crossed ρ poles give a negligible contribution. Substituting into (4.2),

$$\Gamma_{\omega\to\pi\gamma}\approx(1.5\text{ MeV})\times[f_{\pi\omega}(0)]^2, \qquad (4.6)$$

while experimentally

$$\Gamma_{\omega\to\pi\gamma}\approx 1.2\pm0.1\text{ MeV}.$$

Using Eqs. (4.3)–(4.5) to calculate $f_{\omega\pi}(0)$ by the same techniques as before gives

$$f_{\omega\pi}(0)=1.03\pm0.08,$$

and thus

$$\Gamma_{\omega\to\pi\gamma}{}^{\text{theor}}=1.6\pm0.3\text{ MeV},$$

in fair agreement with the experimental value.

In conclusion, it has been demonstrated that isovector form factors can be calculated quite accurately at low momentum transfer using two-pion intermediate states only. The ingredients used included an effective-range formula for the pion form factor, expressions for annihilation amplitudes that satisfy elastic unitarity while treating the principal left-hand cuts properly, and an assumption of universality for ρ-meson couplings.

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High-Energy Nucleon Scattering and Electromagnetic Form Factors

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A field-theoretic model of the exchange of virtual soft neutral vector mesons between nucleons is shown to provide the needed damping to describe pp elastic scattering and the electromagnetic form factors of the proton at high energies and large momentum transfers. The model permits a calculation of the infinite-energy limit of pp scattering, and suggests that $p\bar{p}$ scattering will rise to that limit at very high energies.

I. INTRODUCTION

In this paper we would like to present a high-energy, large-momentum-transfer parametrization of nucleon elastic scattering amplitudes and electromagnetic form factors. The model is a field-theoretic realization of the Yang-Wu idea[1] in which the proton's form factors are related to the infinite-energy limit of the pp scattering amplitude, here generalized to provide a quantitative description of elastic scattering at large but finite energies. The underlying physical mechanism adopted is the exchange of virtual massive neutral vector mesons between the nucleons entering into any hadronic process. Such mesons could be regarded as the quanta of a field coupled to a conserved baryonic current, although this interpretation is not obligatory. The model is defined by extracting the "soft" meson contributions of all such exchanges, leaving a remaining "hard," or nonsoft part unspecified. For zero-mass mesons, it is necessary to consider simultaneously the soft part of virtual and real processes, in order to remove the latter's infrared divergences. For massive mesons, there are no infrared divergences and multiple soft real emissions are strictly meaningful only in the limit of infinitely high nucleon energies. On the other hand, the soft virtual processes must, in principle, contribute to every amplitude, and remain as a possible source of interesting momentum transfer and energy dependence.[2] Neutral mesons are used because their soft effects can be obtained in closed form, without the inhibiting isotopic complications[3] of charged mesons;

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‡ Supported in part by the U. S. Atomic Energy Commission [Report No. AT(30-1)2098].

[1] T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965). Similar and related arguments have been made by H. Abarbanel, S. Drell, and F. Gilman, Phys. Rev. Letters **20**, 280 (1968), and by T. T. Chou and C. N. Yang, *ibid.* **20**, 1213 (1968).

[2] This approach is complementary to that of G. Mack, Phys. Rev. **154**, 1617 (1967), who inferred the behavior of elastic-scattering amplitudes by an examination of the inelastic emission of soft quanta.

[3] The essence of this approach and of all such "soft" methods is the independent emission, real or virtual, of successive quanta. If the mesons are charged, the fact that they may be soft is not sufficient to provide the necessary degree of solubility for the model, since there must still be correlations between them; e.g., a proton cannot emit two positive pions in succession, no matter how soft. A discussion of this point has been given by H. M. Fried, Summer Institute Lectures, University of Colorado, Boulder, 1968 (unpublished).

vector mesons are used because the resulting s, t distributions appear to fit the experiments.

The input information of this model, derived from the basic interaction $\mathcal{L}'=ig\bar{\psi}\gamma_\mu A_\mu\psi$, is described in terms of three constants, g, μ, and μ_c, denoting the (bare) coupling between meson and nucleon fields, the (bare) mass of the vector mesons, and a momentum cutoff needed to distinguish the soft quanta, respectively; the latter will be described in detail shortly. Upon extracting the desired soft portion of the virtual-meson exchanges, we combine these parameters to form a single constant γ which is then determined by comparison with the experimental data. In this way, with one parameter, it is possible to reproduce the gross features (to within $\sim 10\%$ of the experimental values of $\log[(d\sigma/dt)/(d\sigma/dt)_{t=0}]$) of the elastic pp data of Allaby *et al.*[4] for large s and $-t \geqslant 9$ BeV^2, indicating that soft contributions can provide the needed damping at large energies and momentum transfers. In conjunction with a scaling parameter drawn from a low-momentum-transfer description, we can fit the recent Stanford Linear Accelerator Center (SLAC) data[5] for the proton's form factors at $-t \geqslant 7$ BeV^2; one can, in fact, construct an excellent fit to the form factors for all measured momentum transfers. The physical picture which thus emerges is one in which the hard mesons make possible large-momentum-transfer scatterings which the soft mesons tend to damp out.

The model predicts that the normalized $p\bar{p}$ elastic scattering will *rise* to the same infinite-energy limit to which the corresponding pp scattering descends. If the pp system may be said to exhibit s-dependent damping at large $-t$, then the $p\bar{p}$ system should display a corresponding growth. The physical origin of this difference is, in this model, due to the circumstance that if a nucleon interacts with a neutral vector-meson field with coupling constant g, an antinucleon interacts with the same field with charge $-g$. Because the present model neglects interference between soft and hard processes, which is probably where Regge-type behavior arises, it cannot describe the shrinkage and dip-bump patterns appearing at lower energies and momentum transfers; but one might expect that the observed antishrinkage in the $p\bar{p}$ system at lower energies has the same simple origin.

