Regge-Pole Eikonal Theory of Small-Angle Kaon-Nucleon Scattering*

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A quantitative model for small-angle, high-energy kaon-nucleon scattering is presented. The model is a generalization of an eikonal Regge model of Arnold and Blackmon. Improvements on pure Regge models are naturally obtained. We find a natural crossover effect in $K^{\pm}$-p scattering and we predict polarizations in K-N charge-exchange processes. We use two pairs of exchange-degenerate Regge poles (the ρ and A_2 and the ω and P') and a flat Pomeranchon with a residue function given by a product of dipoles. All differential and total cross sections are fitted quite well.

I. INTRODUCTION

It has recently been pointed out by Arnold[1] that many difficulties with Regge-pole phenomenology can be naturally explained by using an optical-model-type modification to the usual pure Regge-pole model. (This has led to the name "hybrid model."[2]) In this combined optical-Regge approach, reactions which are dominated by the exchange of one Regge pole, such as $\pi^- p \to \pi^0 n$ and $\pi^- p \to \eta n$, naturally exhibit polarization. In the case of $\pi^- p \to \pi^0 n$, the polarization calculated agrees well with the data.[3] For the reaction $\pi^- p \to \eta n$, only a few data points have been measured and no definite comparisons can be made.[4] The optical-Regge model also affords a natural explanation for the crossover effects that occur in reactions $\pi^{\pm} p$, $K^{\pm} p$, and pp, $\bar{p}p$ elastic scattering. The pure Regge-pole model can be made to give such crossovers, usually by arbitrarily putting a zero in the residue function of a suitable helicity amplitude. For example, a zero in the helicity-nonflip ρ-exchange amplitude will give a crossover in $\pi^{\pm} p$ scattering. Similarly, $K^{\pm} p$ or pp, $\bar{p}p$ reactions acquire a crossover where the ω-exchange helicity-nonflip amplitude has a zero. This, however, brings up a different difficulty, namely, an apparent violation of factorization for the ω Regge pole.[5] The optical-Regge model removes this difficulty, since the crossover is due to an interference between the Regge-pole amplitude and the absorptive correction cuts.[1–3] The absorptive corrections will vary from one reaction to the next, and there will not be a universal value of t where an ω-exchange amplitude is forced to be zero by factorization.

In this paper, we discuss kaon-nucleon scattering using an eikonal formulation.[6] We find that the optical-Regge model gives a good description of the data. Other calculations[2,7,8] have been done using similar models. Most other models just calculate the first term, the second-order scattering, in the series of corrections, whereas the eikonal formulation gives a precise way for calculating all orders of multiple scattering. The models all give the same second-order scattering but can vary considerably in higher orders. The differences become important at large momentum transfers.

We assume the Pomeranchon is a fixed pole and has a residue function which is a squared dipole. We use two pairs of exchange-degenerate Regge poles, the $I=0$ pair being the ω and the $P'(f^0)$, and the $I=1$ pair being the ρ and the A_2. The trajectories are linear in t with slopes and intercepts close to values read off a Chew-Frautschi plot. In fact, the same trajectories are used here as in Refs. 3 and 4. The residue functions, which are also exchange-degenerate, are chosen to be proportional to α but are otherwise constant. This factor of α is necessary to eliminate the ghost for an even-signature trajectory. Exchange degeneracy requires the factor of α to appear for odd-signature poles as well. This is also known as the choosing-nonsense mechanism. Notice that the ρ is usually made to choose sense. It has been found, however, that a nonsense-choosing ρ gives a better fit to πp data when one uses the optical-Regge model.[3] Finally, only the $I=1$ exchanges have a nonzero helicity-flip amplitude.

