Singularities on the Fermi Surface of Bismuth

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A study of the Fermi surface of bismuth is carried out by means of the magnetoacoustic effect. In particular, the hole Fermi surface is investigated. The main result is a strong indication that the usual dispersion law cannot be valid over the entire **k** space. The experimental results, in fact, seem to fit the hypothesis that there are singular points (saddle points) on the Fermi surface.

1. INTRODUCTION

AMONG various other experimental methods (for example, the de Haas–van Alphen effect and the de Haas–Shubnikov effect) for study of the Fermi surface, much attention has been paid in the past few years to the magnetoacoustic effect.

Such a method consists in the experimental determination of the acoustic absorption coefficient in a sample placed in a magnetic field at a low temperature. The acoustic absorption coefficient α oscillates as the magnetic field strength H changes. The period of such oscillations as a function of H^{-1} gives the value of the cross-sectional area of the Fermi surface orthogonal to the magnetic field direction.

As the magnetic field increases, various regions can be considered in the behavior of the acoustic absorption coefficient. Reneker[1] considers five regions that arise as the magnetic field increases, namely, (a) the magnetoresistance region, (b) the cyclotron resonance region, (c) the geometric resonance region, (d) the saturation region, and (e) the de Haas–van Alphen region. The first four regions are important at relatively small intensities of the magnetic field. The de Haas–van Alphen region is characterized by an oscillating behavior of α, the oscillating part being only a few percent of the total attenuation. From these oscillations the extremal cross-sectional area of the Fermi surface can be evaluated. In addition, the oscillations in the de Haas–van Alphen region are independent of the angle between the magnetic field vector and the sound wave vector.[2]

For stronger magnetic fields and lower temperatures, a new phenomenon arises as a consequence of the Landau levels. This phenomenon consists of sharp peaks in the acoustic attenuation coefficient as a function of the magnetic field strength (giant quantum oscillations). The effect is a quantum effect and was first predicted by Gurevich *et al.*[3] The transition from the de Haas–van Alphen region to the giant quantum oscillation region was investigated by Liu and Toxen[2] and by Mase *et al.*[4] Giant quantum oscillations have been observed by several authors.[5–8]

The giant quantum oscillations differ from the de Haas–van Alphen oscillations mainly in two characteristics. The first is that the variation of α in giant quantum oscillations is much more sizable. The second is that the behavior of the giant quantum oscillations depends on the angle β between the magnetic field vector and the sound wave vector, especially when this angle approaches $\frac{1}{2}\pi$. This latter circumstance produces, in turn, two fundamental effects. The first is the disappearance of the giant oscillations for $\beta=\frac{1}{2}\pi$. The behavior of α for $\beta\sim\frac{1}{2}\pi$ at high magnetic fields is known as the tilt effect and was first classically explained in a qualitative way by Reneker[1] and more rigorously by Spector.[9] From a quantum-mechanical point of view, the disappearance of the oscillations for $\beta=\frac{1}{2}\pi$ is contained in the Gurevich theory [see Eq. (1) below]. The second effect is that from the period of the giant quantum oscillations one can calculate the cross-sectional area of the nonextremal Fermi surface, as will be seen in more detail.

For a better understanding of the various phenomena concerning the acoustic absorption coefficient in a magnetic field, refer to the work of Mase *et al.*[4]

The present work is concerned with giant quantum oscillations in bismuth. That we actually observe giant quantum oscillations is supported by the following arguments:

(a) The amplitudes of oscillations that are observed are an order of magnitude larger than those of the de Haas–van Alphen oscillations.

(b) The sharpness of the so-called tilt effect ensures that the condition $\omega_q\tau\gg1$ is fulfilled.[1,9]

(c) The behavior of oscillations shows a dependence on the angle between the sound wave vector and the magnetic field vector, as is the case for the giant quantum oscillations.

[1] D. H. Reneker, Phys. Rev. **115**, 303 (1959).
[2] S. H. Liu and A. N. Toxen, Phys. Rev. **138**, A487 (1965).
[3] V. L. Gurevich, V. G. Skobov, and A. Firsov, Zh. Eksperim. i Teor. Fiz. **40**, 786 (1961) [English transl.: Soviet Phys.—JETP **13**, 552 (1961)].
[4] S. Mase, Y. Fuimori, and H. Mari, J. Phys. Soc. Japan **21**, 1744 (1966).
[5] P. Korolyuk and T. A. Prushchak, Zh. Eksperim. i Teor. Fiz. **41**, 1689 [English transl.: Soviet Phys.—JETP **14**, 1201 (1962)].
[6] Y. Shaphira and B. Lax, Phys. Rev. Letters **12**, 166 (1964).
[7] A. M. Toxen and S. Tansal, Phys. Rev. **137**, A211 (1965).
[8] M. Giura, R. Marcon, T. Papa, and F. Wanderlingh, Nuovo Cimento **51B**, 150 (1967).
[9] H. Spector, Phys. Rev. **120**, 1261 (1960).

As pointed out before, the main characteristic of such a phenomenon is that the cross-sectional areas of the Fermi surface that one measures are not extremal, as in other similar effects. The distance from the origin, in **k** space, at which the cross-sectional area is measured, is given by[3]

$$k_H = m^* u / \hbar \cos\beta , \qquad (1)$$

where β is the angle between the magnetic field vector **H** and the sound wave vector, u is the sound velocity, and m^* is the effective mass in the **H** direction. Recently, Mase *et al.*[4] obtained in the case of the ellipsoidal (but not the parabolic) energy surface the equation

$$k_H = (m^* u / \hbar \cos\beta)(1 + E_F / E_g) .$$

However, the corrective factor is angle-independent, as Mase *et al.* point out, and is therefore not important for our purposes.

Because of the small value of m^*, the quantity k_H is appreciably different from zero only for β near $\frac{1}{2}\pi$. This condition is generally fulfilled in our experiments. In addition, in order to have sound absorption, k_H must fulfill the condition

$$E_F = (n + \tfrac{1}{2})\hbar\omega_c + \hbar^2 k_H^2 / 2m^* , \qquad (2)$$

where E_F is the Fermi energy, ω_c is the cyclotron frequency, and n is an integer.

The present paper is a report of an investigation on the hole Fermi surface in bismuth, from which an interesting consequence is obtained.

The first model proposed for the Fermi surface in bismuth was given by Schoenberg.[10] In recent years more detailed investigations were carried out.[11] In each case convex closed surfaces (ellipsoids) were considered, some with small departures from ellipticity. Our experimental results, however, seem to indicate that such a model is inadequate.

The main characteristic of the experimental results is that, for particular directions of the magnetic field with respect to the crystallographic axes and the sound wave vector, the giant quantum oscillations of the ultrasound absorption coefficient as a function of magnetic field strength disappear.

