Nonleptonic and Weak-Electromagnetic Decays of Hyperons in the Pole Model*

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An estimate is made for the contribution of the decuplet poles to the weak decays of hyperons using the Rarita-Schwinger formalism for spin-$\frac{3}{2}$ fields. $SU(3)$ symmetry is used in conjunction with the Gourdin-Salin model for the D-B-π and D-B-γ couplings. It is shown that the decuplet contribution is large for the parity-violating amplitudes. For weak-electromagnetic decays, this predicts asymmetry parameters which are at variance with earlier calculations. The decay rate for $\Sigma^+\rightarrow p\gamma$ agrees well with the experimental result.

I. INTRODUCTION

RECENTLY, interest has been shown in estimating the effect of decuplet poles on nonleptonic decays. Okubo[1] and Chan[2] have used the current algebra approach and Loebbaka[3] has used the pole model.

The purpose of this note is to extend the work of Graham and Pakvasa[4] by including the baryon resonant states in the calculation, and to update the theoretical predictions for weak-electromagnetic (WE) decays in the light of more recent experimental data[5] on the nonleptonic (NL) decay modes. The pole model for weak decays of hyperons allows us to treat both NL and WE decays in a unified manner when it is assumed that the effective two-body weak Hamiltonian is the same for both sets of decays. With this approach it is found that the decuplet pole terms significantly alter the predictions for the asymmetry parameters in WE decays. This comes about because a phenomenological fit to the NL data indicates that these pole terms contribute significantly to the parity-violating (pv) amplitudes. That the latter is true for NL decays has been shown by Loebbaka[3] who used the Pauli-Fierz formalism for spin-$\frac{3}{2}$ fields in evaluating the decuplet pole terms. In the following we use the Rarita-Schwinger formalism. While the basic results appear to be the same, as in Ref. 3, for the pv amplitudes in NL decays, our method yields simpler expressions for the decuplet poles and these are more amenable to an application of $SU(3)$ symmetry and, if possible, a theoretical interpretation. The extrapolation procedure off the mass shell of the decuplet particle is shown explicitly.

So far the symmetry properties of the weak Hamiltonian have been studied only in the context of baryon octet and meson poles.[4,6] The present analysis which includes the decuplet poles is at a phenomenological level as far as the decuplet poles are concerned. Symmetry properties of the weak baryon octet-decuplet vertex will be dealt with elsewhere.

We assume that the weak Hamiltonian transforms under $SU(3)$ as a member of the octet representation and that it is invariant under time reversal. We limit further the possible form of the Hamiltonian for the octet poles by demanding that it be invariant under the $TL(2)\times P$ transformation.[6] The effective two-body weak Hamiltonian is then $TL(1)\times P$ invariant. This is consistent with the $\Delta T=\frac{1}{2}$ rule and yields the Lee-Sugawara triangular relation for the NL decay amplitudes

$$\sqrt{3}(\Sigma_0{}^+)=(\Lambda_-{}^0)-2(\Xi_-{}^-).$$

The assumption of symmetry under $TL(1)\times P$ eliminates the meson poles for the pv amplitude.

We now include the contribution of the decuplet poles. Following the procedure adopted by Graham and Pakvasa,[4] the parameters for the effective weak vertex are evaluated from the experimental NL decay data and are used in predicting the decay rates and asymmetry parameters for the WE decays of the hyperons.

II. NONLEPTONIC DECAYS

The NL decays, $B\rightarrow B'+\pi$, are described by the Hamiltonian

$$H_{\rm NL}=\bar{\Psi}_{B'}(A-B\gamma_5)\Psi_B\Phi_\pi,$$

where A is the pv amplitude and B the parity-conserving (pc) one.

In the pole model it is assumed that the amplitudes A and B are saturated by the baryon octet and decuplet poles and meson poles (Fig. 1). The strong vertex in baryon and meson poles is assumed to be $SU(3)$-symmetric and is given by

$$H_S=\sqrt{2}g\{\alpha(\bar{B}_\nu{}^\lambda\gamma_5B_\mu{}^\nu+\bar{B}_\mu{}^\nu\gamma_5B_\nu{}^\lambda)P_\lambda{}^\mu +(1-\alpha)(\bar{B}_\nu{}^\lambda\gamma_5B_\mu{}^\nu-\bar{B}_\mu{}^\nu\gamma_5B_\nu{}^\lambda)P_\lambda{}^\mu\} +\frac{\sqrt{2}\lambda_D}{m_\pi}\left\{\bar{B}_l{}^i\partial_\mu(P_m{}^j)\frac{\epsilon^{klm}}{\sqrt{n}}(\Psi_{ijk})_\mu+\text{H.c.}\right\}. \quad (1)$$

Here g is the usual π-N coupling constant with $g=13.4$. The $SU(3)$ mixing parameter α has the value $\alpha\simeq0.66$. Our assignment of the $SU(3)$ operators for

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[1] S. Okubo, Ann. Phys. (N.Y.) **47**, 351 (1968).
[2] F. C. P. Chan, Phys. Rev. **171**, 1543 (1968).
[3] D. S. Loebbaka, Phys. Rev. **169**, 1121 (1968).
[4] R. H. Graham and S. Pakvasa, Phys. Rev. **140**, B1144 (1965) and references therein to earlier work.
[5] J. P. Berge, *Proceedings of Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1961* (University of California Press, Berkeley, 1967).
[6] S. P. Rosen, Phys. Rev. **137**, B431 (1965).

