PHYSICAL REVIEW VOLUME 179, NUMBER 5 25 MARCH 1969

Generalized Potential and Bootstraps in the N/D Framework*

PHILIP W. COULTER
Department of Physics, University of California, Irvine, California 92664
(Received 11 November 1968)

In the ρ bootstrap, the generalized potential is usually computed by assuming that the ρ resonance is the main feature of the scattering amplitude in the crossed channels, making the narrow-width approximation, and using crossing symmetry to obtain the generalized potential in the direct channel. A specific example is given to illustrate that one cannot expect to obtain meaningful results by using this method.

THE bootstrap problem within the N/D framework has been the subject of many papers.[1] For the purposes of this paper we will talk about the ρ-meson bootstrap.[2] The philosophy behind the calculation is to begin with the assumption that the exchange of the ρ meson generates the generalized potential in the N/D equation which then produces the ρ meson. This is equivalent to assuming that the ρ resonance is the only important feature of $\pi\pi$ scattering in the crossed channels and then using the narrow-width approximation and crossing symmetry to compute the generalized potential. For the ρ bootstrap the hope is that the generalized potential is primarily responsible for producing the ρ resonance and inelastic effects are only necessary to improve the agreement with experiment, i.e., the ρ meson is not due to Castillejo-Dalitz-Dyson (CDD) effects in one of the inelastic channels.[3] Thus one should be able to compute the ρ resonance in a single-channel formalism.

We will investigate the question of whether it is meaningful to attempt to compute the generalized potential in the manner described above and expect to get reasonable results in a single-channel calculation.

The single-channel N/D equations without inelasticity are

$$N(s)=B(s)+\frac{1}{\pi}\int_{s_E}^{\infty}\frac{ds'}{s'-s}\left[B(s')-\frac{s-s_E}{s'-s_E}B(s)\right]\times\rho(s')N(s'),\quad(1)$$

$$D(s)=1-\frac{s-s_E}{\pi}\int_{s_E}^{\infty}\rho(s')N(s')\frac{ds'}{(s'-s_E)(s'-s-i\epsilon)},\quad(2)$$

where the partial-wave amplitude is $A=N/D$, ρ is a kinematic factor, and we make a subtraction in D at threshold s_E. $B(s)$ is the generalized potential.

Instead of computing B from the crossed channels, we will assume that A or the phase shift δ is known on the physical cut and compute B from this. Levinson's theorem tells us that[4,5]

$$\delta(\infty)-\delta(0)=\pi(N_{\text{CDD}}-N_B),$$

where N_{CDD} is the number of CDD zeros in D and N_B is the number of bound states. We assume that no CDD effects are present and that there are no bound states so that $\delta(\infty)=0$ if we take $\delta(0)=0$. In order for the calculation to make any sense, we must let $\delta(\infty)=0$ if we hope to describe a resonance in a single-channel calculation.

We can compute B from the partial-wave dispersion relation

$$A(s)=B(s)+\frac{1}{\pi}\int_{s_E}^{\infty}\frac{\text{Im}A(s')}{s'-s-i\epsilon}ds'$$

or since $A=e^{i\delta}\sin\delta/\rho$,

$$B(s)=\frac{\sin\delta(s)\cos\delta(s)}{\rho(s)}-\frac{1}{\pi}\mathcal{P}\int_{s_B}^{\infty}\frac{\sin^2\delta(s')}{\rho(s')}\frac{ds'}{s'-s},\quad(3)$$

where $\mathcal{P}\int$ denotes principal-value integral. By using Eq. (3) we can compute B in the physical region if δ is known for physical values of s. We have no way to continue B to unphysical s, but that is irrelevant for our purposes since we only need to know B for physical s to solve the N/D equations.

Now suppose that the $l=1$ partial wave has a resonance and the low-energy phase shift is given by the Layson resonant form[6] $\rho=[(s-s_E)/s]^{1/2}(s-s_E)$,

$$\tan\delta=[\gamma(s-s_E)^{3/2}/s][1/(m_R^2-s)]\quad(4)$$

and at some higher energy the phase shift is modified so that it returns to zero. In the spirit of the narrow-width approximation we would expect to be able to compute $B(s)$ approximately by ignoring the high-energy behavior of δ [i.e., we let $\delta(\infty)=\pi$ in computing B]. We would expect to be able to use this B in the N/D equations to at least compute the main features of the resonance. The result of following this procedure is shown in Fig. 1. Surprisingly, we find not

* Supported in part by the National Science Foundation.

[1] F. Zachariasen, Phys. Rev. Letters 7, 112 (1961); G. F. Chew and S. Mandelstam, Nuovo Cimento 19, 752 (1961); F. Zachariasen and C. Zemach, Phys. Rev. 128, 849 (1962); F. Zachariasen, Pacific International Summer School of Physics, Honolulu, 1965 (unpublished).

[2] See the first paper in Ref. 1.

[3] M. Bander, P. W. Coulter, and G. L. Shaw, Phys. Rev. Letters 14, 270 (1965).

[4] J. B. Hartle and C. E. Jones, Ann. Phys. (N. Y.) 38, 348 (1966); Phys. Rev. 140, B90 (1965); Phys. Rev. Letters 14, 801 (1965).

[5] D. Atkinson, K. Dietz, and P. Morgan, Ann. Phys. (N. Y.) 37, 77 (1966); D. Atkinson and D. Morgan, Nuovo Cimento 41, 559 (1966).

