

THE LAWS OF ABSORPTION OF X-RAYS.<sup>1</sup>

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## SYNOPSIS.

*Measurement of Absorption of X-rays.*—The use of a filtered beam of rays can at best give only approximate values of the coefficients, for such a beam is not homogeneous. It is pointed out that logarithmic absorption is no proof of homogeneity. In these experiments homogeneous radiation was secured by using a Bragg spectrometer and the effects of secondary radiation were avoided by placing the absorbing material between the two slits which limited the beam. By taking these and other precautions a precision of 1–2 per cent. was obtained. For wave-lengths below 0.15 Å. it was found better to place the thin analyzing crystal of NaCl nearly at right angles to the beam and to use the radiation reflected from atom planes perpendicular to the ones usually used.

*X-ray Absorption by Ag, Al, Cu, Mo, Pb and H<sub>2</sub>O for Wave-lengths about 0.13 to 0.7 Å.*—The linear relation, established by Hull and Rice, between the *mass-absorption coefficient* and  $\lambda^3$ ,  $\mu/\rho = F\lambda^3 + \sigma/\rho$ , holds accurately for these substances until, as  $\lambda$  is increased, the characteristic *K* limit is approached, when the values of  $\mu/\rho$  are increasingly too small. The linear relation also holds above the *K* limit, *F* having a much smaller value but  $\sigma/\rho$ , the *mass-scattering coefficient* being about the same. The *atomic absorption coefficient* for wave-lengths below *K* is a linear function of  $N^4$ , where *N* is the atomic number, in agreement with Owen's results. The *general law of X-ray absorption* is therefore as follows:  $\mu_a = kN^4\lambda^3 + \sigma_a$  where *k* is a universal constant equal to  $2.29 \times 10^{-27}$  below the *k* limit and to  $0.33 \times 10^{-27}$  above that limit. The experimental curves give no evidence for the existence of the "J" radiation reported by certain observers for Ag, Al, Cu and H<sub>2</sub>O. The absorption for water was measured down to 0.093 Å but shows no indication of an approach to the absorption found for the hard  $\gamma$ -rays of RaC.

## GENERAL STATEMENTS ON ABSORPTION MEASUREMENTS.

AN accurate knowledge of the laws of X-ray absorption by various media is of obvious importance in connection with the problems of atomic structure and of the nature of radiant energy. One may classify all measurements of X-ray absorption coefficients according as the investigator (1) has used a strictly homogeneous beam obtained by reflection from a crystal grating or (2) has attempted to get a more or less homogeneous beam either by employing the characteristic radiation of some target or by filtering out the longer waves from a gross beam by a filter such as aluminum and then assuming that the resultant filtered beam is practically homogeneous. In the latter case the mean wave-

<sup>1</sup> The work herein described was done at the Research Laboratory of the General Electric Co. The writer wishes hereby to make grateful acknowledgment, particularly to Dr. A. W. Hull, for the many courtesies extended and for the laboratory facilities made available.

length of the beam is given. Thus Auren,<sup>1</sup> in studying the absorption coefficients of a large number of compounds, from which to compute the coefficients of the components, filters his radiation (voltage applied to tube not stated) through 1.12, 2.05, 5.00 and 10.00 mm. of aluminum respectively. He gets the mean wave-length of each filtered beam by measuring the absorption coefficient of copper for the beams and compares this coefficient with data from Barkla for the variation of the absorption coefficient of copper as a function of wave-length. The mean wave-length of the filtered beams are .38, .36, .34, and .30 Ångströms, respectively. He concludes, from absorption measurements by use of these wave-lengths, that "although the atomic absorption coefficient in a great number of elements may be proportional to a power of the atomic number (yet) . . . in certain elements there are very marked discontinuities."

In an earlier paper<sup>2</sup> Auren concludes that the atomic absorption coefficient is not proportional to the fourth power of the atomic number of the absorber, contrary to conclusions reached by Bragg and Pierce,<sup>3</sup> and confirmed later by Owen<sup>4</sup> for the absorption of the  $\alpha$  line of Pd obtained by reflection from a crystal.

The use of a filtered beam for absorption measurements may give approximate results so long as one of the characteristic absorption limits of the absorber does not fall within the spectral region covered by the filtered beam; *but at best the results can be only approximations to the true values of the absorption coefficients.*

Another error should be pointed out in absorption measurements involving filtered radiation. Assuming the general absorption law for homogeneous radiation

$$I = I_0 e^{-\mu x}$$

one gets the linear equation between  $\log I$  and  $x$ :

$$\log I = -\mu x + \log I_0.$$

Conversely it has been assumed<sup>5</sup> that a strictly homogeneous radiation has been obtained by filtering when a plot between  $\log I$  and  $x$  is a straight line. This conclusion is not necessarily justified. The linear relation simply means that  $\mu$  has the same value for all components of the radiation, which may be homogeneous or heterogeneous; or in other words that, for the radiation remaining after filtering,  $\mu$  does not change

<sup>1</sup> Phil. Mag., 37, p. 165 (1919).

<sup>2</sup> Phil. Mag., 33, p. 471 (1917).

<sup>3</sup> Phil. Mag., 28, p. 626 (1914).

<sup>4</sup> Proc. Roy. Soc., 94, p. 510 (1919).

<sup>5</sup> E.g., see Rutherford, Barnes, and Richardson, Phil. Mag., 30, p. 339 (1915).

with  $\lambda$ . As is shown by data contained herein and also by Barkla and White<sup>1</sup> there is comparatively little change in the mass absorption coefficient,  $\mu/\rho$ , for water below  $\lambda = .2 \text{ \AA}$ . In the case of Al the change in  $\mu/\rho$  from  $\lambda = .15 \text{ \AA}$ . down to the shortest X-ray wave-lengths yet obtained is not more than 20 per cent. This is the obvious explanation of the linear relation between  $\log I$  and  $x$  after filtering through Al as obtained by Rutherford, Barnes, and Richardson, rather than the one given by these writers, namely that there is no decrease in wave-length in X-radiation by use of voltages above 125,000.

On the contrary absorption measurements in which a strictly homogeneous beam has been obtained by reflection from a crystal grating seem to lead to perfectly definite laws. Thus, Hull and Rice,<sup>2</sup> conclude that the mass absorption coefficient,  $\mu/\rho$ , is given as a function of the wave-length,  $\lambda$ , by an equation of the form

$$\frac{\mu}{\rho} = F\lambda^3 + \frac{\sigma}{\rho}, \quad (1)$$

where  $\sigma/\rho$  is the so-called mass-scattering coefficient, and  $F$  is a constant. These investigators verified this equation for aluminum, copper, and lead from  $.147 \text{ \AA}$ . to  $.392 \text{ \AA}$ . They assumed  $\sigma/\rho$  to be constant and equal to  $.12$ . In this equation the term  $F\lambda^3$  is due to fluorescent radiation. It is frequently written  $\tau/\rho$ , and is then called the mass-fluorescent absorption coefficient. Owen<sup>3</sup> shows that  $\tau/\rho$  is proportional to the cube of the atomic number of the absorber, *i.e.*,

$$\frac{\tau}{\rho} = CN^3 \quad (2)$$

at wave-length  $\lambda = .589 \text{ \AA}$ . But it should be remarked that Owen's results can hold only for those substances whose  $K$  absorption limit is on the long wave-length side of  $\lambda = .589 \text{ \AA}$ . A similar law may possibly hold for the group of elements whose  $K$  absorption limit is on the short wave-length side of  $\lambda = .589 \text{ \AA}$ ., but with a different value of  $C$ .