The introduction of a second parameter in the characterization of the soft quanta permits an even better reproduction of the pp-scattering data, providing a quantitative fit (to within 5% of the experimental values of $\log[d\sigma/dt/(d\sigma/dt)_{t=0}]$) for $s \geqslant 20$ BeV^2 and $\theta_{\text{c.m.}} \geqslant 65°$. In order to describe the origin of this second parameter, it is useful to recall similar operations performed in the extraction of soft virtual quanta in electrodynamics. A graphical treatment may be found in the classic paper of Yennie, Frautschi, and Suura[6]; an heuristic procedure for obtaining the results of this paper is to start from the complete, virtual soft-photon contribution (the "B" integrals of YFS) to any process, give the photon a mass μ, coupling constant g, and to cut off the virtual photon 4-momentum k at μ_c. This last step can be done in a superficially invariant way by cutting off k^2 at μ_c^2, and will be described in detail in Sec. II. In the YFS method, where each fermion line of four-momentum p contributes a factor $(2p_\mu+k_\mu)/(k^2+2p\cdot k)$, a cutoff is not necessary, since the renormalized, momentum-transfer-dependent integral converges and effectively cuts itself off at $\mu_c^2 \cong -t$, for $-t \gg m^2$, where m denotes the fermion mass. One then finds an over-all result for the B integral proportional to $\ln^2(-t)$. We have chosen the somewhat more conservative approach of using the simpler factor $p_\mu/p\cdot k$ and effectively cutting off the integral at lower values of k. This avoids including contributions for soft $k \gg m$, with the result that, for constant μ_c^2 and at large $-t$, we find a result proportional to $\ln(-t)$. With either method, one constructs in this way an exponential dependence upon the appropriate external invariant variables, t, s, $\cdots$ of the form $e^{\gamma[F(t)\pm F(s)\pm\cdots]}$, which after proper renormalization multiplies the remaining, unspecified hard-meson dynamics; the latter is, of course, actually responsible for scattering at large momentum transfers. In our one-parameter calculation, γ is approximately given by

$$\gamma=(g^2/4\pi)(1/2\pi)\ln[1+(\mu_c^2/\mu^2)],$$

and the function $F(t)$ is a once subtracted, elementary integral over two-particle phase space, displayed in Sec. II.

The simplest choice, $\mu_c^2=$ constant, is sufficient to provide an accurate fit to the proton form factors. Rather than the asymptotic behavior $G\sim(-t)^{-\gamma}$ of this analysis, the YFS choice $\mu_c^2 \cong -t$ produces $G\sim(-t)^{-\gamma'\ln(-t)}$, which is of the form of Mack's curve (Ref. 2); our agreement with the data seems to be as good if not better, since our curve of Fig. 1 is almost identical to Mack's but runs slightly higher at large $-t$. The corresponding fit (Fig. 2) to the more accurate pp-scattering data contains systematic deviations from the experiments. We have found that the introduction of a second parameter appearing in the form $\mu_c^2/\mu^2=c_1+c_2/|t|$, or more generally $c_1+c_2/|z|$, where z denotes the appropriate invariant variable, is sufficient to give better agreement with the pp-scattering data (Fig. 3). The form of such dependence is difficult to understand, as is the fact that the effective γ for pp scattering differs from the constant γ used in the proton form

[4] J. V. Allaby, G. Cocconi, A. N. Diddens, A. Klovning, G. Matthiae, E. J. Sacharidis, and A. M. Wetherell, Phys. Letters **25B**, 156 (1967).

[5] D. H. Coward, H. DeStaebler, R. A. Early, J. Litt, A. Minten, L. W. Mo, W. K. H. Panofsky, R. E. Taylor, M. Breidenbach, J. I. Friedman, H. W. Kendall, P. N. Kirk, B. C. Barish, J. Mar, and J. Pine, Phys. Rev. Letters **20**, 292 (1968).

[6] D. Yennie, S. Frautschi, and H. Suura, Ann. Phys. (N. Y.) **13**, 379 (1961), hereafter referred to as YFS.

factor fit.[7] Nevertheless, it is hard to believe that such two-parameter reproduction of the experimental data is merely fortuitous; rather, it suggests that there probably is a heavy, neutral vector meson which strongly decays into a nucleon and antinucleon pair.

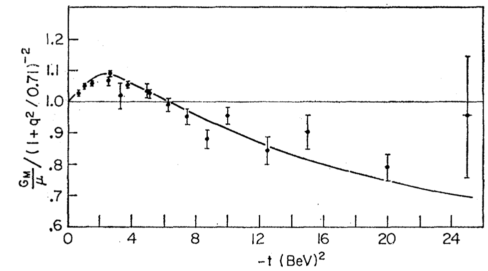

FIG. 1. A plot of $(1-t/0.4)^{-1}e^{1.4F(t)}$ superimposed upon the data points of Coward *et al.* (Ref. 5) and normalized to the dipole fit. On the ordinate, $q^2=-t$ is in BeV^2.

II. DETAILS OF MODEL

The removal of the infrared divergences in electrodynamics is an excellent example of the efficacy of functional methods in those situations where a sufficient degree of solubility exists, and permits the formal, functional solutions for any amplitude to be evaluated without the need for a perturbation expansion. These procedures shall be followed here in order to perform the necessary derivations in the most compact way, and to set the stage for subsequent generalizations. We are here concerned with the soft, virtual quanta exchanged between the nucleon "legs" of the vertex function and scattering amplitude, and for clarity shall sketch in detail the manipulations needed to extract the soft-meson dependence of these processes.

