Model for Low-Energy Meson-Baryon Scattering*†

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(Received 19 September 1968)

An effective Lagrangian, containing relativistic $SU(6)$-invariant three- and four-point interaction terms, is constructed for the scattering of the **35** meson representation by the **56** baryons. The lowest-order matrix elements calculated from the Lagrangian include single-particle exchange terms that account for long-range forces, and four-point interaction terms that approximate short-range forces for low-energy scattering. Because of the imposition of free-field conditions on the scattered particles, the amplitudes maintain a coplanar $U(3)\otimes U(3)$ symmetry, which is then broken by the mass differences in the multiplets. There are three parameters in the model, which are fixed by the S_{11}, S_{31}, and P_{11} scattering lengths for πN elastic scattering. Unitarity is implemented without introducing more parameters, by equating the matrix elements to those of the reaction matrix K. The amplitudes for all other reactions contained in $\mathbf{35}\otimes\mathbf{56}$ scattering are determined thereby. In particular, the cross sections and angular distributions for $\pi N\to\pi N$, $KN\to KN$, $\pi N\to\eta N$, $\pi N\to K\Lambda$, $\pi N\to K\Sigma$, and $\bar{K}N\to\eta\Lambda$ are calculated near their thresholds and compared with experiment. In all cases where comparisons can be made, the model is in fairly good agreement with the data, with the exception of $\pi N\to K\Sigma$. The threshold amplitudes for vector-meson production are also given as a prediction of the model.

I. INTRODUCTION

IN the past several years various methods have been evolved to calculate theoretically the parameters of low-energy meson-nucleon scattering. These methods fall into two general categories: the dispersion-theoretic approach and the algebraic approach.

In the dispersion-theory approach an attempt is made to account for dynamical features by assuming that the analytic structure of the scattering amplitude at low energies is dominated by nearby singularities in the crossed channels.[1] Once these singularities are chosen, the requirements of analyticity, unitarity, and crossing symmetry must be satisfied, at least approximately. There are many approximation schemes that have been developed to satisfy these restrictions, all based on the Mandelstam representation[2] of two-body scattering amplitudes. The schemes generally begin with the assumption that the nearby singularities in the partial-wave amplitudes can be approximated by single poles, representing resonances and bound states. One school of thought then proposes to use only these singularities to determine the input functions in an iteration scheme, the "bootstrap," which will finally lead to self-consistent solutions for the partial-wave amplitudes without initially taking into account the contributions of uncorrelated multiparticle intermediate states.[1] This philosophy has had qualitative success, at least in some of its implementation, in that it generates resonances and bound states having masses and coupling strengths in rough agreement with nature.[3] Without *further* assumptions, however, there have been no quantitatively satisfactory predictions for low-energy differential and integrated cross sections.

Another dispersion-theoretical approach, due to Cini and Fubini,[4] is to attempt to account for the multiparticle intermediate-state contributions to the dispersion integrals, explicitly. This is accomplished by formally expanding the Mandelstam double spectral integrals about the threshold for the reaction under consideration. What results, then, are polynomials in the kinematical variables, s, t, and u, with undetermined coefficients representing the effect of the short range forces, or the "background" terms. The nearby singularities (long-range forces) are still approximated by single poles. The coefficients of the background terms are in principle calculable by using unitarity to produce coupled nonlinear integral equations. In practice, however, this approach provides a phenomenological parametrization for low-energy scattering, in which the undetermined polynomial coefficients are fixed by fitting the experimental data.[5] Any theoretical model that can relate these coefficients to one another, thereby reducing the number of parameters available for fitting cross sections and partial-wave amplitudes, will provide a test of the nature of the background terms. The Cini-Fubini representation, then, can not only explain the gross features of a scattering process, but can phenomenologically account for the details of differential and integrated cross sections at low energies.[6,7]