#### PURPOSE OF THE INVESTIGATION.

In view of the simplicity of the laws expressed by (1) and (2) and of the corresponding importance which they thereby assume, it seemed desirable to extend these measurements over as great a range of wave-lengths as possible, getting measurements of the highest attainable precision, in order to answer the following questions.<sup>3</sup>

<sup>1</sup> Phil. Mag., 34, p. 270 (1915).

<sup>2</sup> Phys. Rev., VIII., p. 326 (1916).

<sup>3</sup> Preliminary reports of the results of this investigation have been given in papers presented to the American Physical Society. See Richtmyer and Grant, Phys. Rev., and

I. Do equations (1) and (2) accurately hold over a wide range of wave-lengths, especially if the measurements be extended to the long wave-length side of the K absorption limit—the latter of course with change in value of the constant  $F$ ?

II. Is the term  $\sigma/\rho$  constant for all substances?

III. Is there, as reported by Barkla and White,<sup>1</sup> Williams,<sup>2</sup> Owen<sup>3</sup> and Dauvillier,<sup>4</sup> a discontinuity in the absorption curve, indicating a “ $J$ ” radiation, similar to the discontinuities that mark the  $K$  and  $L$  radiations?

#### APPARATUS AND METHOD.

The apparatus, patterned after the well-known Bragg X-ray spectrometer, needs no special description. The X-ray tube used was of the Coolidge type, the filament of which was operated by a well-insulated storage battery. For the short wave-length work was used a tube of the radiation type, specially exhausted. For the longer wave-lengths a water-cooled tube was employed, thus giving sufficient energy for accurate measurements in the neighborhood of 1.0 Å. Voltage was applied direct to the tube, without rectification, from the secondary of a high tension transformer, the primary of which was connected to a specially controlled 60 cycle A.C. dynamo. Under normal conditions the variations in voltage, over long periods was never greater than 0.1 per cent. The radiation type tube was operated at as high as 110 kv. r.m.s.

The target was approximately 45 cm. from the NaCl crystal. In line with target and crystal, and between the two, were two micrometer slits with lead jaws limiting the X-ray beam to a few tenths of a millimeter in width. The absorbing material to be studied was placed between these two slits, thus avoiding the troublesome secondary radiation when the absorber is placed next to the ionization chamber.

Above .15 Å. the crystal was used in the ordinary way, *i.e.*, the X-ray beam was reflected from the surface planes of a thin crystal. At shorter wave-lengths the glancing angle becomes rather small, of the order of 1 degree, and considerable trouble was experienced from stray secondary radiation entering the chamber directly. It was found that this was in part avoided if a crystal about three millimeters thick were employed,

Richtmyer, *PHYS. REV.* The writer wishes to express his indebtedness to Dr. Kerr Grant of Adelaide University, South Australia, who collaborated in the early part of this investigation and thereby made many of these measurements possible.

<sup>1</sup> *Phil. Mag.*, 34, p. 270 (1917).

<sup>2</sup> *Proc. Roy. Soc.*, 94, p. 567 (1919).

<sup>3</sup> *Proc. Roy. Soc.*, 94, p. 339 (1919).

<sup>4</sup> *Ann. de ph.*, XIV., p. 49 (1920).

and placed as shown in Fig. 1, thus using the atom planes at right angles to the major surfaces of the crystal. For these wave-lengths the crystal is sufficiently transparent, but cuts off the fluorescent radiation of longer wave-lengths coming from slit edges, crystal mounting, etc.



Fig. 1.

The ionization chamber was of brass, lead covered, approximately 60 cm. long by 10 cm. diameter. Rays were admitted through a thin mica window attached with sealing wax. A micrometer slit, with lead jaws immediately in front of the chamber, protected it from stray radiation. The collecting rod was supported by sulphur insulation, with suitable guard rings. Methyl bromide vapor was found to be most satisfactory for filling the chamber. This substance boils at 4° C. and is rather troublesome to handle but it did not attack the insulation as did some of the other vapors used. The collecting rod was charged to a potential of 200 volts by dry cells.

The ionization current was measured by observing the rate of increase of potential of the insulated system by means of an electroscope of the Bumstead type, the plates of which were charged to + 225 and - 225 volts respectively. The sensitivity was of the order of 200 divisions per volt. The rate of drift of the electroscope leaf was observed by a stop watch.

In order to eliminate observational and instrumental errors as far as possible the following precautions were observed.

1. The intensity of the X-ray beam was so adjusted (by current through filament or by applied voltage) that the rate of drift of the electroscope leaf was of the order of one or two divisions per second, some 60 divisions being covered for each reading. Errors in time measurements were of the order of 0.2 per cent. for each reading. The mean of several readings was taken for each observation. Correction was made for natural leak which was usually less than one per cent. of the ionization current measured.

2. The thickness of the absorber was so chosen that the intensity of the transmitted beam was seldom less than 40 per cent. or more than 65 per cent. of the incident beam.

3. When making an observation at any angle  $\theta$ , and wave-length  $\lambda$ , the voltage applied to the tube was so adjusted that there was not present in the X-ray beam second order energy of wave-length  $\lambda/2$ . This was determined by the equation

$$\lambda_{\max} = \frac{12.3}{V},$$

where  $\lambda_{\max}$  is the wave-length in Ångströms corresponding to the maximum frequency produced by  $V$  kilovolts. For example, if one is working at 0.4 Å. there should not be present in the incident beam any radiation of second order of 0.2 Å. Or

$$V = \frac{12.3}{.2} = 61.5 \text{ kilovolts}$$

is the (peak) voltage just necessary to produce radiation of 0.2 Å. Consequently a voltage somewhat less than this is to be employed. To guard against errors in estimating the maximum permissible voltage, due to distortion of wave form, a check was frequently made by observing whether, with the maximum permissible voltage as estimated, there was any radiation at  $\theta/2$ .<sup>1</sup>

The method of making observations was briefly as follows: At any given (chamber) angle  $2\theta$ , the time required for the gold leaf, after breaking earth connection, to drift from division  $A$  to division  $B$ , say 60 divisions apart, was observed, several readings being taken. The absorbing material was then put in place and several more readings taken. The absorber was removed and the first set repeated. The reciprocals of the mean average time with and without the absorber give quantities proportional to the respective ionization current. The absorption constant,  $\mu$ , was then determined from the usual law

$$I = I_0 e^{-\mu x}.$$

From knowledge of the density of the absorber and its thickness, or of the mass per square centimeter of absorbing material in the path of the X-ray beam, the mass absorption coefficient  $\mu/\rho$  was determined.

The following table will make the observations and method of computation clear.

