the particular fit). The integrals of Eq. (54) have also been numerically calculated and we get

$$\mu'^S = -0.16\,, \quad \mu'^V = 0.95\,. \tag{63}$$

All the above results are summarized in Table I, where we also report the numerical values of the quantities $I_e{}^S$, $I_e{}^V$, $I_m{}^S$, and $I_m{}^V$ of Eqs. (55) and (56).

We note that Models I and II give values of the integrals and of the anomalous moments which are rather close to each other. Model III uses an empirical fit and the problem arises whether the MacDowell symmetry is satisfied at least approximately by such fits. The prediction for μ'^S can be considered satisfactory in all three models, in view of the inaccuracies and approximations introduced. Presumably the small discrepancy comes from errors in the high-energy tail of the photoproduction amplitudes, and it should be possible to obtain better estimates by further refinements. Our general impression is that a better estimate (for μ'^S and especially for μ'^V) requires a better knowledge of photoproduction in the intermediate-energy region (beyond the region of the static model). On the other hand, much work is being devoted at this time in various laboratories to a better understanding of these amplitudes, and we hope that it will soon be possible substantially to improve the results obtained here.

PHYSICAL REVIEW VOLUME 179, NUMBER 5 25 MARCH 1969

Signature of Regge Cuts Coupled to Spinning Particles

DAVID BRANSON

Department of Mathematics, University of Essex, Wivenhoe Park, Colchester, Essex, England

(Received 3 July 1968; revised manuscript received 6 December 1968)

It is proved, by perturbation theory and by the method of Sudakov, that if two Regge poles with trajectories α_1, α_2 and signatures τ_1, τ_2 are exchanged, the resulting Regge branch point at $j=\alpha_1+\alpha_2-1$ appears only in partial-wave amplitudes of signature $\tau_1\tau_2\eta$, where $\eta=-1$ if both Regge poles are fermions and $\eta=+1$ otherwise. An example is given from the case of proton-proton scattering.

I. INTRODUCTION

IN situations where it is impossible to fit experimental data by assuming Regge-pole dominance, it is useful to investigate whether the discrepancy can be accounted for by contributions from branch cuts in the complex angular momentum plane. For this reason (as well as simply to satisfy one's theoretical curiosity), it is important to know which amplitudes receive contributions from particular Regge cuts.

A Regge pole has associated with it definite quantum numbers (e.g., isospin, G parity, parity, signature) and will affect only those amplitudes with an identical set of quantum numbers. It can be shown[1–5] that the exchange of two Regge poles with trajectories α_1 and α_2 will give rise to branch points in the complex j plane; of these branch points, the one lying furthest to the right has a trajectory

$$j=\alpha_1+\alpha_2-1\,, \tag{1}$$

where the arguments of α_1 and α_2 are given by definite rules.[2–4] In order to discover which amplitudes possess such a branch point, we need to know the quantum numbers associated with a two-Reggeon system.

It is clear that internal quantum numbers, such as isospin and G parity, will combine in exactly the same way as if the Reggeons were elementary particles; for example, the exchange of two Pomeranchukons will give a cut in an amplitude with $I=0$ and $G=1$, whereas the exchange of a Pomeranchukon and a pion Regge pole will give a cut in an amplitude with $I=1$ and $G=-1$. Gribov[6] pointed out that one would expect any particular Regge cut to appear in amplitudes of both parities because of the arbitrary orbital angular momentum associated with the two-Reggeon system. There remains the important question of signature.

Mandelstam[7] proved that a partial-wave amplitude involving a state of two elementary particles of spins σ_1 and σ_2 has a singularity in the j plane at

$$j=\sigma_1+\sigma_2-1\,, \tag{2}$$

provided this is a wrong-signature point. For positive-signature amplitudes, the wrong-signature points are the odd integers (or odd integers plus one-half in the case of boson-fermion amplitudes); for negative-signature amplitudes, the wrong-signature points are the even integers (or even integers plus one-half). Thus, if we put $\nu=\frac{1}{2}$ for a boson-fermion amplitude and $\nu=0$ otherwise, we see that the singularity at $j=\sigma_1+\sigma_2-1$

[1] D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento **26**, 896 (1962).
[2] S. Mandelstam, Nuovo Cimento **30**, 1127 (1963); **30**, 1148 (1963).
[3] C. Wilkin, Nuovo Cimento **31**, 377 (1964).
[4] V. N. Gribov, I. Ya. Pomeranchuk, and K. A. Ter-Martirosyan, Phys. Rev. **139**, B184 (1965).
[5] J. C. Polkinghorne, J. Math. Phys. **4**, 1396 (1963).
[6] V. N. Gribov, Yadern. Fiz. **5**, 197 (1967) [English transl.: Soviet J. Nucl. Phys. **5**, 138 (1967)].
[7] S. Mandelstam, Nuovo Cimento **30**, 1113 (1963).

appears in partial-wave amplitudes of signature

$$(-1)^{\sigma_1+\sigma_2-\nu}. \quad (3)$$

If we put ν_1 equal to $0(\frac{1}{2})$ if the particle of spin σ_1 is a boson (fermion), and similarly for ν_2, we can write (3) as

$$(-1)^{\sigma_1-\nu_1}(-1)^{\sigma_2-\nu_2}(-1)^{\nu_1+\nu_2-\nu}. \quad (4)$$

If one considers Regge cuts as the generalization of these singularities arising from the promotion of the elementary particles into Reggeons[4] with trajectories α_1 and α_2, the obvious generalization of (2) and (4) is a cut in the j plane at $j=\alpha_1+\alpha_2-1$ in partial-wave amplitudes of signature

$$\tau_1\tau_2\eta. \quad (5)$$

In this expression, τ_1 and τ_2 are the signatures of the Regge poles (equal to $+1$ for a positive-signature pole and -1 for a negative-signature pole), and $\eta=-1$ if both poles are fermions, and $\eta=+1$ otherwise.

There has been a good deal of confusion in the literature regarding the signature of Regge cuts, but it is now well established that for amplitudes which involve the exchange of boson Regge poles and which have spinless external particles, the cuts have signature given by (5). This has been shown by Polkinghorne[8] using a perturbation theory method, and by Gribov[9] using a method devised by Sudakov.[10] However, it has been shown[11] that if the external particles have spin, then the Amati-Fubini-Stanghellini procedure[1] gives a Regge cut which appears in amplitudes of both signatures. This implies that such cuts with both signatures exist on an unphysical sheet, and it was conjectured that they would be present also on the physical sheet. This paper shows the conjecture to be false. Even when the external particles have spin, the signature of Regge cuts is given by (5).

