Amelioration of Divergence Difficulties in the Theory of Weak Interactions

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(Received 5 August 1968)

A new approach to the problem of the singularity of the weak interactions is presented. Its aim is to provide a theoretical interpretation of the extreme smallness of the violation of selection rules associated with the weak-vector-current operator appearing in the conventional Fermi or intermediate-vector-boson interaction Lagrangian. To illustrate what we have in mind, we note that on account of this singular character, the conventional theories have not yet yielded an understanding of the weakness of strangeness and parity violation in hadronic processes and the weakness of semileptonic neutral decays. We begin with an interaction Lagrangian in which the constituents of the conventional weak current (e.g., strangeness-changing, axial-vector, muonic, etc.) are coupled to possibly distinct local vector operators. This is done in such a way that the effective weak interaction between two currents decomposes into two parts, one having the universality of the weak interaction, the other, called diagonal, acting only between a constituent and itself. It is then possible to transfer the singularity of the weak interaction to the diagonal interaction and to impose any desired degree of symmetry upon the singular part of the diagonal interaction. Two realizations of this approach are presented. Both are intermediate-boson theories involving gradient-coupled spin-0 bosons as well as spin-1 bosons. An important consequence of these theories is that, apart from implying a lower bound, the weak interactions give no indication of the magnitude of the diagonal interactions. Thus while the scattering of μ-neutrinos by electrons should be governed by the conventional universality formula, there is no reason to expect universality to hold for the scattering of e-neutrinos by electrons.

I. INTRODUCTION

ALL known weak processes can be described by a phenomenological interaction Lagrangian

$$L_I = (G/\sqrt{2})(J_\lambda^{(h)} + J_\lambda^{(l)})^\dagger (J_\lambda^{(h)} + J_\lambda^{(l)}), \quad (1.1)$$

where G is the Fermi constant,

$$G = 10^{-5}/M_N^2, \quad (1.2)$$

M_N is the nucleon mass, $J_\lambda^{(l)}$ is the leptonic current,

$$J_\lambda^{(l)} = \bar{\psi}_e i \gamma_\lambda (1+\gamma_5)\psi_{\nu_e} + \bar{\psi}_\mu i \gamma_\lambda (1+\gamma_5)\psi_{\nu_\mu}, \quad (1.3)$$

with $\psi_e, \psi_{\nu_e}, \psi_\mu$, and ψ_{ν_μ} standing for the fields associated with the electron, electron-neutrino, muon, and muon-neutrino, respectively, and the γ's are the usual (Hermitian) Dirac matrices. $J_\lambda^{(h)}$ is the hadronic current operator, the algebra of whose components has been studied so intensively during the past few years.

The Lagrangian (1.1) has been established principally on the basis of a phenomenological analysis of the leptonic and semileptonic weak processes. We have included the usual nonleptonic part contained in the current-current theory even though the strong interactions make a comparable phenomenological analysis very difficult.

The following properties of $J_\lambda^{(h)}$ are, among others, fairly well established, some with great accuracy, some with only moderate accuracy.

(1) $$J_\lambda^{(h)} = V_\lambda^{(h)} - A_\lambda^{(h)}, \quad (1.4)$$

with V a polar and A an axial-vector operator.

(2) $$V_\lambda^{(h)} = \cos\theta \, V_\lambda^{(\Delta S=0, |\Delta I|=1)} + V_\lambda^{(\Delta S=1, |\Delta I|=1/2)} \sin\theta, \quad (1.5)$$

(3) $$A_\lambda^{(h)} = \cos\theta \, A_\lambda^{(\Delta S=0, |\Delta I|=1)} + A_\lambda^{(\Delta S=1, |\Delta I|=1/2)} \sin\theta, \quad (1.6)$$

(4) $$\theta \sim 15^\circ, \quad (1.7)$$

(5) Conserved vector current (CVC): i.e., $V_\lambda^{(\Delta S=0, \Delta I=1)}$ is the charge-lowering isotopic partner

of the isotopic vector part of the electromagnetic hadronic current, $J_\mu^{\text{e.m.},|\Delta I=1|}$; hence, for example, the charge operators

$$Q = \int V_0^{(\Delta S=0,|\Delta I|=1)}d\mathbf{x},$$

$$Q^\dagger = \int V_0^{(\Delta S=0,|\Delta I|=1)\dagger}d\mathbf{x},$$

and

$$Q^3 = \int j_0^{\text{em},|\Delta I|=1}d\mathbf{x}$$

generate the algebra of $SU(2)$.

(6) The axial-vector and vector $\Delta S = 0$ charges generate the algebra $SU(2) \times SU(2)$; possibly the $\Delta S = 0$ and $\Delta S = 1$ charges generate the algebra $SU(3) \times SU(3)$.

We emphasize that (1.4)–(1.6) already implies:

(7) absence of $\Delta S = 2$ or higher currents;
(8) absence of neutral currents;
(9) absence of $\Delta S/\Delta Q = -1$ currents and apart from electromagnetic corrections, absence of currents with $|\Delta I| > 1$.

As long as one treats L_I as a phenomenological interaction to be used only in lowest order, one finds that all leptonic or semileptonic processes can be very well accounted for. However, when one attempts to construct a theory whose first-order interaction will have the desired properties (1)–(9) above, one runs into great difficulty.

There are two straightforward theories which have this property:

(1) the Fermi theory, which looks just like (1.1) but in which one is to take L_I as an interaction Lagrangian and *not* as a pseudopotential;
(2) the intermediate-vector-boson (IVB) theory, in which a charged *vector* boson field X_μ is coupled to the currents according to the Lagrangian

$$L_I = gX_\mu^\dagger(J_\mu^{(h)}+J_\mu^{(l)})+\text{H.c.} \qquad (1.8)$$

In this theory, instead of two currents being coupled at a point, they are coupled in second order according to the effective interaction

$$-g^2\int (J_\mu(x_1)J_\nu^\dagger(x_2))_+\Delta_{\mu\nu}(x_1-x_2), \qquad (1.9)$$

where

$$\Delta_{\mu\nu} = \int \frac{(\delta_{\mu\nu}+q_\mu q_\nu/M_x^2)e^{iq\cdot(x_1-x_2)}}{(q^2+M_x^2)}\frac{d^4q}{(2\pi)^4} \qquad (1.10)$$

and M_x is the mass of the IVB. If M_x is large compared to the range of energies and momentum transfer involved in the process, (1.9) will look very much like

$-L_I$ (1.1), with

$$G/\sqrt{2} = +g^2/M_x^2. \qquad (1.11)$$

Although at present no rigorous consequences have been deduced from either of these theories, arguments given below strongly suggest that they lead to the following difficulties[1-3]:

(1) The prescribed nonleptonic interaction gives rise in general to *strong* violation of parity, isospin, and strangeness conservation.
(2) In semileptonic weak processes, the weak selection rules embodied in properties (1)–(9) are all strongly violated.
(3) The universality embodied in the $SU(2)$ and $SU(2) \times SU(2)$ algebra is strongly violated.

