Phase-Shift Analysis of High-Precision (*p*,*p*) Differential Cross-Section Data at 6.141, 8.097, and 9.918 MeV†

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Recent high-precision (*p*,*p*) differential cross-section data at 6.141, 8.097, and 9.918 MeV from Berkeley have been phase-shift analyzed. The data at 9.918 MeV are not consistent with the data at 6.141 and 8.097 MeV, and they are also inconsistent with other (*p*,*p*) data at nearby energies.

WE have recently published a phase-shift analysis of the (*p*,*p*) data below 400 MeV.[1] With the aid of this analysis, we were able to show[2] that the recent high-precision 19.7-MeV (*p*,*p*) polarization data from Berkeley[3] are not compatible with other (*p*,*p*) data at neighboring energies. A subsequent experimental measurement has borne out our conclusion.[4]

Recently we have completed a reanalysis of the (*p*,*p*) and (*n*,*p*) data from 1 to 450 MeV.[5] The (*p*,*p*) analysis differs from our previous analysis[1] at low energies mainly in the addition of vacuum polarization effects to the $^1S_0$ phase[6] and the addition of (small) magnetic moment effects to the one-pion-exchange phases. With this handling of the vacuum polarization effects in the $^1S_0$ phase,[6] which is combined with the conventional procedures for handling vacuum polarization effects in the higher phases,[7] we now have an energy-dependent continuous parametrization that gives a precision fit to the (*p*,*p*) data from 1.397 [7] to 450 MeV. With this new (*p*,*p*) solution in mind, we would like to present here our analysis of new Berkeley (*p*,*p*) differential cross-section data which have recently become available.[8]

(*p*,*p*) differential cross-section data at 6.141, 8.097, and 9.918 MeV have been measured at the Berkeley 88-in. cyclotron. Vacuum polarization corrections to the data for $l \geq 1$ phases were supplied to us by Slobodrian.[9] These corrections were applied to the data in the same manner as for the lower-energy Wisconsin data.[7] The new Berkeley data are quoted to an absolute precision of about 0.5%. In our analysis of these data,[5] we arbitrarily added an over-all normalization error of 0.1% to the data to aid us in interpreting the results.

The Berkeley experimenters used two different methods to arrive at the data normalization: a discrimination method (denoted D) and a background subtraction method (BGS). The BGS method gave cross-section values that are about 0.5% lower than the D method. The first task for the phase-shift analysis was to select between these two choices for the data. To do this we added both the BGS and the D data sets at 6.141, 8.097, and 9.918 MeV to our (*p*,*p*) data collection from 1 to 450 MeV, which now includes just over 1000 other (*p*,*p*) data.[5] This gave the results shown in Table I. From Table I it is clear that the BGS data are preferred over the D data on the basis of compatibility with the other (*p*,*p*) data. Also, the normalization uncertainty we added (0.1%) has little effect on the BGS data.

There is one disturbing note in Table I. The data at 9.918 MeV have a much larger χ^2 sum than the other data, even for the BGS set. When we deleted the D data and repeated the analysis, we found that the 9.918-MeV BGS data appear tipped with respect to the other differential cross-section data. The small-angle experimental data appear to be too high in value, and the large-angle data appear to be too low. In this analysis (with the D data removed), the χ^2 sum for the 9.918-MeV data was 40.6. The data at scattering angles less than 30° c.m. are on the average 1.3 standard deviations above the theoretical curve, and the data at scattering angles larger than 30° c.m. are on the average 1.2 standard deviations below the theoretical curve. The probability of this being a statistical fluctuation is vanishingly small.

TABLE I. Comparison of the BGS and D data sets (Ref. 8) on the basis of the 1–450-MeV (*p*,*p*) phase-shift analysis (Ref. 5). Vacuum polarization corrections have been applied to the data (Ref. 9). The least-squares sum χ^2 and the (theoretical) renormalization obtained in the solution fit are given for each set.