As will be shown in Sec. 3, and with a detailed analysis of experiments in Sec. 4, this effect seems to be due to the intrinsic structure of the dispersion law of the carriers in bismuth. A possible explanation is the existence of singular points (saddle points) on the Fermi surface. In a saddle point[12] the Landau quantization disappears when the plane orthogonal to the magnetic field direction (cross-sectional plane), placed a distance k_H from the origin, becomes tangent to the Fermi surface in the saddle point itself. It is the possibility of the variation of k_H in the magnetoacoustic effect, as given in Eq. (1), that allows the detection of saddle points on the Fermi surface.

2. EXPERIMENTAL SETUP AND PROCEDURE

The experimental arrangement is described in a previous paper.[8]

The sample is a parallelepiped monocrystal of bismuth, with edges parallel to the binary, bisector, and trigonal axes.

The orientation of the sample in the helium bath can be carefully adjusted by rotation around two mutually orthogonal axes (see Fig. 1).

The magnetic field acting on the sample is the sum of a major field supplied by an electromagnet and an auxiliary field, orthogonal to the first, given by a coil to which a variable current can be supplied (see Fig. 2). In the range of our measurements the main magnetic field supplied by the electromagnet can be considered proportional to the current without appreciable error. This was experimentally verified.

The experiment can be performed in two different ways. In the first, the current flowing in the field coil is proportional to the current driving the electromagnet. In this arrangement the direction of the magnetic field with respect to the sound wave vector is left constant, while the field intensity is continuously changed. A Hall probe detects the magnetic field strength and drives the x axis of an x-y plotter. The amplitude of an echo pulse is recorded on the y axis.

In the second arrangement, the major magnetic field is kept constant, while the current in the auxiliary field coil is continuously changed. In such a case the direction of the total magnetic field changes, while the intensity remains approximately constant. In this arrangement the x axis of the recorder is driven by the auxiliary coil current, i.e., the x displacement is proportional to the angle between the magnetic field vector and the sound wave vector.

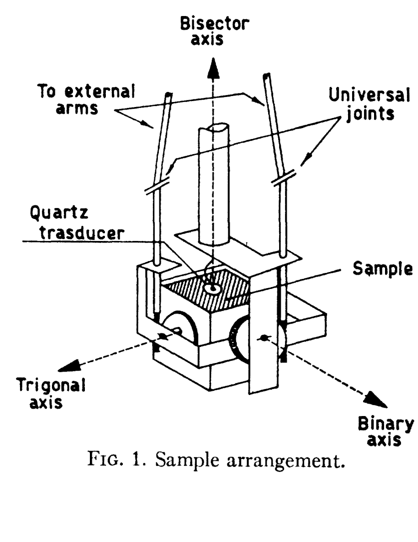

FIG. 1. Sample arrangement.

[10] D. Schoenberg, Proc. Roy. Soc. (London) **A170**, 341 (1939).

[11] See, e.g., Yi-Han Kao, Phys. Rev. **129**, 1122 (1963); R. N. Bhargava, *ibid.* **156**, 785 (1967).

[12] M. Giura and F. Wanderlingh, Phys. Rev. Letters **20**, 445 (1968).

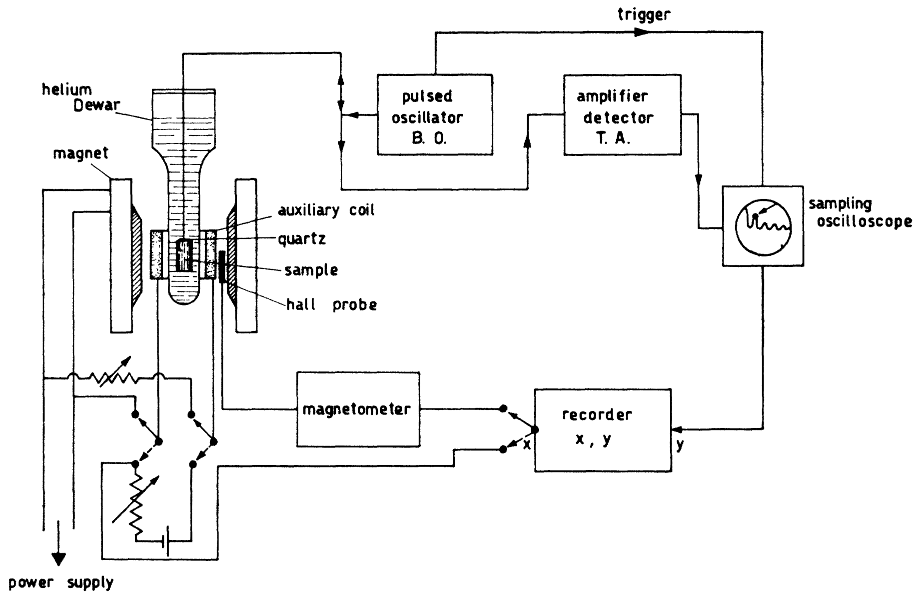

FIG. 2. Experimental setup.

The sound frequency used was 190 Mc/sec; the temperature of the sample was maintained at 1.3°K.

In the previous work[8] we investigated the shape of the Fermi surface of the electrons in bismuth. Our results agree with the ellipsoid model proposed for the Fermi surface.

The major axis of the ellipsoids lies in a plane 7° tilted from the binary-bisector plane. As indicated above, the period of the oscillations of the acoustic absorption coefficient, as a function of the reciprocal of the magnetic field strength, is inversely proportional to the cross-sectional area of the Fermi surface. Therefore, there exist particular directions, close to the trigonal axis, characterized by a large value of the area, and consequently by a very small period of the oscillations due to the electron pockets. In such a case, the existence of a hole pocket should easily be detected as a consequence of the large difference between the periods corresponding to different cross-sectional areas.

In order to investigate such a possibility, a preliminary experiment was performed in which the sound wave vector is directed along the bisector axis, while the direction of the magnetic field vector **H** lies in the plane determined by bisector and trigonal axes. In this experiment the cross-sectional area of the Fermi surface was measured for various orientations of **H**. The results are shown in Fig. 3. Dots represent the experimental results, while the solid line refers to a theoretical calculation based on previous results. As predicted, a small region around the trigonal axis exists in which a new cross-sectional area becomes evident, while the large value corresponding to the electron pocket is beyond the range of the measurements.

Information concerning the hole Fermi surface that can be found by varying the direction of **H** in the range of few degrees around the trigonal axis is reported in Sec. 3.

3. EXPERIMENTAL RESULTS: OSCILLATIONS VERSUS MAGNETIC FIELD INTENSITY

In this section we present the results of experiments performed using the first arrangement described in Sec. 2, namely, that in which the direction of the magnetic field is maintained constant (near to the trigonal axis direction) and the oscillations of the acoustic absorption coefficient are recorded as a function of the magnetic field intensity. From the experimental data, the values of the cross-sectional areas of the hole Fermi surface can be calculated.