In Sec. II, we construct helicity amplitudes that have a definite relation between the signature of j-plane singularities and behavior under crossing. We use these amplitudes in Secs. III and IV to prove our result by two different methods: In Sec. III we use perturbation-theory techniques, and in Sec. IV we employ Gribov's adaptation of Sudakov's method. In Sec. V we take, as an example, the particular proton-proton amplitude that was shown in Ref. 11 to have, on an unphysical sheet, a cut in an amplitude with signature opposite to that given by (5). We show that this cut is not present on the physical sheet.

[8] J. C. Polkinghorne, Nuovo Cimento **56A**, 755 (1968).
[9] V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **53**, 654 (1967) [English transl.: Soviet Phys.—JETP **26**, 414 (1968)].
[10] V. V. Sudakov, Zh. Eksperim. i Teor. Fiz. **30**, 87 (1956) [English transl.: Soviet Phys.—JETP **3**, 65 (1956)].
[11] D. Branson, S. Nussinov, S. B. Treiman, and W. I. Weisberger, Phys. Letters **25B**, 141 (1967).

II. CONSTRUCTION OF HELICITY AMPLITUDES WITH DESIRED PROPERTIES

In terms of the Mandelstam variables s and t, we shall let s be the asymptotic variable, and describe the t-channel reaction as

$$a+b\rightarrow c+d.$$

We use the labels a, b, c, and d to describe both the particles and their helicities. Using the method of Jacob and Wick[12] we decompose the t-channel amplitude into partial waves,

$$f_{cd,ab}(z)=\sum (2J+1)F^J{}_{cd,ab}d^J{}_{\lambda\mu}(z)\,, \quad (6)$$

where $\lambda=a-b$, $\mu=c-d$, and z is the cosine of the scattering angle. (We do not make explicit the dependence on the variable t.)

The d^J functions introduce kinematic singularities in s which we remove in the usual way by defining

$$\bar{f}_{cd,ab}=[\tfrac{1}{2}(1+z)]^{-|\lambda+\mu|/2}[\tfrac{1}{2}(1-z)]^{-|\lambda-\mu|/2}f_{cd,ab}. \quad (7)$$

Since the function $\bar{f}_{cd,ab}$ is free from s kinematic singularities, we may assume it obeys a fixed t-dispersion relation, and Reggeize as described in Refs. 13 and 14. We are thus led to partial-wave helicity amplitudes $F^{\pm}{}_{cd,ab}(t,j)$ for whose precise definition we refer the reader to Ref. 14. The positive-signature amplitude $F^{+}{}_{cd,ab}(j)$ coincides with the physical amplitude $F^J{}_{cd,ab}$ when j is equal to an even integer (an even integer $+\frac{1}{2}$ when $\nu=\frac{1}{2}$), and the negative-signature amplitude $F^{-}{}_{cd,ab}(j)$ coincides with the physical amplitude when j is equal to an odd integer plus ν.

Using (6), (7), and the relation[15]

$$d_{\lambda\mu}(j,z)=(-1)^{j-\lambda}d_{\lambda-\mu}(j,-z)\,,\ \text{for } j=\lambda_{\max},\lambda_{\max}+1,\cdots$$
$$[\lambda_{\max}=\max(|\lambda|,|\mu|),\quad \lambda_{\min}=\min(|\lambda|,|\mu|)],$$

we can therefore write

$$\bar{f}_{cd,ab}=\sum_{j=\lambda_{\max}}^{\infty}(2j+1)[F^{+}{}_{cd,ab}(j)\bar{d}^{+}{}_{\lambda\mu}(j,z)+F^{-}{}_{cd,ab}(j)\bar{d}^{-}{}_{\lambda\mu}(j,z)], \quad (8)$$

where

$$\bar{d}^{\pm}{}_{\lambda\mu}(j,z)=\tfrac{1}{2}[\tfrac{1}{2}(1+z)]^{-|\lambda+\mu|/2}[\tfrac{1}{2}(1-z)]^{-|\lambda-\mu|/2}\times[d_{\lambda\mu}(j,z)\pm(-1)^{\lambda-\nu}d_{\lambda-\mu}(j,-z)]. \quad (9)$$

We may continue the rotation matrices to complex values of j by expressing them in terms of hypergeometric functions[14,15] and perform a Sommerfeld-Watson transformation on (8). From the known

[12] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) **7**, 404 (1954). The d^J function that we use differs from that of Jacob and Wick by a factor $(-1)^{\lambda-\mu}$.
[13] F. Calogero, J. M. Charap, and E. J. Squires, Ann. Phys. (N.Y.) **25**, 325 (1963).
[14] W. Drechsler, Nuovo Cimento **53A**, 115 (1968).
[15] M. Andrews and J. Gunson, J. Math. Phys. **5**, 1391 (1964).

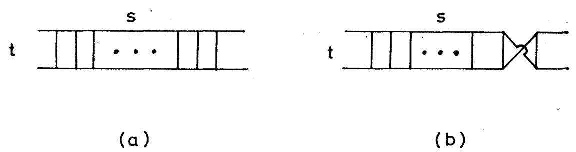

FIG. 1. Ladder diagrams whose sum gives asymptotic behavior (a) s^α and (b) u^α.

asymptotic behavior of the hypergeometric functions[16] for large values of z, we may show that

$$[\tfrac{1}{2}(1+z)]^{-|\lambda+\mu|/2}[\tfrac{1}{2}(1-z)]^{-|\lambda-\mu|/2}d_{\lambda\mu}(j,z)$$
$$\sim\tfrac{1}{2}\,\mathrm{sgn}(\lambda,\mu)\Gamma(2j+1)[\Gamma(j+\lambda_{\max}+1)\Gamma(j-\lambda_{\max}+1)$$
$$\times\Gamma(j+\lambda_{\min}+1)\Gamma(j-\lambda_{\min}+1)]^{-1/2}\times(\tfrac{1}{2}z)^{j-\lambda_{\max}},$$

where

$$\mathrm{sgn}(\lambda,\mu)=1, \quad \text{if } \lambda-\mu\geqslant 0$$
$$=(-1)^{\lambda-\mu}, \quad \text{if } \lambda-\mu<0.$$

It is simple to derive the relation

$$\mathrm{sgn}(\lambda,-\mu)=(-1)^{\lambda_{\max}-\lambda}\,\mathrm{sgn}(\lambda,\mu),$$

and hence, from (9) we have

$$\bar{d}^{\pm}{}_{\lambda\mu}(j,z)\sim\text{const}$$
$$\times[(z)^{j-\lambda_{\max}}\pm(-1)^{\lambda_{\max}-\nu}(-z)^{j-\lambda_{\max}}]. \quad (10)$$

If $F^{\pm}{}_{cd,ab}(j)$ has a singularity at $j=\alpha$, we see from (8) and (10) that $\bar{f}_{cd,ab}$ has a term asymptotically proportional to

$$(z)^{\alpha-\lambda_{\max}}\pm(-1)^{\lambda_{\max}-\nu}(-z)^{\alpha-\lambda_{\max}}.$$

Therefore, such a singularity will give to the function

$$g_{cd,ab}(z)=z^{\lambda_{\max}-\nu}\bar{f}_{cd,ab}(z) \quad (11)$$

an asymptotic behavior

$$z^{\alpha-\nu}\pm(-z)^{\alpha-\nu}. \quad (12)$$

[We note that $\lambda_{\max}-\nu$ is a non-negative integer, so that the factor $z^{\lambda_{\max}-\nu}$ in (11) does not introduce any singularities in s.] Thus, we see that a singularity in a partial-wave amplitude of signature plus (minus) one gives to the function $g_{cd,ab}$ a behavior which is asymptotically even (odd) under the transformation $z\leftrightarrow -z$ (i.e., under $s\leftrightarrow u$).

