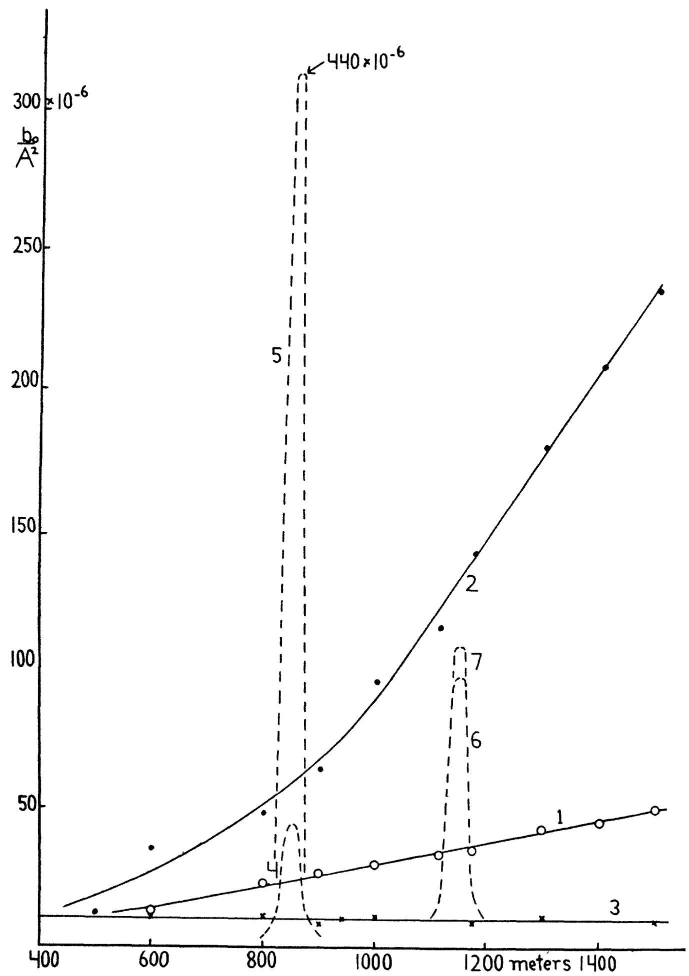

Fig. 6.

4 and 5, Fig. 6. These curves refer to a two-tube and a three-tube amplifier, respectively. The two-tube amplifier consisted of two tubes connected by a high frequency air (wood) core transformer tuned to a wave-length of 850 meters. The transformer was made of No. 36 silk-covered copper wire wound on paraffined wooden spools 3 cm. in diameter, 200 turns on the primary and 250 turns on the secondary. The detector tube of this amplifier was a different one from that of the two-tube resistance-capacity amplifier of curve 1, b_0/A^2 being 4.4×10^{-6}, whereas in the resistance capacity case b_0/A^2 was 10×10^{-6}; so that the curves 1 and 4 are merely an over-all comparison of the two ampli-

fiers. The three-tube amplifier of curve 5 was made by coupling a third tube to the two-tube amplifier by a similar transformer.

A more direct comparison of the performance of resistance-capacity coupling and transformer coupling was carried out in the following way. By means of two double-pole double-throw switches it was arranged so that either the resistance-capacity coupling or the transformer coupling could be used in turn. Fig. 7 shows the connections. When the re-

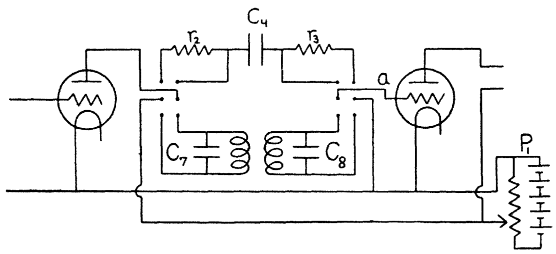

Fig. 7.

sistance-capacity coupling was used the amplifier was the same as that used above for curve 1, Fig. 6. The transformer coupling consisted of a tuned air core transformer. This was made of No. 36 silk-covered copper wire wound on paraffined wooden spools 3 cm. in diameter, 225 turns on the primary and 275 turns on the secondary. The primary was tuned by a small air condenser, c_7, Fig. 7, of capacity 37 $\mu\mu$F, and the secondary by another condenser c_8, of capacity 32 $\mu\mu$F. The transformer was tuned to a wave-length 1,150 meters. The detecting efficiency of this amplifier for various wave-lengths is shown by curve 6, Fig. 6. In order to make this comparison the procedure was as follows: With no impressed voltage A on the grid of the first tube the potential divider P_2, Fig. 2, was adjusted so that the galvanometer rested approximately at zero. Then A was turned on and the deflection of the galvanometer noted. The transformer coupling was then thrown in by means of the double-throw switches. It was found necessary to readjust P_2 by as much as perhaps 0.05 volts, and when A was turned on, the deflection was in the *opposite* direction to what it had been before in the case of the resistance capacity coupling. This meant that the grid potential of the second tube was not the same for the two cases, and hence that the detector tube was operating upon two different points on the grid-plate characteristic curve, one point being on a concave portion of the curve and the other on a convex portion. As a result curves 6 and 1, Fig. 6, do not necessarily show the true comparison between the two methods of coupling. To obtain a better comparison a voltage was inserted at point a, Fig. 7, with the negative terminal connected to the grid. Using the resistance-capacity coupling the voltage at a was adjusted until the galvanometer deflection, due to A, had the same absolute value as before, but in the opposite direction. The voltage at a was 1.10 volts. The deflection was now in the same direction as for the transformer coupling. Further, it was found that only a small readjustment of P_2 was required in changing from one coupling to the other. This meant that the detector was being used at approximately the same operating point in the two cases. The values of the detecting efficiency of the transformer-coupled amplifier were now found to be about ten per cent. greater than the values given in curve 6, Fig. 6, and are shown in curve 7, Fig. 6. Curves 7 and 1 exhibit the comparison between the two methods of coupling under approximately similar conditions. It should be noticed that Fig. 6 portrays in a clear manner the relative merits of a number of amplifiers as far as their detecting efficiency for an unmodulated voltage is concerned.

In conclusion the author finds pleasure in expressing his thanks to Mr. W. G. Brombacher for valuable assistance in carrying out the experimental arrangements.

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