II. SPECIFIC EQUATIONS OF THE MODEL

We consider altogether six reactions: $K^{\pm} p \to K^{\pm} p$, $K^{\pm} n \to K^{\pm} n$, $K^- p \to \bar{K}^0 n$, and $K^+ n \to K^0 p$. We work with the amplitudes G_+ and G_-, which correspond to helicity nonflip and helicity flip in the s channel, respectively. The eikonal formulas for these amplitudes are

$$G_+ = ikW\cos\tfrac{1}{2}\theta \int_0^\infty bdb\, J_0(b\Delta) \times[1-e^{i\chi_0(s,b)}\cos\chi_f(s,b)], \quad (1)$$

$$G_- = kW\int_0^\infty bdb\, J_1(b\Delta)e^{i\chi_0(s,b)}\sin\chi_f(s,b), \quad (2)$$

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where $\Delta=(-t)^{1/2}$, $W=s^{1/2}$, and k is the c.m. momentum. The amplitudes G_+ and G_- are normalized so that

$$\frac{d\sigma}{dt}=\frac{\pi}{k^2s}(|G_+|^2+|G_-|^2) \tag{3}$$

and

$$P(t)=\frac{2\,\mathrm{Im}(G_+G_-^*)}{|G_+|^2+|G_-|^2}. \tag{4}$$

By expanding the amplitudes to first order in χ_0 and χ_f and performing a Fourier-Bessel transform, one finds defining equations for χ_0 and χ_f:

$$\chi_0(s,b)=\int_0^\infty \Delta d\Delta\, J_0(b\Delta)\frac{G_+{}^B(s,t)}{kW\cos\frac{1}{2}\theta}, \tag{5}$$

$$\chi_f(s,b)=\int_0^\infty \Delta d\Delta\, J_1(b\Delta)\frac{G_-{}^B(s,t)}{kW}. \tag{6}$$

Here, the $G_\pm{}^B$ are the Regge-pole approximations to the amplitudes

$$G_+{}^B=G_+{}^P+G_+{}^{P'}+G_+{}^\omega+G_+{}^{A_2}+G_+{}^\rho, \tag{7}$$

$$G_-{}^B=G_-{}^{A_2}+G_-{}^\rho. \tag{8}$$

For the Pomeranchon, we take a fixed pole, with squared dipole momentum transfer dependence

$$G_+{}^P=iCkW\mu^8/(\mu^2-t)^4. \tag{9}$$

The parameters C and μ are adjusted to fit the high-energy scattering data. For the even-signature poles P' and A_2 we take

$$G_+=-\frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha}\alpha b_1\left(\frac{s}{s_0}\right)^\alpha, \tag{10}$$

and for the odd-signature trajectories we use

$$G_+=\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\alpha b_1\left(\frac{s}{s_0}\right)^\alpha. \tag{11}$$

The factor α must appear in the residues of even-signature trajectories to remove the pole at $\alpha=0$ in the signature factor. The assumption of exchange degeneracy for residues then gives the same factor of α in the residue function of odd-signature exchanges. Similarly, for the helicity-flip amplitudes (for $I=1$ exchanges only) we take

$$G_-=\frac{\mp 1-e^{-i\pi\alpha}}{\sin\pi\alpha}\frac{\Delta}{2M}\alpha(b_1-b_2)\left(\frac{s}{s_0}\right)^\alpha. \tag{12}$$

(For $I=0$ exchanges we set $G_-=0$.)

It is convenient to consider sums and differences of exchange-degenerate, opposite-signature Regge-pole terms. Thus, for the $I=1$ exchanges, we have the sum amplitudes $(G^{A_2}+G^\rho)$,

$$G_+=-\frac{e^{-i\pi\alpha}}{\sin\pi\alpha}\alpha(2b_1)\left(\frac{s}{s_0}\right)^\alpha, \tag{13}$$

$$G_-=-\frac{e^{-i\pi\alpha}}{\sin\pi\alpha}\alpha\frac{\Delta}{2M}2(b_1-b_2)\left(\frac{s}{s_0}\right)^\alpha, \tag{14}$$

and the difference amplitudes $(G^{A_2}-G^\rho)$,

$$G_+=-\frac{1}{\sin\pi\alpha}\alpha(2b_1)\left(\frac{s}{s_0}\right)^\alpha, \tag{15}$$

$$G_-=-\frac{1}{\sin\pi\alpha}\alpha\frac{\Delta}{2M}2(b_1-b_2)\left(\frac{s}{s_0}\right)^\alpha. \tag{16}$$

For $I=0$ exchanges, there are analogous equations for $(G_+{}^{P'}\pm G_+{}^\omega)$. Notice that the difference amplitudes are purely real and that the sum amplitudes have a phase $e^{-i\pi\alpha}$.