If we exchange in the t channel two Regge poles, the product of whose signatures is ±1, the result we wish to prove is that the branch point at $j=\alpha_1+\alpha_2-1$ appears only in partial-wave amplitudes of signature $\pm\eta$. From the preceding paragraph, we see that this is equivalent to proving that if

$$g_{cd,ab}\sim As^{\alpha_1+\alpha_2-1-\nu}, \text{ for large positive } s,$$

then

$$g_{cd,ab}\sim \pm\eta Au^{\alpha_1+\alpha_2-1-\nu}, \text{ for large positive } u.$$

[16] Bateman Manuscript Project, *Higher Transcendental Functions*, edited by A. Erdélyi (McGraw-Hill Book Co., New York, 1953), Vol. I, Chap. 2.

III. PERTURBATION-THEORY METHOD

We begin this section by briefly setting down some of the relevant results from the high-energy behavior of the perturbation theory of a neutral scalar particle.

The sum of all ladder diagrams of the form of Fig. 1(a) has, for large s, the behavior $\kappa s^{\alpha(t)}$, where α is a known function.[17,18] It follows that the sum of all diagrams of the form of Fig. 1(b) has asymptotic behavior (for large u) $\kappa u^{\alpha(t)}$. Therefore, if we take all diagrams like Fig. 1(a) and add to them all diagrams like Fig. 1(b), we obtain a combined asymptotic behavior that is *even* under the interchange $s\leftrightarrow u$; it follows simply from Sec. II that the sum of all these diagrams gives a model for a Regge pole of trajectory α and of *positive* signature.

We next review the properties of the diagram represented in Fig. 2 which is the simplest diagram which gives rise to a branch point in the angular-momentum plane.[2–5] This diagram gives a model for the exchange of a Regge pole (without definite signature) and a spinless elementary particle. Setting $\alpha_1=\alpha$, $\alpha_2=0$ in Eq. (1), we expect a branch point at $j=\alpha-1$. Although this situation is simpler than the exchange of two Regge poles, it reproduces all the essential features of the latter. We quote here the results obtained by Polkinghorne.[5,18] If we introduce the usual Feynman parameters (which we denote by β_i) and perform the integrations over the loop momenta, we obtain a function of the form

$$\frac{(-1)^n}{(16\pi^2)^{n+3}}g^{2n+8}(n+3)!\int\frac{d\beta_i\delta(1-\sum\beta_i)C_1{}^{n+2}(\beta_i)}{D_1{}^{n+4}(s,t,\beta_i)}, \quad (13)$$

where there are $(n+1)$ rungs in the ladder, g is the coupling constant, C_1 is a function just of the β_i, and D_1 is a function of the form

$$D_1(s,t,\beta_i)=f(\beta_i)s+g(\beta_i)t+d(\beta_i). \quad (14)$$

Polkinghorne shows that the behavior of this diagram

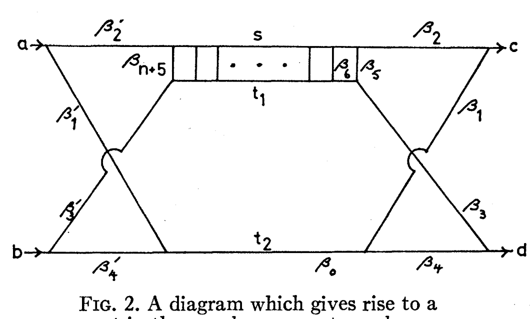

FIG. 2. A diagram which gives rise to a cut in the angular-momentum plane.

[17] J. C. Polkinghorne, J. Math. Phys. 4, 503 (1963).
[18] R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge University Press, New York, 1966), Chap. 3.

for large s is

$$\frac{(-1)^n i g^{2n+8}}{2(4\pi)^{2n+5}} \frac{(\ln s)^{n+2}}{(n+2)s^2} \int \frac{d\beta_i \delta C_1^{n+1}(\beta_i)}{D_1^{n+2}(s,t,\beta_i)}, \quad (15)$$

where

$$\delta = \delta(1-\sum \beta_i)\delta(\beta_1\beta_3-\beta_2\beta_4)\delta(\beta_1'\beta_3'-\beta_2'\beta_4') \times\delta(\beta_1)\delta(\beta_1')\delta(\beta_5)\delta(\beta_6)\cdots\delta(\beta_{n+5})\beta_2\beta_2'; \quad (16)$$

the parameters are labeled as in Fig. 2. The integrand is independent of s since the effect of δ is to set $f(\beta_i)$, the coefficient of s in D_1, equal to zero.

The expression (15) may be rewritten as

$$\frac{g^4}{(4\pi)^2 s^2} \int_{\lambda \leqslant 0} \frac{dt_1 dt_2}{\sqrt{\lambda}} \frac{\{[\alpha(t_1)+1]\ln s\}^{n+2}}{(n+2)!} \frac{1}{t_2-m^2}, \quad (17)$$

where t_1 and t_2 are the squares of the momenta carried, respectively, by the ladder and the scalar particle (of mass m), as shown in Fig. 2, and

$$\lambda = t^2+t_1^2+t_2^2-2tt_1-2tt_2-2t_1t_2.$$

The function $\alpha(t_1)$ is the same function as was mentioned at the beginning of this section, i.e., the trajectory function associated with the sum of ladder diagrams like Fig. 1(a). If we sum (17) over n (i.e., over the number of rungs in the ladder), we obtain

$$\frac{g^4}{(4\pi)^2} \int_{\lambda \leqslant 0} \frac{dt_1 dt_2}{\sqrt{\lambda}} \frac{s^{\alpha(t_1)-1}}{t_2-m^2},$$

behavior characteristic of a branch point at $j=\alpha-1$.

We now turn to the possibility of particles with nonzero spin. We shall suppose at first that the particles in the ladder and the particle with Feynman parameter β_0 are identical neutral scalars; the other particles are arbitrary.

