in the case of a line shape which is actually Lorentzian. Most line shapes which occur may be approximated, though often crudely so, by Gaussian or Lorentzian shapes. Thus $R(T_2)$ may be used as an estimate of the relative error in T_2 due to modulation. Conversely, a small predetermined relative error in T_2 may be used to arrive at a suitable modulation amplitude.

By way of example, consider a line shape which is actually Gaussian. A predetermined minimum relative error of 0.02 in T_2 leads to

$$2/(5T_2)<b<3.8/T_2,$$

as an experimentally acceptable range for the modulation amplitude.

The "folding function" of Spry[26] is contained explicitly in Eq. (A9). Consider the response of the lock-in amplifier to a δ function. The recorded signal is

$$\begin{aligned} f'(\omega) &= -\omega/[b^2-\omega^2]^{1/2}, & |\omega| &\le b \\ &= 0, & |\omega| &> b. \end{aligned} \tag{A10}$$

Within the limitations of the theory, Eq. (A10) is the "folding function." The transition from (A9) back to (23c) represents, in effect, Spry's unfolding procedure.

Figure 7 shows a comparison of (A10) with the recorded signal from a lightly doped sample of water. The width of the water resonance is narrow and for sufficiently large modulation amplitudes will approximate a δ function. The observed response is very well described by Eq. (A10) over the range $|\omega|\le 0.75b$. The deviation of the response from Eq. (A10) near $|\omega|=b$ may be ascribed to the fact that the water resonance has a finite width. The results may be considered as partial experimental verification of Eq. (A9) and also Eq. (23c).

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Energy Loss and Straggling in Silicon by High-Energy Electrons, Positive Pions, and Protons*

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The most probable energy loss and energy-loss straggling are investigated in lithium-drifted silicon for electrons, positive pions, and protons with energies 31.5–767.2 MeV. Very good agreement with theory is obtained with protons and pions. The electron spectra demonstrate effects which can be attributed to the absorption by the detector of a portion of the electron bremsstrahlung radiation. No apparent evidence for a predicted decrease in electron ionization through radiative corrections is observed.

INTRODUCTION

THE high resolution which can be obtained with semiconductor radiation detectors suggests that these devices should be very attractive for use in precise investigations of the ionization process. The deep depletion regions which can be produced by the lithium-drifting process permit the realization of an excellent signal-to-noise ratio, even for charged particles near the minimum ionizing region. If such an investigation is undertaken at relatively high energies, where the density of the ionization column produced by the charged particle remains essentially constant over the dimensions of the active detector region, the same data may be used to reveal details of the trapping, recombination, and other "bulk" properties of the semiconductor material.[1]

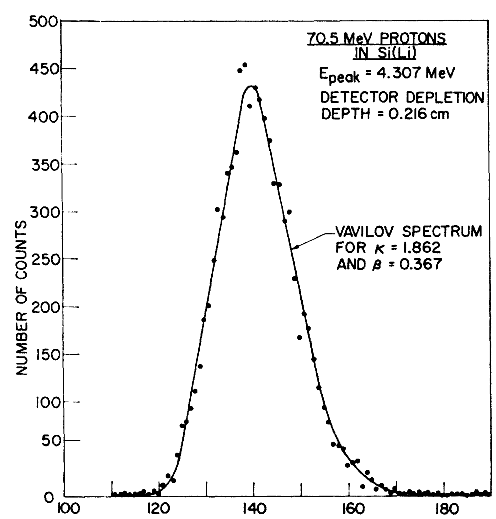

FIG. 1. Experimental and theoretical collision-loss spectra for 70.5-MeV protons in silicon. The parameters κ and β are defined by the kinematics and by the properties of the silicon and the incident particle. The theoretical curve shown is uniquely defined by the experimental parameters, indicating a very good absolute agreement between experiment and theory.

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[1] D. W. Aitken, D. W. Emerson, and H. R. Zulliger, IEEE Trans. Nucl. Sci. NS-15, 456 (1968).