The direction of the magnetic field with respect to the crystallographic axes of the sample can be chosen either by rotating the electromagnet or by regulating the ratio of the current flowing in the electromagnet to the current flowing in the auxiliary coil.

Figure 3 refers to the experimental condition where the sound wave vector lies in the direction of the bisector axis while the direction of the magnetic field is rotated in the bisector-trigonal plane.

Note that in this case, because of the tilt angle of the electron ellipsoids, the curves representing the cross-sectional areas are not symmetrical with respect to the trigonal axis. The direction in which the area reaches its maximum value is at about 7° from the trigonal axis (see Fig. 3). In such an arrangement the investigation of the hole Fermi surface can be performed more extensively at one side of the trigonal axis than at the opposite side, because a superposition of electron and hole periods occurs in the latter case at a smaller angle from the trigonal axis.

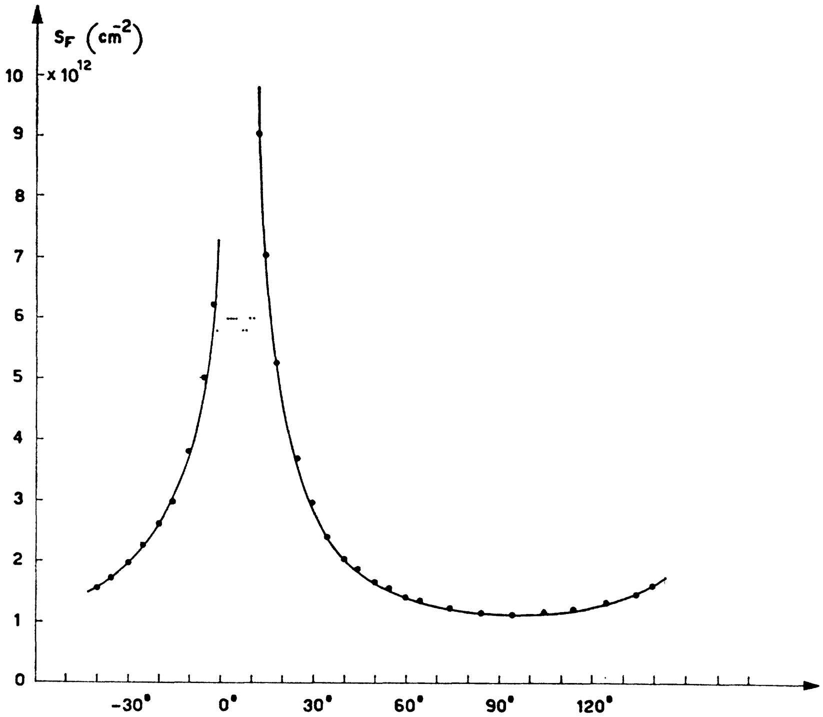

FIG. 3. Fermi-surface cross sections for magnetic field directions in the trigonal-bisector plane. Angles are measured from the trigonal axis.

Figure 4 is an enlargement of the central part of Fig. 3. The solid line represents the cross-sectional area of the electron Fermi surface. Points not falling on the solid line refer to the hole Fermi surface.

The resulting values for the cross-sectional area of the hole pocket are in agreement with the results given by Eckstein and Ketterson.[13] The present experiments, however, exhibit an unexpected result: The oscillations of the acoustic absorption coefficient disappear for some directions of the magnetic field. In Fig. 4 the shaded regions refer to such disappearances.

A similar experiment has been performed with the **H** direction contained in the plane determined by the trigonal and binary axes. In such a case the sound wave vector is directed along the binary axis. Again the disappearance of the oscillations of the acoustic absorption

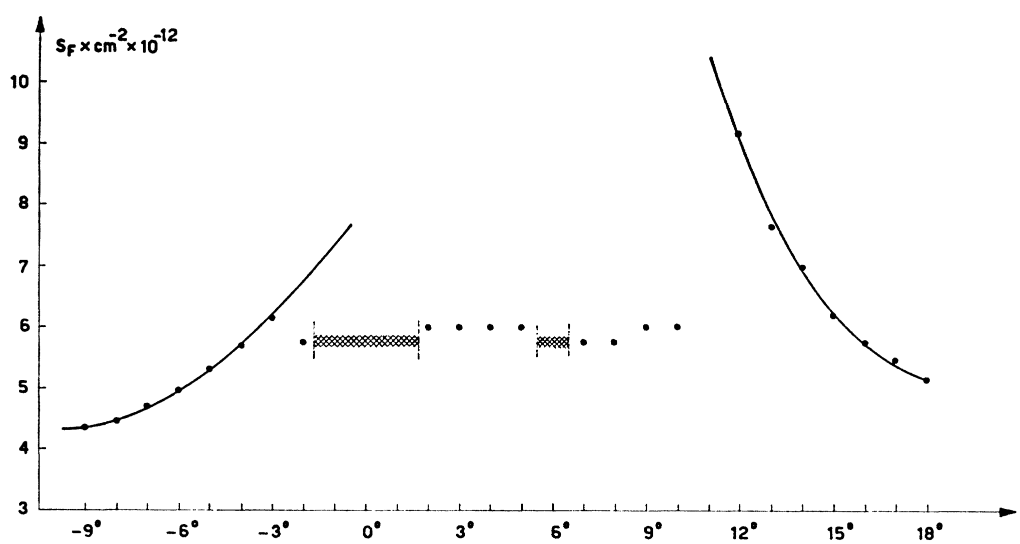

FIG. 4. Enlargement of central part of Fig. 3.

[13] Y. Eckstein and J. Ketterson, Phys. Rev. **137**, 1777 (1965).

coefficient is observed not only for **H** near the direction orthogonal to the sound wave vector but also for two directions symmetrically spaced from the trigonal axis.

In Fig. 5 are shown the directions of disappearance in a stereographic projection. The region outside the circle corresponds to directions of the magnetic field for which the contributions of electron pockets become so large as not to permit an easy interpretation of these data. As an example, in Fig. 6 is shown the behavior of the acoustic absorption coefficient as a function of the magnetic field strength for **H** contained in the trigonal-bisector plane at an angle of 7.5° from the trigonal axis (point A in Fig. 5).

In Fig. 7 we reproduce the behavior of the acoustic absorption coefficient as a function of the magnetic field, as registered by the recorder. The orientations of the field are contained in the trigonal-binary plane and the sound wave vector is directed along the binary axis.

Note that the disappearance of oscillations at 0°, i.e., when the angle between the magnetic field vector and the sound wave vector becomes 90°, is easily understood by means of the following two arguments:

(a) As noted in Sec. 1, the electrons which can interact with phonons are only those characterized by a k_H vector given by

$$k_H = m^*u/\hbar \cos\beta .$$

For $\beta = 90°$, k_H becomes infinite, that is, no electrons can interact with phonons.