Since we are interested in signature, we must consider not Fig. 2 alone, but the sum of Figs. 2 and 3. These two diagrams are identical except for the ladders, which are, respectively, the same as Figs. 1(a) and 1(b). From the remarks at the beginning of this section, it follows that the sum of these diagrams gives a model for the exchange of a *positive*-signature Regge pole and a spinless elementary particle (which in this context we consider to have positive signature). We now prove that such a combination produces a branch point at $j=\alpha-1$

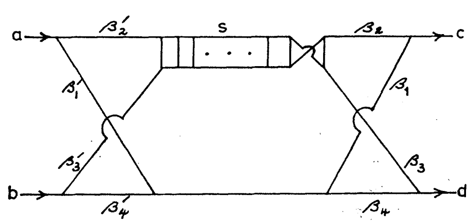

FIG. 3. A diagram which must be added to that of Fig. 2. in order to have a positive-signature Regge-pole insertion.

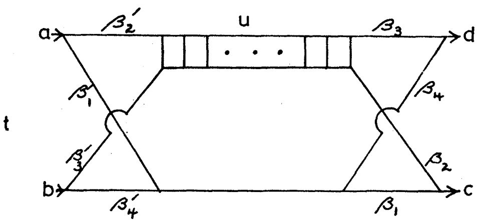

FIG. 4. Figure 3 redrawn with the external particles in a different configuration.

in *positive*-signature partial-wave amplitudes in agreement with (5) (since in this case $\eta=+1$).

From Sec. II and from (15), we see that we need to prove that, in the contribution of Figs. 2 and 3 to $g_{cd,ab}(z)$, the leading term (of order s^{-2}, apart from logarithms) is even under $s \leftrightarrow u$, since it is terms such as this that sum to give the behavior $s^{\alpha-1}$.

The presence of spin modifies the Feynman integral so that the contribution of Fig. 2 to $g_{cd,ab}$ acquires an extra factor, which we call N_1, in the numerator. The factor N_1 may depend on z, but, since we have removed all s kinematic singularities, it must be a polynomial in z. Thus, from (13), we see that the diagram of Fig. 2 contributes to $g_{cd,ab}$ the term (omitting all constant factors)

$$T_1(s,t) = \int \frac{d\beta_i \delta(1-\sum \beta_i) C_1^{n+2}(\beta_i) N_1(s,t,\beta_i)}{D_1^{n+4}(s,t,\beta_i)}, \quad (18)$$

whereas the contribution from Fig. 3 is

$$T_2(s,t) = \int \frac{d\beta_i \delta(1-\sum \beta_i) C_2^{n+2}(\beta_i) N_2(s,t,\beta_i)}{D_2^{n+4}(s,t,\beta_i)}. \quad (19)$$

Let us suppose for the moment that the numerator factors N are independent of s:

$$N_1(s,t,\beta_i) = n_1(t,\beta_i),$$
$$N_2(s,t,\beta_i) = n_2(t,\beta_i).$$

From (15) we see that the coefficient of the s^{-2} term in $T_1(s,t)$ is (omitting constant factors)

$$A = \int \frac{d\beta_i \delta C_1^{n+1} n_1}{D_1^{n+2}(s)}. \quad (20)$$

[In some cases δ may differ slightly from (16); for example, if $n_1\delta$ contains a factor $\beta_1\beta_1'$, the factor $\delta(\beta_1)\delta(\beta_1')$ in (16) is replaced by $C_1/\beta_1\beta_1'C_1'$, where C_1' is C_1 evaluated with $\beta_1=\beta_1'=0$. Such differences will not affect our subsequent argument.]

We may redraw Fig. 3 as in Fig. 4, when we see that this figure differs from Fig. 2 only in that we interchange c and d, β_1 and β_4, β_2 and β_3, and s and u. (We recall that c and d label particle types as well as helicities, and the β_i, as well as being Feynman parameters, also performs the function of labeling particle types. Thus, the line with parameter β_1 represents the same type of particle

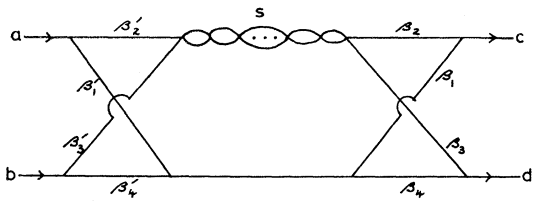

FIG. 5. The effect of setting equal to zero $\beta_5, \beta_6, \cdots, \beta_{n+5}$ in Figs. 2 and 3.

whichever figure it appears in.) It follows that we can find the behavior of $T_2(s,t)$ for large u in exactly the same way as led to (20). The coefficient of the u^{-2} term in $T_2(s,t)$ is [omitting the same constant factors as in (20)]

$$B=\int\frac{d\beta_i\delta' C_2{}^{n+1}n_2}{D_2{}^{n+2}(s)}, \tag{21}$$

where δ' is the result of transforming δ according to $\beta_1\leftrightarrow\beta_4$, and $\beta_2\leftrightarrow\beta_3$.

The factors $\delta(\beta_5)\delta(\beta_6)\cdots\delta(\beta_{n+5})$ in δ imply that we are to contract the corresponding lines to points. We see from Fig. 5 that the effect of this is to make Figs. 2 and 3 identical, which implies that we can set

$$\begin{aligned} C_1&=C_2,\\ D_1(s)&=D_2(s),\\ N_1(s)&=N_2(s), \end{aligned} \tag{22}$$

whenever these terms appear in an integrand multiplied by δ.

We have already remarked that δ sets the coefficient of s in D_1 equal to zero (which is equivalent to setting s itself equal to zero in D_1). We may easily check that under the integral $\delta'=\delta$. Combining these facts we have

$$B=\int\frac{d\beta_i\delta C_1{}^{n+1}n_1}{D_1{}^{n+2}(0)}=A.$$

That is,

$$T_1(s,t)\sim T_2(u,t).$$

Similarly, we may show

$$T_2(s,t)\sim T_1(u,t),$$

so that

$$T_1(s,t)+T_2(s,t)\sim T_1(u,t)+T_2(u,t). \tag{23}$$

We have therefore proved, as required, that the leading term in T_1+T_2 is even under $s\leftrightarrow u$.