* Work supported in part by the U. S. Atomic Energy Commission.
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[2] S. Mandelstam, Phys. Rev. **112**, 1344 (1958).
[3] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962); P. Carruthers, Phys. Rev. **133**, B497 (1964); A. W. Martin and K. C. Wali, *ibid.* **130**, 2455 (1963); R. E. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963).
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[5] J. Bowcock, W. M. Cottingham, and D. Lurié, Nuovo Cimento **16**, 918 (1960).
[6] R. L. Warnock and G. Frye, Phys. Rev. **138**, B947 (1965).
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A completely different method for determining the parameters for low-energy scattering has evolved from the Gell-Mann algebra of weak hadronic currents.[8] Making use of the algebra of currents and the hypothesis of partially conserved axial-vector current (PCAC), Weinberg was able to relate the amplitude for the scattering and emission of an arbitrary number of soft pions to the weak-interaction parameters.[9] In particular, the s-wave scattering lengths for elastic πN scattering were determined[10] and were in good agreement with experiment. This successful application of the current algebra to strong interaction processes has led many investigators to recast the theory into a Lagrangian framework, in which extrapolation to zero-mass mesons is not necessary.[11] A phenomenological Lagrangian is constructed for mesons and baryons by requiring invariance under nonlinear chiral $U(2)\otimes U(2)$ transformations, so that both PCAC and current-algebra relations are satisfied. Although this construction is not unique, the resulting s-wave πN scattering lengths are the same, in all cases, as those given by the zero-mass pion formulas. However, the determinations of the p-wave scattering lengths and s-wave effective-range parameters depend more critically on the particular form of the phenomenological Lagrangian.

In this paper we will present a model for low-energy meson-baryon scattering that combines some of the features of a phenomenological Lagrangian, invariant under a nonchiral $U(3)\otimes U(3)$, with the features of the Cini-Fubini approximation to the double dispersion relations. Our approach then is to write an effective Lagrangian,[12] to be used to second order in the coupling strengths, which contains trilinear interactions among most of the known low-lying hadrons along with four-point interactions which account for the background terms in the dispersion representation, and is invariant under coplanar $U(3)\otimes U(3)$.[13]

By requiring that our effective interaction Lagrangian be formally invariant under transformations of relativistic $SU(6)$ or $U(6,6)$,[14–16] we are able to maintain the desired properties for the scattering amplitudes. We write all relevant trilinear and quadrilinear interactions among the meson **143** representation and the baryon **364** representation of $U(6,6)$. Using this Lagrangian to second order, then, will give us scattering of the pseudoscalar and vector-meson nonets upon the baryon octet and decuplet. As is by now well known, $U(6,6)$ cannot be an invariance group of the full Lagrangian, and consequently of the S matrix.[17] In fact the free-field part of the Lagrangian breaks the symmetry in its kinetic-energy term.[18] Then in computing second-order diagrams, with free-field propagators inserted, the resulting symmetry of the second-order S matrix is no longer $U(6,6)$. The S matrix for two-body scattering, calculated from our effective Lagrangian,[19] is invariant under the coplanar $U(3)\otimes U(3)$ subgroup of $U(6,6)$. However, not all of the coplanar $U(3)\otimes U(3)$-invariant amplitudes have independent parameters in this approach, since there are generally more independent coplanar $U(3)\otimes U(3)$ amplitudes than there are independent $U(6,6)$ invariant three- and four-point interactions in the Lagrangian.

An additional feature in this higher-symmetry approach is the appearance of momentum-dependent factors in the effective Lagrangian which arise through the imposition of symmetry-breaking free-field conditions (Bargmann-Wigner equations) on the external particles. In terms of the Cini-Fubini approximation then, this effective Lagrangian gives the single-pole Born terms, as well as the polynomial background terms, and relates many of the undetermined polynomial coefficients.