The reason these difficulties arise is related to, but not a necessary consequence of, the nonrenormalizability of the theories. We illustrate the mechanism for the IVB theory.

Suppose one calculates the process

$$\nu + \bar\nu \to X^+ + X^-$$

in lowest-order perturbation theory. One finds, for the helicity amplitude $(-\tfrac{1}{2},\tfrac{1}{2}) \to (0,0)$,

$$f_{0,0;-\frac{1}{2},\frac{1}{2}} = (G/4\pi\sqrt{2})W\sin\theta, \quad (W\to\infty) \qquad (1.12)$$

(with W the c.m. energy) corresponding to a partial-wave amplitude

$$F_{0,0;-\frac{1}{2},\frac{1}{2}}^J = (1/12\pi)GW\delta_{J1}. \qquad (1.13)$$

On the other hand, unitarity requires

$$|F^J| < 1/q \cong 2/W, \qquad (1.14)$$

so that the first-order theory embodied in (1.13) becomes inadequate when

$$GW^2 > 24\pi. \qquad (1.15)$$

Thus at energies, and presumably virtual masses, of order $\Lambda^2 \sim 24\pi/G$ the weak interactions must be modified to remain unitary. One mechanism which will give rise to these modifications is the inclusion of terms of all orders in G. We call Λ the weak interaction or unitarity cutoff.

If no smaller cutoff exists in the theory (and there is no obvious one), then it is not unreasonable to estimate singularities by cutting off all divergent integrals at Λ. With this rule, it turns out that for every power of G, the higher-order weak interactions produce a correction of order roughly

$$G\Lambda^2/16\pi^2 \sim 3/2\pi. \qquad (1.16)$$

[1] M. Halpern and G. Segrè, Phys. Rev. Letters **19**, 611 (1967).
[2] B. L. Ioffe and E. P. Shabalin, Yadern. Fiz. **6**, 828 (1967) [English transl.: Soviet J. Nucl. Phys. **6**, 603 (1968)]; R. N. Mohapatra, J. S. Rao, and R. E. Marshak, Phys. Rev. Letters **20**, 1081 (1968).
[3] G. Feinberg and A. Pais, Phys. Rev. **133**, B477 (1964).

Since the weak interactions are selection-rule violating, the effect of virtual weak interactions [as estimated in (1.16)] will be, as claimed above, to violate strong and weak selection rules strongly, the latter specifically including the universality embodied in CVC.

It had been hoped that the softening effect of strong-interaction form factors would remove the apparent singularity of Eq. (1.16), at least for processes involving hadrons. However, it has become clear[1,2] that if the weak interactions are mediated by local currents, no such softening effect can take place except by virtue of an undiscovered current algebra as well as very special strong equations of motion.

It thus appears that the quantum-number-violating part of the weak interactions requires an effective cutoff, $\Lambda' \ll \Lambda$, in order to keep strangeness- and parity-violating processes weak, $\Delta S = 2$ processes doubly weak, and $\Delta S = 1$ neutral currents small. Clearly, Λ' cannot be too different from a few nucleon masses if it is to accomplish these goals.

In Sec. II, we review briefly previously proposed solutions to this problem. In Secs. III, IV, and V, we propose and discuss our own. In Sec. VI we attempt to pin down some free parameters by calculating several weak processes.

II. SOME PREVIOUSLY SUGGESTED RESOLUTIONS OF WEAK-INTERACTION-THEORY DIFFICULTIES

Two types of cures have been suggested for the difficulties we have discussed in Sec. I.

(1) We discuss the less radical first. Here one supposes that the Lagrangian (1.1) or (1.8) holds, so that the underlying interaction still directly involves the vector currents. One may, within this framework, take one of two quite different points of view.

(a) One must renormalize all divergent amplitudes. This involves an infinite number of arbitrary constants, and is therefore aesthetically somewhat unappealing; nevertheless it is not without some predictive value as long as the coupling is weak; for example, the energy and angular dependence of elastic, low-energy, neutrino-neutrino scattering can be accurately calculated once a few arbitrary constants have been adjusted. Experiment tells us that these renormalization constants cannot be estimated, even as to gross order of magnitude, by the kind of argument that led to Eq. (1.16), but must be arbitrarily made small (i.e., second-order weak). Of course, the results so obtained cannot hold at high energies, where the power series *must* fail.

(b) One may hope that the perturbation theory is *totally* misleading, so that a correct non-weak-coupling calculation might cure all the difficulties. The attempts[3] that have been made in this direction have consisted of partial summations of diagrams, which are not convincing, but, of course, such a solution cannot be ruled out.

(2) The second type of cure we call deception (as in conspiracy, evasion, etc.). It consists of denying a fundamental role to the vector currents, which then appear as fortuitous low-energy approximations to the true interaction. Here again at least two different kinds of deception can be practiced.

(a) The Fermi interaction

$$L_I = (G/\sqrt{2})\bar{\psi}_a\gamma_\lambda(1+\gamma_5)\psi_b\bar{\psi}_c\gamma_\lambda(1+\gamma_5)\psi_d \qquad (2.1)$$

can be rewritten by a Fierz transformation[4] as

$$L_I = (2G/\sqrt{2})\bar{\psi}_a(1-\gamma_5)\psi_{\bar{c}}\bar{\psi}_{\bar{b}}(1+\gamma_5)\psi_d, \qquad (2.2)$$

where $\bar{c}$ represents the charge conjugate particle to c, $\bar{b}$ to b. Equation (2.2) evidently suggests exchange of a spin-0 particle, hence a renormalizable theory without singular high-energy behavior. However, in this theory, many scalars must be introduced, universality becomes an accident, and elastic neutrino scattering by neutrons (and hence presumably by protons) is comparable to the observed inelastic neutrino process. The observed[5] small magnitude of neutrino-proton scattering suggests disagreement of this theory with experiment. The experimental investigation of neutrino-neutron scattering together with some additional theoretical analysis of the effects of the strong interaction should permit a more definitive determination.