In the range of values of $\alpha(t)$ that we will be considering $(-\frac{1}{2}\lesssim\alpha\lesssim\frac{1}{2})$ the function $\alpha/\sin\pi\alpha$ is a slowly varying function of t $[0.3\lesssim\alpha(t)/\sin\pi\alpha(t)\lesssim 0.5]$. We thus approximate $(\alpha/\sin\pi\alpha)b_i$ by a constant and calculate χ's for the sums and differences of the Regge-pole amplitudes. For the $I=0$ amplitudes, we have

$$\text{sum:}\quad \chi_0=-C\left(e^{-i\pi}\frac{s}{s_0}\right)^{\alpha_0}\frac{e^{-b^2/2R_s^2}}{R_s^2}; \tag{17}$$

$$\text{difference:}\quad \chi_0=-C\left(\frac{s}{s_0}\right)^{\alpha_0}\frac{e^{-b^2/2R_D^2}}{R_D^2}. \tag{18}$$

Here,

$$C=\langle(\alpha/\sin\pi\alpha)2b_1{}^{I=0}\rangle,$$

$$R_s{}^2=2\alpha'[\ln(s/s_0)-i\pi],$$

and

$$R_D{}^2=2\alpha'\ln(s/s_0).$$

For the $I=1$ amplitudes, we have

$$\text{sum:}\quad \chi_0=-D\left(e^{-i\pi}\frac{s}{s_0}\right)^{\alpha_0}\frac{e^{-b^2/2R_s^2}}{R_s^2}; \tag{19}$$

$$\text{difference:}\quad \chi_0=-D\left(\frac{s}{s_0}\right)^{\alpha_0}\frac{e^{-b^2/2R_D^2}}{R_D^2}; \tag{20}$$

and

$$\text{sum:}\quad \chi_f=-\frac{D-E}{2MkW}\left(e^{-i\pi}\frac{s}{s_0}\right)^{\alpha_0}\frac{b}{R_s{}^4}e^{-b^2/2R_s^2}; \tag{21}$$

$$\text{difference:}\quad \chi_f=-\frac{D-E}{2MkW}\left(\frac{s}{s_0}\right)^{\alpha_0}\frac{b}{R_D{}^4}e^{-b^2/2R_D^2}. \tag{22}$$

Here,

$$D=\langle(\alpha/\sin\pi\alpha)\times 2b_1{}^{I=1}\rangle,\quad E=\langle(\alpha/\sin\pi\alpha)\times 2b_2{}^{I=1}\rangle,$$

and $R_s{}^2$ and $R_D{}^2$ are as defined before, except, of course, that here one uses the value of α' of the $I=1$ trajectories.

The choice of signs for the residues of the Regge-pole terms in G_+ can be determined from the experimental fact that

$$\sigma_T(K^-p)>\sigma_T(K^-n)>\sigma_T(K^+p)\simeq\sigma_T(K^+n).$$

This implies that one must choose

$$K^-p:\quad \chi_0=\chi_0{}^P+\chi_0|_{I=0}{}^{\text{sum}}+\chi_0|_{I=1}{}^{\text{sum}}, \quad (23a)$$

$$K^-n:\quad \chi_0=\chi_0{}^P+\chi_0|_{I=0}{}^{\text{sum}}-\chi_0|_{I=1}{}^{\text{sum}}, \quad (23b)$$

$$K^+p:\quad \chi_0=\chi_0{}^P+\chi_0|_{I=0}{}^{\text{diff}}+\chi_0|_{I=1}{}^{\text{diff}}, \quad (23c)$$

$$K^+n:\quad \chi_0=\chi_0{}^P+\chi_0|_{I=0}{}^{\text{diff}}-\chi_0|_{I=1}{}^{\text{diff}}. \quad (23d)$$

For the helicity-flip amplitude, there is no experimental reason to prefer either the sum or the difference amplitude for K^-N reactions. A recent theoretical idea of Schmid,[9] however, relates the partial-wave analysis of a Regge amplitude to direct-channel resonances. Since the phase of the sum amplitudes is $e^{-i\pi\alpha}$, one might expect circles in an Argand diagram. The phase of the difference amplitudes, however, is real, and thus gives no circles. This suggests using sum amplitudes in G_- for K^-N and difference amplitudes in G_- for K^+N. We have done our fitting with both choices for the G_- amplitude.