Energy (MeV)	No. of data	BGS		D	
		χ^2	Norm.	χ^2	Norm.
6.141	17	18.7	1.002	45.2	1.003
8.097	16	14.8	1.001	45.0	1.003
9.918	17	44.7	0.999	52.0	1.001

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To sharpen this conclusion, we made two phase-shift searches. In the first we analyzed our standard (p,p) 1–450-MeV data set,[5] with the BGS data included but not the D data. In the second, we used the same data selection as in the first, but now we decreased the experimental uncertainties in the BGS 9.918-MeV data by a factor of 10. Thus the second solution features a forced fit to the BGS 9.918-MeV data. A comparison of these two solutions then shows which data are in disagreement with the 9.918-MeV BGS data. This comparison is given in Table II. Some of the (p,p) data (usually because they have large experimental errors) gave essentially the same χ^2 values for both the free and the forced solutions. In Table II we include only the data that showed a marked sensitivity to the forced changing of the phases. From the comparisons in Table II, we can see that the BGS 9.918-MeV data are in direct conflict with the Wisconsin differential cross-section data at 2.425 and 3.037 MeV[7] (at lower energies the effective range extrapolation evidently prevents much of a readjustment in the phases), with the Minnesota differential cross-section data at 25.63[10] and 39.4[11] MeV, and with the Harwell differential cross-section data at 49.4 MeV.[12] The disagreement with the other data shown is statistically significant, including a disagreement with the 6.141- and 8.097-MeV BGS data.[8]

If the 9.918-MeV BGS data[8] are incorrect, then a question arises as to the accuracy of the 6.141- and 8.097-MeV BGS data,[8] which were measured in the same experiment. These data do not show the same questionable shape that is seen in the 9.918-MeV data. The M values (χ^2 average per datum point) for these data are 1.68 and 1.32 at 6.141 and 8.097 MeV, respectively. While these are a little higher than the expected values of 1, they are within the range that we consider as acceptable in our analysis[1] ($M \leq 2$ for a set of data). Thus we have no strong evidence from our analysis for the existence of systematic errors in these data.

TABLE II. Comparison of fits to (p,p) data for the normal phase-shift solution and for a solution that is forced to fit the 9.918-MeV BGS data (Ref. 8). The data are essentially as described in Ref. 1.

Energy (MeV)	Data	χ^2 sum Free	χ^2 sum Forced
1.397	11 σ	9.6	9.5
1.855	13 σ	17.0	17.2
2.425	14 σ	3.1	30.5
3.037	13 σ	11.1	50.1
6.141	17 σ	28.5	45.8
8.097	16 σ	21.2	30.3
9.68	1 σ	1.3	4.4
9.69	26 σ	20.4	27.2
9.918	17 σ	40.6	1167.5[a]
25.63	23 σ	12.4	51.0
27.6	3 A	4.5	11.1
27.6	2 R	0.9	1.6
39.4	27 σ	29.6	131.4
47.5	5 A	8.3	26.1
49.4	28 σ	33.1	105.7
52.34	26 σ	20.9	34.4
68.3	26 σ	32.6	56.6

[a] Errors decreased by a factor of 10.

We should note here that after the conclusion of the present analysis, we were informed by Noyes[13] that on the basis of his analysis of low-energy (p,p) data, which features a potential-model approach to deduce the splitting of the P phases, he has also concluded that the 9.918-MeV (p,p) data[8] must be in error.

The inconsistency of the 6.141- and 8.097-MeV data with the 9.918-MeV data can also be noted by a direct comparison of the phase shifts obtained by fitting to these data with the phase shifts of Ref. 5. Slobodrian, Conzett, Shield, and Tivol[8] (SCST) obtained several phase-shift fits to the data. The fits they give in Tables IV and VI of their paper are physically unrealistic. However, the fits in Tables III and V do correspond to the accepted form of the phase-shift solution at low energies. The phase-shift analysis of SCST was carried out by using only S, P, and D phases. While this is not really correct, the F and higher phases are small enough that the approximation is not a bad one. In Table III, we list the phase-shift solution for the S, P,

TABLE III. Comparison of MacGregor-Arndt-Wright (MAW) phases (Ref. 5) with SCST phases (Ref. 8) for S, P, and D waves.

