Flux Motion in Type-I Superconductors

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A consistent picture of the current-carrying intermediate state of type-I slabs has been obtained from measurements of the coupled motion of flux, measurements of the Ettingshausen effect, and visual observations of diamagnetic powder replicas of the intermediate state. The electrical resistance has been shown to be due to flux flow when the average magnetic field in the sample is sufficiently small, and to Ohmic resistance of static normal domains when the field is close to H_c. In particular, flux-flow resistance occurs in the regime studied originally by Sharvin and recently by Chandrasekhar *et. al.* and by Brandt and Parks.

I. INTRODUCTION

WHEN a bulk type-I superconductor with nonzero demagnetizing coefficient is in a suitably large magnetic field, the field penetrates the bulk, producing an intermediate state consisting of superconducting regions which exclude flux and normal regions containing flux. It is well known that such a state can exhibit electrical resistance, but identifying the resistance mechanism has caused controversy for over 40 yr. The problem is complicated because the electrical resistance mechanism depends on the topology of the intermediate state which, in turn, depends on the geometry, composition, and history of the sample as well as on the applied magnetic field. This investigation is an attempt to resolve some of the controversy. The paper describes experiments on rectangular bulk slabs of Pb and Sn. Three different techniques were used to identify the electrical resistance mechanism for different magnitudes and orientations of the applied magnetic field. We conclude that when the average magnetic field in a slab is sufficiently small (or, alternatively, the fraction of normal material is small) the electrical resistance is due to flux flow; when the field is close to H_c (the normal fraction is close to unity) the electrical resistance is due to the Ohmic resistance of static normal domains.

It is appropriate to review some of the history of electrical resistance in superconductors.[1] Among the first measurements were those made by De Haas, Voogd, and Jonker[2] on wires in a transverse magnetic field. They showed that the change in the electrical resistance from zero to the normal-state value occurred continuously from some suitably large field up to the critical field of the material. This was very strong evidence for the existence of the intermediate state proposed by Gorter and Casimir[3] because it was assumed that the resistance of the wires was due to the Ohmic resistance of the normal regions. Subsequent experiments using magnetic-field probes[4] and, later, diamagnetic[5] and ferromagnetic[6] powders succeeded in determining the structure of the intermediate state, but there was no evidence that the normal regions would be properly aligned with respect to the transport current to produce the observed electrical resistance. Shoenberg[1] suggested that the current itself might align the normal regions properly. This was observed by Shal'nikov[7] in 1957. Using a magnetic powder, he demonstrated that the intermediate state of a type-I cylinder in a transverse field was transformed from a disoriented pattern to regular laminae elongated perpendicular to the cylinder axis by the application of a transport current along the axis.[8] Shal'nikov's experiment apparently disproved the occurrence of a mechanism proposed by Gorter[9] in 1957, according to which alternating laminae of superconducting and normal phase are elongated in the direction of the transport current and move in the perpendicular direction to produce a potential difference. In 1965, Gorter's model was revived by Sharvin,[10] who claimed to have detected current-induced motion of laminae when the angle between the applied field and the sample surface was about 10°. Recently, however, Chandrasekhar, Farrel, and Huang,[11] and Brandt and Parks[12] have questioned Sharvin's conclusions. We will consider the Sharvin controversy in Sec. VI.

The occurrence of still another resistance mechanism in type-I superconductors is possible. The new mechanism is flux flow, proposed by Kim *et al.*[13] in 1963 to

[1] For a more complete description see, e.g., D. Schoenberg, *Superconductivity* (Cambridge University Press, New York, 1960), and F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1950), Vol. I.
[2] W. J. De Haas, J. Voogd, and J. M. Jonker, Physica **1**, 281 (1934).
[3] C. J. Gorter and H. B. G. Casimir, Physica **1**, 306 (1934).
[4] A. G. Meshkovsky and A. I. Shal'nikov, J. Phys. USSR **11**, 1 (1947); Zh. Eksperim. i Teor. Fiz. **17**, 851 (1947).
[5] A. L. Schawlow, Phys. Rev. **101**, 573 (1956); A. L. Schawlow, B. T. Matthias, H. W. Lewis, and G. E. Devlin, *ibid.* **95**, 1344 (1954).
[6] A. I. Shal'nikov and K. A. Tumanov, *Collection Dedicated to the Seventieth Birthday of A. F. Ioffe* (Publishing House of the Academy of Sciences of the USSR, 1950), p. 303; Yu. V. Sharvin and B. M. Balashova, Zh. Eksperim. i Teor. Fiz. **23**, 222 (1952); B. M. Balashova and Yu. V. Sharvin, *ibid.* **31**, 40 (1956) [English transl.: Soviet Phys.—JETP **4**, 54 (1957)].
[7] A. I. Shal'nikov, Zh. Eksperim. i Teor. Fiz. **33**, 1071 (1957) [English transl.: Soviet Phys.—JETP **6**, 827 (1958)].
[8] Similar behavior was recently observed in rectangular slabs by F. Haenssler and L. Rinderer, Helv. Phys. Acta **40**, 659 (1967).
[9] C. J. Gorter, Physica **23**, 45 (1957).
[10] Yu. V. Sharvin, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **2**, 287 (1965) [English transl.: Soviet Phys.—JETP Letters **2**, 183 (1965)].
[11] B. S. Chandrasekhar, D. E. Farrell, and S. Huang, Phys. Rev. Letters **18**, 43 (1967).
[12] B. L. Brandt and R. D. Parks, Phys. Rev. Letters **19**, 163 (1967); **19**, 1216 (E) (1967).
[13] Y. B. Kim, C. F. Hempstead, and A. R. Strand, Phys. Rev. **131**, 2486 (1963); **139**, A1163 (1965).

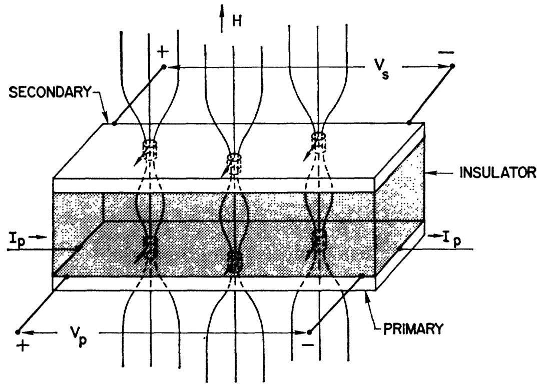

FIG. 1. Schematic drawing of the configuration for observing the coupled motion of flux.

explain the electrical resistance of type-II superconductors in the mixed state. According to this concept a transport current exerts a force on a quantized flux line (Abrikosov vortex), causing it to move perpendicular to the current, and the motion of vortices produces a potential difference between two points which depends only on the rate of passage of flux quanta between the points. Flux flow in type-I superconductors was first demonstrated by observing the coupled motion of flux in two closely adjacent strips.[14,15] Measurements of the Ettingshausen effect in a type-I superconductor[16] and measurements of the noise voltage produced by flux motion[17] have further substantiated the occurrence of flux flow.