III. DISCUSSION OF RESULTS

We have obtained acceptable fits to all available total and differential cross-section data from 5 GeV to the highest measured values.[7–13] For the Pomeranchon, the values that were used for C and μ were

$$C=4.35\ \text{GeV}^{-2},\quad \mu=1.15\ \text{GeV}.$$

s_0 was taken to be 0.3 GeV^2. Having parametrized the two exchange-degenerate trajectories in the form $\alpha(t)=\alpha_0+\alpha' t$, we have chosen

$$\alpha_0=0.45,\quad \alpha'=1.0$$

for the $I=0$ trajectory, and

$$\alpha_0=0.55,\quad \alpha'=0.8$$

for the $I=1$ trajectory. These are the same trajectories as those used by Arnold and Blackmon.[3] The residues were fixed at the values

$$C=1.5,\quad D=0.26,\quad E=6D.$$

Recall that we have defined C, D, and E as average values of t-dependent expressions over the interval $-\frac{1}{2}\lesssim\alpha(t)\lesssim\frac{1}{2}$. Then if we take the average value of $\alpha/\sin\pi\alpha$ as ~ 0.4, we obtain the values for the conventional residues b_1 and b_2. These will be

$$2b_1{}^{P'KK}\simeq 3.75,$$
$$2b_1{}^{\rho KK}\simeq 0.65,$$
$$2b_2{}^{\rho KK}\simeq 6\times(2b_1{}^{\rho KK}).$$

$SU(3)$ relates these residues, for $K^\pm N$ scattering, to the residues for πN scattering

$$b_1{}^{P'\pi\pi}=2b_1{}^{P'KK},$$
$$b_1{}^{\rho\pi\pi}=2b_1{}^{\rho KK},$$
$$b_2{}^{\rho\pi\pi}=2b_2{}^{\rho KK}.$$

We may compare the values thus obtained for the πN residues to the values used by Arnold and Blackmon in fitting πN scattering. The ρ residues are in agreement, but for the P' residue we obtain $b_1{}^{P'\pi\pi}\simeq 6b_1{}^{\rho\pi\pi}$, whereas Arnold and Blackmon used $b_1{}^{P'\pi\pi}=3b_1{}^{\rho\pi\pi}$ (the quark-model relation for exchange-degenerate vector and tensor mesons). However, the P' residue parameter is determined much more precisely in fitting $K^\pm N$ total cross sections than it is in fitting πN cross sections, since the difference between K^+N and K^-N cross sections is fixed by the coupling of the P' (and the exchange-degenerate ω trajectory).

We have obtained fits for the two choices of spin-flip amplitudes, i.e., the case where the ρ and A_2 contributions have the *same* signs in $\bar{K}N$ scattering and the case where the ρ and A_2 contribute with *opposite* signs to $\bar{K}N$ scattering. The fits to the cross sections are not affected by this sign change, in the region of t that we are considering, but the polarizations that are thereby predicted are strongly dependent on the choice of sign.

In Fig. 1(a) we have plotted the differential cross-section data for $K^\pm p$ elastic scattering at the highest available energies along with the fitted curves. The agreement is good up to $-t\sim 1$ GeV^2. It is important to notice that this model predicts a crossover effect in the region of $t\simeq -0.35$ GeV^2, which is consistent with the existing data, although the uncertainties in the data do not allow a definitive comparison. For lower energies (down to $E_{\text{lab}}\sim 5$ GeV), the differential cross sections change very little and the fit remains equally good. Figure 1(b) shows typical differential cross sections for $K^\pm n$ elastic scattering.

Figure 2 compares the $K^-p\to\bar{K}^0n$ differential cross-section data with the fitted values at several energies. The fit is good for all available energies and for momentum transfers up to $-t\simeq 1$ GeV^2. The turnover near the forward direction fixes the magnitude of the helicity-flip amplitude, but not the relative signs of flip and nonflip amplitudes. Notice that there is no dip in the cross section at the point $\alpha=0$. Since we have assumed exchange degeneracy for residue functions at $\alpha=0$, we have both flip and nonflip amplitudes for the ρ-pole term going through zero. However, the A_2-pole term, being even-signature, does not go through zero. Thus, the pole terms show no dip structure near $\alpha=0$.