This model can then make predictions for the low-energy scattering reactions of mesons on baryons belonging to the **35** and **56** representations of $SU(6)$. However, to make the scheme plausible, it is necessary to account for the sizable mass splittings within the representations. This is accomplished at the outset by using the phenomenological mass breaking of Sakita and Wali[16] in the free-field part of the Lagrangian (or equivalently in the free-field equations). The full Lagrangian is then only invariant under isospin transformations, although the number of coupling parameters is still limited to the six parameters of the $U(6,6)$ interactions. In an earlier paper using this model,[19] the parameters were chosen to be constants and were fixed by fitting πN and KN elastic-scattering partial-wave amplitudes at low energies. Under the assumption that the arbitrary functions are constants, we can now test the predictive power of the model for various other scattering processes. Unfortunately, aside from low-energy πN and KN elastic scattering, all other experimentally accessible reactions involve couplings to other inelastic channels, even at low energies (e.g., $\pi N\rightarrow K\Lambda$, $K^-N\rightarrow K^-N$, $\pi N\rightarrow \rho N$). When the coupled-channel effects

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[12] By an "effective Lagrangian" we mean a series of products of interacting fields written in momentum space, with *momentum-dependent* couplings, so that matrix elements between particle states will result in momentum-dependent form factors. The effective Lagrangian is to be used only to lowest nontrivial order in calculating S-matrix elements, since higher orders are already approximately accounted for by the form factors.

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[17] L. O'Raifeartaigh, Phys. Rev. **139**, B1052 (1965).

[18] R. Oehme, Phys. Rev. Letters **14**, 664 (1965); **14**, 866 (1965); P. G. O. Freund, *ibid.* **14**, 803 (1965).

[19] G. R. Goldstein and K. C. Wali, Phys. Rev. **155**, 1762 (1967); hereafter referred to as I.

are in fact large, our model, without unitarity corrections, will fail. If, on the other hand, this method is taken seriously, then its success or failure in describing the low-energy behavior of various inelastic processes, will be an indication of the relative importance of coupled-channel effects, and, where practical, coupled-channel unitarity will be approximated.

In Sec. II, we review the $U(6,6)$ scheme for particle multiplets, the free-field equations for the multiplets and their solutions, the free-particle propagators and the general method for constructing formally invariant interaction Lagrangians. In Sec. III, we show, in general, how S-matrix elements for various processes are invariant under particular subgroups of $U(6,6)$ depending on the kinematical configurations and the number of internal lines. We also classify the particles according to representations of these various subgroups. In Sec. IV we present the effective Lagrangian for meson-baryon scattering and discuss its symmetries, its dynamical structure, and some general consequences of the symmetries. Section V is devoted to the presentation of a simple procedure for approximating multichannel unitarity, and the calculation of total cross sections and angular distributions for several processes near their thresholds: $\pi N\to\pi N$, $KN\to KN$, $\pi N\to\eta N$, $\pi N\to K\Lambda$, $\pi N\to K\Sigma$, $\bar{K}N\to\eta\Lambda$, and vector-meson production. Section VI summarizes the successes and failures of the model. In the Appendix we collect together the invariant amplitudes for vector-meson production.

II. $U(6,6)$, FREE FIELDS, AND PROPAGATORS

In this section we review the relation of representations of the group $U(6,6)$ to physical mesons and baryons.[16] $U(6,6)$ can be characterized as the group of all 12×12 matrices M satisfying the condition

$$M^\dagger\Gamma_4M=\Gamma_4 \tag{2.1}$$

or

$$\Gamma_4M^\dagger\Gamma_4=M^{-1},$$

where Γ_4 is a 12×12 matrix having 6 plus ones and 6 minus ones on the diagonal and zeros for nondiagonal elements. The carrier space for the fundamental (quark) representation is chosen to be the direct product of a three-dimensional with a four-dimensional complex vector space, and the basis for this space is denoted by $\psi_{i\alpha}$, where $i=1, 2, 3, 4$ and $\alpha=1, 2, 3$. In this representation $\Gamma_4=\gamma_4\otimes I$, where γ_4 is the usual Dirac matrix, and I is a 3×3 identity matrix, and the generators can be chosen as the set $\{\gamma\}\otimes\{\lambda\}$, where $\{\gamma\}$ is the set of eight Hermitian Dirac matrices I, γ_4, $i\sigma_{rs}$, $i\gamma_r\gamma_5(r, s=1, 2, 3)$, and eight anti-Hermitian Dirac matrices $i\gamma_r$, σ_{r4}, $i\gamma_5$, $\gamma_4\gamma_5$, and $\{\lambda\}$ is the set of nine 3×3 Gell-Mann matrices.[20] An infinitesimal transformation on ψ is then

$$\psi_{i\alpha}\to\psi_{i\alpha}+i\sum_{A=1}^{16}\sum_{Z=1}^{9}\epsilon_{AZ}(\gamma^A)_i{}^j(\lambda^Z)_\alpha{}^\beta\psi_{j\beta}. \tag{2.2}$$

The contragredient representation is formed by $\bar{\psi}=\psi^\dagger(\gamma_4\otimes I)$, and $\bar{\psi}^{i\alpha}\psi_{i\alpha}$ is an invariant under the transformations (2.2).