We must now consider what happens if the numerator factor N_1 asymptotically depends on s. The function T_1 defined in (18) cannot have an asmyptotic power of s greater than -2 since, if it did, it would lead to a branch point to the right of $j=\alpha-1$. Therefore, if N_1 asymptotically contains a positive power of s, it must also contain a function of the β_i which depresses the asymptotic behavior by the same power. Essentially, the only function of the β_i which depresses the asymptotic power of s is $f(\beta_i)$ defined in (14). Suppose, for example,

$$N_1(s)=f(\beta_i)sn_1(t,\beta_i),$$

i.e.,

$$n_1=N_1(s)/[D_1(s)-D_1(0)]. \tag{24}$$

Then from (18),

$$T_1(s,t)=\int\frac{d\beta_i\delta(1-\sum\beta_i)C_1{}^{n+2}n_1}{D_1{}^{n+3}(s)}-\int\frac{d\beta_i\delta(1-\sum\beta_i)C_1{}^{n+2}n_1D_1(0)}{D_1{}^{n+4}(s)}. \tag{25}$$

We can also write

$$T_2(s,t)=\int\frac{d\beta_i\delta(1-\sum\beta_i)C_2{}^{n+2}n_2}{D_2{}^{n+3}(s)}-\int\frac{d\beta_i\delta(1-\sum\beta_i)C_2{}^{n+2}n_2D_2(0)}{D_2{}^{n+4}(s)}, \tag{26}$$

where

$$n_2=N_2(s)/[D_2(s)-D_2(0)]. \tag{27}$$

As before, we extract the coefficient of the leading terms by multiplying the integrands by δ. It might appear that when we multiply by δ and set $D(s)=D(0)$, the two terms in (25) cancel. However, the constant factors that we have omitted will be different for the two terms. The effect of contracting the rungs of the ladders is given by (22), and hence, by (24) and (27) we can set

$$n_1=n_2,$$

when these terms appear in an integrand multiplied by δ.

Therefore, we may treat the terms in (25) and (26) exactly as in the previous case, reproducing the result (23).

To generalize our result, we now let the exchanged particle with Feynman parameter β_0 (see Fig. 2) have spin σ. (We retain spinless particles in the ladder.) Thus, we are now exchanging a positive-signature Regge pole and a particle which we consider to have signature $(-1)^{\sigma-\nu}$. Setting $\alpha_1=\alpha$, and $\alpha_2=0$ in (1), we see that this combination produces a branch point at $j=\alpha+\sigma-1$. We now prove, in agreement with (5), that this branch point appears in partial-wave amplitudes of signature $(-1)^{\sigma-\nu}(\eta=+1$ in this case also); we, in fact, prove the equivalent result (see Sec. II) that $g_{cd,ab}$ has an asymptotic term of order $s^{\alpha+\sigma-1-\nu}$ which is even (odd) under $s\leftrightarrow u$ according as $(-1)^{\sigma-\nu}$ is positive (negative).

As the presence of the particle of spin σ enhances the asymptotic behavior by $s^{\sigma-\nu}$ relative to the case where all particles are spinless, it follows that the numerator functions N must asymptotically behave as $s^{\sigma-\nu}$ [or as a higher power of s together with compensating $f(\beta_i)$].

We recall that N must be a polynomial in s. So we put

$$N_1(s,t,\beta_i)\sim s^{\sigma-\nu}n_1(t,\beta_i).$$

We saw in (22) that the effect of δ is to set $N_1(s)=N_2(s)$, so that when the coefficient of the leading terms is extracted in the expressions corresponding to (20) and (21), we can set

$$N_2(s)=N_1(s)\sim s^{\sigma-\nu}n_1\sim(-1)^{\sigma-\nu}u^{\sigma-\nu}n_1.$$

Hence, the appropriate modifications of (20) and (21) are (taking into account also the other effects of δ)

$$A=s^{\sigma-\nu}\int\frac{d\beta_i\delta C_1^{n+1}n_1}{D_1^{n+2}(0)},$$

$$B=(-1)^{\sigma-\nu}u^{\sigma-\nu}\int\frac{d\beta_i\delta C_1^{n+1}n_1}{D_1^{n+2}(0)}.$$

It follows that

$$T_1(s,t)\sim(-1)^{\sigma-\nu}T_2(u,t).$$

Similarly,

$$T_2(s,t)\sim(-1)^{\sigma-\nu}T_1(u,t),$$

so we conclude that the contribution of Figs. 2 and 3 to $g_{cd,ab}$ gives an asymptotic term even (odd) under $s\leftrightarrow u$ according as $(-1)^{\sigma-\nu}$ is plus (minus) one, in agreement with (5).

IV. SUDAKOV METHOD

As in Sec. III, we begin by reviewing the spinless case, this time quoting the results of Gribov,[9] which we generalize to the unequal-mass case. We consider the diagram of Fig. 6 for large positive $s=(p_a-p_c)^2$ and fixed $t=(p_a+p_b)^2$. We label the momenta and masses as shown and set $p_a+p_b=q$, so that $t=q^2$. The wavy lines denote Reggeons with trajectories $\alpha_1(k^2)$ and $\alpha_2((q-k)^2)$.

We define p_a' and p_c' by

$$p_a'=p_a+m_a^2p_c/s,\qquad p_c'=p_c+m_c^2p_a/s. \tag{28}$$

The Sudakov method consists in resolving the momenta k, k_1, and k_2 within and perpendicular to the plane defined by p_a and p_c,

$$\begin{aligned}k&=-xp_c'+yp_a'+k_\perp,\\ k_1&=-x_1p_c'+y_1p_a'+k_{1\perp},\\ k_2&=-x_2p_c'+y_2p_a'+k_{2\perp}.\end{aligned} \tag{29}$$

If we likewise resolve q, then we can show that asymptotically

$$q\sim-[(t+m_c^2-m_d^2)p_a+(t+m_a^2-m_b^2)p_c]/s+q_\perp$$

and

$$q^2\sim q_\perp^2.$$

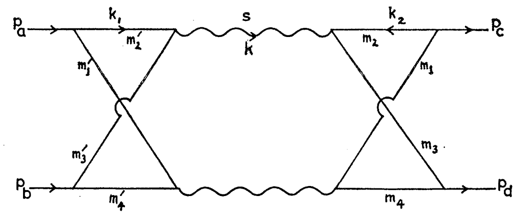

FIG. 6. The diagram under consideration. The wavy lines denote Reggeons.

Gribov assumes that the dominant contribution to the integral comes from that portion of the integration region where the Regge-pole energies $(k_1+k_2)^2$ and $(p_a-p_c-k_1-k_2)^2$ are large, of the order of s, and where the momentum transfers k^2 and $(q-k)^2$ and mass variables k_1^2, $(p_a-k_1)^2$, $(k-k_1)^2$, $(p_a-q+k-k_1)^2$, k_2^2, $(p_c+k_2)^2$, $(k+k_2)^2$, and $(p_c-q+k+k_2)^2$ are of order m^2, where m is a typical mass. In this region it is shown by Gribov that

$$\begin{aligned}&k_\perp^2\sim k_{1\perp}^2\sim k_{2\perp}^2\sim m^2,\\ &x\sim y\sim x_1\sim y_2\sim m^2/s,\\ &x_2\sim y_1\sim 1.\end{aligned} \tag{30}$$