(b) The Fermi interaction can be obtained as a low-energy limit of a *fourth*-order scalar interaction[6]

$$L_I = g'\bar{\psi}_f\varphi(1+\gamma_5)\psi_c, \qquad (2.3)$$

where ψ_c is one of the usual spin-$\frac{1}{2}$ fields, φ is a new spin-0 boson and ψ_f a new spin-$\frac{1}{2}$ field. The Fermi constant will be

$$G/\sqrt{2} \sim g'^4/48\pi^2 M^2, \qquad (2.4)$$

where M is the (assumed common and large) mass of the new particles. The difficulty here is that parity violations first occur in order $g^2/M^2 \sim G^{1/2}/M$; all the higher-order effects discussed in Sec. I may still occur in order GM^2. Thus both $G^{1/2}/M$ *and* GM^2 must be essentially first-order weak. To determine whether these two requirements are actually in conflict would require a more specific strong-interaction model and more detailed calculations. The GM^2 limitation can, however, be weakened by appropriate selection of the strong-interaction model and an elaboration of the system of weakly interacting particles.[7]

III. NEW THEORY OF WEAK INTERACTIONS

In this section we show how one may construct a theory of weak interactions with the following properties.

[4] Y. Tanikawa, Phys. Rev. 108, 1615 (1957); Y. Tanikawa and S. Nakamura, Progr. Theoret. Phys. Suppl. (Kyoto) 37 & 38, 306 (1966).
[5] M. M. Block et al., Phys. Rev. Letters 12, 281 (1964).
[6] W. Kummer and G. Segrè, Nucl. Phys. 64, 585 (1965).
[7] N. Christ, Phys. Rev. 176, 2086 (1968).

(1) The conventional local weak vector and axial-vector currents play the primary role in connecting weak interactions to the known hadrons and leptons.

(2) Estimates of weak-interaction processes (making use of the unitarity cutoff where divergences appear) are in agreement with experiments. That is, in addition to preserving all of the quantitative successes of the leptonic and semileptonic processes, the violation of hadronic selection rules in purely hadronic processes is weak, the violation of weak selection rules is weak, etc. Indeed a theory can be constructed in which (a) any given weak process is finite in the lowest order of weak interactions in which it occurs, and (b) for a well-defined class of weak processes, which includes all so far investigated experimentally, estimates of corrections to lowest-order perturbation theory based upon the unitarity cutoff are small.

By way of introduction we reformulate and slightly generalize the IVB theory in the following way. We rewrite Eq. (1.8) as follows:

$$L_I = g(J_\mu{}^w \mathcal{L}_\mu{}^\dagger + \text{H.c.}), \qquad (3.1)$$

where

$$J_\mu{}^w = J_\mu{}^{(h)} + J_\mu{}^{(l)}.$$

We have simply replaced the IVB by an unspecified local vector operator which is assumed to interact weakly with all known particles. The effective second-order weak interaction is again given by (1.9), with

$$\Delta_{\mu\nu}(p) = i \int \langle v | T^*(\mathcal{L}_\mu{}^\dagger(x), \mathcal{L}_\nu(0)) | v \rangle e^{-ip\cdot x} d^4 x$$

$$= \left(\int \frac{\delta_{\mu\nu}\rho_1(M^2) + (p_\mu p_\nu/M^2)\rho_2(M^2)}{p^2 + M^2} dM^2 + F\delta_{\mu\nu} \right).$$

$$(3.2)$$

In Eq. (3.2) the quantities $\rho_1(M^2)$ and $\rho_2(M^2)$ are defined by the expression

$$\langle v | \mathcal{L}_\mu{}^\dagger(x) \mathcal{L}_\nu(y) | v \rangle$$

$$= \int \frac{d^4 p\, \theta(p_0)}{(2\pi)^3} e^{ip\cdot(x-y)} \int \delta(p^2 + M^2)$$

$$\times \left(\delta_{\mu\nu}\rho_1(M^2) + \frac{p_\mu p_\nu}{M^2}\rho_2(M^2) \right) dM^2. \quad (3.3)$$

T^* means a suitably defined "covariant" time-ordered product, and the constant F is determined by the relation between the true time-ordered product and T^*. It follows from the assumption of a positive-metric Hilbert space that $\rho_2 \geq \rho_1 \geq 0$.

The difficulties that we have been discussing arise from the high-momentum behavior of

$$\Delta_{\mu\nu}(p) \underset{p \to \infty}{\sim} F\delta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \int \frac{\rho_2(M^2)}{M^2} dM^2. \qquad (3.4)$$

Thus, if the high-momentum behavior of $\Delta_{\mu\nu}$ is less singular than that given by the IVB theory or the Fermi theory, it is identically zero.

We now recall the fact that the weak current $J_\mu{}^w$ consists of a linear combination of a number of components as indicated in Eqs. (1.3)–(1.8).

$$J_\mu{}^w = \sum_i \alpha_i J_{\mu i}. \qquad (3.5)$$

The $J_{\mu i}$ are assumed to be some complete set of currents carrying unit charge and specified in a manner appropriate to some symmetry-respecting interaction. Within the context of current theoretical views they could include the complete set of $SU(3) \times SU(3)$ charge-bearing vector and axial-vector currents for the hadrons, as well as the charge-bearing vector and axial-vector currents of the muons and electrons, all regarded as separately indexed entities. The α_i are, of course, determined by the expressions (1.3)–(1.8) (and may, without loss of generality, be assumed real). The over-all normalization of the α_i is determined by the fact that Q^w and $Q^{w\dagger}$ generate an $SU(2)$ algebra. Algebraic requirements on the Q_i determine the scale of the $J_{\mu i}$. The symbol Q, of course, refers to the space integral of the fourth component of the corresponding current.

Equation (3.1) can now be rewritten in the form

$$L_I = g \sum_i J_{\mu i} \mathcal{L}_{\mu i}{}^\dagger + \text{H.c.} \qquad (3.6)$$

The weak interactions in the second-order effective Hamiltonian are thus mediated by the quantity

$$\Delta_{\mu\nu, ij} = i \int \langle v | T^*(\mathcal{L}_{\mu i}{}^\dagger(x), \mathcal{L}_{\nu j}(0)) | v \rangle d^4 x\, e^{-ip\cdot x}. \quad (3.7)$$

Equations (3.6) and (3.1) are identical if $\mathcal{L}_{\mu i} = \alpha_i \mathcal{L}_\mu$, in which case $\Delta_{\mu\nu, ij} = \alpha_i \alpha_j \Delta_{\mu\nu}$. We note that universality of the weak interactions is, in this framework, exhibited by the fact that $\Delta_{\mu\nu, ij}$ depends upon i, j only through the factor $\alpha_i \alpha_j$.