(b) Actually, the sound wave vectors are spread through a small angle $\Delta\beta$ because of the finite size of the transducers:

$$\Delta\beta \simeq \lambda/d , \qquad (3)$$

where λ is the sound wavelength and d is the diameter of

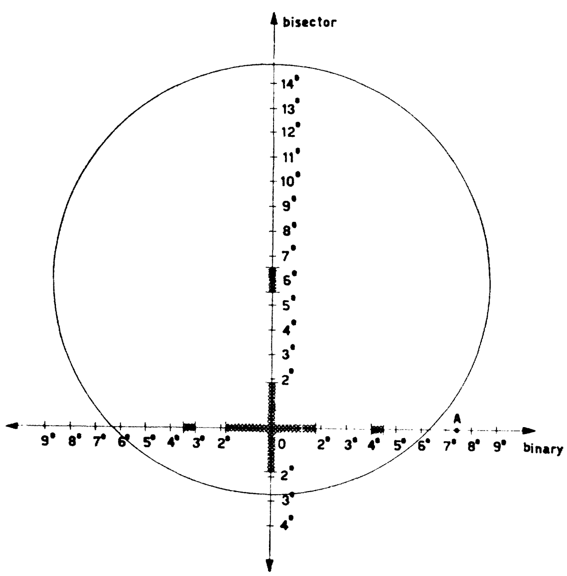

FIG. 5. Map of directions characterized by a disappearance of oscillations.

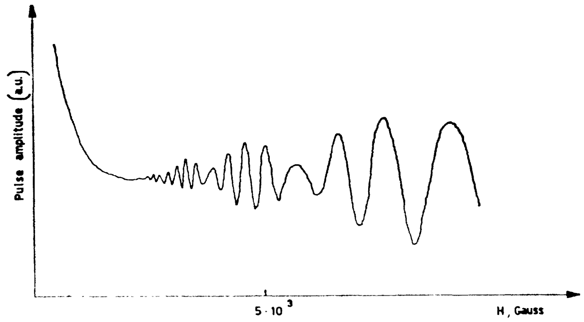

FIG. 6. Echo-pulse amplitude versus magnetic field intensity. The field direction corresponds to the point A in Fig. 5.

the quartz transducer. As a consequence, k_H as given by Eq. (1) is also spread by an amount

$$\Delta k_H = \frac{m^*u \sin\beta}{h \cos^2\beta} \Delta\beta . \qquad (4)$$

The value given by Eq. (4) is to be compared with the spacing between Landau levels, and plays a role similar to the thermal spreading kT.

It is evident that for $\beta \rightarrow 90°$ the diffraction becomes too large, and the quantum oscillations are smeared out.

On the contrary, the disappearance of oscillations for an **H** direction not orthogonal to the sound wave vector can be associated with the structure of the Fermi surface.

The main characteristic of this latter phenomenon consists in the fact that oscillations are detected on the left as well as on the right side of the disappearance directions. This circumstance clearly excludes the possibility that the disappearance of oscillations may be a result of the fact that the k_H value exceeds the k_F value, i.e., that the secant plane lies outside the Fermi surface.

In addition, the value of the Fermi cross-sectional area that can be calculated from the period of such oscillations is about constant (see Figs. 4 and 7). This excludes the possibility of relating the disappearance of the oscillations to an extremely high or low value of the area.

4. EXPERIMENTAL RESULTS: OSCILLATIONS VERSUS k_H

It is to be emphasized that when the angle between the magnetic field vector and the sound wave vector is changed, there is a corresponding change in the distance, measured from the origin in **k** space, at which the Fermi surface is crossed. This means that the data contained in Fig. 5 are not complete. In order to characterize the disappearance of the oscillations, not only the directions must be known but also the distance k_H from the origin.

In other words, from the data reported above the disappearance of the oscillations could be attributed to either a particular direction or to a particular distance