A. Vertex Function

In a theory with nucleon, pion, and photon fields, and containing the assumed neutral vector-meson field as well, the unrenormalized, configuration space, nucleon-photon vertex function, amputated on the photon coordinate z, has, to lowest order in the electric charge e, the exact functional representation

$$V_\mu(y,x;\bar{z})=\exp\left(-\tfrac{1}{2}i\frac{\delta}{\delta\pi}\Delta_c{}^{(\pi)}\frac{\delta}{\delta\pi}\right)\times\exp\left(-\tfrac{1}{2}i\frac{\delta}{\delta A}\Delta_c{}^{(A)}\frac{\delta}{\delta A}\right)G(y,z|\pi,A)\times\tfrac{1}{2}e(1+\tau_3)\gamma_\mu G(z,x|\pi,A)\frac{e^{L[\pi,A]}}{N}\Bigg|_{\pi=A=0},\quad(1)$$

where $\frac{1}{2}e(1+\tau_3)\gamma_\mu$ denotes minimal coupling to the proton, $L(\pi,A)$ is the closed-nucleon-loop functional with associated vacuum normalization constant N, while the $G(a,b|\pi,A)$ are nucleon propagators defined in the presence of c-number pion ($\boldsymbol{\pi}$) and vector-meson (A_μ) fields, as would be the case for the potential theory interaction $\mathcal{L}'=\bar{\psi}[V(\pi)+ig\gamma_\mu A_\mu]\psi$. The use of the physical nucleon mass m in G implies that a mass-renormalization counter-term should be included in the interaction.

The expansion of the functional differentiation operators of (1) yields all the linkages which define the complete set of radiative corrections, produced by the virtual emission of all pions and vector mesons. We wish, however, to isolate the vector-meson exchanges between nucleon legs which would, in the case of zero-mass vector mesons, contribute to the infrared-divergent part of the vertex function. A straightforward way to accomplish this is by an analysis of the Fradkin type,[8] but a somewhat simpler method makes use of an algebraic rearrangement, as follows: We divide the complete interaction $U=V(\pi)+ig\gamma\cdot A$ into a soft part $U_S=ig\gamma\cdot A^{(S)}$, and a hard part, $U_H=V(\pi)+ig\gamma\cdot A^{(H)}$, so that $A_\mu=A_\mu{}^{(S)}+A_\mu{}^{(H)}$, $U=U_S+U_H$. One can, in some arbitrary but definite Lorentz frame, consider the $\tilde{A}_\mu{}^{(S)}(k)$ to be defined with Fourier components k_ν less than some preassigned value, and conversely for the $\tilde{A}_\mu{}^{(H)}(k)$, leaving the relativistic invariance to be enforced at a later stage. As shown in Ref. 3, it does not appear to be necessary to separate out a soft-pion contribution if the soft-pion–nucleon interaction is of chiral form. Using this decomposition, G can be written in the form $G_S(1+UG)=G_S(1-U_HG_S)^{-1}\equiv G_SK$, with $G_S=(m+\gamma\cdot\partial-U_S)^{-1}$; and (1) may be replaced by

$$V_\mu(y,x;\bar{z})=\left[\exp\left(-\tfrac{1}{2}i\frac{\delta}{\delta A^{(S)}}\Delta_c{}^{(A)}\frac{\delta}{\delta A^{(S)}}\right)\right]\times\int G_S(y,w|A^{(S)})V_\mu{}^{(H)}(\bar{w},\bar{u};\bar{z}|A^{(S)})\times G_S(u,x|A^{(S)})\Bigg|_{A^{(S)}=0},\quad(2)$$

where

$$V_\mu{}^{(H)}(\bar{y},\bar{x};\bar{z}|A^{(S)})=\exp\left(-\tfrac{1}{2}i\frac{\delta}{\delta A^{(H)}}\Delta_c{}^{(A)}\frac{\delta}{\delta A^{(H)}}\right)\exp\left(-\tfrac{1}{2}i\frac{\delta}{\delta\pi}\Delta_c{}^{(\pi)}\frac{\delta}{\delta\pi}\right)\times N^{-1}e^{L[\pi,A]}\int K(y,z|A,\pi)\times\tfrac{1}{2}e(1+\tau_3)\gamma_\mu K(z,x|A,\pi)\Bigg|_{A^{(H)}=\pi=0}.\quad(3)$$

[7] For zero-mass neutral vector mesons, these constants would depend upon the real soft-meson emissions, and hence on the details of the experimental arrangements; it may be that a related dependence upon the energy resolution for a large but finite number of emitted low-energy massive mesons is involved here.

[8] E. S. Fradkin, Nucl. Phys. **76**, 588 (1966). The formal functional solutions leading to expressions of the type (1) were first written by J. Schwinger, Harvard Lecture Notes, 1954 (unpublished), and K. Symanzik, Z. Naturforsch. **9**, 809 (1954). An application to soft photons may be found in K. Mahanthappa, Phys. Rev. **126**, 329 (1962). The derivation used here follows that presented by H. Fried, Winter Lectures in Karpacz, 1967 (unpublished), and Ref. 3.