Analysis	Energy (MeV)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$
MAW	6.141	55.26±0.02	2.22±0.02	−1.34±0.01	0.35±0.01	0.08±0.00
	8.097	55.42±0.03	3.02±0.03	−1.81±0.01	0.54±0.01	0.13±0.00
	9.918	55.05±0.04	3.74±0.04	−2.24±0.01	0.72±0.02	0.18±0.00
SCST (Table III)	6.141	55.54	2.36	−1.24	0.20	0.12
	8.097	55.63	3.16	−1.64	0.28	0.12
	9.918	54.78	3.88	−2.12	0.28	0.11
SCST (Table V)	6.141	55.67	2.32	−1.28	0.16	0.08
	8.097	55.91	3.14	−1.67	0.26	0.07
	9.918	55.09	3.83	−2.17	0.23	0.01

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and D phases of our own work (Ref. 5), together with the SCST solutions of Tables III and V as listed in Ref. 8. The shapes and absolute values of the $^1S_0$, $^3P_0$, and $^3P_1$ phases are in good agreement for all of these solutions. However, the SCST solutions show strongly anomalous behavior for the $^3P_2$ and $^1D_2$ phases. From the behavior of phase shifts near threshold, we know that the $^3P_2$ and $^1D_2$ phases must be *monotonically increasing* in value with energy. This is the behavior exhibited by the energy-dependent solution of Ref. 5. The smaller values for $^3P_2$ obtained by SCST are partly a consequence of the fact that they neglected F phases and higher. However, the *decrease* in $^3P_2$ (and also in $^1D_2$) for the SCST solutions in going from 8.097 to 9.918 MeV is completely outside any variations allowed by experimental errors. The errors on the phases of Ref. 5, which are obtained from the parameter error matrix, are really lower limits, and the "true" errors are probably 3 to 5 times as large as the quoted values. But even with this allowance, the *form of the energy dependence* of the phase shifts is clearly established by the work of Ref. 5. Thus the behavior of the SCST phase shifts illustrates qualitatively the discrepancy in differential cross-section shapes at 6.141 and 8.097 as against 9.918 MeV. Our phase-shift studies as described in the present text and illustrated in Table II give a more quantitative statement of the discrepancy. This kind of consistency test is, in our opinion, an excellent illustration of why people are led in the first place to carry out energy-dependent analyses of all of the data in a self-consistent manner.

With regard to our recent analysis of the Berkeley 19.7-MeV (p,p) polarization data,[2] we have carried out one additional test. At an IPPS meeting in England several months ago, McKee gave a talk[14] in which he showed that a number of isolated (p,p) polarization data in the range from roughly 30 to 50 MeV fall well below the values predicted by our phase-shift analysis. The conclusion from his analysis was that perhaps if our energy-dependent (p,p) phase-shift solution were altered to accommodate these data, then possibly it still could be reconciled with the Berkeley polarization data at 19.7 MeV.[3] To test this hypothesis, we forced a solution to behave as prescribed by McKee, by giving very small errors to single polarization points at 30[15] and 46 MeV.[16] These data, which are described in Table I of Ref. 1, have very large experimental errors and were not well fitted in our (p,p) analysis. When we forced a fit to these two data, we indeed found a predicted polarization that went slightly negative at 20 MeV, although not to the large negative values predicted by the Berkeley measurements.[3] However, in the process of making this forced fit to the polarization data, we completely destroyed the fit to other nearby data. For example, we found the following χ^2 increases: A at 27.6 MeV, 4.5 to 37.2; σ at 39.4 MeV, 29.6 to 115.0; σ at 49.4 MeV, 33.1 to 86.0; and A at 47.5 MeV, 8.3 to 99.8. Thus from a statistical point of view, this forced solution is completely unacceptable. Thus we remain with our former conclusion[2] that the Berkeley (p,p) polarization data at 19.7 MeV[3] are incompatible with other nearby (p,p) data.

The (p,p) data are now complete enough, and the analyses comprehensive enough, that the energy-dependent phases should be taken seriously. In our latest (p,p) analysis,[5] we use energy-dependent forms that with 25 elastic phase-shift parameters plus one inelastic parameter give a precision fit (M value of 1.046) to 1076 (p,p) data between 1.397 and 450 MeV. Such a large body of (self-consistent) experimental data puts very severe restrictions on the phase shifts, and the predictive powers of such a solution should be taken seriously. The fact that the Yale group under Breit, using an energy parametrization completely different from ours, arrive at substantially the same phases[17] indicates that the (p,p) data now point to a unique, well-defined solution for any reasonable parametrization of the phases.

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