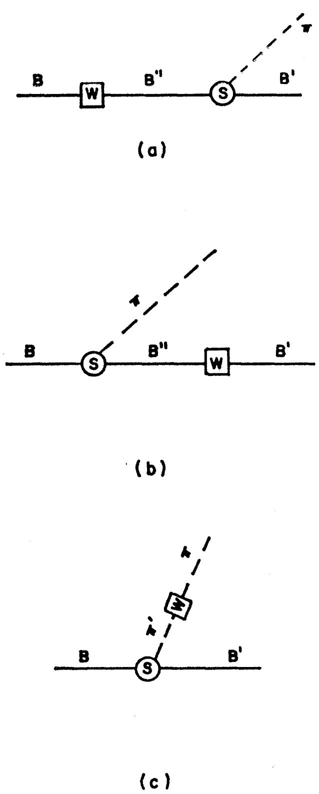

FIG. 1. Pole diagrams contributing to NL decays.

baryons and mesons is the same as that of Carruthers.[7] The decuplet field $(\Psi_{ijk})_\mu$ has the $SU(3)$ indices i, j, and k. Ψ_μ is the spin-$\frac{3}{2}$ field of the decuplet defined, in the Rarita-Schwinger[8] formalism, by

$$(i\gamma\cdot k+m^*)\Psi_\mu=0\,,\qquad \gamma_\mu\Psi_\mu=0=k_\mu\Psi_\mu\,. \tag{2}$$

The propagator for the spin-$\frac{3}{2}$ field is given by[9]

$$\Lambda_{\mu\nu}=\frac{1}{k^2+m^2}\Bigg\{(-ik\cdot\gamma+m^*)\times\left[\delta_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu+\frac{i}{3m^*}(\gamma_\mu k_\nu-k_\mu\gamma_\nu)+\frac{2k_\mu k_\nu}{3m^{*2}}\right]-\frac{(k^2+m^2)}{3m^{*2}}[(ik\cdot\gamma-m^*)\gamma_\mu\gamma_\nu+i(\gamma_\mu k_\nu-\gamma_\nu k_\mu)]\Bigg\}\,. \tag{3}$$

The second term vanishes on the mass shell, and it is assumed that the above form for $\Lambda_{\mu\nu}$ describes adequately the behavior off the mass shell. The Pauli-Fierz propagator used in Ref. 3 does not include any off-shell correction term.

[7] P. Carruthers, *Introduction to Unitary Symmetry* (Wiley-Interscience, Inc., New York, 1966).

[8] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

[9] H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Co., Amsterdam, 1956).

In Eq. (1), the $SU(3)$ coupling of the decuplet to the baryon and meson octets is $((1/\sqrt{n})\bar{B}_l^{i}P_m^{i}\epsilon^{klm})\Psi_{ijk}$, where n is the number of terms in the first factor. We have included a factor $\sqrt{2}$ in the second term in Eq. (1) so that the coupling constant λ_D assumes the usual value obtained from the experimental decay width[10] of N^{*++}. The value is given by

$$\Gamma(N^{*++}\to p\pi^+)=\frac{1}{12\pi}\frac{\lambda_D^2}{m_\pi^2}\frac{p^3(E_p+m_p)}{m_{N^*}}=120\text{ MeV}\,, \tag{4}$$

so that $\lambda_D=2.14$.

The effective two-body weak Hamiltonian for the pc and pv baryon- and meson-octet poles is given by

$$H_W{}^{\mathrm{pc}}=[D(\bar{B}'_\nu{}^2B_3{}^\nu+\bar{B}'_3{}^\nu B_\nu{}^2)+F(\bar{B}'_\nu{}^2B_3{}^\nu-\bar{B}'_3{}^\nu B_\nu{}^2)+cP_\nu{}^2P_3{}^\nu]+\text{H.c.}\cdots\,, \tag{5}$$
$$H_W{}^{\mathrm{pv}}=D'(\bar{B}'_\nu{}^2\gamma_5B_3{}^\nu+\bar{B}'_3{}^\nu\gamma_5B_\nu{}^2)+F'(\bar{B}'_\nu{}^2\gamma_5B_3{}^\nu-\bar{B}'_3{}^\nu\gamma_5B_\nu{}^2)\,.$$

In the tadpole mechanism,[3] the weak vertex is pictured to be an emission of a $K_1{}^0$ followed by the $K_1{}^0$ undergoing a transformation $K_1{}^0\to$ vacuum. In the case of the pv amplitude this mechanism gives a D/F ratio for the weak vertex which is the same as that for the strong vertex. Thus, the parameters D' and F' may be expressed in terms of strong interaction parameters scaled down appropriately by the strength of the $K_1{}^0\to$ vacuum transition. Thus, $D'=f_{K_1}m_Kg\alpha$ and $F'=f_{K_1}m_Kg(1-\alpha)$ where $\alpha\simeq0.66$. Such a simplification does not occur for the pc weak vertex in that the corresponding $SU(3)$ mixing parameter is not related to the one from strong interactions.