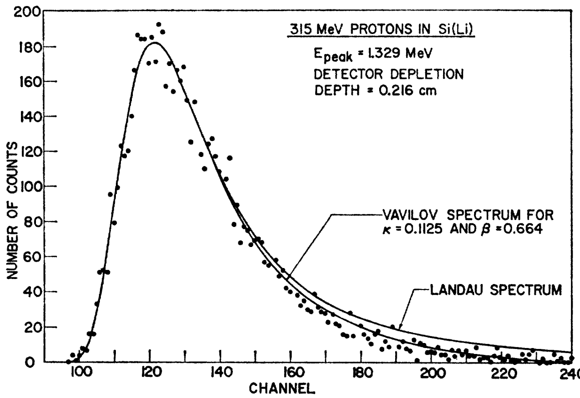

FIG. 2. A comparison of the Vavilov and Landau theories with experiment for 315-MeV protons, showing the convergence of the experimental spectra with increasing particle velocity towards the limiting Landau spectral distribution.

The energy-loss distributions have been calculated in the limiting case for high-particle velocities ($\beta \approx 1$) by Williams[2] and by Landau,[3] approximately for the general case by Symon,[4,5] and more precisely for the general case by Vavilov.[6] Corrections, which do not materially affect the present investigation, have been evaluated by Fano,[7] Hines,[8] and Blunck and Leisegang.[9] Numerous experimental investigations of the energy-loss spectra have demonstrated general agreement with theory.[10–15] The most precise test of the Landau theory would seem to be the recent Stanford study of high-energy muons in NaI(Tl) by Bellamy *et al.*,[14] while the most precise test to date of the more general Vavilov theory is provided by the recent Berkeley investigation of Maccabee *et al.*[15] with high-energy protons, pions, and α in silicon.

Several investigations on the response of silicon detectors to high-energy charged particles have been reported.[11,13,15–19] The earlier works[11,16–19] suffered in general from inadequate statistics or from high background. The more recent papers by Lander *et al.*[13] and by Maccabee *et al.*[15] showed excellent statistics and negligible background.

The presently reported work extends the Berkeley pion and proton observations[15] through intermediate energies, and extends the investigation to high-energy electrons as well, in order to seek further confirmation of the correction to the ionization theory for the density effect,[20,21] and to seek evidence for postulated radiative corrections to the ionization-loss theory.[22] Our investigation also utilized high-energy charged particles to probe the charge collection and bulk material properties of lithium-drifted silicon radiation detectors. An analysis of these properties has been reported elsewhere.[1] The present paper is concerned with the ionization-loss and energy-straggling processes.

EXPERIMENTAL METHOD

A monoenergetic beam of high-energy electrons from the Stanford Mark III Linear Accelerator was used to provide electrons, protons, and positive pions by elastic and inelastic scattering and photoproduction processes in liquid hydrogen and CH_2. The energy spread of the incident beam ($\Delta E/E$) was never more than 1%. The scattered or recoiling particle was analyzed in momentum by a 1 BeV/c double focusing magnetic spectrometer. The application of measured dispersion corrections to the calculated spectrometer trajectories permitted a determination of the momentum of the particle to within about 0.1% of the true value.

Three lithium-drifted silicon detectors were used in the present investigation. For uniformity of presentation, the data illustrated in this paper were all obtained with a detector manufactured by the Technical Measurements Corp., which featured an active surface area of 1.0 cm^2 and a depleted depth of 0.216 $\pm$0.001 cm.

The detector was mounted in an evacuated cryostat near the spectrometer focal plane. A 100-channel scintillation ladder counter and a liquid Čerenkov counter were located behind the detector housing. These "backing" counters permitted direct observation of the particle spectra, and enabled us to "center" these spectra on the silicon detector in order to realize