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The absorptive corrections do not give any sharp dip structure either. The wrong-signature nonsense zeros in the odd-signature flip and nonflip amplitudes get moved

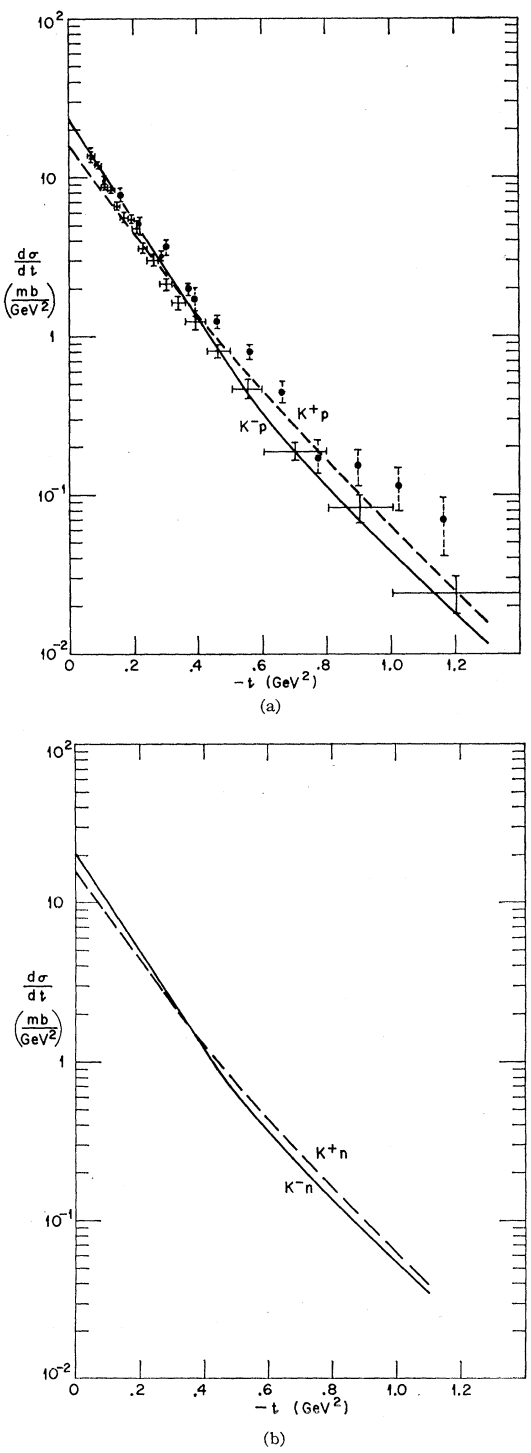

FIG. 1. Differential cross section for (a) $K^{\pm}p$ and (b) $K^{\pm}n$ scattering at $E_L=13.8$ GeV/c. The K^+p data are for energy 14.8 GeV/c and the K^-p data are for energy 10 GeV/c.

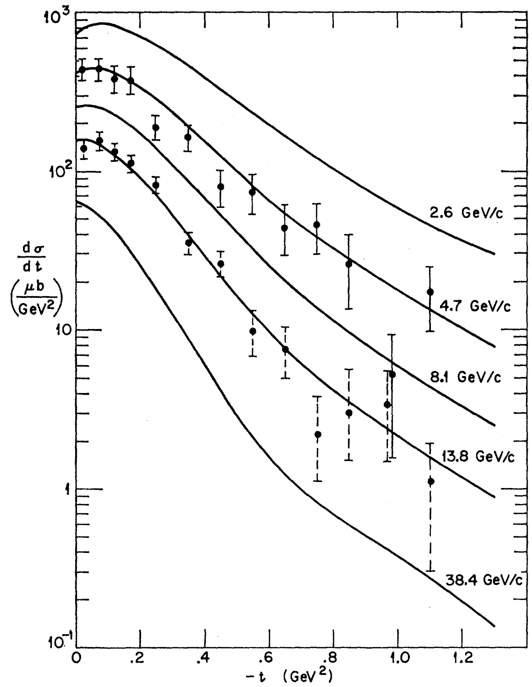

FIG. 2. Differential cross sections for $K^-p\to\bar{K}^0n$. The data are for energies 5.0 and 12.3 GeV/c.

slightly toward $t=0$. The even-signature amplitudes have zeros induced by the absorption, but at larger values of t than the point which corresponds to $\alpha=0$. Thus, the complete amplitudes, poles and absorptive corrections, even- and odd-signature, have only a smooth behavior in t.