Equation (3.6) is, however, more general than Eq. (3.1) and constitutes an appropriate framework for defining a class of theories having the properties described at the beginning of this section. These theories are defined by the requirement that

$$\Delta_{\mu\nu, ij} = \alpha_i \alpha_j \Delta_{\mu\nu}{}^w + \delta_{ij} \Delta_{\mu\nu, i}{}^d, \qquad (3.8)$$

where $\Delta_{\mu\nu}{}^w$ is less singular at high momentum than the expression in Eq. (3.4). We shall refer to $\Delta_{\mu\nu}{}^w$ as the weak-interaction propagator and refer to its high-momentum behavior as nonsingular. The argument leading to Eq. (3.4) applies unchanged to the diagonal parts of $\Delta_{\mu\nu, ij}$ (i.e., $\Delta_{\mu\nu, ii}$), but has no bearing on the high-energy behavior of the nondiagonal parts. We shall indeed find that one may require that the singular behavior be contained entirely in $\Delta_{\mu\nu, i}{}^d$, and, accordingly, that $\Delta_{\mu\nu}{}^w$ can be chosen to be as well behaved as experimental and theoretical considerations suggest.

The $\Delta_{\mu\nu,i}{}^d$ will be referred to as the diagonal propagators and their contributions to the effective Hamiltonian as diagonal interactions. Since all nonvanishing $\Delta_{\mu\nu,i}{}^d$ are required to have the high-momentum behavior of Eq. (3.4), they will be referred to as singular.

Equation (3.8) is the fundamental equation of our theory. The properties described in (2) at the beginning of this section are achieved by imposing appropriate restrictions upon the index dependence of $\Delta_{\mu\nu,i}{}^d$ and the high-momentum behavior of $\Delta_{\mu\nu}{}^w$. One may, for example, go so far as to require that the index dependence of $\Delta_{\mu\nu,i}{}^d$ be so chosen that the diagonal effective interaction Hamiltonian conserves C, P, and T and, when supplemented with a diagonal interaction between neutral currents, have an $SU(3)\times SU(3)\times SU(2)\times SU(2)\times SU(2)\times SU(2)$ symmetry[8] for the combined hadron-lepton system. There is then no danger that singular interactions will contribute to symmetry breaking. The finiteness of lowest-order perturbation theory can be ensured by requiring sufficiently good behavior of $\Delta_{\mu\nu}{}^w$ at high momentum. It is in fact sufficient to require that it vanishes as $1/p^4$.

A little examination of the situation indicates that theories of the type described above can be most easily realized by the introduction of spin-0 and spin-1 intermediate bosons. The effective interaction of spin-1 and gradient-coupled spin-0 bosons take the forms

$$\frac{\delta_{\mu\nu}+(p_\mu p_\nu/m^2)}{p^2+m^2}$$

and

$$\frac{p_\mu p_\nu}{p^2+\mu^2},$$

respectively. For diagonal interactions all such interactions combine with the same sign. For nondiagonal interactions, however, the couplings can be arranged so that different bosons contribute with different signs, leading to cancellation of the singularities. In particular, as exhibited in Sec. IV, supplementing a charged spin-1 boson with gradient-coupled spin-0 bosons can lead to a theory in which the coefficient of the $p_\mu p_\nu$ term of the $\Delta_{\mu\nu}{}^w$ part of the effective interaction falls off as $1/p^4$ for $p^2\gg M_c{}^2$, where M_c is some mass characteristic of the system of bosons. The quadratic cutoff-dependent strangeness and other selection-rule-violating effects discussed in Sec. I then acquire $M_c{}^2$ as an effective cutoff.

IV. MODEL LAGRANGIANS

In this section we exhibit two Lagrangian models which yield an effective interaction having the properties described in connection with Eq. (3.8).

Model I

The weak interactions are assumed to be mediated by a set of charged spin-0 fields φ_i carrying an index i corresponding to the index on the $J_{\mu i}$ and carrying the same electric charge as the current $J_{\mu i}$, and a single charged spin-1 field X_μ. The local operator $\mathcal{L}_{\mu i}(x)$ of Eq. (3.6) is then defined as[9]

$$\mathcal{L}_{\mu i}(x)=\alpha_i X_\mu+(1/\lambda_i)\partial\varphi_i/\partial x_\mu. \tag{4.1}$$

In order that it be possible to impose special symmetry requirements on the diagonal interaction, such as charge independence or $SU(3)$ invariance, we introduce, in addition, a set of neutral Hermitian spin-0 fields φ_i' and an interaction Lagrangian density

$$L_{IN}=g\sum\frac{1}{\lambda_i'}J_{\mu i}'\frac{\partial\varphi_i'}{\partial x_\mu}, \tag{4.2}$$

where $J_{\mu i}'$ are Hermitian neutral counterparts of the $J_{\mu i}$.

It is assumed that the expressions for the $J_{\mu i}$ and $J_{\mu i}'$ do not contain these fields. The dynamics of these fields is determined by the Lagrangian density

$$L_w=-\tfrac{1}{2}\left(\frac{\partial X_\nu}{\partial x_\mu}-\frac{\partial X_\mu}{\partial x_\nu}\right)^\dagger\left(\frac{\partial X_\nu}{\partial x_\mu}-\frac{\partial X_\mu}{\partial x_\nu}\right)-m^2 X_\mu{}^\dagger X_\mu$$

$$-\sum_i\frac{\partial\varphi_i{}^\dagger}{\partial x_\mu}\frac{\partial\varphi_i}{\partial x_\mu}-\mu^2\sum_i\varphi_i{}^\dagger\varphi_i-\tfrac{1}{2}\sum_i\left(\frac{\partial\varphi_i'}{\partial x_\mu}\right)^2$$

$$-\tfrac{1}{2}\mu^2\sum\varphi_i'^2-X_\mu{}^\dagger\sum_i\alpha_i\lambda_i\frac{\partial\varphi_i}{\partial x_\mu}-X_\mu\sum_i\alpha_i\lambda_i\frac{\partial\varphi_i{}^\dagger}{\partial x_\mu}. \tag{4.3}$$

The λ_i and λ_i' are assumed real. We note that the interaction Lagrangian L_I of Eq. (3.6) contains a term identical with Eq. (1.8). One may define in analogy with $J_\mu{}^w$ an electrically charged spin-0 field

$$\varphi^w=\sum_i\alpha_i\lambda_i\varphi_i/(\sum_i\alpha_i{}^2\lambda_i{}^2)^{1/2},$$

and one notes that it is this quantity which is coupled to the X particle in Eq. (4.3).[10]

The propagator is to be determined in the limit $g=0$, in which case only L_w is relevant. It is evident that L_w gives rise to linear equations of motion which can be

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[8] The possibility of constructing a theory in which the symmetry-violating part of an interaction is less singular than is characteristic of quantum field theories has been utilized in connection with electromagnetic effects; see T. D. Lee, Phys. Rev. **171**, 1731 (1968).