[2] E. J. Williams, Proc. Roy. Soc. (London) **A125**, 420 (1929).
[3] L. D. Landau, J. Phys. (USSR) **8**, 201 (1944).
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[6] P. V. Vavilov, Zh. Eksperim. i Teor. Fiz. **32**, 920 (1957) [English transl.: Soviet Phys.—JETP **5**, 749 (1957)].
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[10] For references to earlier work and discussions, see H. A. Bethe and J. Ashkin, in *Experimental Nuclear Physics*, edited by E. Segré (John Wiley & Sons, Inc., New York, 1953), Vol. 1, p. 166; also R. M. Sternheimer, in *Methods of Experimental Physics*, edited by L. C. L. Yuan and C. S. Wu (Academic Press Inc., New York, 1961), Vol. 5A, p. 1.
[11] G. W. Grew, IEEE Trans. Nucl. Sci. **NS-12**, 308 (1965).
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[21] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952); **103**, 511 (1956).
[22] V. N. Tsytovich, Zh. Eksperim. i Teor. Fiz. **42**, 457 (1962); **43**, 1782 (1962) [English transls.: Soviet Phys.—JETP **15**, 320 (1962); **16**, 1260 (1963)].

the maximum signal-to-background ratio. By properly selecting the spectrometer polarity, the kinematical conditions, and the pulse-height range, spectra of the particles of interest were completely separated. No appreciable contamination of any spectrum was encountered.

The detector signal was initially amplified by a Tennelec 100C charge-sensitive preamplifier, placed immediately behind the detector. Further amplification and *RC* pulse shaping with 0.8-msec time constants was accomplished by a Tennelec TC 200 amplifier placed in the counting room at a distance of approximately 350 ft from the detector. The spectra were recorded in a Nuclear Data ND 181FM 1024 channel analyzer, with the analyzer input gated on by the 2-μsec accelerator gun pulse. With this system only a small contribution to the background from rf pickup was encountered. Subsequent use of a cooled FET preamplifier removed the residual rf pickup and improved the resolution. It was demonstrated that neither improvement was necessary for the success of the experiment. The data reported here were obtained while using the 100C preamplifier.

The energy calibration of the entire system was performed with a Cs^{134} source of γ rays (796 keV) and conversion electrons (760 keV). Nevertheless, it cannot be assumed that one may apply the calibration obtained either with the conversion electrons or with the photoelectrons released by the γ rays linearly to an investigation of nonminimum ionizing particles (e.g., protons and pions), although this has generally been assumed by other workers. The present investigation demonstrates that in fact the calibration should not be performed with conversion electrons which do not penetrate uniformly through the detector depletion region, and that the calibration with γ rays can only be applied precisely if the charge collection efficiency corrections are determined directly for the nonminimum ionizing particles.[1] These efficiency corrections were separately determined during the actual high-energy runs for both warm and cold detectors and applied to the experimental data to yield the final energy values reported here. Further details concerning the method of obtaining these corrections and the possible physical significance of the observed nonlinear behavior of the charge collection are reported elsewhere.[1]

A. Energy Loss and Straggling

Figures 1–4 provide a sequence in order of increasing particle velocity, or, alternatively, in order of decreasing magnitude of the energy deposited in the detector. The spectrum approaches a Gaussian distribution in the limit of low particle velocities ($\beta < \sim 0.2$).[15] The spectrum of the 70.5-MeV protons shown in Fig. 1 is still not far removed from this condition. A small high-energy "tail" is beginning to appear, representing those single ionization encounters

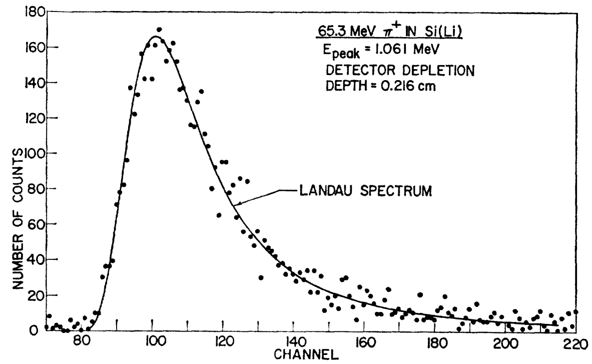

FIG. 3. The 65.3-MeV pions, with a β of 0.732, demonstrate that the Landau limiting conditions have been met within the statistical errors of the observation.

with large energy transfers. Figure 2 demonstrates that for $\beta = 0.664$ the distribution has already nearly approached the Landau limit. For a value of $\beta = 0.742$, represented in Fig. 3 by the 65.3-MeV positive pions, the actual Vavilov and limiting Landau spectra are practically indistinguishable. All of the electron points, such as the one at 458 MeV shown in Fig. 4, represent a particle velocity very nearly equal to c, the speed of electromagnetic radiation in free space. In theory, the Landau limiting conditions should be precisely realized by all of the electron points.