Higher-dimensional representations are constructed by taking irreducible tensor products of the fundamental basis. In particular $\phi_{i\alpha}{}^{j\beta}\sim(\psi_{i\alpha}\bar{\psi}^{j\beta}-\text{trace})$, which is identified with mesons, forms a basis for the **143** representation, and $\psi_{i\alpha,j\beta,k\gamma}\sim(\psi_{i\alpha}\psi_{j\beta}\psi_{k\gamma})$ totally symmetrized, which is identified with the baryons, forms a basis for the **364** representation.

To relate these representations to field operators for physical particles, free-field equations are imposed upon them. For example, the quark field is required to satisfy the Dirac equation, so that the free-quark Lagrangian is

$$\bar{\psi}^{i\alpha}(p)[ip_\mu\gamma_\mu\otimes I+mI\otimes I]_{i\alpha}{}^{j\beta}\psi_{j\beta}(p). \tag{2.3}$$

This not invariant under the $U(6,6)$ transformations in (2.2) since the kinetic-energy term does not commute with the generators. Hence the free quark with four-momentum p is a projection on the basis of the fundamental representation of $U(6,6)$ of the form

$$\psi_{i\alpha}(p)=[2m(m+p_0)]^{-1}(m-ip_\mu\gamma_\mu)_i{}^j\delta_\alpha{}^\beta\psi_{j\beta}. \tag{2.4}$$

Similarly, the free-meson field of momentum k, namely, $\phi_{i\alpha}{}^{j\beta}(k)$, is a projection on the basis for the **143** representation that satisfies the Duffin-Kemmer equation[21] and can be written in the form

$$\phi_{i\alpha}{}^{k\gamma}(k)=[1-(i/m_0)k_\mu\gamma_\mu]_i{}^j\times[(\gamma_5)_j{}^kP_\alpha{}^\gamma(k)+(\gamma_\lambda)_j{}^kV_{\lambda,\alpha}{}^\gamma(k)], \tag{2.5}$$

where $P_\alpha{}^\gamma(k)$ is an element of the pseudoscalar nonet of $SU(3)$ and $V_{\lambda,\alpha}{}^\gamma(k)$ is a member of the vector nonet satisfying the auxiliary condition $k_\lambda V_{\lambda,\alpha}{}^\gamma(k)=0$. Finally, the free-baryon field $\psi_{i\alpha,j\beta,k\gamma}(p)$ is a projection on the **364** basis satisfying the Bargmann-Wigner equation[22] and can be denoted by

$$\begin{aligned}\psi_{i\alpha,j\beta,k\gamma}(p)=&\tfrac{1}{2}[(\gamma_\mu-(i/M_0)\sigma_{\mu\nu}p_\nu)C]_{jk}\psi_{\mu,i,\alpha\beta\gamma}(p)\\&+\tfrac{1}{6}\{[(I-(i/M_0)\gamma_\mu p_\mu)C]_{jk}\psi_{i,\alpha}{}^\delta(p)\epsilon_{\delta\beta\gamma}\\&+[(I-(i/M_0)\gamma_\mu p_\mu)C]_{ki}\psi_{j,\beta}{}^\delta(p)\epsilon_{\delta\gamma\alpha}\\&+[(I-(i/M_0)\gamma_\mu p_\mu)C]_{ij}\psi_{k,\gamma}{}^\delta(p)\epsilon_{\delta\alpha\beta}\},\end{aligned} \tag{2.6}$$

where C is the antisymmetric charge conjugation matrix, $\psi_{i,\alpha}{}^\delta(p)$ is the ith component of the Dirac spinor belonging to the spin-$\frac{1}{2}$ baryon octet of $SU(3)$ and, $\psi_{\mu,i,\alpha\beta\gamma}(p)$ is a component of the Rarita-Schwinger spin-$\frac{3}{2}$ field[23] for the baryon decuplet [with auxiliary conditions $p_\mu\psi_{\mu,i,\alpha\beta\gamma}(p)=0$, $(\gamma_\mu)_i{}^j\psi_{\mu,j,\alpha\beta\gamma}(p)=0$, and total symmetry in the $SU(3)$ indices α, β, and γ].