We assume that the effective two-body decuplet-octet Hamiltonian is in the form:

$$(H_w{}^{\mathrm{pv}})_{10}=G\frac{\sqrt{2}\lambda_D}{m_\pi}[-\partial_\mu\bar{B}_l{}'^{i}\epsilon^{kl2}(\Psi_{i3k})_\mu+\text{H.c.}]$$

and (6)

$$(H_w{}^{\mathrm{pc}})_{10}=G'\frac{\sqrt{2}\lambda_D}{m_\pi}[-\partial_\mu\bar{B}_l{}'^{i}\epsilon^{kl2}\gamma_5(\Psi_{i3k})_\mu+\text{H.c.}]\,.$$

The coupling constants G and G' then correspond to the ones obtained if the tadpole mechanism is invoked since we have included the strong coupling constant λ_D in the above expressions.

To obtain the contribution of the decuplet poles to the physical amplitudes it is necessary to assume that the propagator given earlier approximates the behavior off the mass shell well enough. We also assume that there is no momentum transfer across the weak vertex. Ample use is made of the subsidiary condition in (2) for the spin-$\frac{3}{2}$ field and the Dirac equation for the baryon octet fields. The resultant expressions for the pv and the pc pole terms [excluding the $SU(3)$ coefficients]

[10] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

are:

$$A_{10}(s\text{ channel})=\frac{2\lambda_D^2}{m_\pi^2}G\left[\frac{-m_BE_{B'}(M^*+m_B)}{(m^*)^2}\right],$$
$$A_{10}(u\text{ channel})=\frac{2\lambda_D^2}{m_\pi^2}G\left[\frac{-m_BE_{B'}(M^*+m_{B'})}{(m^*)^2}\right],$$
$$B_{10}(s\text{ channel})=\frac{2\lambda_D^2}{m_\pi^2}G'\left[\frac{-m_BE_{B'}(m^*-m_B)}{(m^*)^2}\right],$$
$$B_{10}(u\text{ channel})=\frac{2\lambda_D^2}{m_\pi^2}G'\left[\frac{-m_BE_{B'}(m^*-m_{B'})}{(m^*)^2}\right].\quad(7)$$

The s and u channels correspond to the diagrams in Figs. 1(a) and (b), respectively. The final expressions and the parameters giving the best fit to experiment are given in Appendix A.

In the tadpole model, with the strong-interaction parameters assumed to be known, the pv amplitudes have only two parameters; they correspond to the strengths of the spurion vertex in the baryon-octet and decuplet poles. The best fit for the four independent amplitudes with the above parameters yields $A(\Sigma_+^+)=0$. The values obtained are $|f_{K_1}m_K|=4.81\times10^{-6}$ MeV and $G=2.84\times10^{-7}$ MeV. The value for $f_{K_1}m_K$ obtained with the Fierz-Pauli formalism by Loebbaka[3] is $f_{K_1}m_K\sim4\times10^{-6}$ MeV. This indicates that the basic result that the decuplet pole makes a significant contribution to the pv amplitude is essentially independent of the formalism used in describing the spin-$\frac{3}{2}$ field.

The pc amplitudes have four parameters which correspond to the D and F couplings for the weak vertex in the baryon pole and the strength of the weak vertex for the decuplet and meson-octet poles. It is found that the contribution of the decuplet pole is small, but not negligibly small, in comparison with the total amplitude. Here the four independent amplitudes Λ_-^0, Ξ_-^-, Σ_+^+, and Σ_-^- are used to solve for the four parameters. We obtain $D=-1.78\times10^{-4}$ MeV, $F=4.17\times10^{-4}$ MeV, $G'=1.50\times10^{-5}$ MeV, and $c=0.36$ MeV.[2] The values for the corresponding parameters (except G') obtained in Ref. 4 are larger. This difference has been traced to the more recent experimental data we are using as input. It will be shown in Sec. III that the pc amplitudes give the dominant contribution to the WE decay rates. The above changes in the values for the parameters also affects the theoretical predictions for decay rates.