It follows that in this region the Feynman denominators can be written in terms of the new variables as

$$(p_a-k_1)^2-m_1'^2\sim x_1(y_1-1)s+(1-y_1)m_a^2+k_{1\perp}^2-m_1'^2, \tag{31a}$$

$$k_1^2-m_2'^2\sim x_1y_1s+k_{1\perp}^2-m_2'^2, \tag{31b}$$

$$(k-k_1)^2-m_3'^2\sim(x_1-x)y_1s+(k_\perp-k_{1\perp})^2-m_3'^2, \tag{31c}$$

$$(p_a-q+k-k_1)^2-m_4'^2\sim(x-x_1)(1-y_1)s-(1-y_1)(q^2-m_b^2)+(q_\perp-k_\perp+k_{1\perp})^2-m_4'^2, \tag{31d}$$

$$(p_c+k_2)^2-m_1^2\sim(x_2-1)y_2s+(1-x_2)m_c^2+k_{2\perp}^2-m_1^2, \tag{31e}$$

$$k_2^2-m_2^2\sim x_2y_2s+k_{2\perp}^2-m_2^2, \tag{31f}$$

$$(k+k_2)^2-m_3^2\sim x_2(y+y_2)s+(k_\perp+k_{2\perp})^2-m_3^2, \tag{31g}$$

$$(p_c-q+k+k_2)^2-m_4^2\sim(x_2-1)(y+y_2)s-(1-x_2)(q^2-m_d^2)+(q_\perp-k_\perp-k_{2\perp})^2-m_4^2. \tag{31h}$$

The momentum transfers become

$$k^2\sim k_\perp^2,\qquad (q-k)^2\sim(q_\perp-k_\perp)^2,$$

and the Regge-pole terms

$$\begin{aligned}&g_1\xi_1(x_2y_1s)^{\alpha_1-\nu_1},\\ &g_2\xi_2[(1-x_2)(1-y_1)s]^{\alpha_2-\nu_2},\end{aligned} \tag{32}$$

where

$$\begin{aligned}\xi_1&=(e^{-i\pi(\alpha_1-\nu_1)}\pm1)/\sin\pi(\alpha_1-\nu_1),\\ \xi_2&=(e^{-i\pi(\alpha_2-\nu_2)}\pm1)/\sin\pi(\alpha_2-\nu_2).\end{aligned} \tag{33}$$

(Since we are here considering spinless particles and boson Regge poles, $\nu_1=\nu_2=0$, but we include ν_1 and ν_2

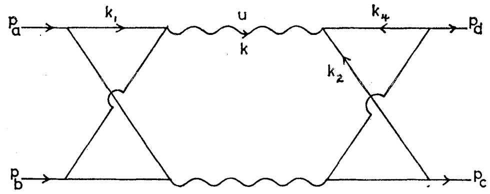

FIG. 7. A diagram identical to that of Fig. 6.

in these expressions as we shall need them later.) The residue functions g_1 and g_2 depend on the momentum-transfer and mass variables of the particles to which the Regge poles are coupled, and each pole has signature ± 1. The Jacobian of the transformation to the integration variables defined in (29) is asymptotically $\frac{1}{8}s^3$.

If we now let the particles carry spin, we must, as mentioned above, allow for the possibility of fermion Regge poles by allowing ν_1, $\nu_2=\frac{1}{2}$ (as well as the values ν_1, $\nu_2=0$ for boson Regge poles). As in Sec. III, the other effect of spin is to modify the contribution of Fig. 6 to $g_{cd,ab}$ by an extra factor N in the numerator. This factor will contain momentum terms which may invalidate[19] Gribov's assumption that the dominant contribution to the integral arises when the mass variables in the end crosses are of the order of m^2. The following argument is valid only when Gribov's assumption is valid.

We see from the preceding paragraphs that when we have spinning particles, we can write the asymptotic contribution of the diagram in Fig. 6 as (omitting constants)

$$\frac{i}{s}\int d^2k_\perp \xi_1\xi_2 s^{\alpha_1+\alpha_2-\nu_1-\nu_2}\int d^2k_{1\perp}d^2k_{2\perp}dxdydx_1dy_1dx_2dy_2 \times s^4 g_1 g_2 (x_2y_1)^{\alpha_1-\nu_1}[(1-x_2)(1-y_1)]^{\alpha_2-\nu_2}\frac{N}{D}, \quad (34)$$

where D is the product of the eight denominator terms (31). If N is a constant (as for example in the spinless case), the second integral in (34) is independent of s as, by (30), we can scale the variables x, y, x_1, and y_2 by s.

If we wish to evaluate the diagram of Fig. 6, not for large s, but for large u, we redraw Fig. 6 as in Fig. 7; then, it is clear that the procedure will be similar to that prescribed by Gribov, but we must interchange p_c and p_d, k_2 and $k_4=-(k+k_2)$, m_1 and m_4, m_2 and m_3. We therefore define

$$\bar{p}_a'=p_a+m_a^2p_d/u,$$
$$\bar{p}_d'=p_d+m_d^2p_a/u, \quad (35)$$

and resolve the momenta k, k_1, and k_4 in and perpendicular to the plane defined by p_a and p_d,

$$k=-\bar{x}\bar{p}_d'+\bar{y}\bar{p}_a'+\bar{k}_\perp,$$
$$k_1=-\bar{x}_1\bar{p}_d'+\bar{y}_1\bar{p}_a'+\bar{k}_{1\perp}, \quad (36)$$
$$k_4=-\bar{x}_2\bar{p}_d'+\bar{y}_2\bar{p}_a'+\bar{k}_{4\perp},$$

and, resolving q similarly,

$$q\sim[(t+m_d^2-m_c^2)p_a+(t+m_a^2-m_b^2)p_d]/u+\bar{q}_\perp$$

and

$$q^2\sim\bar{q}_\perp^2.$$

Therefore, the expression analogous to (34) when u is large will be

$$\frac{i}{u}\int d^2\bar{k}_\perp\bar{\xi}_1\bar{\xi}_2u^{\alpha_1+\alpha_2-\nu_1-\nu_2}\int d^2\bar{k}_{1\perp}d^2\bar{k}_{4\perp}d\bar{x}d\bar{y}d\bar{x}_1d\bar{y}_1d\bar{x}_2d\bar{y}_2 \times u^4\bar{g}_1\bar{g}_2(\bar{x}_2\bar{y}_1)^{\alpha_1-\nu_1}[(1-\bar{x}_2)(1-\bar{y}_1)]^{\alpha_2-\nu_2}\frac{\bar{N}}{\bar{D}}, \quad (37)$$

where

$$\bar{\xi}_1=[1\pm e^{-i\pi(\alpha_1-\nu_1)}]/\sin\pi(\alpha_1-\nu_1),$$
$$\bar{\xi}_2=[1\pm e^{-i\pi(\alpha_2-\nu_2)}]/\sin\pi(\alpha_2-\nu_2). \quad (38)$$

From the comparison of Figs. 6 and 7, and from Eqs. (28), (29), (35), and (36), it is clear that the factors in $\bar{D}$ are obtained from the right-hand sides of (31) by replacing x, y, x_1, y_1, x_2, y_2, $k_\perp$, $k_{1\perp}$, and $q_\perp$, by the corresponding barred quantities, by replacing $k_{2\perp}$ by $\bar{k}_{4\perp}$, by replacing s by u, and by exchanging m_c^2 with m_d^2, m_1^2 with m_4^2, and m_2^2 with m_3^2. The residue functions $\bar{g}_1$ and $\bar{g}_2$ are g_1 and g_2 with their arguments modified in the same way. The relation of $\bar{N}$ to N is discussed below.