The desired soft-vector-meson dependence arises from exchanges between the two G's written explicitly in (2); whatever else may occur, this dependence will always be present and may be exhibited by rewriting (2) in the form

$$V_\mu(y,x;\bar{z})=\exp\left(-i\frac{\delta}{\delta A_1^{(S)}}\Delta_c^{(A)}\frac{\delta}{\delta A_2^{(S)}}\right)\times\int G_S(y,w|A_1^{(S)})\Gamma_\mu^{(H)}(w,u;z)\times G_S(u,x|A_2^{(S)})\Big|_{A_1^{(S)}=A_2^{(S)}=0}. \quad (4)$$

The notation of (2) and (4) suggests that $\Gamma_\mu^{(H)}(y,x;z)$ may be considered as the proper vertex function, containing everything except the desired soft-meson exchanges, and this would be true were not the hard-nucleon self-energy effects still included in $\Gamma_\mu^{(H)}$. Such effects can easily be recognized upon performing a perturbation expansion of (3), and will, if G already contains the physical nucleon mass m, influence our calculation only to the extent of providing extra wave-function renormalizations upon mass-shell amputation of the nucleon legs. It is therefore simplest to neglect all self-energy processes completely, both hard and soft, and hence to drop all associated renormalization constants, performing the necessary vertex renormalization at the end of the calculation.

To obtain the momentum-space vertex function $\tilde{\Gamma}_\mu(p',p)$, with p and p' on their respective mass shells, we require the amputated function

$$(2\pi)^{-4}\int dz\, e^{-ik\cdot z}\int dx\, e^{ip\cdot x}\int dy\, e^{-ip'\cdot y}(m+i\gamma\cdot p')\times V_\mu(y,x;\bar{z})(m+i\gamma\cdot p)$$

or

$$\delta(p'+k-p)\tilde{\Gamma}_\mu(p',p)=(2\pi)^{-4}\int dz\, e^{-ik\cdot z}\int dw\int du\times\left[\exp\left(-i\frac{\delta}{\delta A_1^{(S)}}\Delta_c^{(A)}\frac{\delta}{\delta A_2^{(S)}}\right)\right]\times\int dy\, e^{-ip'\cdot y}(m+i\gamma\cdot p')G_S(y,w|A_2^{(S)})\Gamma_\mu^{(H)}(w,u;z)\times\int dx\, e^{ip\cdot x}G_S(u,x|A_1^{(S)})(m+i\gamma\cdot p)\Big|_{A_1=A_2=0}, \quad (5)$$

and must now calculate the G_S combinations of (5). The basic differential equation for the nucleon propagators

$$[m+\gamma\cdot\partial-ig\gamma\cdot A^{(S)}]G_S=1 \quad (6)$$

is now approximated by replacing each γ_μ by a constant four velocity $-iv_\mu$, where $v^2=-1$; this is the well-known Bloch-Nordsieck model[9] which neglects the radiative recoil of the soft mesons on the emitting nucleons. In the propagator $G_S(u,x|A_1^{(S)})$ we make the replacement $v_\mu\to p_\mu/m$, while in $G_S(y,w|A_2^{(S)})$ we use $v_\mu'\to p_\mu'/m$, as the appropriate four velocities in each nucleon leg. Within such a no-recoil model, the solution to the modified equation

$$[m-iv\cdot(\partial-igA)]G_S=1 \quad (7)$$

may be written in parametric form as

$$G_S(a,b)=i\int_0^\infty d\xi\, e^{-i\xi m}\exp\left(ig\int_0^\xi d\eta\, v_\mu A_\mu(a-\eta v)\right)\times\delta(a-b-\xi v), \quad (8)$$

from which it is not difficult to see that the amputated, mass-shell (m.s.) combinations necessary for (5) may be evaluated as[10]

$$\int dx\, e^{ip\cdot x}G_S(u,x)(m+v\cdot p)\Big|_{\text{m.s.}}=e^{ip\cdot u}\exp\left(ig\int_0^\infty d\xi\, v\cdot A(u-\xi v)\right), \quad (9a)$$

$$\int dy\, e^{-ip'\cdot y}(m+v'\cdot p')G_S(y,w)\Big|_{\text{m.s.}}=e^{-ip'\cdot w}\exp\left(ig\int_0^\infty d\xi\, v'\cdot A(w+\xi v)\right). \quad (9b)$$

The functional operations of (5) can now be easily performed and yield

$$\delta(p'+k-p)\tilde{\Gamma}_\mu(p',p)=(2\pi)^{-4}\int dz\, e^{-ik\cdot z}\int dw\, e^{-ip'\cdot w}\int du\, e^{ip\cdot u}\Gamma_\mu^{(H)}(w,u;z)\times\exp\left(ig^2\int_0^\infty d\xi\int_0^\infty d\eta\, v\cdot v'\Delta_c^{(A)}(u-w-\xi v-\eta v')\right), \quad (10)$$

or

$$\tilde{\Gamma}_\mu(p',p)=\int dx_2\, e^{-ip'\cdot x_2}\int dx_1\, e^{ip\cdot x_1}\tilde{\Gamma}_\mu^{(H)}(x_2,x_1)\exp\left(ig^2\int\int_0^\infty d\xi d\eta\, v\cdot v'\Delta_c^{(A)}(x_1-x_2-\xi v-\eta v')\right), \quad (11)$$

[9] F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937).
[10] A discussion of the operations involved here may be found in H. Fried and G. Erickson, J. Math. Phys. 6, 414 (1965).

where we have used the fact that $\Gamma_\mu^{(H)}$ can only depend upon the differences of its three coordinates, here chosen as $\Gamma_\mu^{(H)}(x_2,x_1)=\Gamma_\mu^{(H)}(w-z,u-z)$. All of the soft-meson dependence now arises from the exponential factor of (11), which may be written in the form

$$ig^2\int_0^\infty d\xi\int_0^\infty d\eta\, v\cdot v'\Delta_c^{(A)}$$

$$=-i\frac{g^2}{(2\pi)^4}(p\cdot p')\int\frac{d^4k}{k^2+\mu^2}\frac{e^{ik\cdot(x_1-x_2)}}{(k\cdot p-i\epsilon)(k\cdot p'-i\epsilon)}. \quad (12)$$