*Determination of  $\mu$  for Molybdenum.*

*30.3 kilovolts; 1.70 milliamperes.*

Absorber: Molybdenum No. 1 + No. 2;  $\rho x = .0358$  gms. per sq. cm. Time to charge from division 20 to division 100:

|                               | Absorber out | Absorber in | Absorber out |
|-------------------------------|--------------|-------------|--------------|
|                               | 32.0 sec.    | 64.8 sec.   | 31.8 sec.    |
|                               | 32.2         | 64.2        | 31.6         |
|                               | 31.8         | 64.2        | 31.6         |
| Mean time.....                |              | 64.4        | 31.85        |
| 1/T (proportional to current) |              | .0314       | .0155        |
| Corrected for leakage.....    |              | .0311       | .0152        |

$$\frac{\mu}{\rho} = \frac{1}{\rho X} \log_e \frac{I_0}{I} = \frac{2.30 \times .311}{.0358} = 20.0 \text{ gms.}$$

<sup>1</sup> Assuming  $\theta = \sin \theta$  in the equation  $n\lambda = 2d \sin \theta$ .

The center of the direct beam was located on the divided circle by ionization measurements and the wave-length corresponding to any chamber angle  $2\theta$  was computed from the well known equation

$$\mu\lambda = 2d \sin \theta,$$

where  $d$  was taken as  $2.814 \times 10^{-8}$  Å. The calibration was always checked by setting on the  $K_{\alpha}$  lines of tungsten.

TABLE I.

| $\lambda$ | $\mu/\rho$ for   |       |      |       |      |      |
|-----------|------------------|-------|------|-------|------|------|
|           | H <sub>2</sub> O | Al    | Cu   | Mo    | Ag   | Pb   |
| .09 Å     | .165             |       |      |       |      |      |
| .120      | .175             |       |      |       |      |      |
| .135      | .179             | .193  | .59  | 1.51  |      |      |
| .184      | .199             | .239  | 1.25 | 3.27  | 4.26 | 3.20 |
| .209      | .207             | .276  | 1.74 | 4.55  | 5.94 | 5.11 |
| .234      | .220             | .328  | 2.31 | 6.15  | 9.50 | 7.00 |
| .258      | .234             | .393  | 3.00 | 8.10  | 12.5 | 9.35 |
| .283      | .249             | .469  | 3.87 | 10.75 | 15.6 | 12.0 |
| .308      | .266             | .567  | 4.87 | 13.5  | 19.5 | 15.5 |
| .332      | .284             | .670  | 6.02 | 16.7  | 23.6 | 19.3 |
| .356      | .306             | .795  | 7.29 | 19.8  | 28.5 | 23.7 |
| .368      | .319             | .861  | 8.10 | 21.7  | 31.0 | 26.3 |
| .381      | .335             | .940  | 8.85 | 23.7  | 34.0 | 29.0 |
| .393      | .348             | 1.017 | 9.64 | 25.6  | 37.0 | 31.9 |
| .405      | .358             | 1.095 | 10.2 | 27.5  | 39.2 | 34.5 |
| .417      | .376             | 1.174 | 11.3 | 29.5  | 43.7 | 37.9 |
| .430      | .394             | 1.285 | 12.3 | 31.5  | 47.8 | 41.4 |
| .453      |                  | 1.477 | 13.4 | 37.3  | 56.0 | 47.0 |
| .479      |                  | 1.706 | 16.6 | 42.5  |      | 55.0 |
| .503      |                  | 1.95  | 19.0 | 49.4  | 11.5 | 60.8 |
| .551      |                  | 2.49  | 24.9 | 63.8  | 15.1 | 73.0 |
| .600      |                  | 3.18  | 31.6 | 80.7  | 19.6 |      |
| .650      |                  | 4.05  | 39.5 | 15.1  | 24.3 |      |
| .715      |                  | 5.32  | 52.8 | 19.9  |      |      |
| .743      |                  | 5.83  |      | 22.6  |      |      |
| .798      |                  |       |      | 27.2  |      |      |
| .847      |                  |       |      | 32.3  |      |      |
| .895      |                  |       |      | 36.0  |      |      |
| .943      |                  |       |      | 43.4  |      |      |

## DATA AND RESULTS.

The following substances have been studied: water, aluminum, copper, molybdenum, silver, and lead, over wave-lengths ranging from  $.093 \text{ \AA}$ . to  $.95 \text{ \AA}$ . Owing partly to experimental difficulties, however, not all of these substances have been studied over the entire range. Thus measurements on lead were not made at wave-lengths greater than  $.550 \text{ \AA}$ . because of the difficulty of making homogeneous lead foil of the order of  $.001$  in. thick, to say nothing of the difficulty of measuring, to 1 per cent., the thickness after it is made. And in passing, it may be remarked that the securing of homogeneous absorbing films and of measuring their mass per square centimeter, constituted one of the serious difficulties of this investigation.

Table I. and Figs. 2 to 8 show the values of  $\mu/\rho$  as a function of wave-length for the substances studied. In the table, the data given for aluminum are actual observed points. Because of the fact that measurements were not in general made at the same wave-length settings for all the substances, some of the values given for the substances other than aluminum were read from curves plotted through observed points. Figures 2-8 however are plotted from actual observations, and show  $\mu/\rho$  plotted as a function of both  $\lambda$  and  $\lambda^3$ . With few exceptions the values of  $\mu/\rho$  are accurate to one per cent., as may be judged from the smoothness of the curves.

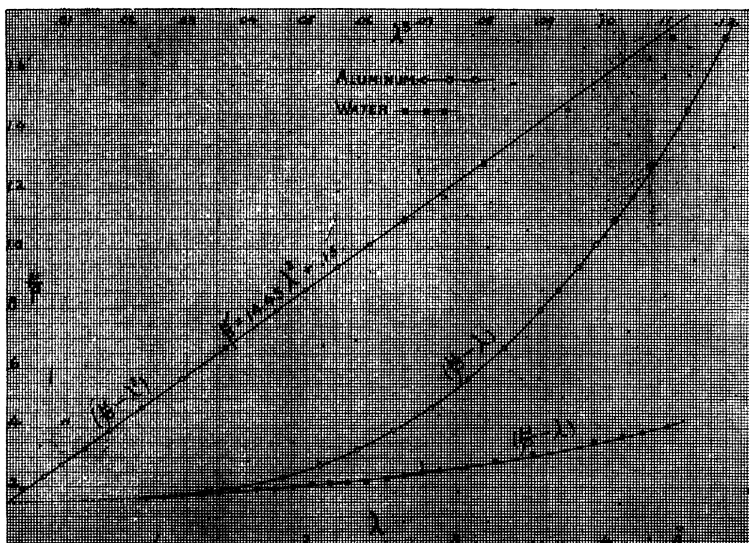


Fig. 2.



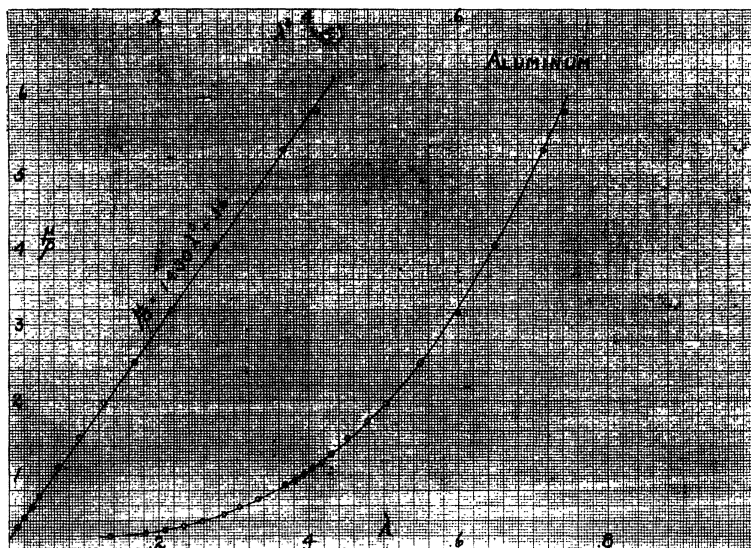


Fig. 3.