Figures 3–6 show our results for the polarization in elastic scattering. In one case, Figs. 3 and 4, we have used the sum (difference) of Regge-pole amplitudes [Eqs. (21) and (22)] for the helicity-flip amplitude in K^-N (K^+N) scattering. This is the same choice of signs for the Regge residues used in the helicity-nonflip amplitude. This choice will hereafter be referred to as the uniform-sign model. Figures 5 and 6, on the other hand, show the results of using the difference (sum) of Regge-pole amplitudes for the helicity-flip amplitude in K^-N (K^+N) scattering. This choice will hereafter be referred to as the mixed-sign model, since flip and nonflip amplitudes have different choices of signs for the residues.

The uniform-sign model and the mixed-sign model give similar predictions for the polarization in elastic scattering for $0.3\lesssim -t\lesssim 0.9$ GeV^2. In this region, K^-p polarizations are positive and K^-n polarizations are negative. For $-t<0.3$ GeV^2, the mixed-sign model yields polarizations which continue smoothly to zero at $-t=0$, while the uniform-sign model gives polarizations which have a zero around $-t\simeq 0.2$ GeV^2. For $-t>0.9$ GeV^2, the mixed-model polarizations are falling smoothly, while the uniform-model polarizations go through zero around $-t=0.9$ GeV^2 and become large again with

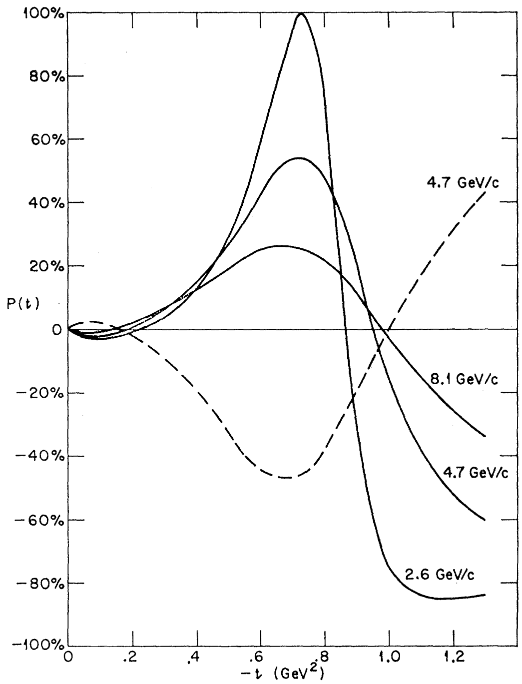

FIG. 3. Polarization for K^-p (solid line) and K^-n (dashed) elastic scattering. The signs of the Regge residues in the helicity-flip amplitude are the same as the helicity-nonflip amplitude.

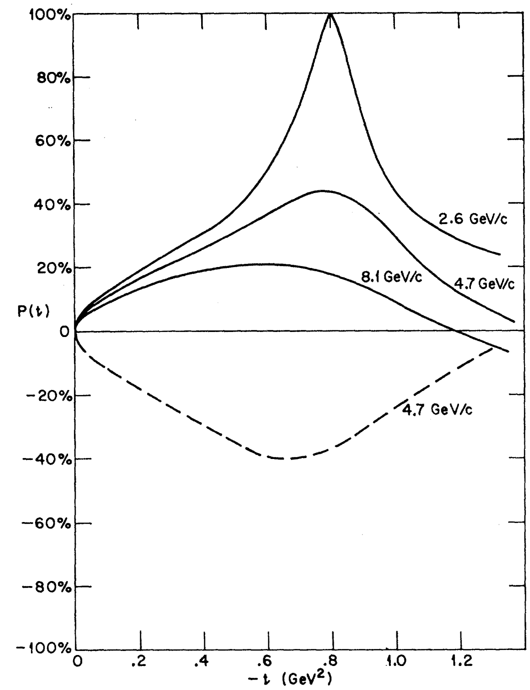

FIG. 5. Polarization for K^-p (solid line) and K^-n (dashed) elastic scattering. The signs of the Regge residues in the helicity-flip amplitude are different from the signs of the nonflip amplitude (see text).