Measurements of the coupled motion of flux have been expanded to include a variety of samples and orientations of the magnetic field. These results are discussed in Sec. II. In Sec. III, we review the Ettingshausen-effect measurements and compare them with data on the coupled motion of flux. In Sec. IV, we describe a study of flux flow in which the intermediate state in rectangular slabs of Sn was displayed with diamagnetic powder. To display moving domains, the sample was vibrated to keep the powder in continuous equilibrium with the moving magnetic field. Typical results were recorded with motion pictures[18]; selected still pictures are shown. In Sec. V, the results of the three techniques are correlated to yield a consistent picture of the current-carrying intermediate state of a superconducting slab, and in Sec. VI we consider the mechanism of electrical resistance in the geometry studied by Sharvin[10] and by others.[11,12]

II. COUPLED MOTION OF FLUX

A. Experimental Details

The first conclusive proof of flux flow in type-I superconductors was the observation of the coupled motion of flux.[14] The experimental arrangement is illustrated schematically in Fig. 1. The configuration was a sandwich of two superconductors (primary and secondary) separated by a thin insulating layer. A sandwich was placed in a magnetic field, current was passed through the primary, and the potential drops in the primary and secondary were measured. If the current caused motion of vortices across the primary, and the two superconductors were in close enough proximity, then vortices were also pulled across the secondary. The simultaneous motion of vortices was evidenced by equal simultaneous potential drops in the primary and secondary. Operated in this fashion, the sandwich was a dc transformer.

The origin of the force which couples the vortices is the increase in magnetic-field energy which is necessary to move a vortex in the primary out of alignment with a vortex in the secondary. The force depends upon the thickness of the insulator and on the size and separation of the vortices. If the vortices are very close together or the insulator is very thick, so that the magnetic field at the center of the insulator is nearly homogeneous, moving the vortices out of alignment does not increase the field energy substantially and the coupling force will be small. The force will increase as the field at the center of the insulator becomes more inhomogeneous. Vortices in the secondary will move when the coupling force is large enough to overcome pinning. In the experiments reported here the primary was a type-I foil. The vortices were therefore quite large and well separated, and coupling was seen with insulator thicknesses as large as 20 000 Å.

A typical sandwich is shown in Fig. 2. A strip of foil was attached to a Lucite substrate (not shown) and the ends of the foil were bent to conform to a step cut in the ends of the substrate. A small foil tab, electrically insulated from the strip, was attached above each step.

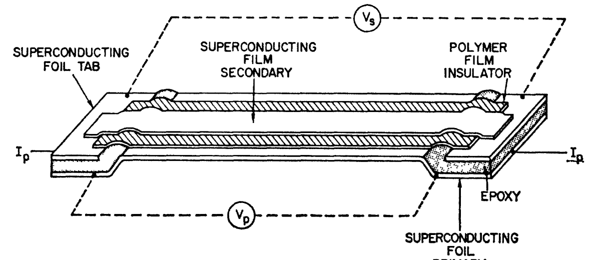

FIG. 2. Typical dc transformer. The sandwich consists of a type-I foil primary, an evaporated polymer film insulator, and an evaporated type-I superconducting secondary.

[14] P. R. Solomon, Phys. Rev. Letters **16**, 50 (1966); in *Proceedings of the Tenth International Conference on Low-Temperature Physics, Moscow, 1966*, edited by M. P. Malkov (Proizvodstrenno-Izdatel'skii Kombinat, VINITI, Moscow, 1967).

[15] A similar experiment, performed independently, demonstrated the occurrence of flux flow in type-II superconductors; see I. Giaever, Phys. Rev. Letters **15**, 825 (1965).

[16] See, e.g., P. R. Solomon and F. A. Otter, Jr., Phys. Rev. **164**, 608 (1967).

[17] G. J. Van Gurp, Phys. Letters **24A**, 528 (1967).

[18] P. R. Solomon, in *Proceedings of the Eleventh International Conference on Low-Temperature Physics*, edited by J. F. Allen, D. M. Finlayson, and D. M. McCall (St. Andrews, Scotland, 1968).

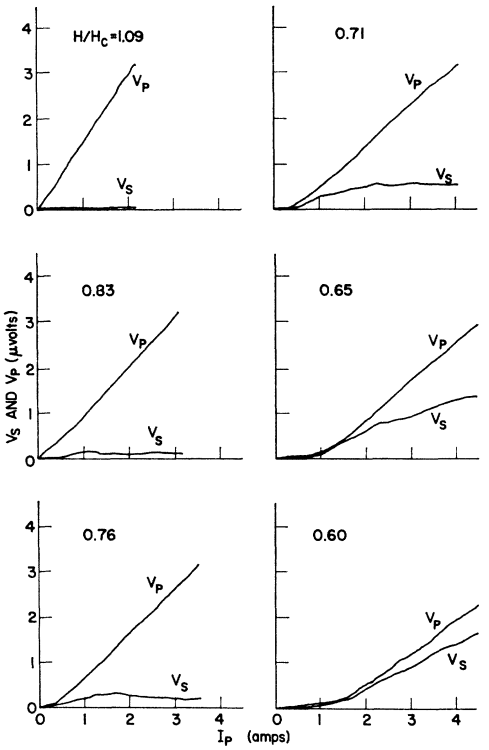

FIG. 3. Typical curves of primary and secondary voltage for a Pb-Pb sandwich as a function of primary current for various values of reduced magnetic field at 4.2°K.

The foil tab was positioned so that its top surface lined up with the top surface of the center of the strip. The insulating film (a layer of electron beam polymerized diglycidyl ether of bisphenol-A)[19] was evaporated over the strip leaving the foil tabs partially bare. The secondary superconductor was evaporated over the insulating layer to connect the foil tabs. Leads were attached to the foil tabs and the primary strip. Shorts, which usually occurred between the primary and secondary, could generally be cleared by discharging a capacitor across the short. This procedure worked even after the sandwich had been cooled to liquid-helium temperatures. The primary foils ranged in thickness from 0.025 to 2.0 mm. The secondary films were between 2000 and 70 000 Å thick, and the insulators were between 500 and 20 000 Å thick. The thickness of the insulator was determined from the frequency shift of a quartz-crystal oscillator upon which the polymer was simultaneously evaporated. The quartz crystal was periodically calibrated by measuring with an interferrometer the thickness of the insulating layer deposited on the test sample. The relative thicknesses of the insulating layers of several samples were also determined by measuring the capacitance of each sandwich. These data were in good agreement with the thicknesses determined from the calibrated quartz-crystal oscillator.

For each sandwich, the potential drops in the primary and secondary were measured as a function of the current in the primary for various values of the magnetic field. The potential drops were measured simultaneously with two Astrodata nanovoltmeters. The dc outputs of the voltmeters were recorded on the Y axes of two X-Y recorders. The X axes of both recorders measured current in the primary. Typical curves for a magnetic field applied normal to the broad face of the sample are shown in Fig. 3. The curves, which are for a sample with a Pb primary and secondary, are superimposed for convenience. The coupling was strongest at low magnetic fields. As expected, there is no coupling above the critical field. For all samples on which measurements were made, the resistance between primary and secondary was observed to be in excess of $10^3\,\Omega$ even while a voltage in the secondary was present. It was also demonstrated that while a voltage drop appeared in the secondary, the secondary was capable of carrying current without any change in the voltage drop.