III. WEAK-ELECTROMAGNETIC DECAYS

These decays are characterized by the phenomenological Hamiltonian[4,11]

$$H_{\rm WE}=\tfrac{1}{2}i\Psi_{B'}(C+D\gamma_5)\sigma_{\mu\nu}\Psi_BF_{\mu\nu}.$$

The decay rate and the asymmetry parameters are given by

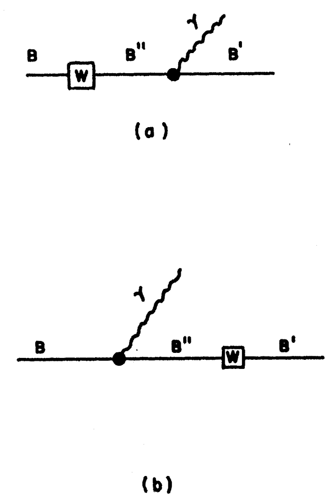

FIG. 2. Pole diagrams contributing to WE magnetic decays.

$$\Gamma=\frac{1}{8\pi}\left(\frac{m_B^2-m_{B'}^2}{m_B}\right)^3(|C|^2+|D|^2),\quad(8)$$
$$\alpha=2\,{\rm Re}(C^*D)/(|C|^2+|D|^2).$$

The pc amplitude C and the pv amplitude D are assumed to be saturated by the baryon-octet and decuplet poles in the pole model. Our assumption is that the weak vertex in the pole diagrams (Fig. 2) is the same as that for the NL decays. For the baryon poles, the electromagnetic vertex is described by

$$H_E=ie\bar{B}'\gamma_\mu BA_\mu+\tfrac{1}{2}ie\bar{B}'(\mu_B/M_B)\sigma_{\mu\nu}BF_{\mu\nu}.\quad(9)$$

Here μ_B are the anomalous magnetic moments of the baryons. The baryon decuplet-octet coupling is given by

$$H_{DB\gamma}=\frac{eC_2}{m_\pi}(\bar{B}\gamma_\mu\gamma_5\Psi_\nu F_{\mu\nu}+\text{H.c.}).\quad(10)$$

Only the magnetic coupling has been used in the above. Analysis of pion-photoproduction data in the Gourdin-Salin[12] model indicates that this is the dominant one. The value of the coupling constant[13] is $C_2=0.345$.

The decuplet-pole terms are evaluated by going off the mass shell of the decuplet particle, since we again assume that there is no momentum transfer across the weak vertex. We choose the rest frame of the initial baryon as the frame of reference and use the Dirac equation and momentum conservation to simplify the expressions. The final result is not manifestly gauge-invariant, because of the above procedure adopted in estimating the decuplet poles. However, the method has

[11] R. E. Behrends, Phys. Rev. **111**, 1691 (1958).

[12] M. Gourdin and P. Salin, Nuovo Cimento **27**, 193 (1963).

[13] S. Fubini, G. Furlan, C. Rosetti, Nuovo Cimento 43, 161 (1966). This value of the coupling constant C_2 differs slightly from the one in Ref. 12 and is based on later data.

simplicity to recommend it. The total amplitude, with the initial baryon at rest is

$$M(B\to B'\gamma)=-i\bar{u}_{B'}(C+D\gamma_5)(\gamma_\mu k_\mu)(\gamma_\nu\epsilon_\nu)u_B$$
$$=-\bar{u}_{B'}[C(m_B+m_{B'})+D(m_B-m_{B'})\gamma_5]\gamma_\nu\epsilon_\nu u_B\,.$$

We are also able to express the decuplet pole terms in the form

$$M_{10}(B\to B'\gamma)=\bar{u}_{B'}[\bar{C}(\gamma\cdot\epsilon)+\bar{D}\gamma_5(\gamma\cdot\epsilon)]u_B\,,$$

where $\bar{C}$ and $\bar{D}$ are kinematical expressions involving masses of particles and coupling constants. Then the decuplet contribution to the amplitudes C and D can be identified. The baryon pole terms pose no such difficulty. The results are in Appendix B.

Numerical evaluation has been carried out by using the weak-vertex parameters estimated from the NL decay data. It is found that the pc amplitude essentially determines the decay rate. Inclusion of the decuplet pole in the theory does not modify the decay rates since the pc amplitude is not affected much by it. However, it does dominate the pv amplitude and this changes the signs of the asymmetry parameters in some of the decays.

Experimental information is available for the decay rate[14] of $\Sigma^+\to p\gamma$. The experimental value for the branching ratio $\Gamma(\Sigma^+\to p\gamma)/\Gamma(\Sigma^+\to p\pi^0)$ is $0.37\pm0.08\%$. When use is made of the $SU(3)$-predicted values for the anomalous magnetic moments of the baryons, the above branching ratio, with $\mu_p=\mu_{\Sigma^+}$, is 0.13%. This does not compare favorably with experiment. The value obtained in Ref. 4, with older experimental numbers as input, is larger by a factor $\sim$2.