The theoretical spectra shown in Figs. 1 and 2 are derived from a numerical solution of the Vavilov theory.[6,23] The Vavilov distributions are described by the parameters $\beta = v/c$, where v is the particle velocity, and

$$\kappa = 0.30058 \frac{mc^2}{\beta^2} \frac{Z}{A} \frac{s}{E_{\max}}, \tag{1}$$

where m is the mass of the electron; A and Z are the atomic weight and atomic number, respectively, of the material through which the particle is passing; s is the thickness of the material in grams per square centimeter; and $E_{\max}$ is the maximum amount of energy that can be transferred to an atomic electron in a single collision with the incident particle. $E_{\max}$ is given by

$$E_{\max} = \frac{2mc^2\beta^2}{1-\beta^2} \left[1 + \frac{2m}{M(1-\beta^2)^{1/2}} + \left(\frac{m}{M} \right)^2 \right]^{-1}, \tag{2}$$

where M is the mass of the incident particle.

[23] S. M. Seltzer and M. J. Berger, National Academy of Sciences –National Research Council Publication No. 1133, Nuclear Science Series Report No. 39, p. 187, 1964 (unpublished). For convenience in comparison of the Seltzer and Berger material with the notation of Sternheimer used in Ref. 10, note that $\kappa \equiv \zeta / E_{\max} \equiv A_{St} s / \beta^2 E_{\max}$, where ζ is the parameter used by Seltzer and Berger, and A_{St} is the constant defined by Sternheimer in his discussion of the density effect [also shown in Eq. (4) above]. s and β are as defined for Eq. (1) above. $E_{\max}$ is defined in Eq. (2).

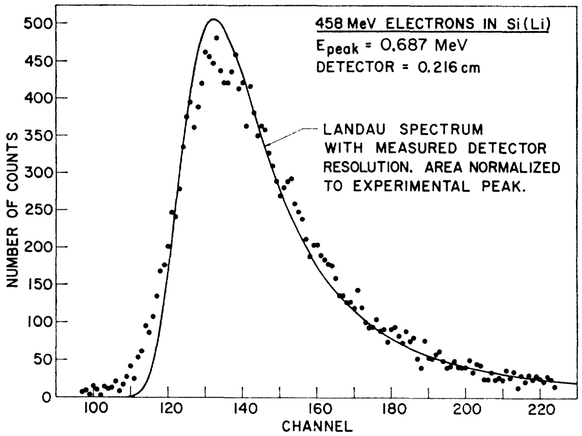

FIG. 4. The Landau limiting conditions are well met in theory by high-energy electrons, and yet the comparison of theory with experiment is unsatisfactory. The experimental peak is broader than the predicted spectrum, and is also shifted an average of 4.5% towards high energies. For this illustration the theoretical and experimental peak channels were normalized. The shift towards higher energies and the broadening can be understood on the basis of the absorption in the detector of a portion of the radiation produced by the electron as it passes through the silicon.

Seltzer and Berger[23] have tabulated the Vavilov distribution for protons, kaons, pions, and muons for energies 1.0 MeV–10 GeV. For approximate comparisons with experiment one can interpolate from the Seltzer-Berger data. For precise comparisons one must either investigate the points tabulated by Seltzer and Berger, or else one must calculate the Vavilov distributions for the actual data points. The Vavilov distributions were calculated directly for the data points included in the present investigation.

The most significant observation to be drawn from this brief discussion is that the parameters β and κ uniquely define a spectrum for a given particle type and energy and a given material. If the areas of the theoretical and experimental spectra are constrained to be equal, then the comparison between theory and experiment is made absolutely, without the benefit of further adjustable parameters. The Vavilov spectra plotted in Figs. 1 and 2 demonstrate a very satisfactory agreement between theory and experiment for the ionization losses by high-energy protons in silicon.