Hysteresis was sometimes observed in the primary and secondary voltages. The effect, which was most prevalent in the Pb samples, is shown in Fig. 4. The curves were taken after the intermediate state was established by increasing the magnetic field from zero to the required value at constant temperature and zero current. If current was passed through the primary for a second time, the curves for increasing and decreasing current were identical with the initial curve for decreasing current (i.e., the hysteresis disappeared). The hysteresis could be reestablished by turning the magnetic field off and then on again. In Sn, the hysteresis effects were smaller but could be observed for repeated passages of the current without changing the magnetic field. Some similar effects have previously been reported.[20] The hysteresis seems to be caused by the rearrangement of the topology of the intermediate state. In the example in Fig. 4, the appearance of voltage in the secondary suggests that the original topology in the primary was being moved, and the disappearance of the voltage

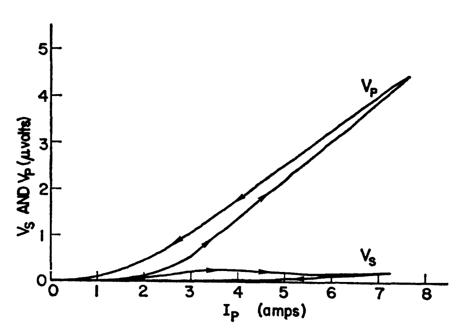

FIG. 4. Hysteresis in primary and secondary voltages in a Pb-Pb sandwich in 540 G at 1.3°K.

[19] R. A. Mazzarella and D. J. Quinn, J. Electrochem. Soc. (to be published).

[20] E. R. Andrew, Proc. Roy. Soc. (London) **A194**, 80 (1948); **A194**, 98 (1948).

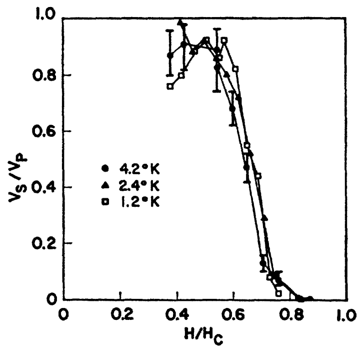

FIG. 5. Coupling parameter for a Pb-Pb sandwich as a function of reduced field at several temperatures. The data points for a fixed temperature are connected by straight lines for convenience. Voltage-versus-current curves for this sample are shown in Fig. 3.

suggests that, after rearrangement, the topology in the primary was static in the presence of current. We also observed the voltage in the primary and secondary as functions of time after a current was suddenly turned on in the primary. We observed that originally the secondary and primary voltages were equal. The secondary voltage subsequently decayed to zero in a time equal to the time it would take a flux bundle to move across the primary under the influence of the applied current. The influence of current in rearranging the intermediate-state topology was directly observed with diamagnetic powder (see Sec. IV).

The sandwiches are essentially dc transformers whose coupling may be varied by varying the applied magnetic field. Voltage step up transformers have been made by evaporating as many as seven secondaries side by side on a single primary and series connecting them head to tail.

B. Results

Curves of the kind shown in Fig. 3 were recorded for more than 40 samples, mostly Pb and Sn. As long as the primary current did not greatly exceed the critical current for depinning, the ratio of V_s to V_p was relatively independent of primary current. It is instructive to plot the average ratio of V_s to V_p for currents near the critical current as a function of reduced magnetic field. (This ratio is called the coupling parameter.) Consider first the case of magnetic field applied normal to the broad face of the sample.

TABLE I. Sample characteristics.

Curve	Sample	Temp. (°K)	Sn primary Width (mm)	Sn primary Thickness (mm)	Polymer insulator thickness (Å)	Sn secondary thickness (Å)
A	TF38	3.3	4.5	0.18	2 000	10 000
B	TF38	2.6	4.5	0.18	2 000	10 000
C	TF38	2.0	4.5	0.18	2 000	10 000
D	TF38	1.25	4.5	0.18	2 000	10 000
E	TF27B	2.6	6.0	2.0	2 000	13 000
F	TF27A	2.6	5.0	1.0	2 000	13 000
G	TF24	2.6	4.5	0.25	20 000	2 000
H	TF26	2.6	4.0	0.025	2 000	12 000
I	TF25	2.6	4.5	0.64	2 000	2 000
J	TF23	2.6	4.8	0.25	10 000	2 000
K	TF22	2.6	4.5	0.25	5 000	70 000
L	TF19	2.6	4.5	0.25	2 000	20 000
M	TF41-7	2.6	2.0	0.25	2 000	10 000

The coupling parameter values obtained from the curves in Fig. 3 (for a Pb-Pb sandwich) are plotted

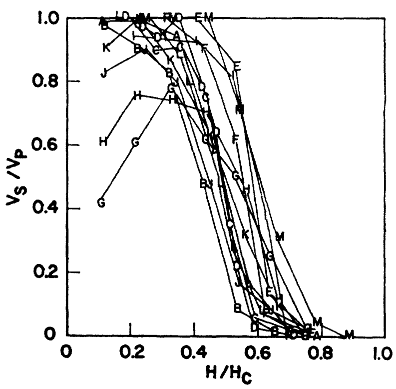

FIG. 6. Coupling parameter for several Sn-Sn sandwiches as a function of reduced field. The data points for a given set of conditions are joined by straight lines for convenience. Table I lists the conditions under which the data were taken.

in Fig. 5 along with data for other reduced fields and temperatures. For ease in identification, a set of points is connected by straight lines. The behavior of the sandwich at the three values of temperature is similar. The important features of the curves are the closeness of the coupling parameter to unity at low magnetic fields, the closeness of the coupling parameter to zero at fields near H_c, and the abrupt transition at an intermediate field h_α (defined as the reduced field at which the coupling parameter drops to $\frac{1}{2}$ its maximum value). Coupling curves were also obtained in perpendicular magnetic fields for Pb-Pb sandwiches with different insulator thicknesses. In all cases the curves were similar to those in Fig. 5, except that for samples with insulator thicknesses of more than 10 000 Å the coupling parameter at low fields was considerably less than 1. All the curves, however, showed a drastic drop in coupling at the same h_α (which for Pb is about $\frac{2}{3}$).

Coupling curves were also obtained for a variety of Sn-Sn sandwiches having widely varying dimensions. Figure 6 shows the coupling curves for sandwiches in perpendicular fields. The sandwich characteristics are listed in Table I. For Sn-Sn sandwiches, h_α is also relatively independent of temperature and sample geometry, and is about $\frac{1}{2}$.

The coupling curves give information about the topology of the intermediate state and mechanism of electrical resistance in the primary. Figure 7 is a summary of what the coupling curves reveal. The similarity of the primary and secondary voltage drops for magnetic fields less than h_α is proof that in this regime the electrical resistance in the primary is substantially due to vortex flow, and the intermediate state must contain simply connected normal regions as shown in Fig. 7. Above h_α the coupling drops to zero. Then, either vortices flow in the primary but do not couple to vortices in the secondary, or else there is no vortex flow. If there is no vortex flow, then the resistance in the primary is most likely due to the Ohmic resistance of static normal domains which extend across the primary blocking any completely superconducting path. Possible topologies for the intermediate state above h_α are, therefore, those containing simply connected normal regions, alternating normal and superconducting laminae, and simply connected superconducting regions.