The results improve when "mass-corrected" anomalous magnetic moments are used. Use of the mass-corrected moments finds support in the recent experimental measurement[15] of the Σ^+ magnetic moment (total) which was found to be $\mu_{\Sigma^+}=(2.1\pm1.0)\mu_N$. Using $\mu_{\Sigma^+}=1.79(m_p/m_{\Sigma^+})\simeq1.4$, we obtain $\Gamma(\Sigma^+\to p\gamma)=2.87\times10^7$ sec^{-1} yielding a branching ratio 0.44%. This is in excellent agreement with experiment. The decay rates for other WE decays are given in Appendix B. The rate for $\Xi^-\to\Sigma^-\gamma$ is predicted to be small (also see Ref. 4). The rates for $\Lambda\to n\gamma$ and $\Xi^0\to\Lambda\gamma$ are almost as large as that for $\Sigma^0\to p\gamma$. However, the rate for $\Xi^0\to\Sigma^0\gamma$ is much smaller.

IV. CONCLUSION

We have used the Rarita-Schwinger formalism to include the decuplet poles in a pole model for NL and WE decays. We have shown that the decuplet poles do indeed make a significant contribution to the pv amplitudes. From a dispersion-theoretic point of view the decuplet poles are sufficiently close to the physical region to affect the physical amplitudes, so that it is not surprising to find their contribution to be large. We have not included the Y_0^* poles because these will contribute to the Σ decays only and entail the introduction of one more parameter.[1,2]

For the pv NL decays our conclusions are identical to those obtained by Loebbaka,[3] who, however, used the Pauli-Fierz formalism for the spin-$\frac{3}{2}$ baryons.

For the WE decays we have used mass-corrected anomalous magnetic moments for the baryons. Experimental evidence tends to support this approach. The results for WE decays are all predictions.

A measurement of the asymmetry parameter for the $\Sigma^+\to p\gamma$ decay could settle the question of whether the decuplet poles do affect the pv amplitude. It could also decide whether our procedure for estimating the decuplet-pole terms is indeed the correct one since the sign of the asymmetry parameter changes when these terms are included in the theory.

ACKNOWLEDGMENTS

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APPENDIX A

I. Phase Conventions

$$B_\nu{}^\mu\equiv\begin{bmatrix}\Sigma^0/\sqrt{2}+\Lambda^0/\sqrt{6} & \Sigma^+ & p\\ \Sigma^- & -\Sigma^0/\sqrt{2}+\Lambda^0/\sqrt{6} & n\\ -\Xi^- & \Xi^0 & -2\Lambda^0/\sqrt{6}\end{bmatrix},$$

$$P_\nu{}^\mu=\begin{bmatrix}\pi^0/\sqrt{2}+\eta^0/\sqrt{6} & \pi^+ & K^+\\ \pi^- & -\pi^0/\sqrt{2}+\eta^0/\sqrt{6} & K^0\\ K^- & \bar{K}^0 & -2\eta^0/\sqrt{6}\end{bmatrix}.$$

$SU(3)$ coupling of the decuplet to the baryon and meson octet is given by $\Psi^{ijk}(n^{-1/2}\epsilon_{klm}B_i{}^lP_j{}^m)$, where n is the number of terms inside the parenthesis. For example N^{*++} couples to $(1/\sqrt{2})(\Sigma^+K^+-p\pi^+)$.

The electromagnetic coupling of the decuplet to the baryon octet is given by $\Psi^{ijk}n^{-1/2}\epsilon_{klm}B_i{}^lT_j{}^m$, where $T_j{}^m\equiv T_1{}^1$. We assume that the electromagnetic operator is a $T_1{}^1$ member of the $SU(3)$ octet. The coupling constant for $N^{*+}\to\gamma p$ is known from photoproduction data. We use the following normalization:

$$\langle\gamma p|N^{*+}\rangle=1=\langle\gamma n|N^{*0}\rangle\,.$$

Then

$$\langle\gamma\Xi^0|\Xi^{*0}\rangle=\langle\gamma\Sigma^+|Y_1{}^{*+}\rangle=-1\,,$$
$$\langle\gamma\Lambda|Y_1{}^{*0}\rangle=-\tfrac{1}{2}\sqrt{3}\,;\quad\langle\gamma\Sigma^0|Y_1{}^{*0}\rangle=\tfrac{1}{2}\,.$$

[14] M. Bazin *et al.*, Phys. Rev. Letters **14**, 154 (1965).
[15] T. S. Mast *et al.*, Phys. Rev. Letters **20**, 1312 (1968).

II. Amplitudes for NL Decays

1. Parity-violating amplitudes

Octet poles:

$$A_8(\Lambda_-^0)=\frac{g}{\sqrt{3}}\left[-\frac{(D'+3F')}{\Lambda+N}+\frac{(D'-F')2\alpha}{\Sigma+N}\right],$$

$$A_8(\Xi_-^-)=\frac{g}{\sqrt{3}}\left[-\frac{(D'+F')2\alpha}{\Xi+\Sigma}-\frac{(D'-3F')(1-2\alpha)}{\Xi+\Lambda}\right],$$

$$A_8(\Sigma_+^+)=g\sqrt{2}\left[\frac{(D'-F')(2-\alpha)}{\Sigma+N}-\frac{(D'+3F')\alpha}{3(\Lambda+N)}\right],$$

$$A_8(\Sigma_0^+)=g\left[\frac{D'-F'}{\Sigma+N}(3-2\alpha)\right],$$

$$A_8(\Sigma_-^-)=g\sqrt{2}\left[-\frac{(D'-F')(1-\alpha)}{\Sigma+N}-\frac{(D'+3F')\alpha}{3(\Lambda+N)}\right].$$

If the tadpole mechanism is assumed to be valid we may set $D'=gf_{K_1}m_K\alpha$ and $F'=gf_{K_1}m_K(1-\alpha)$. The particle symbols represent the particle masses. We use $g=13.4$, $\alpha\simeq0.66$.