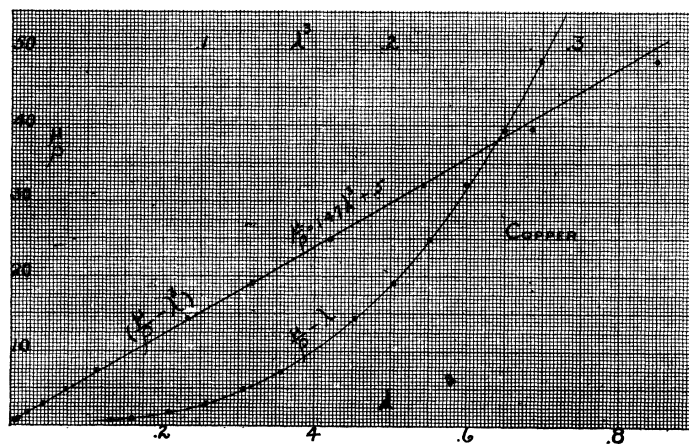


Fig. 4.

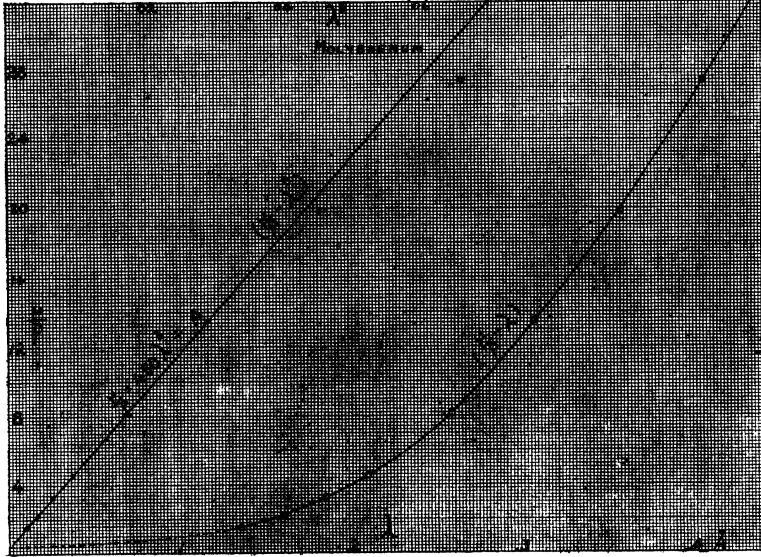


Fig. 5.

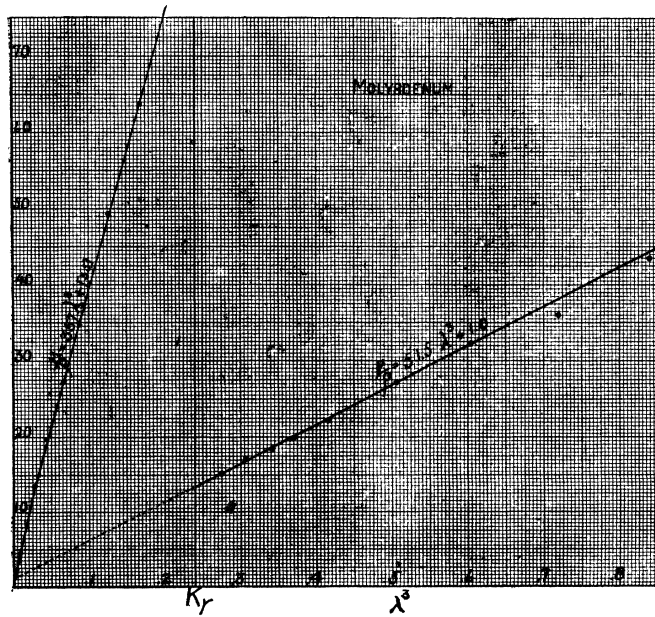


Fig. 6.

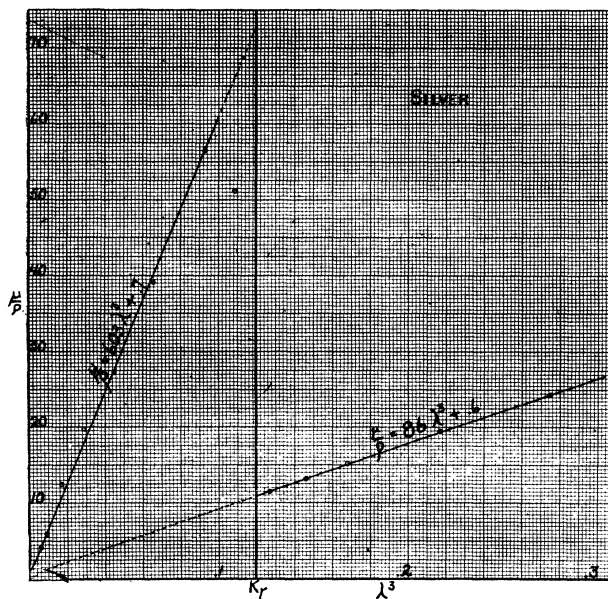


Fig. 7.

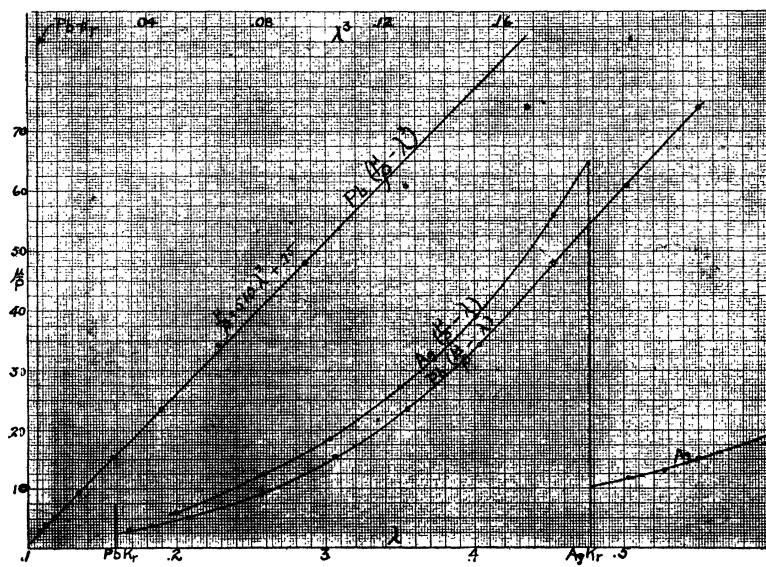


Fig. 8.