The fact that h_α does not depend on temperature or on the thickness and separation of the superconductors suggests that at high field the two topologies consistent with Ohmic resistance are most likely. If the voltage drop in a primary at high fields were due to vortex flow and if the absence of a voltage drop in the secondary were simply due to a lack of coupling strength, then

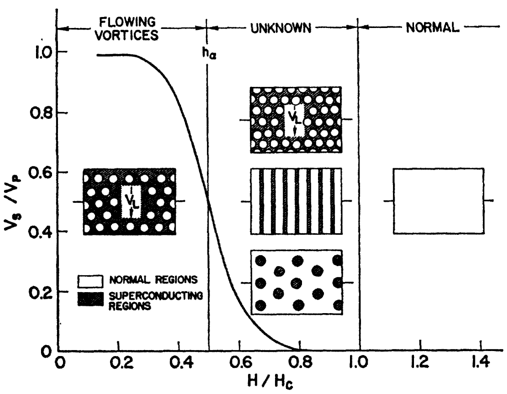

FIG. 7. Summary of results from the experiments on the coupled motion of flux. The figure shows a typical curve of the coupling parameter versus reduced magnetic field and what the curve indicates about the topology of the current-carrying intermediate state.

it is reasonable to assume that the coupling could be increased by using a thinner insulator or increasing the temperature (to decrease the pinning forces in the secondary). The data show that none of these procedures has much influence on the coupling curves. It appears that the shape of the coupling curves does not reflect the magnetic-field dependence of the coupling strength but rather the magnetic-field dependence of the fraction $f(B)$ of the primary voltage which is due to flux flow. The data indicate, therefore, that at high field there is no flux flow in the primary and that the electrical resistance must be due to the Ohmic resistance of static normal domains.

The measurements made by Van Gurp[17] also indicate a decrease in $f(B)$ with increasing magnetic field. He measured the noise voltage of a type-I slab in the resistive intermediate state. He observed noise associated with flux flow at low magnetic fields but no such noise at magnetic fields near H_c.

It is interesting that the current-voltage curves for type-I slabs give very little evidence that there is any change in the resistance mechanism. The only indication is that the critical-current value is generally quite small above h_α and only starts to get large below h_α. Above h_α there are presumably no entirely superconducting paths to conduct a supercurrent. The current-voltage curves for type-I slabs are quite similar to the corresponding curves for type-II superconductors. This similarity prompted Chandrasekhar *et al.*[21] to propose a model in which flux flow occurs across the whole sample at low magnetic field and across a narrower and narrower portion at the center of the sample (the edges being normal) as the magnetic field is increased. This model does not agree with observations of coupling in sandwiches with secondaries evaporated side by side on the same primary.

The coupling experiment was also performed in a magnetic field inclined from the normal. The motivation was to establish whether or not flux flow occurs in the regime studied by Sharvin,[10] Chandrasekhar, Farrell, and Huang,[11] and Brandt and Parks.[12] The sample had a pure Sn primary with $R_{300°K}/R_{4.2°K}=9\times10^3$ and width, thickness, and length of 5.0, 0.64, and 75 mm, respectively. The results are shown in Fig. 8. When the magnetic field was less than 30° from the normal to the sample surface, there was very little change in the coupling from what was observed in normal fields (Fig. 5). When the field was inclined more than 30° from the normal, the flux-flow regime was observed to extend to significantly higher applied fields. At 20° from the sample surface (the regime studied by Brandt and Parks[12]), V_s/V_p was greater than 0.50 at $H/H_c=0.90$.

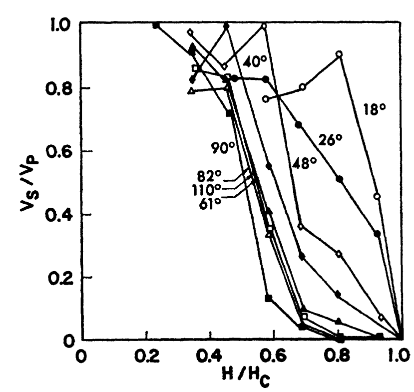

FIG. 8. Coupling parameter for a Sn-Sn sandwich (TF25) as a function of reduced magnetic field for several directions of the applied field at 2.75°K. The angles were measured between the applied field and the sample surface. The data points are joined by straight lines for convenience.

[21] B. S. Chandrasekhar, I. J. Dinewitz, and D. E. Farrell, Phys. Letters 20, 321 (1966).

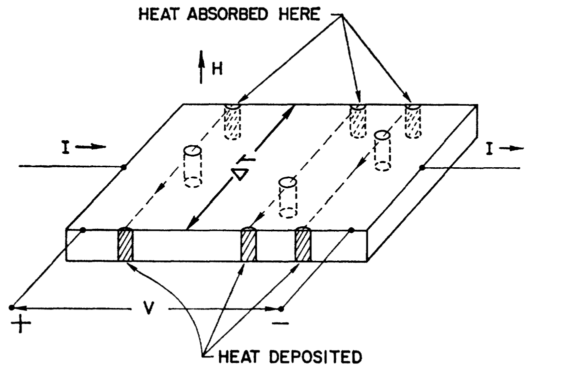

FIG. 9. Schematic diagram depicting the mechanism for the Ettingshausen effect in a superconductor. The diagram shows heat being absorbed by vortices which are created on one side of the sample and heat being deposited by vortices which are being destroyed on the other.

III. ETTINGSHAUSEN EFFECT

Another experiment which provides information about the resistance of a superconductor in the intermediate state is a measurement of the Ettingshausen effect. We have made such measurements in type-I strips similar to the primaries used in the coupling experiment. The experimental details are given in Ref. 16.

The Ettingshausen effect is a transverse temperature gradient produced by passing a current through a sample situated in a magnetic field. Figure 9 shows the geometry used. When the sample is a superconductor in the intermediate state, current-induced vortex motion gives rise to heat flow. Crudely speaking, a vortex forming at one edge of the superconductor absorbs heat from the lattice (to create a normal domain) and then gives the heat back to the lattice when it disappears on the other edge. The Ettingshausen effect produced by this mechanism is substantially greater than that observed in the absence of vortex flow. In Fig. 10, we have plotted $K\Delta T/V$ divided by its maximum value at a given temperature as a function of reduced magnetic field for a sample of Sn+0.05 at.% In with a thickness,

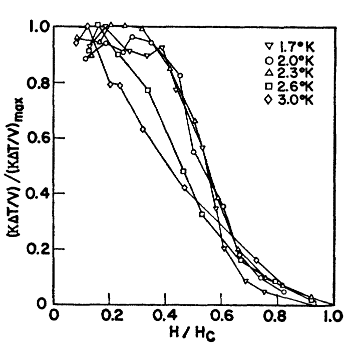

FIG. 10. Ettingshausen effect as a function of reduced field in a slab of Sn+0.05 at.% In at several values of temperature.

length, and width of 0.46, 40, and 10 mm, respectively. K, ΔT, and V are the thermal conductivity (measured when flux lines are static), temperature difference across the sample, and voltage drop, respectively. $K\Delta T/V$ is proportional to the amount of heat carried per flux quantum times $f(B)$, the magnetic-field-dependent fraction of the voltage which is due to flux flow. The curves, especially those for the three lowest temperatures, are very similar to those in Fig. 6. This suggests that both sets of curves have the shape of $f(B)$ and supports the hypothesis that the resistance mechanism changes from flux flow at low magnetic fields to the Ohmic resistance of static normal domains at high fields. The shape of the two higher temperature curves in Fig. 10 may reflect magnetic-field dependence of the quantity of heat carried per flux quantum.