[6] W. M. Layson, Nuovo Cimento 27, 724 (1963).

even a slight hint of resonant behavior in the computed amplitude. In Fig. 2 we include the high-energy behavior of $\delta(s)$ in computing $B(s)$. Now we find that by using this $B(s)$ in solving the N/D equations we are able to duplicate the input phase shift. Thus it is absolutely necessary to include the high-energy behavior of the phase shift in computing a resonance which is not produced by CDD effects.

Computing the generalized potential from the phase shift has nothing to do with bootstraps. However, the conclusions we draw are relevant to the usual bootstrap procedure. When one makes the narrow-width approximation in the crossed channels it is automatically assumed that $\delta(\infty)=\pi$. Setting $\delta(\infty)=\pi$ implies the presence of CDD effects. The generalized potential obtained in this way is then used to solve the N/D equations in the direct channel in the hopes of reproducing resonant behavior in which no CDD effects are present. This procedure is clearly inconsistent and can only lead to meaningful results in the event that the higher-energy behavior of δ does not significantly affect the behavior of the generalized potential. By computing the generalized potential from the behavior of the phase shift in the direct channel, we have explicitly demonstrated that the generalized potential depends very strongly on the high-energy behavior and the resonance cannot be reproduced unless we let $\delta(s)\to 0$ as $s\to\infty$. One cannot hope to obtain a reasonable expression for $B(s)$ by making the narrow-width approximation in the crossed channels and further mutilating the generalized potential term by imposing an approximate form of crossing symmetry. Any resemblance between the potential computed in this manner and the potential which one would compute from a complete knowledge of the (complex) phase shift must be purely accidental.

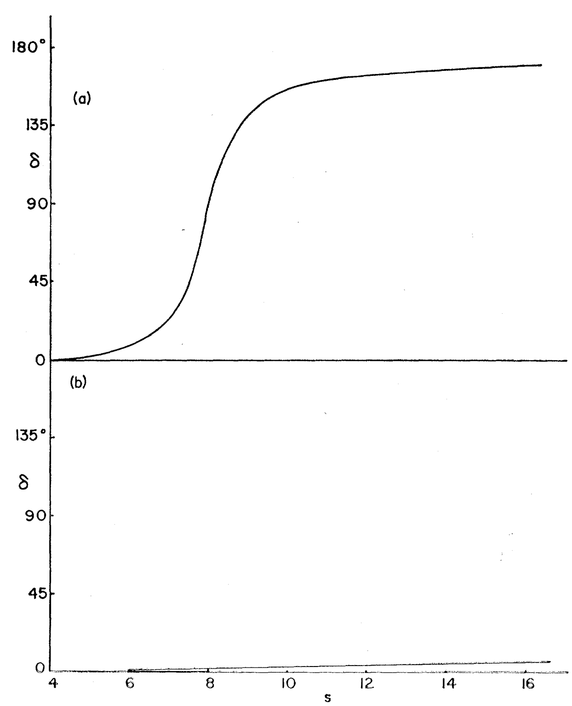

FIG. 1. (a) Input phase shift used to compute $B(s)$. The parameters are $\gamma=0.6$, $s_E=4$, and $m_R^2=8$. (b) Output phase shift found from the N/D equations.

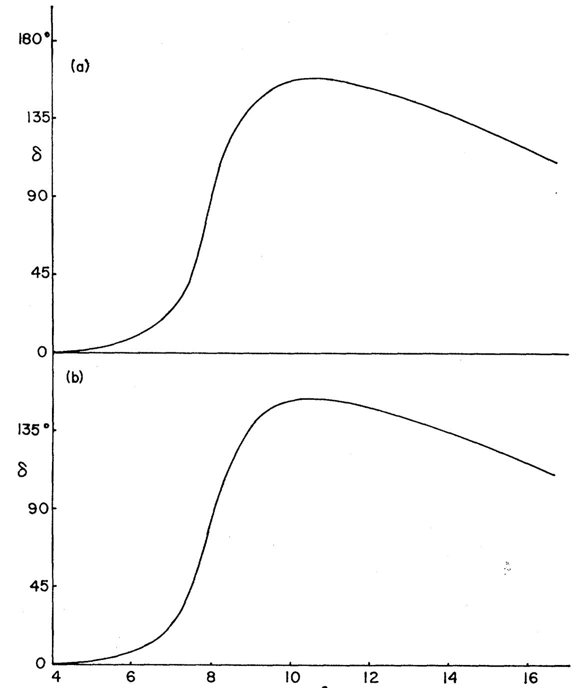

FIG. 2. (a) Input phase shift used to compute $B(s)$. The parameters are the same as in Fig. 1, except that now we explicitly include the behavior $\delta\to 0$ as $s\to\infty$ in computing $B(s)$. (b) Output phase shift found from the N/D equations.

If we attempt to include finite-width effects by using a Breit-Wigner form for the phase shift in the crossed channels, we are still faced with the problem that $\delta(s)\to\pi$ as $s\to\infty$. Instead of making the narrow-width approximation in computing B from the crossed channels, we might try using a form for the phase shift which includes the asymptotic behavior $\delta(\infty)=0$. Using crossing symmetry to obtain $B(s)$ in the direct channel still introduces a high-energy divergence so that we must use a cutoff in solving the N/D equations. If one tries this for the ρ meson, it is impossible to adjust the cutoff to obtain a solution describing the ρ in the direct channel in anything but a very qualitative sense (the resonance is still much too broad). The resonance is probably produced as a function of the cutoff rather than from the B computed in this manner.

These calculations indicate that commonly used methods to compute the generalized potential are clearly inadequate. Until we have a more accurate means of computing $B(s)$ from scattering in the crossed channels it is impossible to carry out a meaningful bootstrap calculation.