[9] We note that the presence of derivative-coupled scalars suggests the possibility of introducing CP violation by means of a strong, nonderivative (and therefore nonsingular) coupling of the same fields.

[10] We observe, in addition, that if the λ_i are chosen equal to one another, then φ^w may be thought of as having the same orientation in component space as $J_\mu{}^w$. While this analogy between φ^w and $J_\mu{}^w$ is suggestive, we do not know whether it is of real significance.

solved exactly. The spectral functions defined by

$$\langle v| \mathcal{L}_{\mu i}(x)^\dagger \mathcal{L}_{\nu j}(y)|v\rangle$$

$$= \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-y)}\theta(p_0)\int \delta(p^2+M^2)$$

$$\times [\delta_{\mu\nu}\rho_{1ij}(M^2)+p_\mu p_\nu \rho_{2ij}(M^2)]dM^2 \quad (4.4)$$

can therefore be found explicitly and are given by

$$\rho_{1ij}=\delta(m^2-M^2)\alpha_i\alpha_j \quad (4.5)$$

$$\rho_{2ij}=\frac{\alpha_i\alpha_j}{m^2}\left(\delta(m^2-M^2)+\frac{(1-\alpha)}{\alpha}\delta(\mu_1^2-M^2)\right.$$

$$\left.-\frac{1}{\alpha}\delta(\mu^2-M^2)\right)+\frac{\delta_{ij}}{\lambda_i^2}\delta(\mu^2-M^2), \quad (4.6)$$

with

$$\alpha=(\sum_i \lambda_i^2\alpha_i\alpha_i)/m^2>0 \quad (4.7)$$

and

$$\mu_1^2=\mu^2/(1-\alpha). \quad (4.8)$$

We note that the Lagrangian density L_w leads to a positive-energy Hamiltonian if and only if $\alpha<1$. One easily sees that if this inequality is satisfied, the spectral density matrices satisfy the requirement that ρ_{1ij} is positive indefinite and $\rho_{2ij}-M^2\rho_{1ij}$ is positive indefinite and nonzero as required by the positivity of the Hilbert-space metric.

The vacuum expectation value of the true time-ordered product is given by the general formula

$$\langle v| T[\mathcal{L}_{\mu i}^\dagger(x),\mathcal{L}_{\nu j}(y)]|v\rangle =\frac{-i}{(2\pi)^4}\int d^4p\, e^{ip\cdot(x-y)}$$

$$\times\left(\frac{\delta_{\mu\nu}\rho_{1ij}(M^2)+p_\mu p_\nu \rho_{2ij}(M^2)}{p^2+M^2}-\delta_{\mu4}\delta_{\nu4}\rho_{2ij}(M^2)\right)dM^2. \quad (4.9)$$

It is obvious from Eqs. (4.5) and (4.6) that this expression splits into two terms, one proportional to $\alpha_i\alpha_i$, the other to δ_{ij}. Furthermore, since

$$\int \rho_{2ij}(M^2)dM^2=\frac{\delta_{ij}}{|\lambda_i|^2}, \quad (4.10)$$

only the diagonal part is noncovariant. The determination from first principles of the covariant propagator to be used in a Feynman-diagram representation of perturbation theory requires a more explicit specification of the rest of the theory (i.e., of the hadron-lepton Lagrangian). We shall assume, as is the case for a number of theories, that the covariant propagator is obtained by omitting the $\delta_{\mu4}\delta_{\nu4}\delta_{ij}$ terms in Eq. (4.8). While the correct covariant propagator may involve a diagonal contact term in some reasonable theories, its presence would not affect the general conclusions that we reach.

It then follows that the weak propagator is given by

$$\Delta_{\mu\nu}{}^w(p)=\frac{\delta_{\mu\nu}}{p^2+m^2}$$

$$+\frac{p_\mu p_\nu}{m^2}\left(\frac{1}{p^2+m^2}+\frac{(1-\alpha)}{\alpha}\frac{1}{p^2+\mu_1^2}-\frac{1}{\alpha}\frac{1}{p^2+\mu^2}\right) \quad (4.11)$$

and the diagonal propagator $\Delta_{\mu\nu,i}{}^d$ by

$$\Delta_{\mu\nu,i}{}^d(p)=\frac{p_\mu p_\nu}{\lambda_i^2}\frac{1}{p^2+\mu^2}. \quad (4.12)$$

It is evident that the *weak* propagator now behaves as $1/p^2$ at large momentum in contrast to its previous singular behavior. The diagonal propagator is independent of the α_i (apart from the inequality $\alpha<1$). It follows that the diagonal interaction, when supplemented by the contribution from L_{IN}, retains any symmetry property possessed by the total Lagrangian, including L_I and L_w in the limit $\alpha_i=0$. Thus one can choose the λ_i,λ_i' so that C, P, T, hypercharge, and isotopic spin are strictly conserved in this limit. One may, of course, impose even more symmetry on the diagonal interaction, such as $U(3)\times U(3)$ for the hadrons and $SU(2)\times SU(2)$ for the leptons, if it seems appropriate. One may also wish to impose less; for example, there is no known reason (see Sec. V) to impose parity conservation upon the leptonic part of the diagonal interaction, and one may therefore wish to avoid the introduction of right-handed neutrinos. This can be accomplished by regarding the $V-A$ combination for the leptons as carrying a single index in Eq. (3.5). We also remark that it is not necessary to take all of the spin-0 masses equal. We have done so for computational simplicity.

The high-momentum behavior of $\Delta_{\mu\nu}{}^w$ in the model discussed above is sufficiently regular to avoid all of the contradictions of weak-interaction theory which arise from the use of unitarity cutoff to estimate divergent expressions. It is, however, not sufficient to make lowest-order perturbation-theory calculations finite. While it is not obviously necessary that they be finite, as will be discussed in Sec. V, we wish to show that a more regular behavior is readily achieved.