IV. INTERMEDIATE STATE DISPLAYED WITH DIAMAGNETIC POWDER

It is possible to display the topology of the intermediate state with a magnetic powder.[22] The technique, first employed by Shal'nikov, Tumanov, Balashova, and Sharvin,[6] who used ferromagnetic powder, and Schawlow,[5] who used diamagnetic powder, takes advantage of the inhomogeneous magnetic field produced by the arrangement of normal and superconducting domains. The diamagnetic powder, for example, seeks the areas of low magnetic field and, therefore, accumulates over the superconducting regions. The pattern of dark, powder-covered, areas and shiny metal areas devoid of powder produces a good replica of the intermediate state at the sample surface.

We have investigated the intermediate state in slabs of Sn using superconducting Pb powder. A powder obtained from Alcan Metals Powders, Inc., gave good results. It consisted of smooth grains between 2 and 10 μ in diam. Fresh powder was quite lively in a magnetic field, but it was discovered that aged powder (presumably having a thicker oxide and a smaller diamagnetic volume) was much more inert and tended to clump. It was also discovered that use of a buzzer to vibrate the sample helped to keep the powder in equilibrium with the inhomogeneous magnetic field and resulted in a more reliable replica of the intermediate state. The buzzer also made it possible for the powder to repilcate slowly moving domains, and such sequences have been recorded with motion pictures.[18] Some selected still pictures of the intermediate state in a slightly impure

[22] Recently, some remarkably high resolution replicas of the intermediate state and mixed state have been obtained by H. Träuble and U. Essmann [Phys. Status Solidi 18, 813 (1966)], who have refined the ferromagnetic-powder technique. Even more promising for studying the dynamics of the intermediate state is the work of H. Kirchner [Phys. Letters 26A, 651 (1968)], who has refined the magneto-optic rotation technique originated by P. B. Alers [Phys. Rev. 105, 104 (1957)] and used extensively by De Sorbo [see, e.g., W. De Sorbo and W. A. Healy, Cryogenics 4, 257 (1964)].

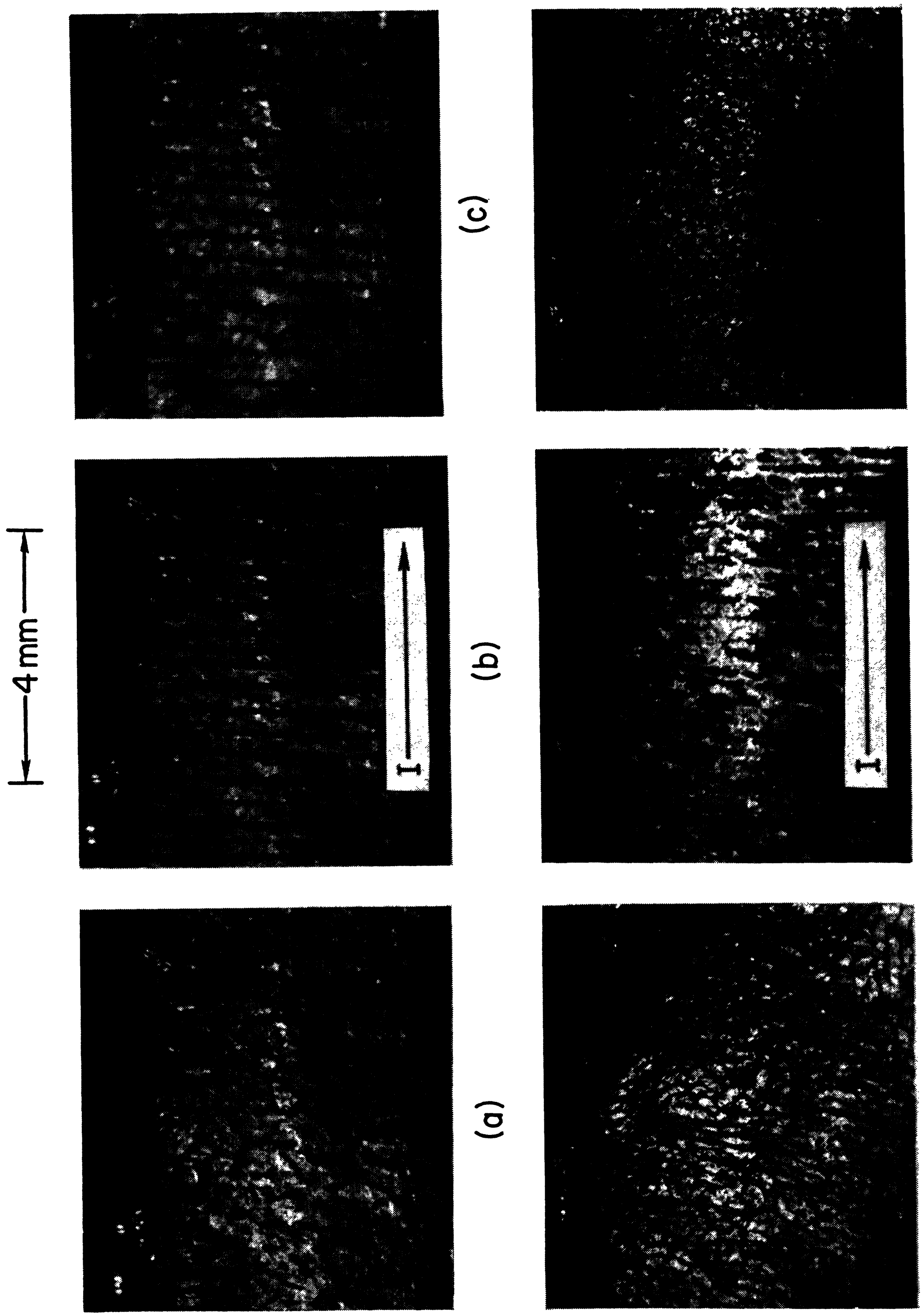

FIG. 11. Diamagnetic-powder replicas of the intermediate state in bulk Sn+0.05 at.% In. (a)–(c) show the intermediate state in a reduced field of 0.75 before, during, and after the application of a transport current of 20 A, respectively. (d)–(f) show the intermediate state in a reduced field of 0.45 before, during, and after the application of a transport current of 30 A, respectively.

Sn slab (30×6×1 mm) in a perpendicular magnetic field are shown in Fig. 11. The bright areas are normal and the dark (powder covered) areas are superconducting.

Figures 11(a)–11(c) were taken when H/H_c was 0.75. They are typical of the high field observations. Figure 11(a) was taken after the intermediate state had been established by lowering the field at constant temperature without current (the buzzer was used to redistribute the powder). The intermediate state consisted of isolated superconducting regions surrounded by normal material. When the current was passed through the sample in the direction indicated in Fig. 11(b), a voltage was observed (there was no measurable critical current). This is consistent with the observation that there are no completely superconducting paths in the indicated direction. As the current was increased the superconducting regions became elongated perpendicular to the current. Figure 11(b) was taken while 20 A were passing through the sample. At this current the realignment of the domains was complete. The powder revealed a static laminar pattern. These observations are similar to what was observed by Shal'nikov[7] and recently by Haenssler and Rinderer.[8] The observations agree with what the measurements of the coupled motion of flux and the Ettingshausen effect indicate: At high magnetic fields there is no flux flow, and the resistance is due to the Ohmic resistance of static normal domains. It should be noted that the drastic reorganization of the topology is consistent with the observation of hysteresis in the resistance described in Sec. II. When the current was turned off [Fig. 11(c)], the pattern remained roughly laminar. At higher fields, the superconducting domains did not extend completely across the sample but were elongated in the direction perpendicular to the current.