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[23] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

Thus the imposition of free-field conditions on the basis for the **143** representation of $U(6,6)$ results in a degenerate multiplet of mesons (2.5) containing the pseudoscalar nonet and the vector nonet with common mass which no longer transforms according to an irreducible representation of $U(6,6)$. To give these mesons their physical masses and break the degeneracy, m_0 in (2.5) is replaced by an $SU(3)$ and spin-dependent operator $m_{i\alpha}{}^{j\beta}$. Similarly, there results a degenerate baryon multiplet, (2.6), containing the spin-$\frac{1}{2}$ octet and the spin-$\frac{3}{2}$ decuplet with common mass M_0; mass splittings can be introduced by replacing M_0 with an $SU(3)$-dependent operator $M_{\alpha\beta\gamma}$, constructed so that the physical masses result. These mass operators will be used in the actual calculations, but for the general discussion following we will ignore the mass splittings unless stated otherwise.

The propagators for the multiplets of free fields (2.5) and (2.6) are obtained by finding Green's functions for the field equations. For the meson field there is a unique propagator; for the baryon field there is no *unique* Green's function, although all choices give the same residue of the pole in momentum space.[24] As we found in I, the nonunique part, the nonpole terms in the propagators, could be chosen in such a way that when the four-momentum tends to zero the entire propagator vanishes. This requirement ensures that the propagators contain only terms that break the $U(6,6)$ symmetry—i.e., the $i\gamma_\mu p_\mu$ terms in the propagators are not $U(6,6)$ covariants. Then the resulting meson "symmetry-breaking propagator" is

$$\langle\phi_A{}^B,\bar{\phi}_C{}^D\rangle_+=\frac{1}{8m^2}\left[-\delta_A{}^D\delta_C{}^B+\frac{(-ik_\mu\gamma_\mu+m)_A{}^D(+ik_\lambda\gamma_\lambda+m)_C{}^B}{m^2+k^2}\right]. \quad (2.7)$$

To obtain the propagator for pseudoscalar mesons merely contract $\langle\phi_{AB},\bar{\phi}_C{}^D\rangle_+$ with $(\gamma_5\otimes I)_B{}^A(\gamma_5\otimes I)_D{}^C$, and similarly for vector mesons contract with $(\gamma_\mu\otimes I)_B{}^A\times(\gamma_\lambda\otimes I)_D{}^C$. Note that contracting with $\delta_B{}^A\delta_D{}^C$ yields zero, which means that there is no propagation of a fictitious scalar meson (this is an important feature of our choice for the meson propagator that is not shared by the other approaches in Ref. 24). For the baryon field the "symmetry-breaking propagator" is

$$\langle\psi_{ABC},\bar{\psi}^{DEF}\rangle_+=\frac{1}{12M^2}\sum_{P(ABC)}\left\{\frac{(-ip_\mu\gamma_\mu+M)_A{}^D(-ip_\nu\gamma_\nu+M)_B{}^E(-ip_\lambda\gamma_\lambda+M)_C{}^F}{M^2+p^2}-\frac{1}{12}\sum_{P(DEF)}(M-3ip_\mu\gamma_\mu)_A{}^D\delta_B{}^E\delta_C{}^F\right\}, \quad (2.8)$$

where the summations are over-all permutations. The M in the second summation is necessary to ensure vanishing at $p_\mu\to 0$, the $-3ip_\mu\gamma_\mu$ term is necessary to ensure that there is no propagation of a fictitious spin-$\frac{1}{2}$ unitary singlet. By contraction with appropriately symmetrized combinations of γ matrices and the charge-conjugation matrix C, we can obtain the baryon octet spin-$\frac{1}{2}$ propagator and the decuplet spin-$\frac{3}{2}$ propagator.