Were $\mu^2=0$, Eq. (12) would exhibit the familiar infrared divergence at very small values of k_μ, with the $e^{ik\cdot(x_1-x_2)}$ phase contributing nothing to the divergent behavior. On the other hand, for $\mu^2\neq0$ there is no infrared divergence, but this phase may be expected to be small when $k\cdot x_{1,2}<1$, with typical $x_{1,2}$ values dependent upon the structure of $\Gamma_\mu^{(H)}$. The most conservative estimate is that $x_{1,2}\sim\mu_0^{-1}$, where μ_0 represents the smallest (e.g., pion) mass exchanged in the construction of $\Gamma_\mu^{(H)}$; but at very large values of momentum transfer $-t$, one might expect $x_{1,2}\sim|t|^{-1/2}$, and hence larger values of $k_\mu^{\max}$ would be permissible before the phase deviates significantly from unity. The approximation of replacing this phase by unity decouples soft and hard effects, permitting $\tilde{\Gamma}_\mu(p',p)$ to differ from $\tilde{\Gamma}_\mu^{(H)}(p',p)$ by the multiplicative factor of interest here; corrections to this approximation serve to define interference, or correlations, between the soft and hard dependence, and will be discussed in a separate note.

Renormalization may be imposed, as in Ref. 6, by simply subtracting from the resulting exponential factor its value at zero momentum transfer; for this reason the $k_\mu k_\nu/\mu^2$ part of the vector-meson propagator has been omitted above, since its contribution would be removed by renormalization. One finds

$$\tilde{\Gamma}_\mu(p',p)\approx\tilde{\Gamma}_\mu^{(H)}(p',p)e^{\gamma F(t)}, \quad (13)$$

where

$$F(t)=t\int_{4m^2}^\infty\frac{dt'}{t'}\frac{1}{t'-t}\left(1-\frac{2m^2}{t'}\right)\left(1-\frac{4m^2}{t'}\right)^{-1/2}, \quad (14)$$

and γ is the constant mentioned previously. It is at this point that one must specify how the constant μ_c is to enter, so that overly large values of k_μ can be circumvented in the phase-approximated integral of (12), while relativistic invariance is maintained. To enforce invariance, we introduce an effective cutoff, at $k^2\sim\mu_c^2$, into the calculation, but then integrate over all k_μ. Were we dealing with a simple integral, e.g., of form

$$\int\frac{d^4k}{k^2+\mu^2}\phi(k^2),$$

where ϕ is limited only by the restriction that passage to a convergent Euclidean integration is possible, there would be no difficulty in effectively limiting the k_μ in this way, since after the k_0 contour is rotated, we obtain, with $\lambda=k^2$, just

$$\pi^2 i\int_0^\infty\frac{\lambda d\lambda}{\lambda+\mu^2}\phi(\lambda);$$

and the insertion under the integral of the real, positive damping factor $e^{-\beta\lambda}$ serves effectively to cut off all k_μ at $\sim\beta^{-1/2}$. Since our integral has a more complicated structure, we introduce the cutoff in a two-step fashion, by (i) inserting the factor $e^{-i\alpha k^2}$ under the integral, with α real and positive, and (ii) after evaluation performing the continuation $\alpha\to-i\beta$, where β is real and $\beta^{-1}=\mu_c^2$. This prescription is certainly not unique; but it is simple,[11] covariant, and provides reasonable results which can be compared with those of YFS in the large-momentum-transfer limit.

For negative t, F is real and negative and is given by

$$1-\frac{2x+1}{[x(x+1)]^{1/2}}\ln(\sqrt{x}+(x+1)^{1/2}), \quad x\equiv-t/4m^2,$$

with the limiting forms $t/3m^2$ and $1-\ln(-t/m^2)$ for small and large $-t$, respectively. Because of the multiplicative form of (13), the soft-meson dependence $e^{\gamma F(t)}$ will appear in every nucleon form factor. We shall discuss the detailed fits to the proton's form factors in Sec. III, and only remark here that a good fit can be obtained to the SLAC data at large momentum transfer in the form $G(t)=ce^{\gamma F(t)}$, using the scaling constant $c=0.16$ and choosing $\gamma=2.4$.

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[11] It is simplest to combine the p and p' denominators, and then write parametric representations for all denominators, so that the phase-approximated integral of (12) becomes

$$I=-ig^2(2\pi)^{-4}(4p\cdot p')(-i)\int_0^1dx\int_0^\infty cdc\int_0^\infty da\, e^{-ia\mu^2}\int d^4k \times\exp[-i(a+\alpha)k^2-2ick\cdot\bar{p}],$$

where $\bar{p}=xp+(1-x)p'$ and $-\bar{p}^2=m^2+x(1-x)(-t)>0$ for $t<4m^2$. The integral over k is an elementary Gaussian, and may be followed by integration over the c variable to yield $I=\gamma f(t)$, where $f(t)=p\cdot p'\int_0^1dx/[-\bar{p}]^2$ is real and negative for physical t, and

$$\gamma=\frac{g^2}{4\pi}\frac{1}{2\pi}\int_0^\infty\frac{da}{a+\alpha}e^{-ia\mu^2}\to\frac{g^2}{4\pi}\frac{1}{2\pi}\int_0^\infty\frac{db}{b+\beta}e^{-b\mu^2},$$