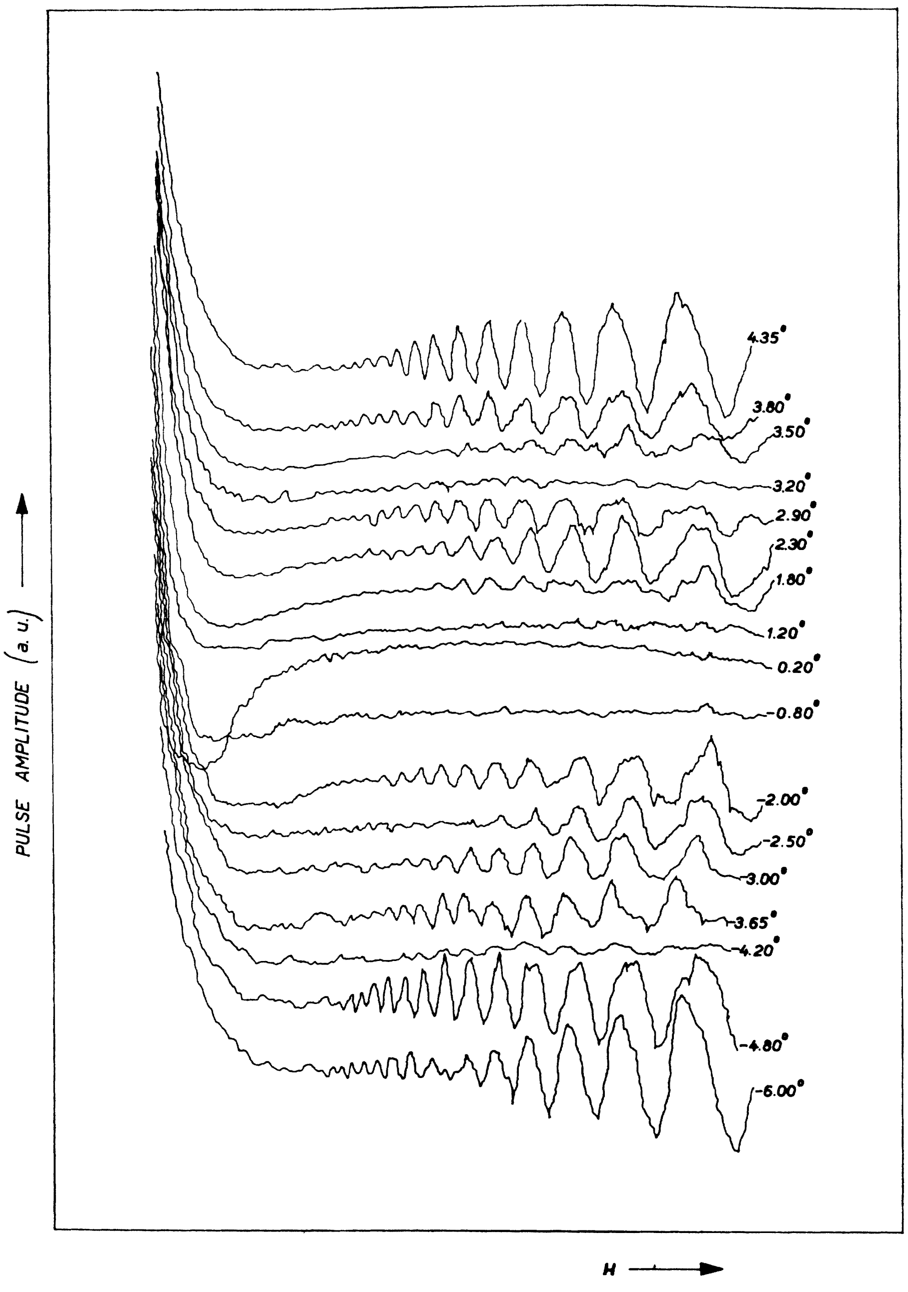

FIG. 7. Echo-pulse amplitude as a function of magnetic field intensity for various orientations of the magnetic field in the trigonal-binary plane; the sound wave vector lies along the binary axis.

from the origin in the **k** space. In order to clarify this point, a second series of experiments was performed in which the magnetic field strength is kept constant while the magnetic field direction is continuously changed (see Sec. 2). With this arrangement the acoustic absorption coefficient may be recorded as a function of the angle β between the sound wave vector and the magnetic field vector (i.e., as a function of the **H** orientation with respect to the crystallographic axis). The behavior of the acoustic absorption coefficient is a rather complicated one. In fact, as the angle β changes, either the cyclotron mass, or the effective mass, or the k_H value changes. In Sec. 5 a more detailed discussion of this situation is made. In any case, an oscillating behavior should be expected because of the fulfillment of the resonance relations [Eqs. (1) and (2)] at various values of the Landau quantum number n (see Figs. 11 and 12).

In Fig. 8 are shown the results relative to the following situation: (a) The sound wave vector is directed along the bisector axis; (b) the magnetic field strength is fixed at 5 kG; and (c) the **H** direction changes in the trigonal-bisector plane.

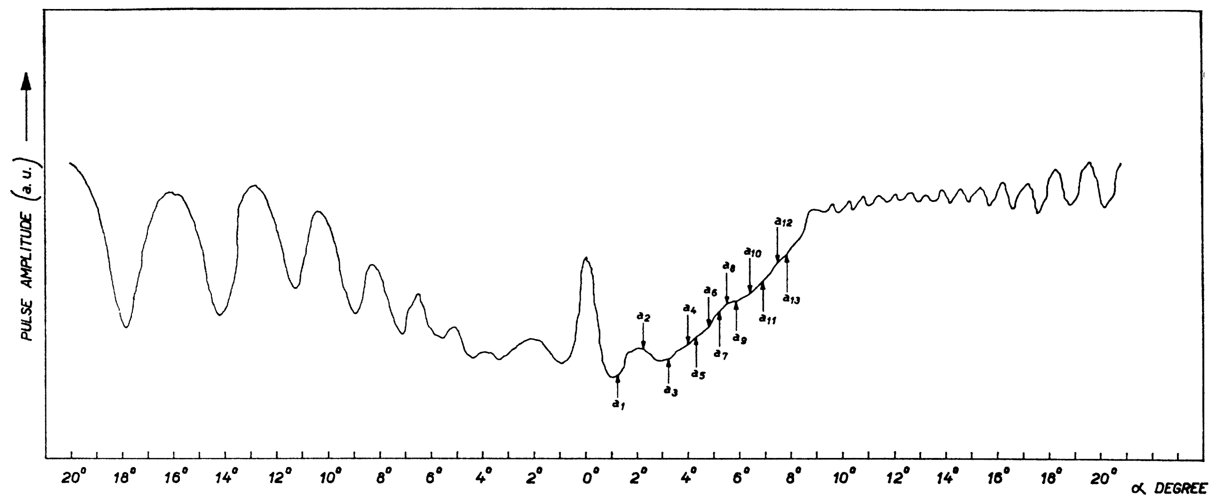

FIG. 8. Echo-pulse amplitude as a function of the angle α between the trigonal axis and the magnetic field direction. The magnetic field lies in the trigonal-bisector plane, and the sound wave vector is along the bisector axis. The magnetic field strength is 5 kG.

The abscissa α in Fig. 8 refers to the angle between the trigonal axis and the **H** direction ($\alpha=\frac{1}{2}\pi-\beta$), while the ordinate gives the echo-pulse amplitude.

Two specific details are to be noted. The first is a decrease of the absorption coefficient when **H** becomes perpendicular to the sound wave vector ($\alpha=0°$). Note that the ordinate of Fig. 8 is proportional to the amplitude of the echo pulse, and therefore an increase in this amplitude corresponds to a decrease in the absorption coefficient. This fact can be easily explained by means of the arguments (a) and (b) of Sec. 3.

The detailed behavior of the absorption coefficient for the magnetic field direction close to the normal to the sound wave vector has been investigated from a classical point of view by Reneker[1] and by Spector.[9] Their predictions agree well with our experimental results. The absorption coefficient increases as the magnetic field is tilted with respect to the direction normal to the sound wave vector, reaching a maximum value for a tilt angle of about 1° (see Fig. 10 of this paper).

The second characteristic shown in Fig. 8 is the almost monotonic behavior to the right of the central peak from $\alpha=3°$ to $\alpha=8°$. Outside of this region the acoustic absorption coefficient shows an oscillating behavior, each peak being produced by the fulfillment of the resonance conditions. This supports the possibility of a loss of Landau quantization for $3°<\alpha<8°$ (see Fig. 5). In order to clarify this point another set of measurements were made as described in Sec. 3. The values of the angle α are indicated in Fig. 8 by the letters $a_1, a_2, \cdots, a_{13}$. The results are listed on the left-hand side of Fig. 9. The oscillations disappear at the point a_9, i.e., at the center of the region in which the acoustic absorption coefficient is monotonic as a function of α. It seems, therefore, that the disappearance of oscillations is related to the monotonic behavior in the pulse-amplitude–angle-α curve. As mentioned above, the distance k_H, in **k** space, at which the Fermi surface is crossed, depends on the angle between the magnetic field vector and the sound wave vector.

The disappearance of giant quantum oscillations at 6° is to be related not only to such a particular orientation but also to a particular value of k_H. In order to verify this point an additional set of measurements was performed in which the sound wave vector was directed along the trigonal axis (instead of along the bisector axis) while the magnetic field vector remains in the trigonal bisector plane. The results are listed on the right-hand side of Fig. 9. The orientations of the magnetic field, with respect to the crystallographic axes, are the same as for the curves of the left-hand side.