## CONCLUSIONS.

I. *Variation of  $\mu/\rho$  with Wave-length and with Atomic Number of Absorber.*—It will be seen in general that the curves plotted with  $\mu/\rho$  as a function of  $\lambda^3$  are, at the shorter wave-lengths, straight lines; but that as one goes toward longer wave-lengths, the values of  $\mu/\rho$  as predicted by this straight line are increasingly too great. Thus in Fig. 2 the equation

$$\left(\frac{\mu}{\rho}\right)_{Al} = 14.45\lambda^3 + .15$$

represents  $\mu/\rho$  very accurately up to about .4 Å., but above that value of  $\lambda$  values of  $\mu/\rho$  are less than would be expected. In Fig. 3, which is an extension of  $\mu/\rho$  for aluminum to longer wave-lengths the same tendency is noticed. The equation

$$\left(\frac{\mu}{\rho}\right)_{Al} = 14.30\lambda^3 + 16$$

gives values of  $\mu/\rho$  slightly too high at .4 Å. and slightly too low at .7 Å.

Reference to copper, Fig. 4, shows the same tendency to concavity toward the  $\lambda^3$  axis. This concavity is still more marked in the case of substances of higher atomic number, as is shown by Figs. 5 and 7 for molybdenum and silver respectively.

Figures 6, 7 and 8 show the values of  $\mu/\rho$  for molybdenum, silver, and lead respectively on the long wave-length side of the  $K$  absorption line. Here again the linear relation between  $\mu/\rho$  and  $\lambda^3$  seems to hold rather accurately, except that in the case of lead the last two observations fall very much below the straight line.

It may be concluded therefore that the general equation

$$\frac{\mu}{\rho} = F\lambda^3 + \frac{\sigma}{\rho} \quad (1)$$

represents the law of absorption, provided one is not approaching, from the short wave-length side, a characteristic absorption band. To express the law accurately there must be deducted from the right hand side of (1) a term involving some unknown function of  $\lambda/\lambda_K$  where  $\lambda_K$  is the wave-length corresponding to the  $K$  absorption limit. Keeping in mind this limitation the following equations give the mass-absorption coefficient for the several substances studied, and the range over which the equation holds.

In order to ascertain how  $\mu/\rho$  varies with the atomic number of the absorber it is necessary to discuss separately the two terms  $F\lambda^3$  and  $\sigma/\rho$ . Since it is the atom as a whole which acts as a unit in absorption it is

TABLE II.

*On Short Wave-length Side of the Respective K Absorption Limit.*

|                            |                                   |            |
|----------------------------|-----------------------------------|------------|
| Aluminum.....              | $\mu/\rho = 14.45\lambda^3 + .15$ | .1Å to .4Å |
|                            | $\mu/\rho = 14.30\lambda^3 + .16$ | .4 to .7   |
| Copper.....                | $\mu/\rho = 147\lambda^3 + .5$    | .1 to .6   |
| Molybdenum ..              | $\mu/\rho = 450\lambda^3 + .4$    | .1 to 35   |
| Silver <sup>15</sup> ..... | $\mu/\rho = 603\lambda^3 + .7$    | .1 to .4   |

*On Long Wave-length Side of the K Absorption Limit.*

|                           |                                 |  |
|---------------------------|---------------------------------|--|
| Molybdenum ..             | $\mu/\rho = 51.5\lambda^3 + 1$  |  |
| Silver <sup>1</sup> ..... | $\mu/\rho = 86\lambda^3 + .6$   |  |
| Lead.....                 | $\mu/\rho = 510\lambda^3 + .75$ |  |

$\lambda$  is expressed in Ångström units.

first necessary to reduce values of  $\mu/\rho$ , in which absorption is given for a layer of material thick enough to contain 1 gram per sq. cm., to the atomic absorption coefficient,  $\mu_a$ , for a layer containing one atom per sq. cm. This is done by dividing both sides of equation (1) by  $n/A$ , the number of atoms per gram, where  $n$  is Avogadro's constant  $6.06 \times 10^{23}$ , and  $A$  is the atomic weight. This gives

$$\mu_a = \frac{\mu/\rho}{n/A} = \frac{A}{n} \cdot F\lambda^3 + \frac{A}{n} \cdot \frac{\sigma}{\rho}, \quad (3)$$

$$= F_a\lambda^3 + \sigma_a \quad (4)$$

where  $F_a = F(A/n)$ , and  $\sigma_a = (\sigma/\rho) \cdot (A/n)$  is the atomic scattering coefficient.

Figure 9, plotted from the last two columns of Table III. shows the relation between  $F_a$  and the atomic number of the absorber. The slope

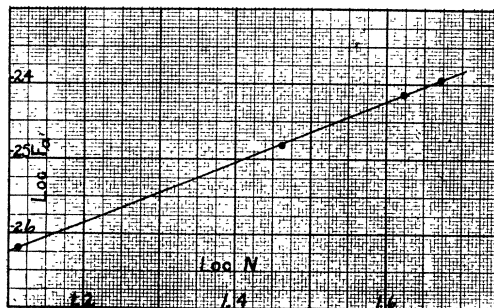


Fig. 9.

<sup>1</sup>Data on Silver is approximate, since measurements were not made as carefully as on other substances.

TABLE III.

| Substance. | At. Number. | $F$ .             | $F_a = F A/n$ .        | Log $N$ . | Log $F_a$ . |
|------------|-------------|-------------------|------------------------|-----------|-------------|
| Al.....    | 13          | 14.45             | $64.5 \times 10^{-23}$ | 1.114     | -27.810     |
| Cu.....    | 29          | 147               | 1,550                  | 1.462     | -25.190     |
| Mo.....    | 42          | 450               | 7,130                  | 1.623     | -25.854     |
| Ag.....    | 47          | 650 <sup>16</sup> | 11,600                 | 1.672     | -24.053     |

of the curve is 4.00. We may accordingly write

$$F_a = kN^4. \quad (5)$$

The mean value of  $k$  for these four elements is  $2.29 \times 10^{-27}$ . Putting this value of  $F_a$  in equation (4) gives

$$\mu_a = 2.29 \times 10^{-27} N^4 \lambda^3 + \sigma_a \quad (6)$$

as the value of the atomic absorption coefficient as a function of atomic number and wave-length, *provided* the values of  $\lambda$  are well removed (*i.e.*, on the short wave-length side) from the  $K$  characteristic absorption limit. Although of numerical significance only, it may be observed that this value of  $k$  is about one third of Planck's constant  $h$ . So that for keeping the equation in mind we may write, approximately

$$\mu_a = \frac{h}{3} N^4 \lambda^3 + \sigma_a. \quad (6')$$

Equation (5) agrees with the conclusions of Owen,<sup>5</sup> who, however, worked at only one wave-length.

It must be kept in mind, as pointed out above, that (6) does not represent the value of  $\mu_a$  as the absorption limit is approached. To make the equation complete we should have to write

$$\mu_a = kN^4 \lambda^3 + \sigma_a - f\left(\frac{\lambda}{\lambda_K}\right), \quad (7)$$

where  $f(\lambda/\lambda_K)$  is a function, at present unknown, but which increases, perhaps rapidly, as the  $K$  absorption limit  $\lambda_K$  is approached.