a positive constant, when the continuation $\alpha\to-i\beta$ is made. Approximating the b integral by $\int_0^{\mu^{-2}}db/(b+\beta)$ leads to the estimate of γ quoted in the Introduction. A more complicated result, but one having the same phase, is obtained by use of the YFS procedure, in which our $(2k\cdot p)^{-1}(2k\cdot p')^{-1}$ is replaced by $(k^2+2k\cdot p)^{-1}(k^2+2k\cdot p')^{-1}$ and no cutoff is required. (More precisely, one subtraction will be necessary when extra k dependence is inserted, for reasons of gauge invariance, into the numerator of the integral; but this complication is irrelevant to the present discussion.) That integral has the parametric representation

$$\frac{g^2}{4\pi}(p\cdot p')\int_0^1dx\int_0^1ydy[\mu^2(1-y)-y^2\bar{p}^2]^{-1}$$

and is also real and negative for physical t.

B. Nucleon-Nucleon Scattering

We very briefly sketch the quite similar steps involved in extracting the soft-meson dependence of the pp-scattering amplitude. In this process there are four nucleon legs to be approximated by four Bloch-Nordsieck Green's functions, each represented by (8) with the appropriate mass-shell amputations given by (9). Again, we extract only the cross-linkages between each pair of nucleon legs, and so obtain an equation analogous to (10) describing the elastic scattering amplitude for the reaction $p_1+p_2\to p_1'+p_2'$

$$\delta(p_1+p_2-p_1'-p_2')\tilde{M}(p_1,p_2,p_1',p_2') = (2\pi)^{-4}\int du_1 e^{ip_1\cdot u_1}\int du_2 e^{ip_2\cdot u_2}\int dw_1 e^{-ip_1' w_1} \times\int dw_2 e^{-ip_2' w_2} M^{(H)}(u_1,u_2,w_1,w_2)e^{\Phi}, \quad (15)$$

where $M^{(H)}$ denotes that part of the amplitude constructed from hard-meson processes, and

$$\Phi=ig^2\int_0^\infty\int_0^\infty d\xi d\eta\{v_1\cdot v_2\Delta_c(u_1-u_2-\xi v_1+\eta v_2)+v_1'\cdot v_2'\Delta_c(w_1-w_2+\xi v_1'-\eta v_2')+v_1\cdot v_1'\Delta_c(u_1-w_1-\xi v_1-\eta v_1') + v_2\cdot v_2'\Delta_c(u_2-w_2-\xi v_2-\eta v_2')+v_1\cdot v_2'\Delta_c(u_1-w_2-\xi v_1-\eta v_2')+v_2\cdot v_1'\Delta_c(u_2-w_1-\xi v_2-\eta v_1')\}. \quad (16)$$

The leading, multiplicative, soft-meson-exchange dependence is now obtained by neglecting the phase terms in each integrand of (16), and evaluating each integral in a manner identical to that which lead from (12) to (13), with the result

$$\Phi(s,t,u)=2\gamma[F(t)+F(u)-F(s)], \quad (17)$$

where $s+t+u=4m^2$. Renormalization has again been performed in passing from (16) to (17) by removing the zero-momentum-transfer dependence of the phase-approximated exponential of (16); that is, before renormalization, Φ has the form $2\gamma[f(t)+f(u)-f(s)]$, with the renormalized answer given in terms of $F(t)=f(t)-f(0)$. The sign change of $F(s)$ is due simply to the algebraic difference in the definitions of $t=-(p_1-p_1')^2$, $u=-(p_1-p_2')^2$, and $s=-(p_1+p_2)^2$, but its consequence is very important. The soft dependence of (17) will produce in the differential cross section the multiplicative factor

$$e^{4\gamma[F(t)+F(u)-\mathrm{Re}F(s)]}. \quad (18)$$

For $s>4m^2$, $F(s)$ is complex, with $\mathrm{Re}F(s)=F(4m^2-s)$; and the difference

$$F(u)-\mathrm{Re}F(s)=F(4m^2-s-t)-F(4m^2-s)$$

vanishes as $s\to\infty$ for fixed t. In this limit one is left with just the factor $e^{4\gamma F(t)}$, which for large $-t$ and a value of γ close to that needed for the proton's form factor, is effectively proportional to $[G(t)]^4$. This is the manner in which the Yang-Wu idea is realized in this model. Equation (17) is, incidentally, symmetric under the interchange of initial or final protons ($u\leftrightarrow t$, $s\leftrightarrow s$), and hence the necessary asymmetry of the amplitude must be contained in $\tilde{M}^{(H)}$.

For $p\bar{p}$ scattering the only deviation from the preceding analysis is a change of sign, $g\to -g$, for the $\bar{p}$ coupling to the neutral vector-meson field. This has the effect of interchanging the $F(u)$ and $F(s)$ terms, yielding, in place of (18), the factor

$$e^{4\gamma[F(t)+\mathrm{Re}F(s)-F(u)]}, \quad (19)$$

which obeys the same $s\to\infty$ limit. However, since $F(u)-\mathrm{Re}F(s)\sim\ln(1+t/s)^{-1}$ for $s\gg -t\gg m^2$, and $F(t)$ is always negative, (18) produces a pp cross section which decreases to its $s=\infty$ limit, while that of (19) increases to the same value. Without attempting an estimate of $M^{(H)}$ for either case, it is impossible to make a firm prediction, but if the large $(s, -t)$ pp