No disappearance of oscillations is found either for **H** directed along the trigonal axis or for **H** tilted 6° from the trigonal axis. In the latter case the angle between sound wave vector and the magnetic field vector is 6° (instead of 84°) and the cross-sectional areas on the Fermi surface are therefore practically extremal. Extremal orbits observed at 6° do not show a loss of giant quantum oscillations, while nonextremal orbits at the same angle show such a loss.

With the procedures described in Sec. 4, a set of measurements was made in which the plane containing the **H** vector was rotated around the bisector axis in steps of 10° up to 90°. The monotonic behavior tends to disappear as the plane containing the **H** vector is tilted from the trigonal axis. In Fig. 10 are the results relative to 90°, that is, with the **H** vector contained in the binary-bisector plane. The abscissa refers to the angle between the **H** vector and the binary axis. No monotonic region is present. In addition, the general behavior is almost symmetrical with respect to the value of $\alpha=0$.

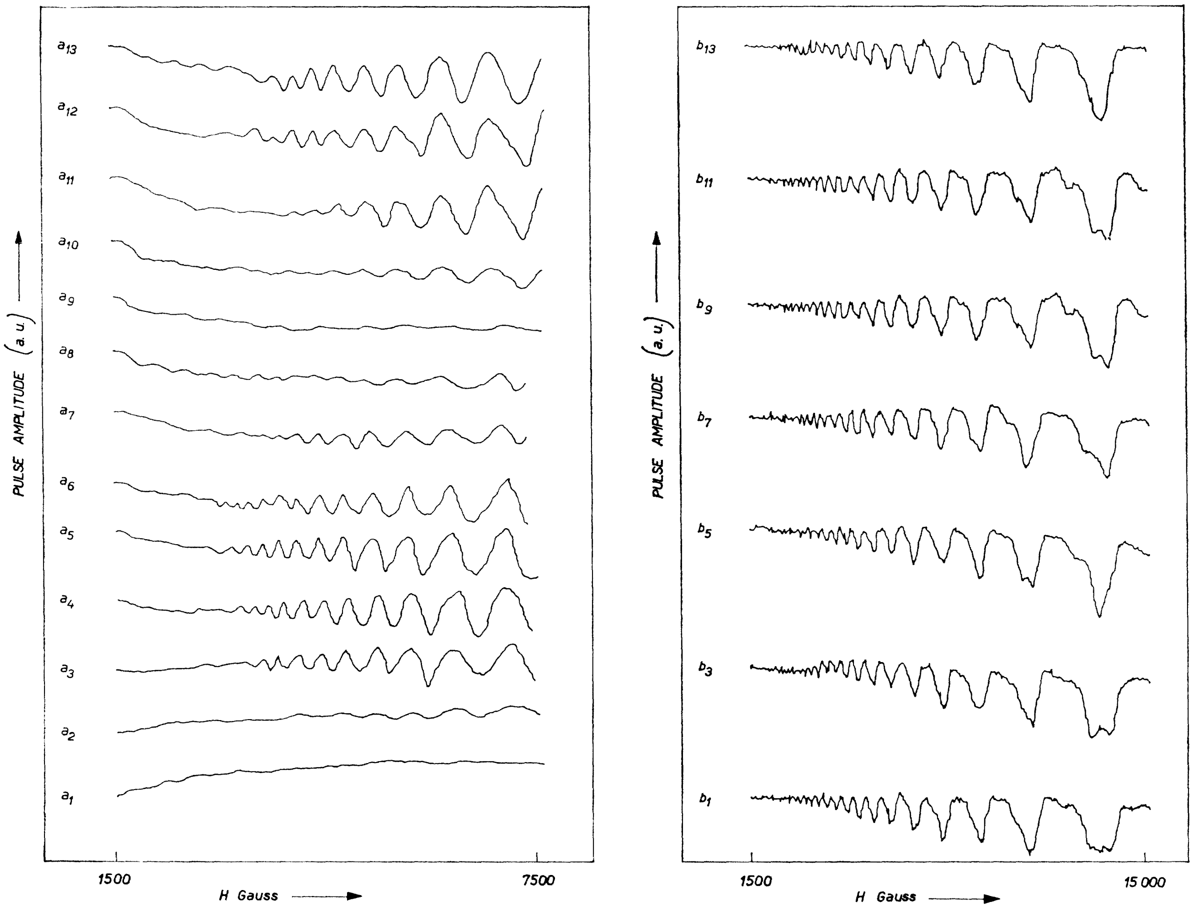

FIG. 9. Echo-pulse amplitude versus magnetic field intensity. The magnetic field lies in the trigonal-bisector plane. Left-hand side: sound wave vector along the bisector axis (see Fig. 8). Right-hand side: sound wave vector along the trigonal axis.

5. DISCUSSION OF EXPERIMENTAL RESULTS

A. Numerical Data for the Fermi Surface

Apart from the question of the disappearance of oscillations, a certain number of conclusions can be drawn from the measurements described above.

First, with regard to the electron pocket, we can confirm the value of the effective masses calculated in the preceding paper.[8] A more precise value of the tilt angle can be computed because of better orientation and alignment of the sample (see Figs. 3 and 4). The major

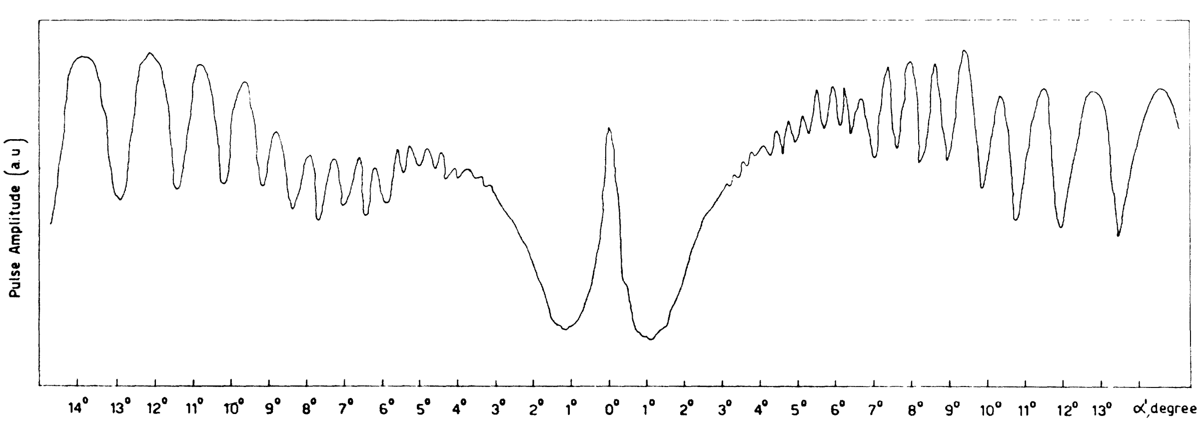

FIG. 10. Echo-pulse amplitude versus the angle α' between the binary axis and the magnetic field direction. The magnetic field vector lies in the bisector-binary plane, and the sound wave vector is along the bisector axis. The magnetic field strength is 5 kG.

axis of the electron ellipsoids is tilted 7° with respect to the binary-bisector plane.