The Landau spectra can be evaluated by taking advantage of the numerical calculations of the Landau distribution prepared by Börsch-Supan.[24] The calculated spectra are shown in Figs. 2–4 for protons, pions, and electrons, respectively. In Fig. 2 the Landau limit is not yet reached, so that the Landau distribution does not agree with experiment. In Fig. 3 the Landau distribution provides a good fit to the experimental data points.

It was stated above that the Landau limit is precisely realized in theory for all of the high-energy electron points. However, Fig. 4 demonstrates that the apparent agreement between theory and experiment for the electron points is not very good. The theoretical spectrum is higher and narrower than the experimental spectrum. Since the areas of the theoretical and experimental spectra are normalized, the theoretical peak is higher because it is narrower. The discrepancy in width is the important observation here. In addition, it will be shown in the next section (and in Fig. 6) that the experimental peak position is shifted about $4\frac{1}{2}\%$ towards energies higher than the calculated value of the most probable energy loss. For Fig. 4 the spectra were normalized at the peak.

The interpretation of this discrepancy is provided by the radiation spectrum produced by the electron as it passes through the silicon. At 458 MeV in silicon, for example, the electron is losing approximately ten times as much energy by radiation as it is through collision losses. Some of this radiation will interact in the silicon, causing a greater amount of energy to be deposited in the detector than one would calculate on the basis of collision theory alone. This would shift the most probable and average energy-loss values towards higher energies, in agreement with the observations.

The electron spectrum would be broadened by the superposition of the radiation absorption spectrum upon the collision-loss spectrum. A first approximation to this broadening was provided by utilizing the Gaussian distribution introduced into our computer program to permit the inclusion of a finite detector resolution in the calculations of the predicted spectral shapes. The magnitude of this contribution was varied in steps of 3% up to 24%. Figure 5 demonstrates that the closest agreement between experiment and theory is obtained with a 12% Gaussian contribution folded into the calculated spectral distribution. The agreement between experiment and theory is considerably better in Fig. 5 than it is in Fig. 4. We do not take the Gaussian shape seriously, though, since the 12% superposition is not particularly sensitive to the precise shape of the peaked radiation absorption distribution.

Additional radiative contributions appear in the background spectra. An almost uniform background was observed under the experimental electron spectra which did not appear with the other particle spectra. This background limited the peak-to-valley ratio of the measured electron spectra to about 9:1. By removing the target from the path of the accelerator beam while leaving the spectrometer entrance slits open, and then by closing the spectrometer entrance slits with the target in place, we were able to demonstrate that very little of the background spectrum entered through the concrete walls shielding the detector housing. This background is interpreted as arising from interactions in the detector of background radiation produced by electrons scattering on the walls of the spectrometer or scattering on material in the vicinity of the detector. This background component

[24] W. Börsch-Supan, J. Res. Natl. Bur. Std. **65B**, 357 (1957).

would be absent when heavier particles are being selected by the spectrometer, in agreement with our observations.

A valuable experimental confirmation of these observations is provided by the earlier and similar conclusions of Bellamy *et al.*[14] They obtained very good agreement between theory and experiment for 1.0- to 10.5-GeV/c muons in NaI(Tl), but 10.5-GeV electrons produced a spectrum that was too wide and about 7% too high, in qualitative agreement with the observations reported in the present paper. The greater discrepancy is to be expected at the higher energies, as the radiation density produced by the electron in the material is correspondingly greater.

B. Most Probable Energy Loss

The most probable energy loss, which corresponds to the peak in the measured experimental distribution, can be calculated from the Landau theory.[3] In the notation of Sternheimer,[10] this is given by

$$\epsilon_{\rm prob}=\frac{As}{\beta^2}\left[B+1.06+2\ln\left(\frac{P}{Mc}\right)+\ln\left(\frac{As}{\beta^2}\right)-\beta^2-\delta-U\right],\quad(3)$$

where

$$A=\frac{2\pi ne^4}{mc^2\rho}=0.1536\left(\frac{Z}{A_0}\right)\frac{\rm MeV}{\rm g\ cm^{-2}},\quad(4)$$

$$B=\ln[mc^2(10^6\ {\rm eV})/I^2].\quad(5)$$

P, M, and βC represent the momentum, mass, and velocity, respectively, of the incident particle; m and e refer to the mass and charge of the electron; n is the number of atomic electrons in a cubic centimeter of the material through which the particle is passing; Z and A_0, respectively, represent the atomic number and weight of the material, with the subscript now placed on the atomic weight to prevent confusion with the Sternheimer constant A; and I represents the mean excitation potential of the material. U is the shell correction term, needed to take into account the participation of only the outer atomic electrons for very low particle velocities. U is negligible for the parameters of the present investigation.[25,26]