Figures 11(d)–11(f) show results for $H/H_c=0.45$. They are typical of the low-field observations. Figure 11(d) was made after the magnetic field was lowered from above H_c at constant temperature. The superconducting regions were connected, and a small current could be passed through the sample without producing any voltage. For small currents the pattern remained static but as the current was increased some domains started to move. With roughly 20 A in the sample, all the domains were moving, and a voltage drop in the sample was observed. Figure 11(e) was made with 30 A in the sample. The moving domains created furrows in the powder. The motion of the domains was evidenced by the motion of loose powder grains[23] in the direction of flux line motion. The motion of powder was observed with the buzzer on or off. With the buzzer on, the moving domains could actually sweep the sample clean of powder in a period of 10 sec or so. This is quite different from the observations of Haenssler and Rinderer[8] and confirms the existence of a dynamic intermediate state. When the current was turned off [Fig. 11(f)], the powder revealed that the intermediate state was a honeycomb pattern consisting of an array of normal spots in a superconducting background. With the reapplication of a current just slightly above the critical current, this honeycomb structure was seen to move slowly sideways across the sample while shearing and distorting. For sufficiently small current, flux flow could actually be observed.

An estimate of the size of the moving normal spots can be obtained from the width of the furrows in Fig. 11(e) or from the size of the normal spots in Fig. 11(f). Both are on the order of 150 μ.

Observations were also made when the magnetic field was inclined away from the normal. The superconductor was a pure Sn slab, with $R_{300°K}/R_{4.2°K} = 7\times10^3$ and width, thickness, and length of 9.5, 0.45, and 35 mm, respectively. The results can be summarized by saying that for any angle of applied field, flux flow was observed if the normal component of field was less than the field at which flux flow ceased in the field perpendicular case. That is, the parallel component of field did not affect the occurrence of flux flow. The smaller the angle between the field and the sample surface, the higher in applied field was flux flow observed. In the regime studied by Brandt and Parks (the magnetic field at 20° from the sample surface with the projection on the sample surface parallel to the current), motion was observed at $H/H_c=0.90$, and in the regime studied by Sharvin (the magnetic field was about 10° from the sample surface and the projection on the sample surface was parallel to the current), motion was evident above $H/H_c=0.95$. No motion of powder was observed above H_c. The results agree with the coupling experiment in an inclined field. The exact nature of the flux-flow state when the field was almost parallel to the sample surface could not be ascertained, because the powder tended to form long spindles along the direction of the field (much as would a ferromagnetic powder) and so diminished the spatial resolution. It seemed, however, that the dynamic motion was that of small normal domains and not of very long lamina. The size of the normal domains must have at least several hundred μ's since the powder spindles which moved were of that length. It was also apparent that there were many domains which did not move.

V. DISCUSSION

Our results and Van Gurp's noise measurements[17] provide a consistent picture of the current-carrying intermediate state in bulk type-I slabs. The picture is that the electrical resistance is due to flux flow at low magnetic fields and due to the Ohmic resistance of static normal domains at high fields. The reduced field h_α, at which the resistance mechanism changes, is smallest (about $\frac{1}{2}$ for Sn and $\frac{2}{3}$ for Pb) for magnetic

[23] Van Gurp (Ref. 17) has also observed the motion of powder which he interprets as evidence of the motion of vortices.

fields applied perpendicular to the broad surface of the slab and largest (approaching unity) for magnetic fields which are almost parallel to the broad surface. The results apply to both thick and thin slabs, suggesting that resistance mechanism changes may occur in other geometries, perhaps in cylinders.

The problem of why the resistance mechanism changes as it does is difficult, and we can only offer some heuristic arguments. In what follows, we shall argue that the intermediate state established at a fixed field and temperature after a sufficiently large transport current is passed through a superconductor (we will call this the current-established state) corresponds to a local minimum of the free energy, and that small transport currents passing through a superconductor in this state perturb it only slightly. The mechanism of electrical resistance at a given temperature and magnetic field would, therefore, be determined by the topology of the current-established state. If that state consists mostly of simply connected superconducting regions in a normal matrix, then the resistive state which least perturbs the topology is one in which the normal regions exhibit Ohmic resistance. If the current-established state consists mostly of simply connected normal regions in a superconducting matrix, then the resistive state which least perturbs the topology is one in which the whole topology moves sideways to the current, i.e., the flux-flow state. Finally, we hypothesize that the current-established state is actually the equilibrium state, and the problem is reduced to predicting the topology of the equilibrium intermediate state.

The relationship of the electrical resistance mechanism to the topology of the current-established state was demonstrated with the diamagnetic-powder technique. For example, Fig. 11(c) shows a current-established state at a field in which flux flow does not occur. As can be seen, this state favors simply connected superconducting regions. Figure 11(f) shows a current-established state for a field in which flux flow occurs, and this state favors simply connected normal regions. Similar results were observed at other fields and temperatures.

To justify the hypothesis that the current-established state is the equilibrium state we point out the importance of the transport current in changing the intermediate-state topology. This feature is most strikingly exhibited with the diamagnetic powder and less strikingly by the hysteresis in the current-voltage curves. It is well known that the topology of the intermediate state is very sensitive to the way it is established. Haenssler and Rinderer[8] have observed variations in the intermediate-state topology which depend on whether the transition to the intermediate state is from the superconducting state or from the normal state and whether the transition takes place at fixed field or fixed temperature. Establishing an intermediate state by passing a large current through a sample at a fixed field and temperature is another way of establishing a reproducible intermediate state. A wide variation in intermediate-state topologies is possible because the free energy is relatively insensitive to the detailed structure, and pinning of the domains makes it possible to have many metastable states. The passage of a current can detach the domains from the pinning sight and might, therefore, establish the state with minimum free energy. An analogy which can be made is that passing a transport current through a superconductor with a haphazardly established intermediate state has a tendency to lower the free energy of that state in the same sense that agitating a bucket which has been haphazardly filled with rocks has a tendency to lower the free energy of the rocks.

Calculations of the free energy for various intermediate-state topologies have been performed by several authors.[20,24–27] Particularly relevant is the work of Andrew,[20] who calculated the free energy of branching laminae and of branching normal threads and found that at low magnetic fields the state with lowest free energy contains normal threads and at high fields contains laminae. Also, Landau[24] calculated the minimum free energy for a laminar intermediate state but pointed out that at low magnetic fields a state containing normal threads would have a lower free energy.