The value of the cross-sectional area nearly orthogonal to the trigonal axis can be calculated for the hole pocket from experimental results (see Fig. 4). A value between 5.5×10^{12} and 6×10^{12} cm^{-2} was obtained for the cross-sectional area for angles smaller than 7° with respect to the trigonal axis. Such a value can be compared to that obtained by Eckstein and Ketterson[13] for extremal cross-sectional areas of the hole Fermi surface. From their data a value of about 6×10^{12} cm^{-2} is found. It is to be taken into account that the present results refer to nonextremal cross sections. That the values of the extremal and nonextremal cross-sectional areas are about the same seems to indicate that the hole pocket exhibits a prolate shape almost up to the k_H value with which we are concerned (see Fig. 12).

For cross-sectional areas made at angles larger than 7° with respect to the trigonal axis, the effects due to electron and hole pockets are superimposed (see Fig. 6). The value of the two periods can be separated and the hole pocket cross-sectional area can be determined. The general behavior shows an increase of the value of the area as the angle between **H** and the trigonal axis increases. Under such conditions the cross-sectional areas become extremal. These results agree with those obtained by Eckstein and Ketterson.[13]

B. Disappearance of Giant Quantum Oscillations with Magnetic Field Intensity

Certain directions of the magnetic field intensity are characterized by the disappearance of the giant quantum oscillations of the acoustic absorption coefficient as a function of the magnetic field strength.

This disappearance can be attributed to various reasons.

(a) Oscillations can be smeared out either because of an increase of temperature or because of an increase of the indeterminacy of the direction of the sound wave vector [Eq. (4)].

(b) Oscillations can disappear if the value of k_H becomes so large that the plane does not intersect the Fermi surface [Eq. (1)].

(c) Oscillations will not be detected if the period becomes too large or too small.

(d) Oscillations can disappear if a particular structure of the dispersion law implies a disappearance of Landau quantization.

The disappearance of oscillations for directions of the magnetic field intensity nearly perpendicular to the sound wave vector can be attributed to conditions contained in statement (b) above. This effect can also be attributed to the indeterminacy of the direction of the sound wave vector.

The disappearance of oscillations observed outside of this region can be explained by statement (d) above. The possibility that oscillations can disappear if the value of k_H becomes so large that the plane will not intersect the Fermi surface does not occur. This is because k_H is a continuous increasing function of the angle between the magnetic field vector and the sound wave vector from 0 to $\frac{1}{2}\pi$. Thus, if the disappearance begins at an angle $\beta<\frac{1}{2}\pi$, it should continue up to $\beta=\frac{1}{2}\pi$. Experimental results do not support this assumption. Moreover, the possibility that oscillations will not be detected if the period becomes too large or too small can be excluded because of the nearly constant values of the period before and after the disappearance of the giant quantum oscillations.

Note that, notwithstanding the disappearance of the oscillations, the experimental curves still show a dependence of the echo-pulse amplitude on the magnetic field strength (see Fig. 9). This latter observation seems to exclude the possibility of a drop in the electron-phonon interaction as the reason for the disappearance of oscillations.

The possibility of a disappearance of Landau quantization due to the structure of the dispersion law $E=E(k)$ was first proposed by Lifshitz and Kaganov[14] and later by Baldareschi and Bassani.[15] More recently, the relation between the dispersion law and Landau quantization has been investigated in the framework of the magnetoacoustic effects.[12]

The dispersion law $E=E(k)$ can be expanded in **k** space in the neighborhood of a singular point 0, in quadratic form:

$$E(k)=E(0)+\tfrac{1}{2}\hbar^2\left(\frac{k_1^2}{m_1}+\frac{k_2^2}{m_2}+\frac{k_3^2}{m_3}\right), \qquad (5)$$

where the origin of the frame of reference is taken at the singular point.

If $E(0)$ is either a maximum or a minimum, then all the coefficients m_i have the same sign. However, if $E(0)$ is a conic double point, then two of the coefficients m_i have signs opposite to the third. In such a case, the surfaces $E=$ const for $E>E(0)$ exhibit a negative curvature, and saddle points exist over these surfaces.

Note that the cyclotron frequency that determines the spacing between Landau levels is a property of the entire electron orbit in **k** space:

$$\omega_c=\frac{2\pi eH}{c\hbar}\oint\frac{dl}{v_\perp},$$

where dl is the elementary arc of the orbit and $v_\perp$ is the component of the electron velocity in the plane of the orbit orthogonal to magnetic field.

Therefore, the Landau quantization can disappear either if the electron orbits become infinite or if, at a

[14] J. U. Lifshitz and N. I. Kaganov, Usp. Fiz. Nauk **69**, 419 (1959) [English transl.: Soviet Phys.—Usp. **2**, 831 (1960)].

[15] A. Baldareschi and F. Bassani, Phys. Rev. Letters **19**, 66 (1967).

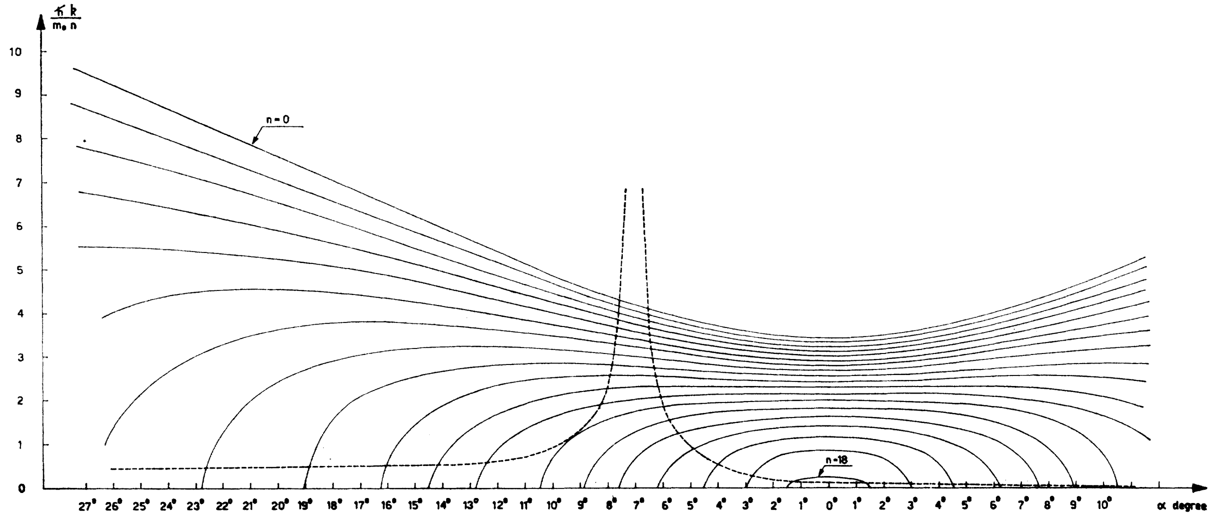

FIG. 11. Spacing of electron Landau levels in **k** space [Eq. (2)] as a function of the angle α between the magnetic field direction and trigonal axis (solid lines). Dashed line refers to k_H as given by Eq. (1).