δ is the correction for the density effect produced by local polarization of the material by the incident particle. This has been evaluated for silicon by Sternheimer,[20] based upon his earlier developments of the theory.[21] Sternheimer derived a phenomenological form for δ, given by

$$\delta=4.606X+C+a(X_1-X)^m,\quad X_0<X<X_1\quad(6)$$

$$\delta=4.606X+C,\quad X>X_1\quad(7)$$

[25] M. C. Walske, Phys. Rev. **88**, 1283 (1952).
[26] M. C. Walske, Phys. Rev. **101**, 940 (1956).

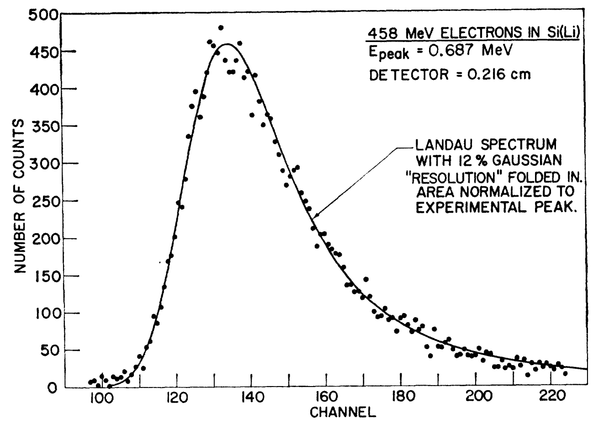

FIG. 5. The same experimental spectrum shown in Fig. 4, but with a theoretical fit formed by the mixture of a 12% Gaussian contribution with the Landau spectrum, as a first approximation towards the inclusion of the radiation absorption spectrum. The theoretical spectrum which results is not very sensitive to the actual shape of the radiative contribution. The agreement with experiment is satisfactory.

where $X=\log_{10}(P/Mc)$, with P and M the momentum and rest mass of the incident particle. For the present investigation, the following values have been used: $I=172$ eV, $A=7.66\times10^{-2}$, $B=16.66$, $C=-4.38$, $a=8.74\times10^{-2}$, $m=3.586$, $X_0=0.10$, $X_1=3$, and $\rho=2.33$ g/cm^3. For the detector used in obtaining the data displayed in Figs. 1–6, $s\equiv\rho t=0.503$ g/cm^2.

The solid curves in Fig. 6 represent the calculated values for the most probable energy loss, based on the theory outlined above. The proton and pion data agree well with the theoretical curves. No consistent difference is observed either as a function of energy or of temperature, showing that the procedures developed for energy calibration and efficiency corrections (these differ for the warm and cold cases) are uniformly applicable.

In the highly relativistic region, the density correction compensates exactly for the relativistic rise. As a consequence the same value, 0.716 MeV, is calculated for the energy loss through ionization by the electrons over the full energy range investigated. This is represented by the straight line plotted in Fig. 6. Within statistics, the electron points do indeed follow a straight line, thus providing additional confirmation of the Sternheimer density correction. The average value of the electron points deviates from the theoretical value by about 4.5%. This uniform deviation is apparently the result of the radiative contributions discussed in the previous section.

