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Regge Trajectories of Mesons in the Quark Model*

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We examine a model in which mesons are produced as quasibound states in the quark-antiquark system by using the multichannel ND^{-1} equations. The ρ meson is considered as a particular example. The parameters of the model can be adjusted to produce the ρ meson with the experimentally observed mass and width. We continue the solution as a function of l to compute the Regge trajectory of the ρ. In contrast to attempts to compute the ρ trajectory by using a single-channel formalism in which the "force" producing the ρ is the exchange of the ρ meson, we are able to compute trajectories which are approximately linear and have $\alpha_\rho(0)\approx 0.5$. We present a reason for this difference and examine the model dependence of the results. The apparently equal slopes of the known Regge trajectories can be explained within this model if the particles on the trajectories are produced as quasibound states in the quark-antiquark system.

I. INTRODUCTION

THE quark model has been very successful in accounting for the $SU(3)$ multiplet structure of the mesons as well as the meson mass spectrum.[1] In this model the mesons are produced as bound states in the quark-antiquark system. We will study the consequences of this model for the ρ meson. The results can clearly be generalized to other mesons.

Many attempts have been made to compute the experimentally observed features of the ρ mesons by using the N/D equations.[2] The usual assumption is that the primary force which generates the ρ meson is due to the exchange of the ρ itself. By using this approach and a cutoff procedure, one is able to obtain self-consistent values for the mass and width of the ρ, but the results are in very poor agreement with experiment. Fulco, Shaw, and Wong[3] attempted to improve the results by using the multichannel ND^{-1} equations to include the $\pi\omega$ and $K\bar{K}$ channels. They found that to obtain the correct ρ mass meant that the ρ width must be five times the experimental value. The Frye-Warnock[4] equations were also used to include inelastic effects with the result that a ρ width could be produced which is about $2\frac{1}{2}$ times the experimental value for strong absorption.[5] Attempts were also made to compute the Regge trajectory of the ρ meson by solving the N/D equations for noninteger l.[5,6] The result is that $\alpha_\rho(0)\gtrsim 0.95$, in contrast to the experimental result $\alpha_\rho(0)\approx 0.5$.

The calculations presented here are done by using the two-channel ND^{-1} equations and assuming that the ρ meson is produced as a bound state of the higher-mass channel. The higher-mass channel is assumed to consist of two equal-mass scalar particles ϕ, and the force between these particles is assumed to be produced by the exchange of a scalar particle. We also considered forces due to the exchange of a vector particle and conclude that the results obtained are not dependent on the spin of the exchanged particle.

It is clear that one can obtain an arbitrarily narrow resonance in the $\pi\pi$ channel near the bound state in the $\phi\phi$ channel by choosing the interchannel coupling small enough.[7] Thus it is not surprising that we can compute a ρ mass and width in good agreement with experiment. We are more interested in computing the Regge trajectory of the ρ which one finds by using this model. The trajectory now is determined primarily by the forces in the $\phi\phi$ channel and we find that it is easy to produce almost linear trajectories with an $\alpha_\rho(0)$ much smaller than one can produce by the single-channel calculation using ρ exchange as the dominant force. The slope of the trajectory seems to be determined primarily by the ratio of the ϕ mass to the mass of the exchanged particle. As this ratio increases, $\alpha_\rho(0)$ decreases. The slope is insensitive to small variations ($\ll 4m_\phi^2$) in the bound-state mass. If the ϕ can be identified as a quark, the apparently equal slopes of the known trajectories can be explained by this model if the particles in the trajectories are quasibound states of the quark-antiquark system.

II. FORMALISM

We use the multichannel ND^{-1} equations

$$N_l(s)=B_l(s)+\frac{1}{\pi}\int_{\mathrm{P.C.}}\frac{ds'}{s'-s}\times\left[B_l(s')-\frac{s-s_1}{s'-s_1}B_l(s)\right]\rho_l(s')N_l(s')\,,\quad(1)$$

$$D_l(s)=1-\frac{s-s_1}{\pi}\int_{\mathrm{P.C.}}\rho_l(s')N_l(s')\times\frac{ds'}{(s'-s_1)(s'-s-i\epsilon)}\,,\quad(2)$$

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[1] See, for example, R. H. Dalitz, in *Proceedings of the Second Hawaii Topical Conference in Particle Physics* (University of Hawaii Press, Honolulu, 1968).

[2] F. Zachariasen, Phys. Rev. Letters **7**, 112 (1961); F. Zachariasen and C. Zemach, Phys. Rev. **128**, 1939 (1962).

[3] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).

[4] G. Frye and R. L. Warnock, Phys. Rev. **130**, 478 (1963).

[5] P. W. Coulter and G. L. Shaw, Phys. Rev. **138**, B1273 (1965).

[6] M. Bander and G. L. Shaw, Phys. Rev. **135**, B267 (1964).