Given the meson and baryon fields and propagators, possible interactions among them can be obtained by forming products of the fields and propagators in which all indices are fully contracted and the total four-momentum is conserved. Products of this type, containing no propagators, will be *formally* $U(6,6)$-invariant—until the free-field conditions are imposed to determine physical S-matrix elements. These latter interactions will result in contact interactions with form factors, for the free fields. Products containing propagators will not be $U(6,6)$-invariant, even without free-field conditions imposed.

In Sec. III we will clarify these statements and investigate the subgroups of $U(6,6)$ under which the various kinds of interaction terms are invariant.

III. HIERARCHY OF $U(6,6)$ SUBGROUPS IN TWO-BODY SCATTERING

It has been emphasized that $U(6,6)$ cannot be an invariance group for two-body scattering amplitudes.[17] In an effective Lagrangian this point is made manifest by the presence of $U(6,6)$ symmetry-breaking terms in the free-particle Lagrangian and in the free-particle propagators. In both cases the appearance of $\gamma_\mu p_\mu$ terms destroys the full invariance, even for completely degenerate masses. When the mass breakings are included, the full Lagrangian is only invariant under isotopic spin transformations (Lorentz invariance is, of course, maintained). However, when the masses are degenerate, the effective Lagrangian will be invariant under larger subgroups of $U(6,6)$ which depend on the particular kinematical configuration of the external particles. In this section we will display the heirarchy of $U(6,6)$ subgroups[25] that are invariances of the full effective Lagrangian (with mass degenerate multiplets) and the scattering amplitude for two-body scattering. We will first clarify the situation by considering mathematical quarks and their interactions.

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[25] R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965); H. Harari and H. J. Lipkin, Phys. Rev. **140**, B1617 (1965); R. Oehme, in *Preludes* (Wiley-Interscience, Inc., New York, 1966), p. 143.

In Sec. II we presented the free "quark" field as the projection on the basis for the fundamental representation of $U(6,6)$ satisfying the Dirac equation (2.4). It was shown that the free Lagrangian (2.3) was not invariant under $U(6,6)$.

If we now ask, under what *subgroup* of $U(6,6)$ is (2.3) an invariant, we are led to the subgroup defined by only those elements M of $U(6,6)$ which satisfy

$$M^{-1}(p_\mu\gamma_\mu\otimes I)M=(p_\mu\gamma_\mu\otimes I)\,, \tag{3.1}$$

as well as condition (2.1) above. Those elements form a $U(6)\otimes U(6)$ group which we will call the *momentum* p $U(6)\otimes U(6)$, since the elements will depend on the choice of momentum. If the quark is at rest $[p_\mu=(0,im)]$, the subgroup is called the *rest* $U(6)\otimes U(6)$ which is generated by the Hermitian set $\{I,\gamma_4,i\sigma_{rs},i\gamma_r\gamma_5\}\otimes\{\lambda\}$ and the quarks (antiquarks) transform according to the [6,1] ([1,6*]) irreducible representation. The same *rest* $U(6)\otimes U(6)$ will leave the free "**143**" meson Lagrangian and the free "**364**" baryon Lagrangian invariant when the particles are at rest. The degenerate mesons in (2.5) transform irreducibley according to the [6,6*] representation and degenerate baryons in (2.6) (antibaryons) according to the [56,1] ([1,56*]) irreducible representation of the *rest* $U(6)\otimes U(6)$ when the respective particles are at rest. [For convenience we will continue to denote the mesons by "**143**", and the baryons by "**364**" although the free fields do not transform according to these, or any, *irreducible* representations of $U(6,6)$.] More generally if we construct a basis for any irreducible representation of $U(6,6)$ as an irreducible tensor product of bases for fundamental representations, and then impose Bargmann-Wigner conditions on this basis [for example, $\psi_{i\alpha,j\beta,\ldots}(p)$, with the conditions $(ip_\mu\gamma_\mu+M)_i{}^{i'}\psi_{i'\alpha,j\beta,\ldots}(p)=(ip_\mu\gamma_\mu+M)_j{}^{j'}\psi_{i\alpha,j'\beta,\ldots}(p)=\cdots=0$], the associated field will transform according to an irreducible representation of the *momentum* p $U(6)\otimes U(6)$. [We will refer to these fields as "broken $U(6,6)$" fields.]