Decuplet poles:

$$A_{10}(\Lambda_-^0)=\left(2G\frac{\lambda_D^2}{m_\pi^2}\right)\left(\frac{\Lambda E_p}{2\sqrt{6}}\right)\left(\frac{Y_1^*+N}{(Y_1^*)^2}\right),$$

$$A_{10}(\Xi_-^-)=\left(2G\frac{\lambda_D^2}{m_\pi^2}\right)\left(\frac{\Xi E_\Lambda}{2\sqrt{6}}\right)\left[\frac{Y_1^*+\Xi}{(Y_1^*)^2}+\frac{\Xi^*+\Lambda}{(\Xi^*)^2}\right],$$

$$A_{10}(\Sigma_+^+)=\left(2G\frac{\lambda_D^2}{m_\pi^2}\right)\frac{\Sigma E_n}{6}\left[\frac{N^*+\Sigma}{(N^*)^2}-\frac{Y_1^*+N}{2(Y_1^*)^2}\right],$$

$$A_{10}(\Sigma_0^+)=-\left(2G\frac{\lambda_D^2}{m_\pi^2}\right)\frac{\Sigma E_p}{3\sqrt{2}}\left[\frac{N^*+\Sigma}{(N^*)^2}+\frac{Y_1^*+N}{2(Y_1^*)^2}\right],$$

$$A_{10}(\Sigma_-^-)=\left(2G\frac{\lambda_D^2}{m_\pi^2}\right)\Sigma E_n\left[\frac{N^*+\Sigma}{2(N^*)^2}+\frac{Y_1^*+N}{12(Y_1^*)^2}\right].$$

Here E_B are the energies of the daughter baryons. The coupling constant λ_D/m_π is obtained from $N^*\to N\pi$ decay width.

2. Parity-Conserving Amplitudes

Octet pole terms:

$$B_8(\Lambda_-^0)=\frac{g}{\sqrt{3}}\left[\frac{D+3F}{\Lambda-N}+\left(\frac{D-F}{\Sigma-N}\right)2\alpha\right]+\frac{gc(2\alpha-3)}{\sqrt{3}(m_K^2-m_\pi^2)},$$

$$B_8(\Xi_-^-)=\frac{g}{\sqrt{3}}\left[\frac{(D+F)2\alpha}{\Xi-\Sigma}-\frac{(D-3F)(1-2\alpha)}{\Xi-\Lambda}\right]+\frac{gc(4\alpha-3)}{\sqrt{3}(m_K^2-m_\pi^2)},$$

TABLE I. Experimental data (Ref. 5).

Decay	$A/10^5(\hbar/m_\pi)^{1/2}$	$B/10^5(\hbar/m_\pi)^{1/2}$
Λ_-^0	1.551 ± 0.024	11.045 ± 0.495
Ξ_-^-	2.022 ± 0.029	-6.627 ± 0.57
Σ_+^+	0.008 ± 0.034	19.081 ± 0.347
Σ_0^+	-1.558 ± 0.142	11.71 ± 1.88
Σ_-^-	1.861 ± 0.017	-0.152 ± 0.386

$$B_8(\Sigma_+^+)=\sqrt{2}g\left[\frac{-\alpha(D-F)}{\Sigma-N}-\frac{(D+3F)\alpha}{3(\Lambda-N)}\right],$$

$$B_8(\Sigma_0^+)=g\left[\frac{(D-F)(1-2\alpha)}{\Sigma-N}\right]-\frac{(2\alpha-1)gc}{m_K^2-m_\pi^2},$$

$$B_8(\Sigma_-^-)=\sqrt{2}g\left[-\frac{(D-F)(1-\alpha)}{\Sigma-N}-\frac{(D+3F)\alpha}{3(\Lambda-N)}\right]+\frac{\sqrt{2}gc(2\alpha-1)}{m_K^2-m_\pi^2}.$$