[7] M. Bander, P. W. Coulter, and G. L. Shaw, Phys, Rev. Letters **14**, 270 (1965).

where $B(s)$ is the generalized potential and N, B, D, and ρ are 2×2 matrices in our problem. ρ is a diagonal matrix of the form $\rho_i(s)\theta(s-s_i)$, where $s_i=4m_i^2$ is the threshold for the ith channel. The integration is over the physical cut (P.C.).

For the $\pi\pi$ channel we make the usual assumption that B_{11} is due primarily to ρ exchange

$$B_{11}{}^l(s)=G_{11}\frac{2s+m_\rho{}^2-4}{(s-4)^{l+1}}Q_l\left(1+\frac{2m\rho^2}{s-4}\right)(\tfrac{1}{4}s)^{\alpha_\rho(0)-1},\quad (3)$$

where we set $m_\pi=1$ and m_ρ is mass of the ρ. We let the kinematical factor ρ_i have the form

$$\rho_i(s)=\left(\frac{s-s_i}{s}\right)^{1/2}(s-s_i)^l \quad (4)$$

for $l\leq 1$ in order to obtain the desired threshold behavior. The additional factor $(\frac{1}{4}s)^{\alpha_\rho(0)-1}$ is inserted to remove the high-energy divergence.[6] $G_{11}=0.9$ for a ρ width of 120 MeV.

If the force in the $\phi\phi$ channel is due to scalar exchange, then

$$B_{22}{}^l(s)=\frac{G_{22}}{(s-s_2)^{l+1}}Q_l\left(1+\frac{2m_2{}^2}{s-s_2}\right),\quad (5)$$

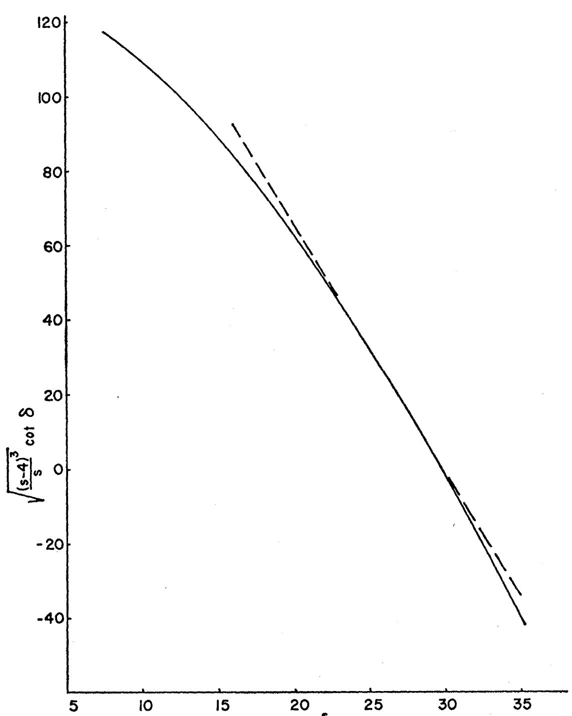

FIG. 1. $[(s-4)^3/s]^{1/2}\cot\delta$ as a function of s. The input parameters are $m_\rho=5.4$, $G_{11}=0.9$, $m_\phi=3.5$, $G_{22}=16\,200$, $m_2=7$, $m_1=5$, $G_{12}=775$. The dashed line is the result one would obtain if the phase is pure Breit-Wigner.

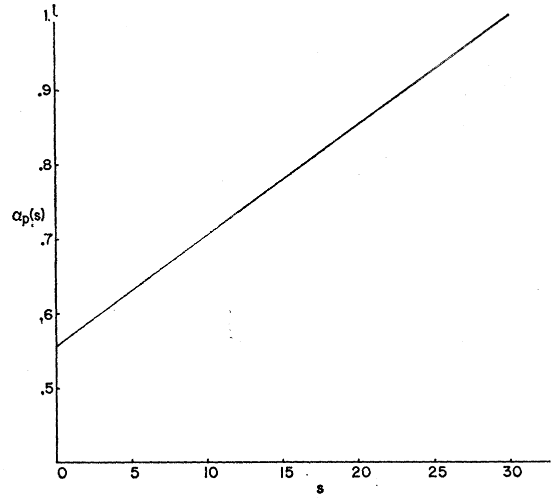

FIG. 2. α_ρ as a function of s. The parameters are the same as in Fig. 1.

where m_2 is the mass of the exchanged particle and $s_2=4m_\phi{}^2$.

If the force for the reaction $\pi\pi\rightarrow\phi\phi$ is also assumed to be dominated by the exchange of a scalar particle with mass m_1, then

$$B_{12}{}^l(s)=\frac{G_{12}}{(q_1q_2)^{l+1}}Q_l\left(\frac{s+2m_1{}^2-\Sigma}{4q_1q_2}\right),\quad (6)$$

where

$$q_i=[\tfrac{1}{4}(s-s_i)]^{1/2}\quad\text{and}\quad\Sigma=2+2m_\phi{}^2.$$

We shall see that in this model the important function is $Q_l(1+2m_2{}^2/(s-s_2))$ and the forms for the other potential terms are unimportant.