An inspection of Table II. and the curves for molybdenum, silver, and lead shows that on the long wave-length side of their characteristic  $K$  absorption limits an equation of the form

$$\frac{\mu}{\rho} = F' \lambda^3 + \frac{\sigma}{\rho} \quad (8)$$

expresses the relation between  $\mu/\rho$  and  $\lambda$ . Data is not sufficiently accurate (on account of the very thin absorbing sheets necessary) to deter-

<sup>1</sup> Taken from the short wave-length data for Ag.

mine how  $F_a'$  depends on atomic number. But a plot, similar to that for  $F_a$  shows that

$$F_a' = k'N^{4.4}.$$

All that can be said is that the exponent of  $N$  is at least of the same order as that in the equation for  $F_a$ .

II. *The Value of the Mass Scattering Coefficient.*—From Fig. 2 it is seen that the value of the mass scattering coefficient of aluminum,  $\sigma/\rho$ , is .15. Data for the remaining elements is not sufficiently accurate to determine values of  $\sigma/\rho$  except to say that there seems to be a tendency for an increase in  $\sigma/\rho$  with increasing atomic number of absorber. Much more data at very short wave-lengths is necessary to determine how  $\sigma/\rho$  varies with  $N$ .

But it is to be noted, that, so far as data goes, the value of  $\sigma/\rho$  as determined from data on silver and molybdenum is to a first approximation the same on the long wave-length side of the  $K$  absorption limit as on the short wave-length side, for both curves cut the  $\sigma/\rho$  axis at approximately the same point.

The importance of this is shown in Fig. 10, in which the value of  $\mu/\rho$  is shown diagrammatically as a function of  $\lambda^3$ . Allow X-rays of wave-length  $\lambda_1$  to fall on a *thin* sheet of the absorber whose characteristic  $K$  limit is  $\lambda_K$ . The vertical line  $ad$  is proportional to the total energy removed from the transmitted beam. Of this energy an amount proportional to  $ab$  is re-radiated as  $K$  fluorescent radiation; an amount proportional to  $bc$ , as  $L$  fluorescent radiation; and an amount proportional to  $cd$

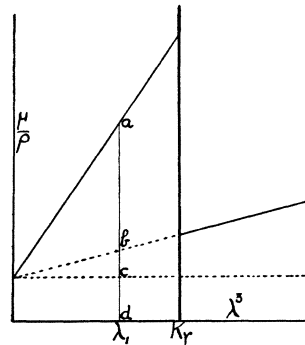


Fig. 10.

is scattered without change of wave-length. Subject therefore to the correction term mentioned in equation (7) the mass absorption coefficient for a given substance is given by

$$\frac{\mu}{\rho} = K\lambda^3 \int_0^K + L\lambda^3 \int_0^L + \frac{\sigma}{\rho}. \quad (9)$$

The term  $K\lambda^3 \int_0^K$  is to be used from  $\lambda = 0$  to the  $K$  absorption limit and determines the  $K$  fluorescent radiation; the term  $L\lambda^3 \int_0^L$  is to be used up to the first  $L$  absorption limit.  $L$  itself is probably made up of several terms.

In spite of the correction term mentioned in equation (7) these laws of X-ray absorption are exceedingly simple as compared with the corre-

sponding laws in the ultra-violet and visible region of the spectrum. It may not be too much to hope that a gradual extension of these laws to longer and longer wave-lengths may lead to some clue as to absorption of visible radiation.

III. *Is there Evidence of a Discontinuity in Absorption Curves Indicative of a Characteristic "J" Radiation?*—Table IV. gives the location of the discontinuities in the curves for  $\mu/\rho$  as observed by various investigators.

TABLE IV.

| N       | Element. | Barkla and White. <sup>10</sup> | Williams. <sup>11</sup> | Owen. <sup>12</sup> | Dauvillier. <sup>13</sup> |
|---------|----------|---------------------------------|-------------------------|---------------------|---------------------------|
| 6.....  | C        | .42 Å.                          | —                       | —                   | —                         |
| 7.....  | O        | .39                             | —                       | —                   | —                         |
| 13..... | Al       | .37                             | .49 Å.                  | —                   | .358                      |
| 14..... | Si       | —                               | —                       | .493                | —                         |
| 29..... | Cu       | —                               | .448                    | —                   | —                         |
| 35..... | Br       | —                               | —                       | —                   | .227                      |
| 47..... | Ag       | —                               | —                       | —                   | .178                      |

Barkla and White's data is shown in Fig. 11.<sup>17</sup> They assume that the absorption of water is due mainly to oxygen; and of paraffin mainly to carbon. In the present paper no data is shown for carbon, but reference to Fig. 2 will show that there is no discontinuity in water at .39 Å.

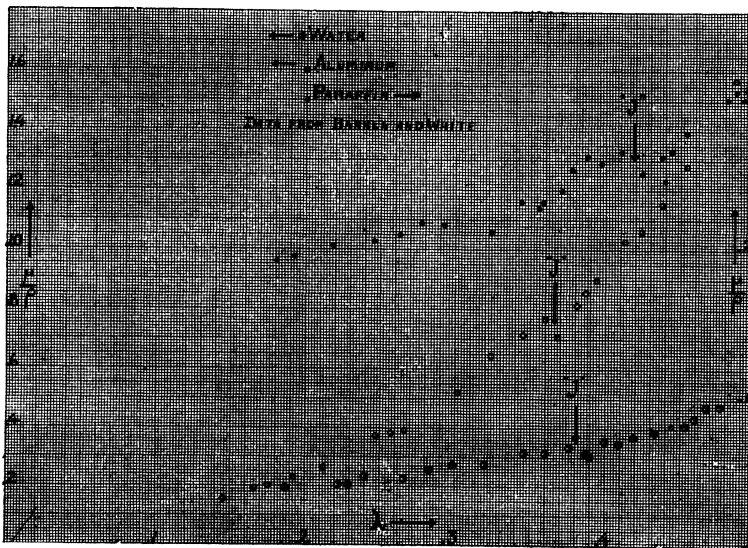


Fig. 11.

<sup>17</sup> The plotting is my own. F.K.R.



Williams concluded that Barkla's discontinuity for aluminum was at too short a wave-length, and should be at  $.49 \text{ \AA}$ . Fig. 3 shows no discontinuity in aluminum at this wave-length, nor is there any apparent discontinuity in copper at  $.448 \text{ \AA}$ . as will be seen from Fig. 4. It is suggested that these apparent discontinuities were due to a rather large experimental error, combined with an accidental grouping of points. Thus Barkla and White's data would show apparent discontinuities at several other wave-lengths than those mentioned.

Dauvillier's apparent discontinuity in aluminum at  $.358$  seems to be due to the  $K$  absorption limit of iodine in his ionization chamber, for no discontinuity is to be observed at this point in Fig. 2. His " $J$ " for bromine is identical with Duane's  $\alpha_3$  line for tungsten. I have been unable to trace the origin of his suggestion of a " $J$ " for silver at  $.178 \text{ \AA}$ . But Dauvillier states that if the square root of the frequencies for these three " $J$ 's" be plotted, after Mosley, against atomic number, a straight line results. Such a graph predicts a discontinuity for copper at  $.250 \text{ \AA}$ . and for molybdenum at  $.196 \text{ \AA}$ . An inspection of Figs. 4 and 5 will show that there are no discontinuities for these elements at these points. The conclusion is that there is no evidence in favor of discontinuities in absorption curves indicating a radiation in the X-ray region shorter than the  $K$  series.