If we eliminate k, k_1, and k_2 between (29) and (36), we obtain (retaining only the terms that are dominant at large s or large u)

$$xs\sim\bar{x}u,$$
$$ys\sim-\bar{y}u-2\bar{k}_\perp\cdot\bar{q}_\perp,$$
$$x_1s\sim\bar{x}_1u,$$
$$y_1\sim\bar{y}_1,$$
$$x_2\sim\bar{x}_2, \quad (39)$$
$$y_2s\sim\bar{y}_2u+\bar{y}u-\bar{x}_2q^2+2(\bar{k}_\perp+\bar{k}_{4\perp})\cdot\bar{q}_\perp,$$
$$k_\perp\sim\bar{k}_\perp,$$
$$k_{1\perp}\sim\bar{k}_{1\perp},$$
$$k_{2\perp}\sim-\bar{k}_\perp-\bar{k}_{4\perp}+\bar{x}_2\bar{q}_\perp.$$

[$k_\perp$ is a two-dimensional vector perpendicular to p_a and p_c; $\bar{k}_\perp$ is a two-dimensional vector perpendicular to p_a and p_d. In changing from s to u, we replace p_c by p_d. Therefore, $k_\perp$ in (34) and $\bar{k}_\perp$ in (37), in fact, span the same two-dimensional space, so that the last three equations of (39) are meaningful.] We can easily check that if we apply (39) to D, given by (31), we obtain $\bar{D}$. The numerator factor introduced by spin is first evalu-

[19] I am grateful to Professor S. B. Treiman for stressing this point.

ated in the t channel and then continued to large s to produce N and to large u to produce $\bar{N}$. Since we have removed all s kinematic singularities, and since the transformation (39) contains no s singularities, it follows that $\bar{N}$ is obtained from N simply by applying (39).

We remarked above that if N is a constant, the second integral in (34) is independent of s, so that (34) as a whole has in that case, a power of s equal to $\alpha_1+\alpha_2-1-\nu_1-\nu_2$. But we saw in Sec. II that a branch point at $j=\alpha_1+\alpha_2-1$ leads to an asymptotic behavior of $g_{cd,ab}$ of order $s^{\alpha_1+\alpha_2-1-\nu}$. It follows that N must contribute to the asymptotic behavior a factor $s^{\nu_1+\nu_2-\nu}$. [The factor N may contain further powers of s, but in order to prevent the appearance of a branch point to the right of $j=\alpha_1+\alpha_2-1$, we see from (30) that any such s must be accompanied by x, y, x_1, or y_2 and will disappear when we scale these variables.] Making this factor explicit, we may therefore write (34) as

$$\frac{i}{s}\int d^2k_\perp\xi_1\xi_2 s^{\alpha_1+\alpha_2-1-\nu_2}s^{\nu_1+\nu_2-\nu}I = i\int d^2k_\perp\xi_1\xi_2 s^{\alpha_1+\alpha_2-1-\nu}I\,, \quad (40)$$

where I is independent of s. We may similarly write (37) as

$$\frac{i}{u}\int d^2\bar{k}_\perp\bar{\xi}_1\bar{\xi}_2 u^{\alpha_1+\alpha_2-\nu_1-\nu_2}s^{\nu_1+\nu_2-\nu}\bar{I} \sim (-1)^{\nu_1+\nu_2-\nu}i\int d^2\bar{k}_\perp\bar{\xi}_1\bar{\xi}_2 u^{\alpha_1+\alpha_2-1-\nu}\bar{I}\,, \quad (41)$$

where $\bar{I}$ is also independent of s and is obtained from I by (39). Comparing (40) and (41), we see that they differ by the factor $(-1)^{\nu_1+\nu_2-\nu}=\eta$; they differ in that s and u are interchanged, $\xi_1\xi_2$ is replaced by $\bar{\xi}_1\bar{\xi}_2$, and otherwise the integrand of (41) is obtained from the integrand in (40) by the change of variable given by (39). This change of variable has no effect on the value of the integral as the Jacobian of the transformation is 1. From (33) and (38), we see that $\xi_1\xi_2$ and $\bar{\xi}_1\bar{\xi}_2$ differ by a factor ±1 according as the two poles have like (unlike) signature. Hence, if τ_1 and τ_2 are the signatures of the two poles, we conclude that the contribution of Fig. 6 to $g_{cd,ab}$ has a term of order $s^{\alpha_1+\alpha_2-1-\nu}$ whose coefficient varies under $s\leftrightarrow u$ crossing by a factor $\tau_1\tau_2\eta$. This is the result (5) that we wished to prove.

V. EXAMPLE

In this section, we show how the results of Secs. III and IV apply to the particular example that was shown in Ref. 11 to disobey the signature rule given by (5) in the Amati-Fubini-Stanghellini approximation.[1] We consider proton-proton scattering in the s channel, so that the t-channel states are proton-antiproton pairs. In this case, the charge conjugation operator is equivalent to -1 times the particle-exchange operator for identical particles, so that the t-channel partial-wave helicity state

$$(|J,M,\tfrac{1}{2},-\tfrac{1}{2}\rangle-|J,M,-\tfrac{1}{2},\tfrac{1}{2}\rangle)$$

is an eigenstate of C with eigenvalue[12] equal to $(-1)^{J+1}$. Throughout this section we restrict ourselves to the exchange of Regge poles (or elementary particles) each having $C=+1$. Thus we are restricted to $C=+1$ states, and it follows that the combination of partial-wave helicity states

$$H^J=F^J{}_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}}-F^J{}_{-\frac{1}{2}\frac{1}{2},\frac{1}{2}-\frac{1}{2}}$$

vanishes unless $(-1)^J=-1$; i.e., the positive-signature continuation of H^J is identically zero.