Model II

The set of charged and neutral spin-0 fields φ_i,φ_i' are now supplemented by a set of charged and neutral spin-1 fields $V_{\mu i},V_{\mu i}'$. The charged, unindexed spin-1 field X_μ is omitted. The local operator $\mathcal{L}_{\mu i}$ is then defined as

$$\mathcal{L}_{\mu i}(x)=\gamma_i V_{\mu i}+(1/\lambda_i)\partial\varphi_i/\partial x_\mu \quad (4.13)$$

and the neutral interaction by

$$L_{IN}=g\sum_i J_{\mu i}'\left(\gamma_i'V_{\mu i}'+\frac{1}{\lambda_i'}\frac{\partial\varphi_i'}{\partial x_\mu}\right). \quad (4.14)$$

It is convenient to split that part of the Lagrangian involving φ_i, φ_i' and $V_{\mu i}, V_{\mu i}'$ only into two parts:

$$L_w = L_{w,\text{free}} + L_{w,I} \qquad (4.15)$$

with

$$L_{w,\text{free}} = -\tfrac{1}{2}\sum_i \left(\frac{\partial V_{\nu i}}{\partial x_\mu} - \frac{\partial V_{\mu i}}{\partial x_\nu}\right)^\dagger \left(\frac{\partial V_{\nu i}}{\partial x_\mu} - \frac{\partial V_{\mu i}}{\partial x_\nu}\right)$$

$$-\tfrac{1}{4}\sum_i \left(\frac{\partial V_{\nu i}'}{\partial x_\mu} - \frac{\partial V_{\mu i}'}{\partial x_\nu}\right)^2 - m^2 \sum V_{\mu i}^\dagger V_{\mu i}$$

$$-\tfrac{1}{2}m^2 \sum V_{\mu i}' V_{\mu i}' - \sum_i \frac{\partial \varphi_i^\dagger}{\partial x_\mu}\frac{\partial \varphi_i}{\partial x_\mu}$$

$$-\tfrac{1}{2}\sum_i \left(\frac{\partial \varphi_i'}{\partial x_\mu}\right)^2 - \mu^2 \sum \varphi_i^\dagger \varphi_i - \tfrac{1}{2}\mu^2 \sum \varphi_i'^2 \qquad (4.16)$$

and

$$L_{wI} = (1/m^2)K_\mu^\dagger K_\mu, \qquad (4.17)$$

with

$$K_\mu = \sum_i \alpha_i \left(\frac{m^2}{\gamma_i}V_{\mu i} - \lambda_i \frac{\partial \varphi_i}{\partial x_\mu}\right). \qquad (4.18)$$

In this model, L_I and L_{IN} make no direct reference to what is usually thought of as the weak interaction. The constants $\gamma_i, \gamma_i', \lambda_i$, and λ_i' will typically be chosen so as to yield a symmetry-respecting Lagrangian in the absence of L_{wI}. The universal weak interaction appears as a relatively strong Fermi-like interaction between the "currents" $K_\mu^\dagger$ and K_μ, which is then weakened by the buffering action of the relatively weak coupling of the $V_{\mu i}$ and φ_i to the $J_{\mu i}$.

The equations of motion generated by L_w are again linear, so that one can compute the spectral functions defined by Eq. (4.4) exactly. One finds instead of (4.5)

$$\rho_{1ij} = \alpha_i \alpha_j \frac{m^2}{m^2 - m_1^2}[\delta(m_1^2 - M^2) - \delta(m^2 - M^2)]$$
$$+ \delta_{ij}\gamma_i^2 \delta(m^2 - M^2), \qquad (4.19)$$

and instead of (4.6)

$$\rho_{2ij} = \frac{\alpha_i \alpha_j}{m_1^2}\left(\frac{1}{m^2 - m_1^2}[m^2\delta(m_1^2 - M^2) - m_1^2\delta(m^2 - M^2)]\right.$$

$$+ \frac{1}{\mu_1^2 - \mu^2}[\mu^2\delta(\mu_1^2 - M^2) - \mu_1^2\delta(\mu^2 - M^2)]\right)$$

$$+ \delta_{ij}\left(\frac{\gamma_i^2}{m^2}\delta(m^2 - M^2) + \frac{1}{\lambda_i^2}\delta(\mu^2 - M^2)\right), \qquad (4.20)$$

where

$$m_1^2 = m^2\left(1 - \sum_i \frac{\alpha_i^2}{\gamma_i^2}\right) \qquad (4.21)$$

and

$$\mu_1^2 = \mu^2 m_1^2 / (m_1^2 - \sum \alpha_i^2 \lambda_i^2). \qquad (4.22)$$

The spectral densities satisfy the positive-metric requirements provided m_1^2 and μ_1^2 are positive.

Evidently,

$$\int \rho_{1ij}dM^2 = \delta_{ij}\gamma_i^2,$$

$$\int \rho_{2ij}dM^2 = \delta_{ij}\left(\frac{\gamma_i^2}{m^2} + \frac{1}{\lambda_i^2}\right),$$

$$\int M^2\rho_{2ij}dM^2 = \delta_{ij}\left(\gamma_i^2 + \frac{\mu^2}{\lambda_i^2}\right). \qquad (4.23)$$

It is clear from (4.19) and (4.20) that the diagonal interaction is independent of L_{Iw} and hence that the diagonal interaction retains all symmetry properties which hold in the absence of L_{Iw}. Equation (4.23) together with Eq. (4.9) implies that the weak-interaction propagator behaves as $1/p^4$ at high momentum.

V. THE DIAGONAL INTERACTION AND ITS INFLUENCE UPON OTHER INTERACTIONS

One of the most characteristic features of the class of theories discussed in this section is the fact that the diagonal and weak interactions are to a large extent independent of one another. Thus, apart from inequalities such as $\alpha < 1$ in model I, which imply a minimum strength for the diagonal interaction, no natural connection emerges between the magnitudes of the α_i and the parameters which determine the strength of the diagonal interaction. The theoretical treatment that we have given does assume that the interaction Lagrangian is sufficiently weak to give relevance to an analysis in terms of perturbation theory, and we will continue to make this assumption in our subsequent analysis. We have not, however, found any intrinsic way of characterizing its strength. Indeed, the possibility that some of the extra particles introduced in our model might be strongly coupled cannot be excluded, although some modification of the theory presented seems necessary in that case. The comment of Ref. 10 may have some relevance in this connection.

It follows from the above observation that the universal weak interactions are less inclusive than is usually assumed. It is, of course, well known that what we would call diagonal interactions among the hadrons are not governed by the weak interactions. On the other hand, it has been conventional to assume that the scattering of electron neutrinos by electrons is so governed. According to our theory, however, no connection between this process and the weak interactions should be assumed. A similar remark applies to the experimentally less accessible scattering of μ neutrinos by muons. The experimental investigation of these questions would be especially relevant for theories of this type. In considering the feasibility of such experiments, the possibility that these processes may be

substantially stronger than is typically assumed should not be overlooked.