To understand why the topology should be a function of magnetic field, we consider the various contributions to the free energy of the intermediate state. For simplicity, we treat an ellipsoidal sample so that the average magnetization is distributed uniformly. We assume that the domain size is small compared to the size of the sample and large compared to the thickness of the normal-superconducting boundary. With these assumptions, the free energy may be written

$$F=F_{\text{norm}}+F_1+F_2+F_3+F_4+F_5+F_6,$$

where F_1 is the negative of the condensation energy of the superconducting regions, F_2 is the magnetic-field energy in the bulk of the sample neglecting rounding of the domains at the surface, F_3 is the magnetic-field energy outside the sample assuming that the magnetization of the ellipsoid is smeared out evenly, F_4 is the wall energy associated with a normal-superconducting boundary, F_5 is the magnetic-field energy correction to F_3 associated with the coarse-grained nature of the magnetization, and F_6 is the corrections to the magnetic-field energy, the domain-wall energy, and the condensations energy due to the rounding of the domains at the sample surface. $F_{\text{norm}}+F_1+F_2+F_3$ is the largest part of the free energy. The sum depends only on the applied

[24] L. D. Landau, J. Phys. USSR **7**, 99 (1943); Phys. Z. Sowjet. **11**, 129 (1937).
[25] C. G. Kuper, Phil. Mag. **42**, 961 (1951).
[26] E. M. Lifshitz and Yu. V. Sharvin, Dokl. Akad. Nauk SSSR **79**, 783 (1951).
[27] T. E. Faber, Proc. Roy. Soc. (London) **A248**, 460 (1958).

magnetic field and the shape of the sample and is independent of the intermediate-state topology. With the assumption that $H=H_c$ in the bulk of the sample, the normal fraction of the sample is uniquely related to the geometry and applied field. The problem reduces to choosing the topology which minimizes $F_4+F_5+F_6$. At very high magnetic fields where there is very little superconducting material, a topology containing simply connected superconducting regions (say, superconducting threads) would minimize the free energy by significantly reducing F_4, the domain-wall energy. At very low magnetic fields simply connected normal regions would minimize the free energy.

To get more specific details, we work out a simple model for the case of an ellipsoid with a very large demagnetizing factor in the direction of the applied field. This corresponds to the thin slabs in perpendicular fields which we have studied. Let us compare the topology containing cylindrical superconducting threads to that of cylindrical normal threads. As we said, the superconducting-thread topology has the lowest free energy at high fields, and the normal-thread topology has the lowest free energy at low fields. We look for the magnetic field at which the two topologies have the same minimum free energy. We make the simplifying assumption that the domain walls are all parallel to the applied field and that the shielding currents in the superconductor do not vary in the direction of applied field. Then $F_6=0$. It is also easy to show that the magnetic-field energy F_5 for the normal-thread topology at some reduced field h_α is the same as F_5 for the congruent superconducting-thread topology at the field $1-h$. Assuming that the boundary-wall energy F_4 is independent of the direction of curvature, the sum of F_4 and F_5 is identical for both the superconducting-thread and normal-thread topologies at $h=\frac{1}{2}$. This field corresponds to h_α if these are the only allowable topologies. This is in reasonable agreement with the measured values of h_α which are $\frac{1}{2}$ for Sn and $\frac{2}{3}$ for Pb. With a more realistic model which includes a nonzero F_6, one would expect simply connected normal regions to occur in higher applied fields in Pb than in Sn since Pb has a significantly smaller domain-wall energy.

That flux flow should occur at higher applied fields when the angle between the field and sample surface decreases is consistent with the discussion presented above. The parts of the free energy which determine the topology depend primarily on the magnetic field within the sample (or, alternatively, on the fraction of normal material) and not directly on the applied field. For a given applied field, the normal fraction decreases as the angle between the field and the sample surface decreases. As a rough approximation the normal volume is proportional to the perpendicular component of reduced field, so the resistance mechanism should change when the perpendicular component reaches h_α. This is in reasonable agreement with the data of Fig. 8, and with the observations with diamagnetic powder.

VI. SHARVIN EXPERIMENT

What bearing do these measurements have on the controversy concerning the Sharvin experiment? Sharvin studied the resistive state of a single-crystal Sn disk situated in a magnetic field applied almost parallel to the surface of the disk.[10] He had previously observed that the intermediate state in this configuration consisted of alternating normal and superconducting laminae elongated parallel to the projection of the field on the sample surface.[28] Sharvin observed that when current was passed through the disk in the direction of elongation of the lamina, periodic oscillation occurred in the contact resistance of a small point probe attached to the surface of the disk. Assuming that the resistance of the point contact was strongly dependent on the state of the material in the region of the contact, the oscillation were interpreted as evidence for the motion of the laminae. Chandrasekhar, Farrel, and Huang[11] showed that the oscillation in the contact resistance could also be caused by an inherent instability in the state of the probe itself, having nothing at all to do with flux flow. The present work adds yet a third possible interpretation, that the oscillation are caused by motion of domains other than the long lamina that Sharvin observed under static conditions.

Brandt and Parks[12] tried to prove that Sharvin's interpretation was incorrect. For their own experimental conditions, they showed that some domains did not move and hence that the resistance was not due to long moving lamina. Our powder observations, made under similar experimental conditions, are consistent with the observations that some domains do not move. When current was passed through the sample, it could be seen that while there was abundant motion of domains, many major features remained fixed. The measurements of Brandt and Parks may have revealed only these major features. Their time resolution was perhaps not fast enough to catch the dynamic motion. Their published data do in fact show some differences in the domain patterns in successive observations. Brandt and Parks did not prove, however, that Sharvin's interpretation of his own work was incorrect because there were important differences in experimental conditions (Sharvin used a single-crystal disk with the field inclined at an angle of about 10° to the sample surface, and Brandt and Parks used a polycrystal slab with the field inclined at an angle of 20° to the sample surface). In fact, Sharvin mentions that it may be the

[28] Yu. V. Sharvin, Zh. Eksperim. i Teor. Fiz. **33**, 1341 (1957) [English transl.: Soviet Phys.—JETP **6**, 1031 (1958)].

crystalline anisotropy of the surface energy which tended to keep the lamina aligned.

Which of the three interpretations of Sharvin's oscillations is correct remains an open question. It is probable, however, that there was some kind of domain motion both in Sharvin's experiment and in the Brandt-Parks experiment.

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Phenomenology of Shape Effects in Amperian Magnetic Systems with Application to Superconductors

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This paper is concerned with the effects of sample shape and orientation on the magnetic properties of superconductors. The absence of adequate theoretical formulas is probably the main reason that these effects have been largely ignored in the past. The present discussion begins with the formulation of a thermodynamic potential appropriate to an Amperian system and expressed in a form adapted to the Landau theory of phase transitions. The shape dependence is expressed explicitly in terms of the usual demagnetization factors, and the dependence on H_0 and T (the applied magnetic field and temperature) is expressed implicitly through the magnetization which is defined uniquely and self-consistently in terms of the currents and the magnetic moment. The formulation finds a natural application in the description of superconductors and leads to a vector generalization of Abrikosov's constituitive relation for the superconducting mixed state. Explicit formulas are derived for the magnetic moment per unit volume, specific heat, force, and torque for the Meissner state and mixed state of a spheroid of arbitrary shape and orientation in an applied field. It is found that the sample shape is always important for the Meissner state, and is important for the mixed state whenever the inequality $(2\kappa^2-1)\beta\gg1$ is not satisfied. Here κ is the Ginzburg-Landau parameter, and β is expected to have the value 1.16 for fluxoid lattices carrying a single flux quantum per unit cell.

INTRODUCTION

It is well known that sample shape influences the so-called demagnetization field and thereby substantially influences the magnetic and thermal properties of ferromagnetics and strongly paramagnetic materials when measured in an applied field. In contrast, shape effects have been largely ignored in studies of superconductors, a somewhat surprising situation in view of the knowledge that the superconductor's magnetization is often not negligible compared to the applied field, a fact which may be taken as a criterion for the occurrence of shape effects.