We now consider the following combination of t-channel helicity amplitudes,

$$\begin{aligned}h(z)&=f_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}}(z)+f_{-\frac{1}{2}\frac{1}{2},\frac{1}{2}-\frac{1}{2}}(z)\\&=\sum(2J+1)[F^J{}_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}}d^J{}_{11}(z)+F^J{}_{-\frac{1}{2}\frac{1}{2},\frac{1}{2}-\frac{1}{2}}d^J{}_{1-1}(z)].\end{aligned}$$

For large z, $d^J{}_{11}$ and $d^J{}_{1-1}$ both have leading terms of order z^J, with the coefficients of these terms differing only in sign. Therefore, for large z, $h(z)$ depends on the difference of the partial-wave amplitudes, i.e., it depends on H^J. Hence, if the analytic continuation of H^J contains a singularity at $j=\alpha$, then $h(z)$ will have a term in its asymptotic behavior of order z^α. Recalling that the positive-signature continuation of H^J is identically zero, we now demonstrate that in cases where (5) predicts a branch point at $j=\alpha$ in positive-signature amplitudes, the coefficient of the z^α term in $h(z)$ is zero. (Reference 11 pointed out that the coefficient is nonzero in the Amati-Fubini-Stanghellini approximation.)

A. Perturbation-Theory Method

Consider the diagram of Fig. 2 and let the particles associated with the Feynman parameters β_1, β_4, β_1', and β_4' be protons and all other internal particles be identical neutral scalars (with $C=+1$). Then the sum of Figs. 2 and 3 represents the exchange of a positive-signature Regge pole and a spinless particle. This combination, according to (5), produces a branch point at $j=\alpha-1$ in positive-signature amplitudes. We therefore do not expect a term of order $z^{\alpha-1}$ in $h(z)$.

If we label the momenta as shown in Fig. 6, the numerator introduced into the Feynman integral for Fig. 2 due to the presence of spins is

$$\begin{aligned}&\bar{u}(p_d)[\gamma\cdot(p_d-k-k_2)+M][\gamma\cdot(-p_c-k_2)+M]v(p_c)\\&\quad\times\bar{v}(p_b)[\gamma\cdot(-p_b+k-k_1)+M]\\&\quad\quad\times[\gamma\cdot(p_a-k_1)+M]u(p_a)\,,\end{aligned}$$

where u and v are proton and antiproton spinors, respectively, and M is the proton mass. If we evaluate the contribution of this to $h(z)$ in the t-channel center of mass, where the incident particles move along the z axis and the outgoing particles in the xz plane, we obtain

$$-8M^2[(\surd t)(k^y+2k_2^y)-k^0k_2^y+k^yk_2^0]\times[(\surd t)(k^y-2k_1^y)+k^0k_1^y-k^yk_1^0]. \quad (42)$$

The next step is to find the transformation needed to cast the denominator of the Feynman integral into a form suitable for symmetric integration,[20] to apply the same transformation to the numerator, and to perform the symmetric integration, leaving us with an integral over Feynman parameters. This is a very lengthy procedure, so we merely quote the result.

The coefficient of $(\ln s)^{n+2}/s^2$ (which gives rise to behavior $s^{\alpha-1}$ when we sum all such diagrams) coming from the contribution of Fig. 2 to $h(z)$ is by (20) proportional to

$$A=\int\frac{d\beta_i\delta C_1{}^{n+1}n_1}{D_1{}^{n+2}(0)},$$

where the effect of δ, defined by (16), is to set

$$C_1=[\beta_0(\beta_2+\beta_3)(\beta_2'+\beta_3')+\beta_2\beta_3(\beta_2'+\beta_3')+\beta_2'\beta_3'(\beta_2+\beta_3)]\phi_1+(\beta_2+\beta_3)(\beta_2'+\beta_3')\phi_2,$$
$$D_1(0)=t[\beta_0C_1-\beta_0^2(\beta_2+\beta_3)(\beta_2'+\beta_3')\phi_1]-\sum\beta_im_i^2C_1,$$
$$n_1=D_1(0)\phi_1(\beta_2-\beta_3)(\beta_2'-\beta_3')/C_1^2,$$

where ϕ_1 and ϕ_2 are functions of only those Feynman parameters associated with the sides of the ladder. If we make the following transformation of variables in the integral:

$$\beta_1\leftrightarrow\beta_4,\quad \beta_2\leftrightarrow\beta_3,$$

we see that

$$\delta\to\delta,\quad C_1\to C_1,\quad D_1(0)\to D_1(0),$$

but

$$n_1\to -n_1.$$

Therefore, $A=-A=0$.

[20] See Chap. 1 of Ref. 18.

B. Sudakov Method

We consider the same situation now for the diagram of Fig. 6. Let us exchange two positive-signature boson Regge poles with trajectories α_1 and α_2 (with $C=+1$) and, for simplicity, suppose that the protons have only a scalar coupling to the Regge pole. We take the particles in the end crosses to be the same as in the perturbation-theory discussion above. Then, again, the positive-signature continuation of H^J is identically zero, whereas (5) predicts a branch point at $j=\alpha_1+\alpha_2-1$ in positive-signature amplitudes. Therefore, we expect the coefficient of $s^{\alpha_1+\alpha_2-1}$ in $h(z)$ to be zero. The integral corresponding to the contribution of Fig. 6 to $h(z)$ will be, for large s, (34) with N equal to (42). If we extract the leading behavior of (42) by use of (29) and (30), we obtain

$$-8M^2\{(\surd t)[(1-\tfrac{1}{2}x_2)k_\perp{}^y+2k_{2\perp}{}^y]+k_\perp{}^yk_{2\perp}{}^0-k_\perp{}^0k_{2\perp}{}^y\}\times\{(\surd t)[(1-\tfrac{1}{2}y_1)k_\perp{}^y-2k_{1\perp}{}^y]+k_\perp{}^0k_{1\perp}{}^y-k_\perp{}^yk_{1\perp}{}^0\}. \quad (43)$$

With this expression for N, (34) is not obviously zero. However, let us apply the following change of variable [derived from (39)]:

$$x,\ x_1,\ y_1,\ x_2,\ k_\perp,\ k_{1\perp}\ \text{unchanged},$$
$$y\to -y-2k_\perp\cdot q_\perp/s,$$
$$y_2\to y_2+y-x_2q^2/s+2(k_\perp+k_{2\perp})\cdot q_\perp/s,$$
$$k_{2\perp}\to -k_\perp-k_{2\perp}+x_2q_\perp.$$

In (31) we now have $m_2=m_3$ and $m_1=m_4=m_c=m_d=M$, so we can easily check that this change of variable leaves (34) unchanged, apart from its effect on N. The second bracket in (43) is also unaltered, but the first bracket changes sign (since $q_\perp{}^y=0$, $q_\perp{}^0\sim\surd t$). Hence, the integral (34) is, in this case, zero, in agreement with (5).

ACKNOWLEDGMENTS

I am very grateful to Dr. I. T. Drummond, Professor J. C. Polkinghorne, and Professor S. B. Treiman for helpful discussions. Most of this work was done while I held a Science Research Council Research Fellowship at Cambridge University.