Because of the singular nature of the diagonal interaction, what one means by its strength is not entirely unambiguous. Repeating the arguments of Sec. I, one can show that unitarity considerations imply that perturbation theory must break down when, say, in the case of our models, $g^2q^2/24\pi\lambda^2$ or $g^2\gamma^2q^2/24\pi m^2$ are larger than unity. As a basis for discussion we shall take the view that the diagonal interaction does not lead to an inconsistent theory, but rather to a theory for which conventional perturbation theory is invalid no matter how weak the coupling constant. We shall assume, however, that order-of-magnitude estimates can be correctly made by evaluating perturbation-theory expressions and, when necessary, applying the cutoff, which is determined by the energy at which the Born approximation exceeds the unitarity bound. On this basis, the electron-neutrino-electron interaction at low energies is described phenomenologically by a Fermi interaction of order g^2/λ^2, $g^2\gamma^2/m^2$, or more generally $\int(\rho_{1ij}/M^2)dM^2$, $\int\rho_{2ij}dM^2$.

We now turn to the question of the effect of the diagonal interactions upon other interactions. Because the diagonal interaction yields self-energy and vertex integrals which are typically quadratically divergent in lowest order, and which diverge like Λ^{2n} in nth-order weak, corrections to these quantities are of order unity. In the case of the hadrons and the strong interactions, these effects can, of course, be simply amalgamated with the observed strong interactions. If the diagonal interaction respects all strong-interaction symmetries, then these large modifications will also do so. It may, however, be unnecessary to require the full symmetry for the diagonal interaction, because it is possible to construct special models of the strong interactions for which a partial symmetry of the diagonal interaction is sufficient to guarantee that symmetry violations be small. An example of such a theory is a triplet spin-$\frac{1}{2}$ model with strong coupling mediated by a vector unitary singlet and $SU(3)\times SU(3)$ broken only by mass terms. One may then regard the usual $V-A$ combination as a single indexed entity. Strangeness-changing and strangeness-nonchanging currents must be separately indexed and a neutral interaction between strangeness-changing currents must be introduced in order to avoid strong violations of charge independence. The only symmetry required is between the charged and neutral interactions of the strangeness-changing currents. The diagonal interaction is thus strangeness-conserving, but conserves neither isotopic spin nor parity; however, the gauge invariance of the unitary vector coupled to baryon number ensures that the strong effect of the isotopic spin and parity violation cancels in any process, leaving only a weak residue.

Because the diagonal interaction carries electric charge, electromagnetic effects can be discussed only within the framework of some specific model. We shall confine ourselves to a few general remarks here. First, it appears that the imposition of charge conservation will guarantee the usual $Z_1=Z_2$ relation which preserves the universality of the electromagnetic coupling. It also has the consequence that the most singular parity-violating effects in the electromagnetic vertex, which arise if the diagonal interaction is chosen to violate parity, are cancelled by similar effects in the propagator. Consequently, parity violation in the diagonal interaction for leptons leads only to weak parity violation in leptonic processes. Second, the diagonal interaction and its associated charged particles can be expected to have small but observable effects on the various quantum electrodynamics experiments, the magnetic moment of the muon being the most likely candidate. With reference to any specific model, existing experiments allow one to impose limits on some of the coupling coefficients appearing in the diagonal interaction.

We digress to summarize the situation with respect to parity violation in the diagonal interaction by listing three reasonable possibilities. (1) The entire diagonal interaction conserves parity. This guarantees that all parity violation arises from the nonsingular weak interaction and eliminates the possibility of larger-than-weak parity violation. It requires the introduction of right-handed neutrinos. We note that these can be introduced in a way which preserves the vanishing of neutrino masses. The right-handed neutrinos are not produced in weak decays nor are they produced in the scattering of left-handed neutrinos on hadrons or leptons. They are produced, however, in reactions like $e^+ + e^- \rightarrow \nu + \bar{\nu}$ and consequently can have astrophysical implications. (2) The diagonal interaction among the hadrons only conserves parity. In the case of the leptons the vector and axial-vector currents are not separately indexed, the usual $V-A$ combination being regarded as a single current. In this case, the diagonal interaction among the leptons is parity-violating. Parity violation does, however, continue to be weak, that is, as weak as the diagonal interaction. This choice has the possible advantages of eliminating the introduction of right-handed neutrinos and reducing the number of intermediate mesons required. Its principal disadvantage is the lack of symmetry in the treatment of electrons vis-à-vis the hadrons. (3) The vector and axial-vector currents appear in the combination $V-A$ everywhere. As compared to (2) above, this has the additional advantage of restoring some symmetry between hadrons and leptons and further reducing the number of required intermediate mesons. On the basis of the estimation methods that we are using, this choice can, however, lead to strong parity violation in hadronic processes. We are thus led to require a special strong-interaction model for which large violations vanish.

We turn now to the effect of the diagonal interaction upon the nonsingular weak interactions. We note first
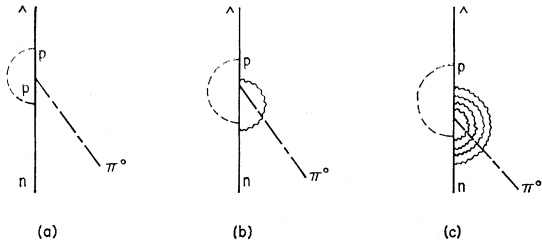
FIG. 1. Typical Feynman diagrams for the process $\Lambda \to \pi^0 + n$. The effects of strong interactions have been omitted. Diagonal propagator, wavy lines; nonsingular weak propagator, dashed lines.

of all that processes mediated by weak propagators alone are at worst logarithmically divergent and are finite for propagators as well behaved as those of model II. The order of the process is equal to the number of weak propagators which appear. For a given process there is a minimum number of such propagators which must appear. Contributions containing only this minimum number yield the lowest-order perturbation-theory result. The inclusion of the strong and electromagnetic interactions presumably do not change the situation qualitatively. We illustrate these remarks by the process $\Lambda \to \pi^0 + n$ (Fig. 1) and $\Sigma^+ \to p + \mu^+ + \mu^-$ (Fig. 2), with hadronic strong interactions ignored. The neutral lepton pair decay of the Σ^+ is clearly second-order weak and finite. The Λ decay is first-order weak and, for model I, logarithmically divergent. Within the framework of our cutoff approach such a divergence is acceptable and not in disagreement with experiment.

The large effects of the diagonal interaction are of the form of Z_1 vertex modifications and Z_2 propagator modifications. These do not change the scale of the weak interactions but can lead to order unity corrections to the weak-interaction coupling constant. Phenomenologically, one can deal with this problem by assuming that the conventional coupling constants of the weak interactions refer to the interaction after the renormalization due to the diagonal interaction has been carried out. From a more fundamental viewpoint it would be preferable to formulate the theory in such a way that the large renormalizations are a universal factor. The evident similarity between electrons and muons makes equal renormalization for these particles quite natural.
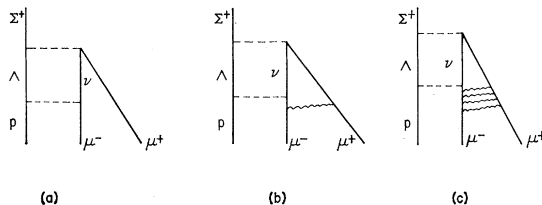

FIG. 2. Typical Feynman diagrams for the process $\Sigma^+ \to \varphi + \mu^+ + \mu^-$. The effects of strong interactions have been omitted. Diagonal propagator, wavy lines; nonsingular weak propagator, dashed lines.