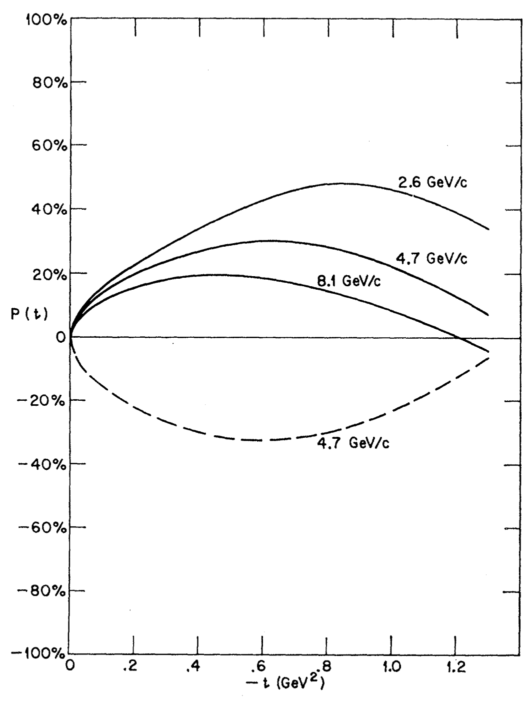

FIG. 4. Polarization for K^+p (solid line) and K^+n (dashed) elastic scattering. The same signs are used as in Fig. 3.

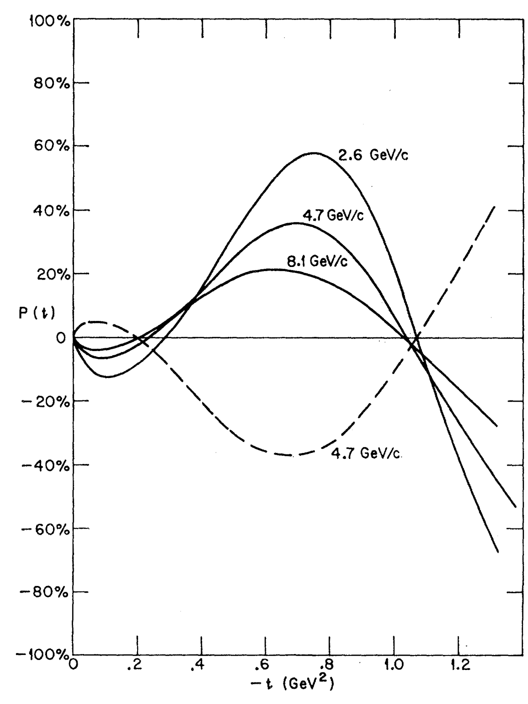

FIG. 6. Polarization for K^+p (solid line) and K^+n (dashed) elastic scattering. The same signs are used as in Fig. 5.

opposite sign. If one tries to compare our calculation with the low-energy data of Daum *et al.*,[14] one might be tempted to prefer the uniform-sign model. However, the data of Daum *et al.* are at too low an energy and the zero in polarization (at $t=-0.9$ GeV^2) corresponds to a scattering angle of 45°–60°. Extrapolating an eikonal calculation to such a region is quite dangerous.