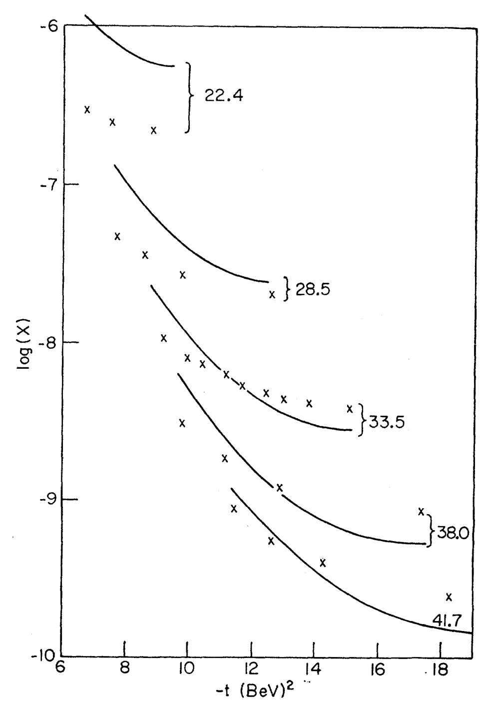

FIG. 2. A plot of $\exp\{4\gamma[F(t)+F(4m^2-s-t)-F(4m^2-s)]\}$ superimposed upon the elastic pp cross sections (crosses) of Allaby *et al.* (Ref. 4) for five values of s (BeV^2). $X(s,t)=(d\sigma/dt)/(d\sigma/dt)_{t=0}$ and $\gamma=3.7$. The base of the logarithms is 10.

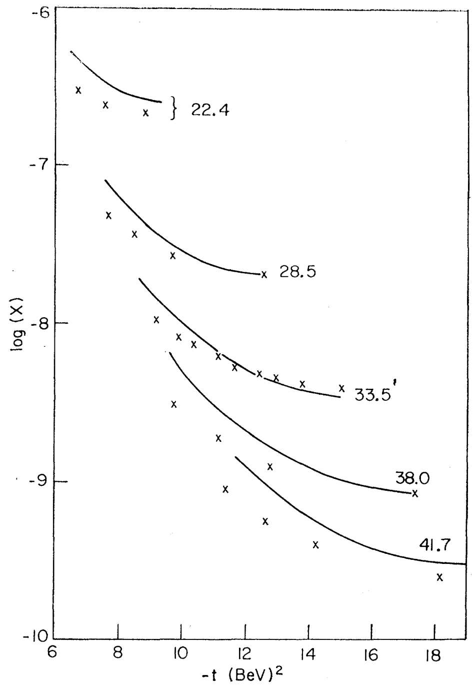

FIG. 3. A plot of $\exp\{4[\gamma(t)F(t)+\gamma(u)F(u)-\gamma(s)F(4m^2-s)]\}$ superimposed on the elastic pp cross sections of Ref. 4 for five values of s (BeV^2). $\gamma(t)=\gamma_0+\gamma_1/|t|$, where $\gamma_0=3\frac{1}{3}$ and $\gamma_1/4m^2=0.71$. $X(s,t)=(d\sigma/dt)/(d\sigma/dt)_{t=0}$. The base of the logarithms is 10.

scattering can be described by (18), one might expect (19) to be relevant to similar $p\bar{p}$ scattering. From this point of view, it is encouraging that, experimentally, the $p\bar{p}$ cross section appears to dip below that of pp with increasing s, $-t$. It would be most interesting to measure the behavior of the $p\bar{p}$ cross section at fixed, large $-t$, and varying s.

The model provides a natural source of dependence on the Krisch[12] variable $\beta^2 p_\perp^2=(ut/s)$ in the wide-angle situation where each variable is separately large. This combination occurs, in (18), because of the asymptotic logarithmic behavior of F, yielding $(ut/s)^{-4\gamma}$, which could be used to fit the very high end of the Krisch plot.

When the two-parameter fit described in the Introduction is desired, it should be remembered that μ_c^2/μ^2, and hence γ, takes on the dependence of the particular F it multiplies. Hence the exponent of (18) is replaced by $4[\gamma(t)F(t)+\gamma(u)F(u)-\gamma(s)F(4m^2-s)]$. The approximation

$$\gamma\sim(g^2/4\pi)(1/2\pi)\ln(1+\mu_c^2/\mu^2),$$

together with the choice $\mu_c^2/\mu^2=c_1+c_2/|z|$ then leads to

$$e^{4\gamma_0[F(t)+F(u)-F(4m^2-s)]+4\gamma_1[F(t)/|t|+F(u)/|u|-F(4m^2-s)/s]}, \tag{20}$$

[12] C. W. Akerlof, R. H. Hieber, A. D. Krisch, K. W. Edwards, L. G. Ratner, and K. Ruddick, Phys. Rev. **159**, 1138 (1967).

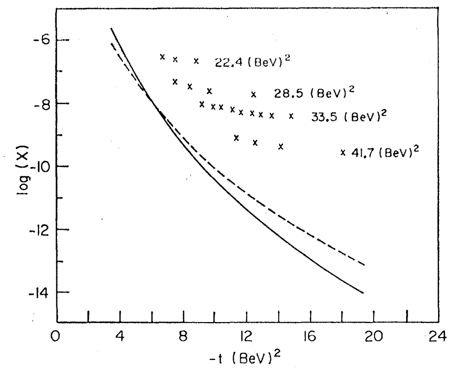

FIG. 4. A plot of the $s=\infty$ limit, $e^{4\gamma(t)F(t)}$, for pp scattering. The solid curve shows $\gamma(t)=3.7$; the dashed curve shows $\gamma(t)=\gamma_0+\gamma_1/|t|$ with $\gamma_0=3\frac{1}{3}$ and $\gamma_1/4m^2=0.71$. Experimental points from Ref. 4 are shown for orientation. The base of the logarithms is 10.

where

$$\gamma_0=(g^2/4\pi)(1/2\pi)\ln(1+c_1),$$
$$\gamma_1=(g^2/4\pi)(1/2\pi)c_2/(1+c_1),$$

and it has been supposed that $(c_2/|z|)\ll 1+c_1$ for the range of variables z considered here. Such a choice cannot change the normalization of $X\equiv(d\sigma/dt)/(d\sigma/dt)_{t=0}$, since, for $t\sim 0$, the exponent of (18) becomes $\sim 4\gamma(t)F(t)$, and $F(t)$ vanishes linearly with t. For small $-t$ the assumed $\ln(1+c_1+c_2/|t|)$ dependence of γ is probably not true, with a constant $\sim m^2$ probably replacing $-t$ inside the logarithm.