#### THE MASS ABSORPTION COEFFICIENT OF WATER AT VERY SHORT WAVE-LENGTHS.

It is known that the value of  $\mu/\rho$  for the hard rays from radium  $C$  is of the order of  $.05$  to  $.08$  for various substances, much lower than would be expected by extrapolation to  $\lambda = 0$  of the curves, say, in Fig. 2 for water and aluminum. The wave-length of these rays from radium  $C$  is not known but has been estimated by Rutherford and Andrade to be less than  $.01 \text{ \AA}$ . An attempt was made to see whether, in the case of water, there was any evidence of a downward bend in the curve, at very short wave-lengths, toward the value given for the rays from radium  $C$ . Although measurements were pushed down to  $.093 \text{ \AA}$ . there was no evidence of a downward bend. It is as if the mechanism operative at  $.01 \text{ \AA}$ . must differ materially from that which is responsible for the scattering of the much shorter rays from radium  $C$ . If we assume that the innermost group of electrons around the nucleus is responsible for the emission and absorption of  $K$  radiation, we must look to the nucleus itself for the emission and absorption (scattering) of still shorter groups of rays as the hard rays from radium  $C$  are thought to be. Rutherford has recently found evidence pointing to a structure of the nucleus, of elements

up to oxygen, as made up of both positive and negative charges. Such a system should obviously both radiate and absorb. Perhaps further knowledge of the nucleus may throw light on the question of absorption of very short waves.

## SUMMARY.

1. It has been shown that the mass absorption coefficient of Ag, Al, Cu, Mo, Pb and H<sub>2</sub>O for X-rays is given in each case by an equation of the form

$$\frac{\mu}{\rho} = F\lambda^3 + \frac{\sigma}{\rho}, \quad (1)$$

so long as the values of  $\lambda$  are considerably smaller than the  $K$  absorption limit  $\lambda_K$ , and that as this limit is approached a term of the form  $f(\lambda/\lambda_K)$  must be subtracted. This function is unknown.

2. Above the  $K$  absorption limit an equation of the same type holds, namely

$$\frac{\mu}{\rho} = F'\lambda^3 + \frac{\sigma}{\rho}. \quad (8)$$

3. If the atomic absorption coefficient  $\mu_a$  be computed, equation (1) becomes

$$\mu_a = F_a\lambda^3 + \sigma_a \quad (4)$$

and it has been shown that  $F_a$  is proportional to the fourth power of the atomic number, *i.e.*,

$$F_a = kN^4,$$

where the numerical value of  $k$  is  $2.29 \times 10^{-27}$ . This combines with (1) to give the atomic absorption coefficient of substances (from Al to Ag at least) by the equation

$$\mu_a = 2.29 \times 10^{-27} N^4 \lambda^3 + \sigma_a.$$

4. The value of the mass scattering coefficient seems to be independent of wave-length, since to a first approximation at least the value of  $\sigma/\rho$  is the same, for each substance, in equations (8) and (1).

5. There is no evidence, on the basis of absorption measurements, for the existence of a "J" radiation.

6. The value of  $\mu/\rho$  for water, down as far as .093 Å. shows no evidence of an approach to the corresponding value for the hard rays from radium C.

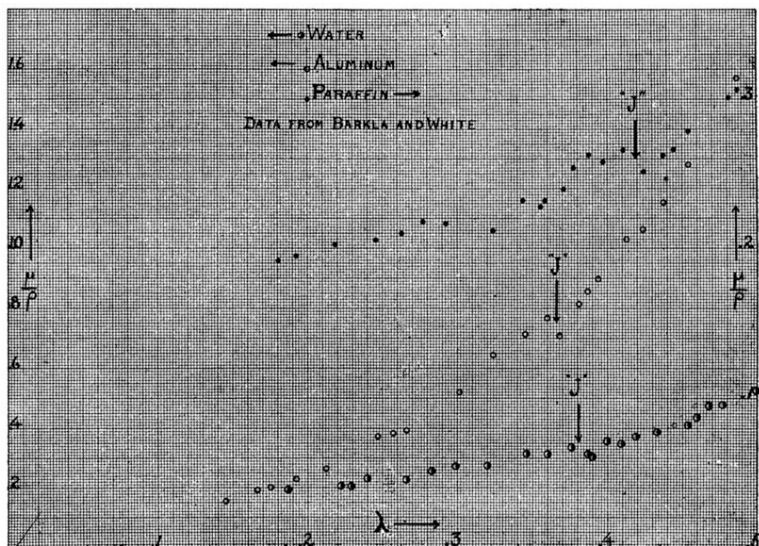


Fig. 11.

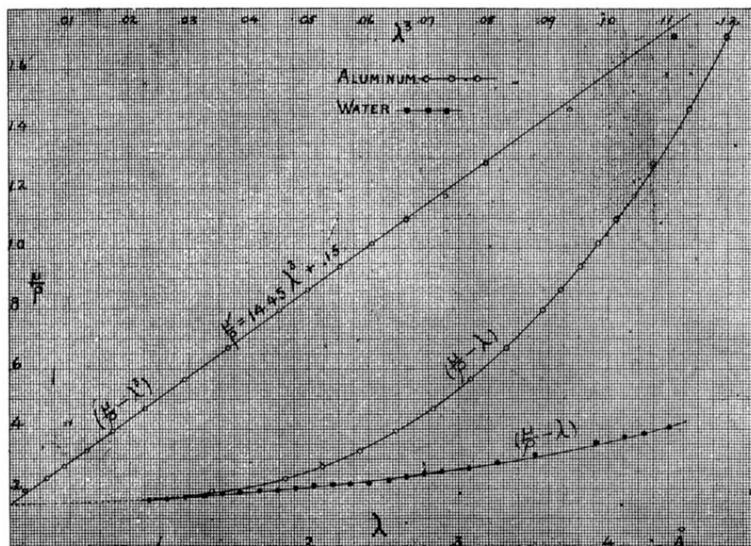


Fig. 2.

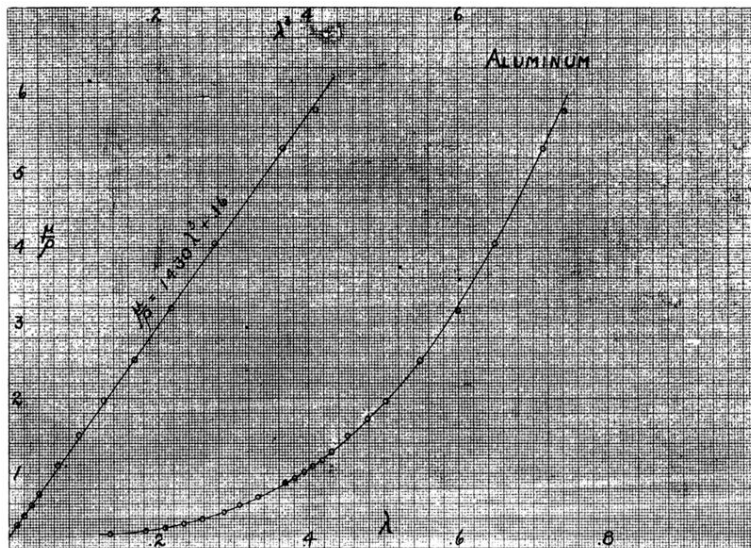


Fig. 3.