Another important conclusion can be drawn from the electron data. Tsytovich[22] calculated that radiative corrections to the collision loss theory should lead to a reduction in the ionization loss of 5–10% for very high values of the particle velocity. This "saturation" value is reached when the ratio $P\beta c/Mc^2$ is somewhere between 600 and 1000. This corresponds to an electron

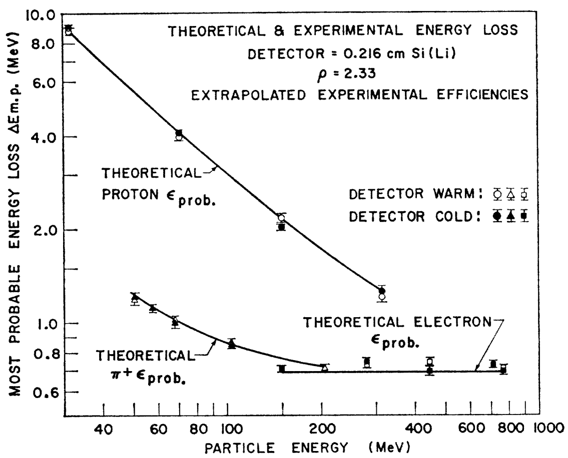

FIG. 6. Comparison of the energy corresponding to the peaks in the experimental distributions with the calculated most probable energy-loss values in silicon. Satisfactory agreement is achieved with the protons and pions, showing also no consistent difference as a function of detector temperature. The electron data points are raised by the absorption of bremsstrahlung radiation in the detector to a value which is on the average about 4.5% higher than the predicted value. The relativistic rise is exactly compensated by the density correction. The data reveal no evidence for radiative corrections to the collision-loss theory.

energy in the range 300–500 MeV. The Tsytovich theory would therefore predict that our 150-MeV electron point should lie rather close to the Sternheimer theoretical value, but that the higher-energy points should show a rather abrupt drop to an average value which is 5–10% below the theoretical line. No such behavior is observed. Statistical fluctuations make it appear that, if anything, the average value increases somewhat at the higher energies. Although the absorption of radiation by the detector causes the experimental average value to be apparently greater than the theoretical value, a 5% abrupt reduction in the experimental electron values for energies above about 300 MeV should clearly have been observed within the statistics of this investigation.

These conclusions may help to resolve an apparent contradiction which has emerged from previous observations by other workers. Investigations with very high-energy cosmic-ray muons by Crispin and Hayman[27] and by Ashton and Simpson[28] have revealed no evidence for the predicted Tsytovich effect, although the energy region covered by these workers should include the fairly pronounced "transition" region predicted by Tsytovich. On the other hand, droplet density counts by Zhdanov *et al.*[29] and by Alekseeva *et al.*[30] for high-energy electrons in emulsions aparently revealed evidence in support of the predicted Tsytovich correction. We suggest that the presently reported data and the data of Bellamy *et al.*[14] present rather conclusive evidence that the Tsytovich correction is not observed with electrons either.

CONCLUSIONS

The experimental spectra observed for high-energy protons and pions verify to a high degree of accuracy the collision-loss theories of Vavilov and Landau. By making reasonable assumptions about the interaction of the electron and background radiation in the detector, the electron spectra also agree within the experimental limits with the theoretical spectra. Similarly, the observed values for the most probable energy loss by protons and pions in silicon agree with the predicted values within an average experimental tolerance of about 1%. It appears that the discrepancy observed between the experimental and theoretical values for the most probable energy loss by electrons in silicon can be accounted for satisfactorily by taking into account the absorption of some of the electron-produced radiation by the detector. It is concluded that there is no evidence to support the Tsytovich radiative corrections to the collision-loss theories.

ACKNOWLEDGMENTS

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[27] A. Crispin and P. J. Hayman, Proc. Phys. Soc. (London) **83**, 1051 (1964).

[28] F. Ashton and D. A. Simpson, Phys. Letters **16**, 78 (1965).

[29] G. B. Zhdanov, M. I. Tretyakova, V. N. Tsytovich, and M. V. Sherbakova, Zh. Eksperim. i Teor. Fiz. **43**, 342 (1962) [English transl.: Soviet Phys.—JETP **16**, 245 (1963)].

[30] K. I. Alekseeva, G. B. Zhdanov, M. I. Tretyakova, and M. V. Sherbakova, Zh. Eksperim. i Teor. Fiz. **44**, 1864 (1963) [English transl.: Soviet Phys.—JETP **17**, 1254 (1963)].