Decuplet pole terms:

$$B_{10}(\Lambda_-^0)=\left(2G'\frac{\lambda_D^2}{m_\pi^2}\right)\frac{\Lambda E_p}{2\sqrt{6}}\left(\frac{Y_1^*-N}{(Y_1^*)^2}\right),$$

$$B_{10}(\Xi_-^-)=\left(2G'\frac{\lambda_D^2}{m_\pi^2}\right)\frac{\Xi E_\Lambda}{2\sqrt{6}}\left(\frac{Y_1^*-\Xi}{(Y_1^*)^2}+\frac{\Xi^*-\Lambda}{(\Xi^*)^2}\right),$$

$$B_{10}(\Sigma_+^+)=\left(2G'\frac{\lambda_D^2}{m_\pi^2}\right)\frac{\Sigma E_n}{6}\left[\frac{N^*-\Sigma}{(N^*)^2}-\frac{Y_1^*-N}{2(Y_1^*)^2}\right],$$

$$B_{10}(\Sigma_0^+)=-\left(2G'\frac{\lambda_D^2}{m_\pi^2}\right)\frac{\Sigma E_p}{3\sqrt{2}}\left[\frac{N^*-\Sigma}{(N^*)^2}+\frac{Y_1^*-N}{2(Y_1^*)^2}\right],$$

$$B_{10}(\Sigma_-^-)=\left(2G'\frac{\lambda_D^2}{m_\pi^2}\right)\Sigma E_n\left[\frac{N^*-\Sigma}{2(N^*)^2}+\frac{Y_1^*-N}{12(Y_1^*)^2}\right].$$

III. Determination of Parameters

The experimental data used are given in Table I. The parameters determined are given below.

The pc amplitude parameters are $D=-1.78\times10^{-4}$ MeV, $F=4.18\times10^{-4}$ MeV, $G'=1.51\times10^{-5}$ MeV, and $c=0.363$ MeV^2.

The best fit for pv amplitudes is obtained for $f_{K_1}m_K=-4.81\times10^{-6}$ MeV, $G=2.84\times10^{-7}$ MeV. We obtain [in units of $10^{-5}(\hbar/m_\pi)^{1/2}$]: $A(\Lambda_-^0)=1.582$, $A(\Xi_-^-)=1.768$, $A(\Sigma_+^+)=0.0002$, $A(\Sigma_-^-)=1.92$.

APPENDIX B

I. Weak-Electromagnetic Decays

1. Parity-Conserving Amplitudes

Baryon-octet poles:

$$C_8(\Lambda\to n\gamma)=\left(\frac{e\mu_n}{2N}-\frac{e\mu_\Lambda}{2\Lambda}\right)\frac{D+3F}{(\sqrt{6})(\Lambda-N)}-\frac{e\mu_{\Sigma\Lambda}}{\Sigma+\Lambda}\frac{(D-F)/\sqrt{2}}{(\Sigma-N)},$$

$$C_8(\Sigma^+\to p\gamma)=\left(\frac{e\mu_{\Sigma^+}}{2\Sigma}-\frac{e\mu_p}{2N}\right)\left(\frac{D-F}{\Sigma-N}\right),$$

$$C_8(\Xi^-\to\Sigma^-\gamma)=\left(\frac{e\mu_{\Sigma^-}}{2\Sigma}-\frac{e\mu_{\Xi^-}}{2\Xi}\right)\left(\frac{D+F}{\Xi-\Sigma}\right),$$

$$C_8(\Xi^0\to\Sigma^0\gamma)=\frac{e\mu_{\Sigma\Lambda}}{\Sigma+\Lambda}\frac{(D-3F)/\sqrt{6}}{\Xi-\Lambda}+\left(\frac{e\mu_{\Sigma^0}}{2\Sigma}-\frac{e\mu_{\Xi^0}}{2\Xi}\right)\frac{D+F}{\Xi-\Sigma},$$

$$C_8(\Xi^0\to\Lambda^0\gamma)=\frac{e\mu_{\Sigma\Lambda}}{\Sigma+\Lambda}\frac{D+F}{\Xi-\Sigma}+\left(\frac{e\mu_\Lambda}{2\Lambda}-\frac{e\mu_{\Xi^0}}{2\Xi}\right)\frac{(D-3F)/\sqrt{6}}{\Xi-\Lambda}.$$

Here μ_B are the anomalous magnetic moments of the baryons. Also $\mu_{\Sigma\Lambda}$ is the transition magnetic moment in the decay $\Sigma\to\Lambda\gamma$.

Decuplet-pole terms:

$$C_{10}(\Lambda\to n\gamma)=-\beta'\left(\frac{\Lambda-N}{8}\right)\left(\frac{Y_1^*-N}{(Y_1^*)^2}\right),$$

$$C_{10}(\Sigma^+\to p\gamma)=\beta'\frac{\Sigma-N}{2\sqrt{6}}\left[\frac{N^*-\Sigma}{(N^*)^2}-\frac{Y_1^*-N}{(Y_1^*)^2}\right],$$

$$C_{10}(\Xi^-\to\Sigma-\gamma)=0,$$

$$C_{10}(\Xi^0\to\Sigma^0\gamma)=\beta'\frac{\Xi^0-\Sigma^0}{4\sqrt{3}}\left[\frac{Y_1^*-\Xi}{2(Y_1^*)^2}+\frac{\Xi^*-\Sigma}{(\Xi^*)^2}\right],$$

$$C_{10}(\Xi^0\to\Lambda^0\gamma)=\beta'\frac{\Xi-\Lambda}{4}\left[-\left(\frac{Y_1^*-\Xi}{2(Y_1^*)^2}\right)+\frac{\Xi^*-\Lambda}{(\Xi^*)^2}\right].$$

Here

$$\beta'=\left(\frac{\sqrt{2}\lambda_D}{m_\pi}G'\right)\left(\frac{eC_2}{m_\pi}\right).$$

The decuplet pole does not contribute to $\Xi^-\to\Sigma^-\gamma$ because of U-spin conservation.