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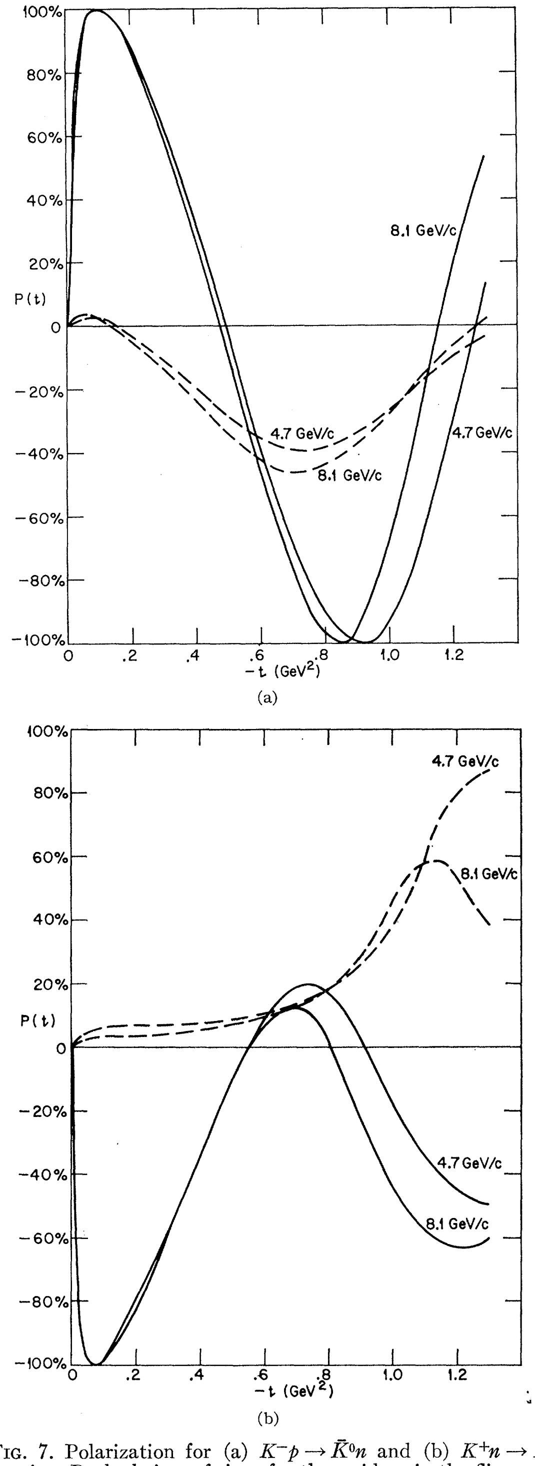

FIG. 7. Polarization for (a) $K^-p \rightarrow \bar{K}^0n$ and (b) $K^+n \rightarrow K^0p$ scattering. Both choices of signs for the residues in the flip amplitude are plotted with the mixed-sign case (uniform-sign case) in solid (dashed) line.

The charge-exchange reactions $K^-p \rightarrow \bar{K}^0n$ and $K^+n \rightarrow K^0p$ offer an unambiguous way of deciding which model—uniform signs or mixed signs—is chosen in nature. The uniform-sign model yields a generally small polarization which is due to interference between poles and absorptive correction. (In a pure pole model, this choice of signs of Regge residues would give no polarization even though there are two trajectories.) The mixed-sign model gives polarizations which reach 100% near $t=0$. This is primarily a pure pole effect as discussed by Arnold and Logan.[15] The effects of the absorption is most noticeable at larger values of $-t$. Our calculation shows [Fig. 7(a)] the K^-p charge-exchange polarization reaching -100% at $-t\simeq 0.9$ GeV^2, whereas Arnold and Logan [see Fig. 2(b) of Ref. 15] find a polarization of -50% in the same region. The polarization becomes larger than the calculation of Arnold and Logan because the absorptive correction is beginning to produce a broad diffractive minimum in the helicity-flip amplitude. K^+n charge-exchange polarization shows the same qualitative features as K^-p charge exchange. The mixed-sign model gives large polarizations near $t=0$, while the other model gives a small polarization. Until more measurements of polarizations are made, however, it will be impossible to prefer one choice of signs over the other.

IV. CONCLUSIONS

We have shown that an optical-Regge model, with assumptions of exchange degeneracy for trajectories and residues, is consistent with all data for KN processes. Additional evidence for this model has been presented in the work on πN processes.[3,4] Assuming that the uniform-sign model is chosen by nature—and this is the sign choice preferred theoretically and, to a lesser extent, experimentally—three experiments would be crucial in demonstrating the consistency of this approach. In particular, the calculated polarizations for the three charge-exchange reactions $\pi^-p \rightarrow \pi^0n$, $\pi^-p \rightarrow \eta n$, and $K^-p \rightarrow \bar{K}^0n$ all show small positive polarization near $t=0$ and then a large negative excursion at larger values of $-t$. If these general features are not reproduced by the data, either exchange degeneracy or the present form for the absorptive corrections must be given up.

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