This state of affairs doubtlessly stems mainly from the fact that at least as regards the type-II mixed state, until recently, no theoretical expression existed to account for sample shape in magnetothermal measurements. Moreover, some recent formulas have been based on inappropriate thermodynamic arguments. The experimentalist, uncertain in this situation, usually sought to restrict his measurements to long slender specimens oriented along the field where presumably demagnetization effects should vanish. Very little information has been derived from force and torque experiments where, particularly in the latter case, the shape and orientation are dominant factors.

The absence of a convincing treatment of shape effects for superconductors stems at least partially from the fact that whereas superconductors are manifestly Amperian systems, the main framework of understanding for other strongly magnetic materials is the localized-spin model. Despite certain similarities, there are fundamental differences between the two systems. In the localized-spin case, where the magnetization arises solely from ordered localized moments, the dipole-dipole coupling between spins is responsible for the demagnetization effects. This comes about basically because of the essential long-range nature of the dipole forces which is evidenced by the familiar dipole sum

$$\Omega(\mathbf{r}_l)=-\frac{1}{N}\sum_{k}^{N}(1-3\cos^2\phi_{kl})r_{kl}^{-3}, \qquad (1)$$

where $r_{kl}\equiv|\mathbf{r}_k-\mathbf{r}_l|$. In a static configuration, for example, the sum measures the contribution to the interior field at the point $\mathbf{r}_l$ due to dipoles distributed at points $\mathbf{r}_k$ of a lattice. The value of the sum is clearly

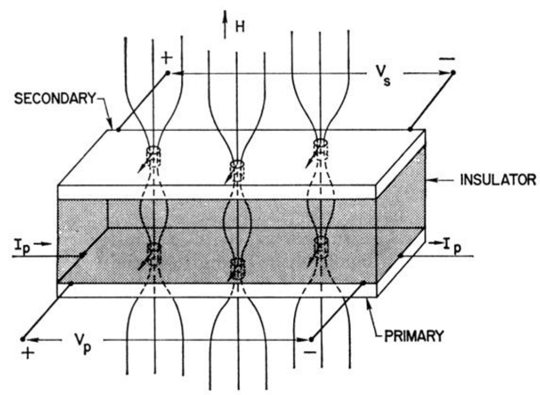

FIG. 1. Schematic drawing of the configuration for observing the coupled motion of flux.

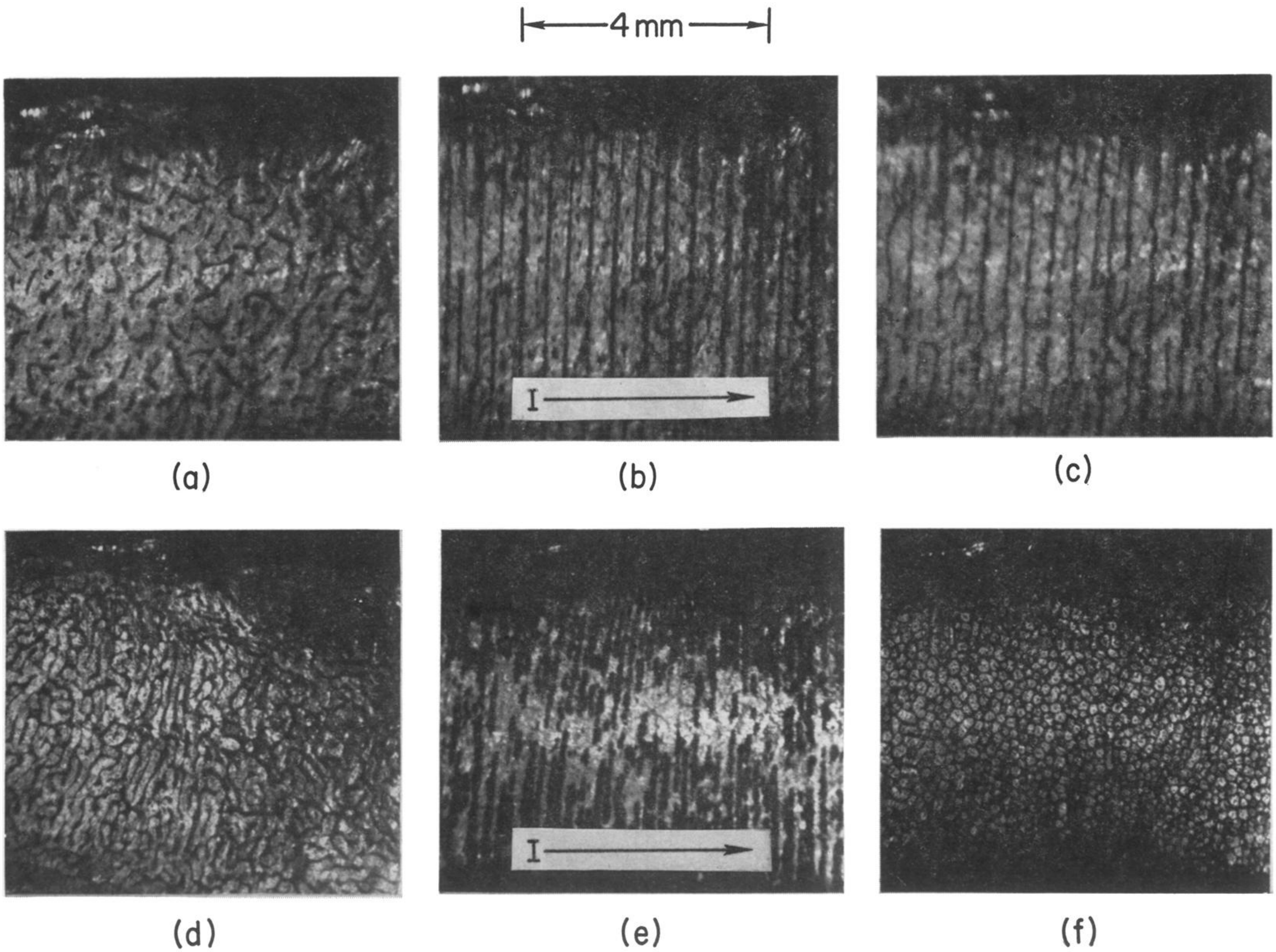

FIG. 11. Diamagnetic-powder replicas of the intermediate state in bulk Sn+0.05 at.% In. (a)–(c) show the intermediate state in a reduced field of 0.75 before, during, and after the application of a transport current of 20 A, respectively. (d)–(f) show the intermediate state in a reduced field of 0.45 before, during, and after the application of a transport current of 30 A, respectively.

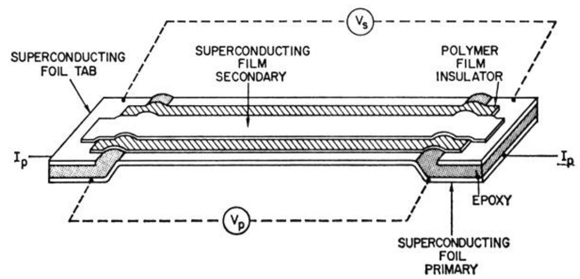

FIG. 2. Typical dc transformer. The sandwich consists of a type-I foil primary, an evaporated polymer film insulator, and an evaporated type-I superconducting secondary.