In order to include the hadrons, one could require a similarity of structure at high energies for the hadrons and leptons. For example, one might postulate the existence of a heavy neutral electron and a heavy neutral muon to form leptonic triplets in analogy with an assumed fundamental hadronic triplet. One might also take the view that on account of the weak-interaction angle the establishment of universality is not so precise as to eliminate the admissibility of a small (but large compared to weak) difference between the hadronic and leptonic renormalization factors.

We remark that strong-interaction current conservation may continue to play its customary role with respect to the renormalization effects of the strong interactions. So long as the strong interactions couple entirely exteriorly to diagonal-interaction modified vertices, the customary arguments are unchanged. Diagrams in which the strong interactions invade the diagonal-interaction vertex structure are assumed to be weak because of a damping effect of the strong interactions. This is equivalent to the assumption that the dominant singularity of structures like

$$T^*(J_{\mu_1}(x_1), \cdots, J_{\mu_n}(x_n)),$$

where the J_{μ_i} are hadronic currents, occurring when all coordinates are close to one another compared to hadronic distances, are independent of the strong interactions.

Typical nonvertex corrections to weak interactions due to the diagonal interactions are illustrated in Figs. 1(b), 1(c), 2(b), and 2(c). On the basis of an elementary denominator count, Fig. 2(b) is seen to be logarithmically divergent, hence yielding a correction of relative order $g^2 \ln(\Lambda^2/m^2)$. The diagrams of Fig. 2(c) are of order $g^2(g^2\Lambda^2/m^2)^{n-1}$, where $n-1$ is the number of diagonal interchanges. Thus all higher-order corrections to the neutral decay of the Σ^+ are "first-order" weak compared to the lowest-order process. That is, the corrections do not continue to decrease with increasing order. The process Fig. 1(b) is of order $(g^2\Lambda^2/m^2)/\ln(\Lambda^2/m^2)$ relative to the lowest-order process for model I. Since the numerator is presumably of order unity, the lowest-order process may still dominate. On the other hand, for more convergent models the relative order is $g^2 \ln(\Lambda^2/m^2)$. The higher-order processes of Fig. 1(c) are of relative order $(g^2\Lambda^2/m^2)^n/\ln(\Lambda^2/m^2)$ for model I and of order $g^2(g^2\Lambda^2/m^2)^{n-1}$ for more convergent models.

These examples illustrate the following state of affairs. (1) If the weak-interaction propagator falls off as $1/p^4$ or faster at large momentum, then every weak-interaction process is finite in the lowest order n in which it occurs and of order g^{2n} in the weak-coupling constant. If no diagonal interaction is involved in the lowest-order process, then higher-order corrections to it due to the diagonal interaction are weaker by a factor $g^2 \ln(\Lambda^2/m^2)$ or g^2 independent of the order to which the diagonal interaction occurs. (2) If the weak-interaction

propagator falls off as $1/p^2$ at higher momentum, then some processes may be of order $g^{2n} \ln(\Lambda^2/m^2)$ in lowest order. For these processes, higher-order corrections due to the diagonal interaction are reduced only by a factor $1/\ln(\Lambda^2/m^2)$. While we have not carried out a systematic and complete investigation, we conjecture that these properties are general.

VI. APPLICATIONS

We consider now two partial lifetimes which give fairly stringent limits on the masses of the intermediate bosons. These are

(1) $\tau_{K^0 \to \mu^+ + \mu^-} > 2 \times 10^6 \tau_{K^+ \to \mu^+ + \nu}$

(2) $\tau_{K_s \to 2\pi} = 0.9 \times 10^{-10}$ sec.

We consider also the $K_1 - K_2$ mass difference

$$m_{K_2} - m_{K_1} = 0.5(1/\tau_{K_s}).$$

We remind the reader that processes which are quadratically divergent in the conventional IVB theory become finite (or logarithmically dependent on the weak cutoff) in our theory, with the quadratic divergence replaced by the square of a mass which is some weighted average of the intermediate boson masses. If this mass is large compared to the characteristic hadron mass scale, then the formerly divergent terms continue to dominate.

The most reliable upper limit is obtained from process (1). The weak propagator is written as

$$\Delta_{\mu\nu}{}^w = \delta_{\mu\nu}\Delta_1(q^2) + q_\mu q_\nu \Delta_2(s^2). \qquad (6.1)$$

We have found that when Δ_1 and $q^2\Delta_2{}^2$ in (6.1) have comparable high-q^2 behavior, Δ_1 dominates. If Δ_1 has the form $1/(q^2+M^2)$, as in our model I, we obtain an upper limit for M which depends on the algebra of the weak currents: For the quark model, for example, Mohapatra *et al.* find $M \lesssim 38$ BeV, whereas the LWZ[11] model gives $M \le 50$ BeV.

Process (2) has been considered in the partially conserved axial-vector current approximation by Glashow, Schnitzer, and Weinberg.[12] Their calculation may be taken over for our theory, where again we ignore the $q_\mu q_\nu$ contribution. They find

$$M \sim 8 \text{ BeV},$$

which should probably also be taken as an upper limit, since the $K \to 2\pi$ rate may well be a much more rapidly growing function of M^2 than is indicated by their calculation. In particular, if the second Weinberg sum rule fails to hold, the rate grows quadratically with M rather than logarithmically (for fixed G).

We mention briefly the problem of the $K_1 - K_2$ mass difference, which has also been treated by all the authors of Ref. 2, and by Olesen.[13] The calculation depends on the evaluation of strong-interaction matrix elements of products of currents, so that it is fairly model-sensitive. A typical contribution is quadratically dependent on M, and yields a value $M \lesssim 3$ or 4 BeV. One can only view these results as order-of-magnitude estimates; nevertheless, it is hard to see how a much larger value of M could be tolerated.

In summary, if the class of models proposed here is correct, we would expect to find intermediate weak bosons somewhere in the mass range 2–8 BeV.

Once a cutoff of that magnitude has been established, it becomes particularly interesting to investigate the nature of the most singular terms. For example, as has been observed by Mathur and Olesen,[14] most models give the most singular term in the nonleptonic decays as a commutator, which may well have octet properties and therefore simply account for the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule.

ACKNOWLEDGMENTS

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