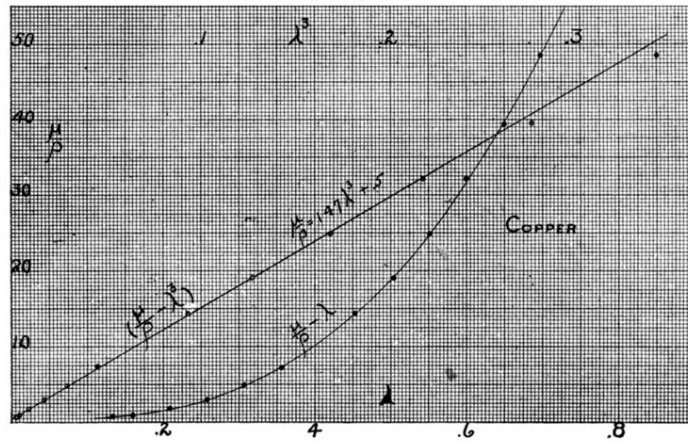


Fig. 4.

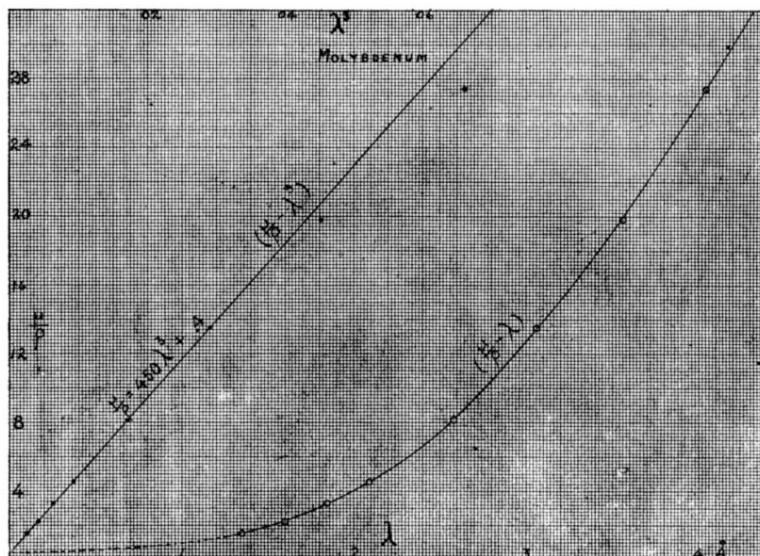


Fig. 5.

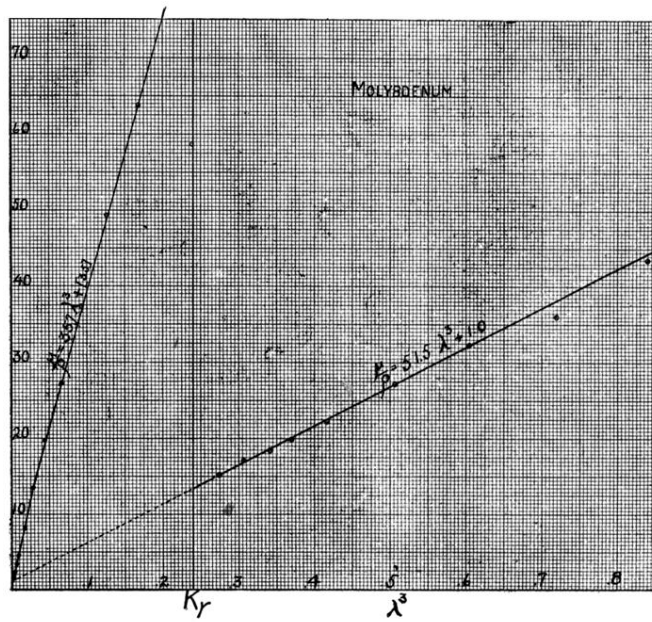


Fig. 6.



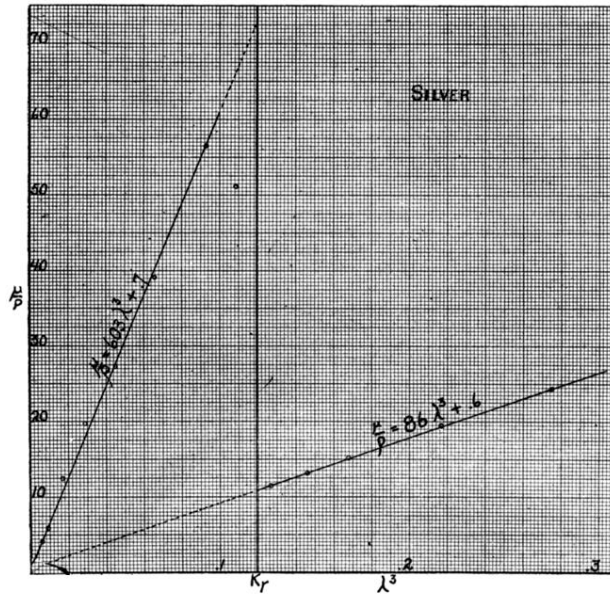


Fig. 7.

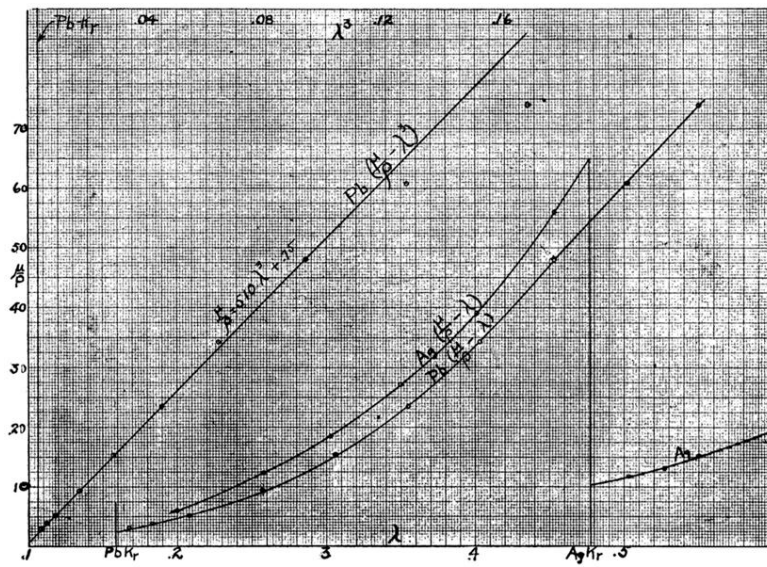


Fig. 8.

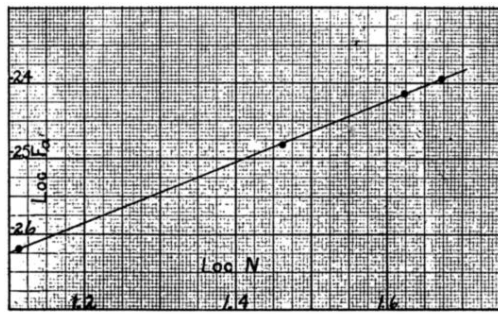


Fig. 9.