2. *Parity-violating amplitudes*

Octet pole terms:

$$D_8(\Lambda\to n\gamma)=-\left(\frac{e\mu_n}{2N}+\frac{e\mu_\Lambda}{2\Lambda}\right)\frac{(D'+3F')/\sqrt{6}}{\Lambda+N}-\frac{e\mu_{\Sigma\Lambda}}{\Sigma+\Lambda}\frac{(D'-F')}{\sqrt{2}(\Sigma+N)},$$

TABLE II. Weak-electromagnetic decay rates and asymmetry parameters.

Decay	$C/e(\hbar/m_\pi)^{1/2}$	$D/e(\hbar/m_\pi)^{1/2}$	Decay rate Γ (sec^{-1})	Asymmetry parameter α
$\Lambda\to n\gamma$	$-\ 3.05\times10^2$	-3.03	6.53×10^6	0.02
$\Sigma^+\to p\gamma$	3.93×10^2	-8.94	2.87×10^7	-0.045
$\Xi^-\to\Sigma^-\gamma$	6.42	-0.06	1.14×10^3	-0.02
$\Xi^0\to\Sigma^0\gamma$	-35.5	-0.90	3.41×10^4	-0.05
$\Xi^0\to\Lambda^0\gamma$	3.13×10^2	8.48	1.03×10^7	0.054

$$D_8(\Sigma^+\to p\gamma)=\left(\frac{e\mu_{\Sigma^+}}{2\Sigma}+\frac{e\mu_p}{2N}\right)\frac{D'-F'}{\Sigma+N},$$

$$D_8(\Xi^-\to\Sigma-\gamma)=-\left(\frac{e\mu_{\Sigma^-}}{2\Sigma}+\frac{e\mu_{\Xi^-}}{2\Xi}\right)\frac{D'+F'}{\Sigma+\Xi},$$

$$D_8(\Xi^0\to\Sigma^0\gamma)=\frac{-e\mu_{\Sigma\Lambda}}{\Sigma+\Lambda}\frac{(D'-3F')/\sqrt{6}}{\Lambda+\Xi}-\left(\frac{e\mu_{\Sigma^0}}{2\Sigma}+\frac{e\mu_{\Xi^0}}{2\Xi}\right)\frac{D'+F'}{\Xi+\Sigma},$$

$$D_8(\Xi^0\to\Lambda\gamma)=\frac{-e\mu_{\Sigma\Lambda}}{\Sigma+\Lambda}\frac{D'+F'}{\Xi+\Sigma}-\left(\frac{e\mu_\Lambda}{2\Lambda}+\frac{e\mu_{\Xi^0}}{2\Xi}\right)\frac{(D'-3F')/\sqrt{6}}{\Xi+\Lambda}.$$

Decuplet pole terms:

$$D_{10}(\Lambda\to n\gamma)=-\beta\frac{\Lambda+N}{8}\frac{Y_1^*+N}{(Y_1^*)^2},$$

$$D_{10}(\Sigma^+\to p\gamma)=-\beta\frac{\Sigma+N}{2\sqrt{6}}\left[\frac{N^*+\Sigma}{(N^*)^2}+\frac{Y_1^*+N}{(Y_1^*)^2}\right],$$

$$D_{10}(\Xi^-\to\Sigma^-\gamma)=0,$$

$$D_{10}(\Xi^0\to\Sigma^0\gamma)=\beta\frac{\Xi+\Sigma}{4\sqrt{3}}\left[-\frac{Y_1^*+\Xi}{2(Y_1^*)^2}+\frac{\Xi^*+\Sigma}{(\Xi^*)^2}\right],$$

$$D_{10}(\Xi^0\to\Lambda^0\gamma)=\beta\frac{\Xi+\Lambda}{4}\left[\frac{Y_1^*+\Xi}{2(Y_1^*)^2}+\frac{\Xi^*+\Sigma}{(\Xi^*)^2}\right],$$

where

$$\beta=\left(\frac{\sqrt{2}\lambda_D}{m_\pi}G\right)\left(\frac{eC_2}{m_\pi}\right).$$

II. Predictions for Amplitudes, Decay Rates and Asymmetry Parameters

The predictions for WE decay amplitudes, decay rates and asymmetry parameters are summarized in Table II. The branching ratio $\Gamma(\Sigma^+\to p\gamma)/\Gamma(\Sigma^+\to p\pi^0)=0.44\%$.