III. CALCULATIONS AND DISCUSSION

The N/D equations were solved by using matrix inversion to compute N.[8] We used 80 mesh points in the $\pi\pi$ channel and 40 mesh points in the $\phi\phi$ channel. In Fig. 1 we plot the quantity $[(s-4)^3/s]^{1/2}\cot\delta$ for the $\pi\pi$ channel where we use $G_{11}=0.9$ and adjust the $\phi\phi$ channel parameters to produce a bound state near the ρ mass. G_{12} is then chosen to give the approximate ρ width. In Fig. 2 we see that $\alpha_\rho(s)$ is approximately linear and $\alpha_\rho(0)=0.56$. If we decrease the size of the potential term in the $\pi\pi$ channel by choosing $G_{11}=0.1$, the effect is to eliminate the curvature seen in Fig. 1 and make the phase shift what one would obtain from a pure Breit-Wigner fit. This also makes $\alpha_\rho(0)=0.60$ so that the effect of the direct-channel potential term on α_ρ is small.

The behavior of $\alpha_\rho(s)$ would seem to depend most strongly on the behavior of the bound state in the $\phi\phi$

[8] See Ref. 2 for a discussion of the solution of the equations.

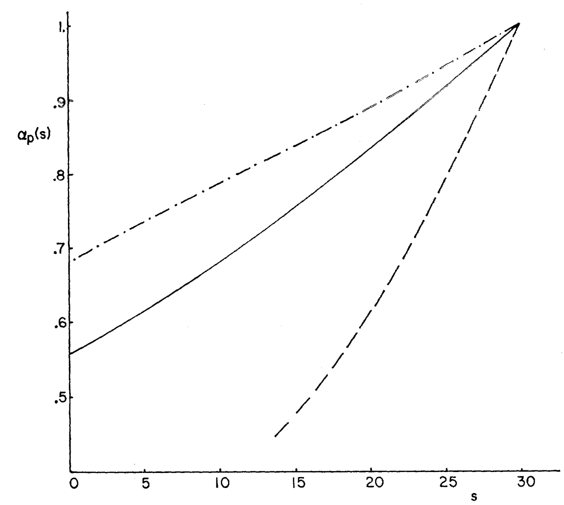

FIG. 3. Regge trajectory of a bound state. For the solid line $m_\phi=3.5$, $m_2=7$, $G_{22}=15\,000$; for the dashed line $m_\phi=3.5$, $m_2=5$, $G_{22}=31\,000$; and for the dash-dot line $m_\phi=3.5$, $m_2=8.5$, $G_{22}=15\,000$.

channel. In Fig. 3 we show the variation in the bound-state position as a function of l to justify this. We also exhibit the dependence of the trajectory as a function of m_2 and we find that $\alpha_\rho(0)$ decreases as the ratio m_ϕ/m_2 increases. The form for B_{22} is the same as the Born term for two particles interacting by a Yukawa potential. We can interpret m_2^{-1} as the range of interaction. If the ϕ is a quark, the range of interaction is certainly very small and it would not be surprising to find m_2 of the order of magnitude of m_ϕ.

The ϕ mass we use is $m_\phi=3.5m_\pi$. We were forced to use a small ϕ mass in the two-channel calculation for the sake of numerical accuracy. If we only look at the ϕ channel in the absence of interaction, as in Fig. 3, the scale of the ϕ mass is arbitrary. We find that for a bound state well below $4m_\phi^2$, the slope of the trajectory is not very sensitive to the position of the bound state. This is certainly true for bound states produced over an energy range which is small by comparison to $4m_\phi^2$. As we approach $4m_\phi^2$, however, we would expect the trajectories to turn over.

Even though the $\alpha_\rho(s)$ we obtain depends strongly on the masses m_ϕ and m_2, we can learn something from the calculation. A bound-state model of the ρ resonance can reproduce the shape and width of the ρ and give a linear trajectory for $\alpha_\rho(s)$ with a reasonable slope. It is unlikely that any model of the ρ which relies on the potential term [Eq. (3)] due primarily to ρ exchange to produce the ρ meson can reproduce the ρ Regge trajectory. The variation with l is contained primarily in the term $Q_l(1+2m_\rho^2/(s-4))$. This calculation indicates that the ratio m_π/m_ρ is just too small to give the desired Regge slope. Thus it seems that either the potential term due to ρ exchange is entirely unrealistic or else the ρ must be produced as a quasibound state in some higher-mass channel. In the bound-state model the trajectory would be almost independent of the width and mass of the resonance. It would be easy to explain the nearly equal slopes of the known trajectories if the particles are produced as quasibound states in a quark-antiquark system, since the slope would be determined primarily by the quark mass and range of interaction.

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