III. COMPARISON WITH EXPERIMENT

A. Nucleon Electromagnetic Form Factor

From (13) one expects the proton's form factors to consist of the soft term $e^{\gamma F(t)}$ multiplying an unspecified hard-meson contribution $H(t)$ which can be thought of as due to an appropriate collection of vector mesons and represented by simple poles. For example, one can use $H(t)=1+at(\frac{2}{3}m_\rho^2-t)^{-1}$ for the isovector form factor, which leads to $G\sim(1-a)e^{\gamma F(t)}$ for $-t\gg m_\rho^2$. We find good agreement with the SLAC experiments using $\gamma=2.4$ and $a=0.84$. An excellent two-parameter (one hard, one soft) fit at all momentum transfers is given by[13] $G=(1-t/m_0^2)^{-1}e^{\gamma F(t)}$ with $\gamma=1.4$ and $m_0^2=0.4$ BeV$^2\approx\frac{2}{3}m_\rho^2$, as plotted in Fig. 1, normalized to the dipole fit, $(1-t/0.71)^{-2}$. The three-parameter form, $H(t)e^{2.4F(t)}$, when normalized to the dipole fit, is virtually identical to the curve shown in Fig. 1.

B. Elastic pp Scattering

In Fig. 2 we have superimposed, using $\gamma=3.7$, the one-parameter curves of Eq. (18) on the experimental data of Allaby *et al.* for five different values of incident energy. The figure exhibits small systematic variations

[13] Both single-pole forms for $H(t)$ appear to require a vector meson of about $\frac{2}{3}m_\rho^2$ to fit the low-$|t|$ experiments. See, e.g., J. S. Ball and D. Y. Wong, Phys. Rev. **130**, 2112 (1963).

from the pp-scattering data which we attribute to residual s and t dependence of the nonsoft part of the matrix element. In particular, for fixed energy and large $-t$ the curve on which the experimental points lie is flatter than the corresponding theoretical curve. This indicates neglected t dependence. Furthermore, as s increases for fixed angle, smaller values of γ are needed to reproduce the experimental data. A measure of the latter behavior is obtained by the observation that one needs a γ of 2.5 to get the observed slope of the high-energy segment of the $\theta_{\text{c.m.}}=90°$ plot.[12] The agreement with the data is thus only qualitative, with an error of the order of 10% in $\log[(d\sigma/dt)/(d\sigma/dt)_{t=0}]$; but it may be remembered that only one parameter has been used.

In Fig. 3 we have plotted the two-parameter fit to the pp data using the values $\gamma_0=3\frac{1}{3}$ and $\gamma_1/4m^2=0.71$.[14] The agreement with experiment is improved both quantitatively (theory agrees with experiment to within about 5% of $\log[(d\sigma/dt)/(d\sigma/dt)_{t=0}]$) and qualitatively. In particular the theory now reproduces the high-energy part of the $\theta_{\text{c.m.}}=90°$ curve.

Finally in Fig. 4 we display the $s=\infty$ predictions for one parameter (solid line) and for two parameters (dashed line). The curves resemble that of Yang and Chou[1] but without any suggestion of zeros in the differential cross section. We point out again that the $s=\infty$ limit of pp-scattering differential cross section has the form of the fourth power of the form factor but with different values of γ. The relevant constant for the form factor is $\gamma=2.4$ whereas $\gamma_{pp}\sim3.7$.

It is also possible to fit the high-momentum-transfer portion of the proton form factor with two or three soft parameters corresponding to the analysis of the pp-scattering case. The difference between form factor and pp-scattering parameters remains, however, and no further insight is obtained by this procedure.

IV. SUMMARY

We have here proposed a simple model to achieve the damping observed at large energies and momentum transfers in certain baryonic processes. The model is obtained by assuming the existence of a fundamental massive, neutral vector-meson field coupled to the nucleons, and extracting the soft part of virtual-meson exchanges between nucleon legs. The present numerical estimates provide an excellent reproduction of the data with just two parameters over a wide range of energies and momentum transfers. On the basis of this analysis, we expect that the $p\bar{p}$ cross sections will continue to fall below those of pp scattering as the momentum transfer is increased; and that, for sufficiently large s and $-t$ (away from the dip-bump regions) the $p\bar{p}$ differential cross section will rise with increasing energy, and approach the $s=\infty$ limit of pp scattering.

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[14] We require the expansion of

$$\gamma(t)=(g^2/4\pi)(1/2\pi)\ln(1+c_1+c_2/|t|)$$

to be valid for $-t\geqslant 7$ BeV^2. The demand that the first two terms of its expansion be a good approximation to $\gamma(t)$ together with the requirement $\gamma_1/4m^2\approx0.71$ lead to the restriction $g^2/4\pi\geqslant20$. The third parameter is thus not completely arbitrary, and we see that the nucleon–neutral-vector-meson interaction is very strong indeed.