particular point of the orbit, $v_\perp$ vanishes. The first case occurs when the expansion of Eq. (5) maintains its validity over the entire **k** space. In this case, a cone exists centered at the singular point, such that for the **H** directions inside this cone, non-Landau quantization arises.[15] However, if the Fermi surface $E=E_F$ is actually a closed one (and this presumably is the case for bismuth), Landau quantization disappears only if the electron orbits cross a point at which $v_\perp$ becomes zero. This occurs when the secant plane that crosses the

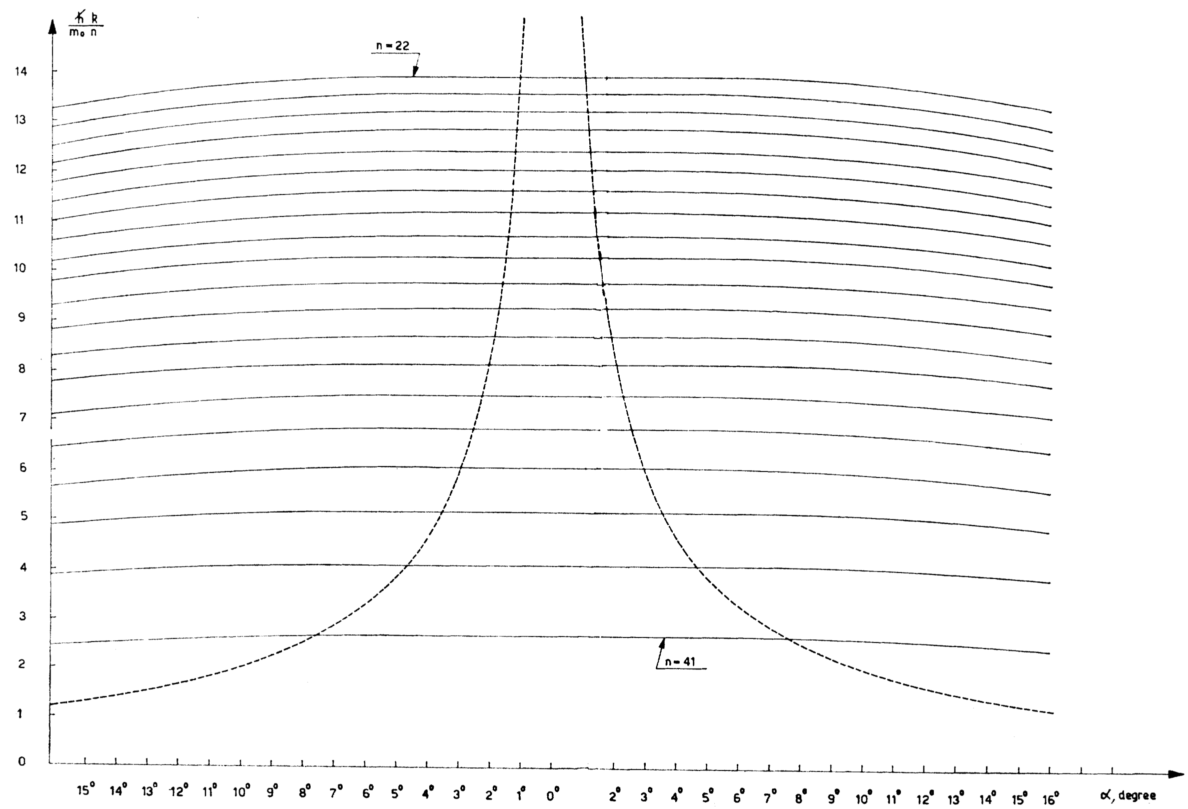

FIG. 12. Spacing of hole Landau levels in **k** space as a function of the angle α between magnetic field direction and trigonal axis (solid lines). Dashed line refers to k_H as given by Eq. (1).

Fermi surface at k_H becomes tangent to the surface at the saddle point. In such a case a particular direction of **H** and a particular value of k_H exist at which the Landau levels will disappear.[12]

The present experimental results seem to be explained in this framework, as can be shown by the following considerations:

(a) Disappearance of oscillations does not take place in a cone; it takes place only for particular critical directions of **H**.

(b) Oscillations exist if the **H** direction is changed by only 0.5° with respect to the critical direction.

(c) For a given orientation of **H** with respect to the crystallographic axes, oscillations can or cannot exist depending on the angle between **H** and the sound wave vector (see Fig. 9).

The last two observations strongly suggest that the major role is played by the k_H value, which is a varying function of the angle β between **H** and the sound wave vector for β near $\frac{1}{2}\pi$, as it is in our case.

C. Oscillations as a Function of k_H

As outlined in Sec. 4, oscillations of the acoustic absorption coefficient can be recorded as a function of the angle between the magnetic field vector and the sound wave vector. Such oscillations are smeared out in the proximity of the presumed saddle point. In Figs. 11 and 12 are shown the behavior of the Landau levels (solid line) and the k_H value (dashed line) as a function of the angle α between the **H** direction and the trigonal axis. The sound wave vector is along the bisector axis. The calculation of the Landau levels and of the k_H value is made by using models of the Fermi surface.[8,10] The intersection of the dashed line with the solid line indicates the condition of resonance at which a peak in the absorption coefficient should be expected [Eqs. (1) and (2)].

However, the experimental results (Fig. 8) do not fit this model. This indicates the possible existence of singularities over the Fermi surface at which the usual model loses its validity.

6. CONCLUSIONS

Experimental results seems to suggest a new feature of the dispersion law for carriers in bismuth. These results can be explained if the existence of saddle points on the Fermi surface is taken into account. This would require a change in the models of the Fermi surface presently used.

The existence of saddle points on the Fermi surface implies the existence of a singular point (conic double point) in the **k** space, i.e., a point at which the dispersion law must be expanded in a quadratic form with new suitable coefficients. As a consequence, the exact location of the singular points in **k** space becomes very difficult because of the dependence of k_H in the dispersion law. Moreover, present data are not sufficient for the choice of an appropriate topology for the Fermi surface.

More measurements are in progress for a detailed investigation of the actual dispersion law throughout the entire **k** space and for an evaluation of the effective-mass tensor near the singular point.

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