We consider now, an effective interaction Lagrangian constructed out of products of "broken $U(6,6)$" fields, in which all indices are fully contracted, for example, a four-quark interaction

$$\mathcal{L}_I=\lambda\bar\psi^{i\alpha}(k')\psi_{i\alpha}(k)\bar\psi^{j\beta}(p')\psi_{j\beta}(p)\,. \tag{3.2}$$

Then any S-matrix elements calculated from such an interaction will be *at most* invariant under a $U(6)\otimes U(6)$ group. In fact, this maximum symmetry will only be realized when all of the external and internal lines of the associated Feynman diagrams can be brought to rest simultaneously by a single Lorentz transformation, for only in that circumstance will all the "broken $U(6,6)$" fields transform irreducibly under the *same* momentum $U(6)\otimes U(6)$. In our example, if we use (3.2) to lowest order in λ to calculate quark-quark scattering, illustrated in Fig. 1(a), the S matrix will be $U(6)\otimes U(6)$ invariant only when the quarks scatter at rest, i.e., at

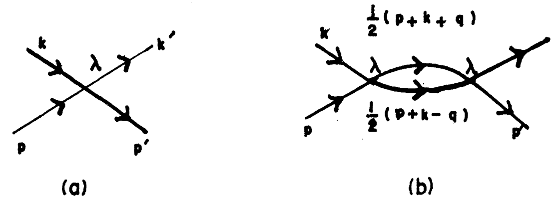

FIG. 1. Feynman diagrams for a four-quark contact interaction, specified by Eq. (3.2) in the text, calculated to (a) first order in λ, (b) second order in λ.

threshold. However, the λ^2 term in the S matrix [Fig. 1(b)], will not be $U(6)\otimes U(6)$-invariant for *any* kinematical configuration, since there is an integration over all four-momenta for the internal quark lines. The same statement applies for all higher orders in λ.

The next simplest kinematical configuration for the Feynman diagrams is one in which there are only *two independent* four-momenta, as in the forward direction for two-body elastic scattering. To determine which subgroup of $U(6,6)$ is the maximum invariance group for this configuration we first consider the product of two free-quark fields with different momenta p and k, and use (2.4) to write

$$\bar\psi^{i\alpha}(k)\psi_{i\alpha}(p)\sim\bar\psi^{j\alpha}(m-ik_\mu\gamma_\mu)_j{}^i(m-ip_\lambda\gamma_\lambda)_i{}^k\psi_{k\alpha}\,. \tag{3.3}$$

This will be invariant under that subgroup of $U(6,6)$, the elements of which satisfy conditions (2.1) and (3.1) above, as well as the condition

$$M^{-1}(k_\mu\gamma_\mu\otimes I)M=(k_\mu\gamma_\mu\otimes I)\,. \tag{3.4}$$

The subgroup so defined is a $U(6)$ group. If we consider the particular Lorentz frame in which $p=(0,im)$ and k is in the Z direction, the conditions (2.1), (3.1), and (3.4) can be replaced by $M^{-1}=M^\dagger$, $M^\dagger(\gamma_4\otimes I)M=(\gamma_4\otimes I)$, and $M^\dagger(\gamma_3\otimes I)M=(\gamma_3\otimes I)$, and the resulting group is $U(6)_W$.[25] The generators of this $U(6)_W$ are

$$\{I,i\sigma_{12},i\gamma_1\gamma_5,i\gamma_2\gamma_5\}\otimes\{\lambda\}\,.$$

The quarks transform according to the **6** representation of $U(6)_W$, the "**143**" mesons according to the reducible **1**$\oplus$**35**, and the "**364**" baryons according to the **56**. Then an S-matrix element calculated from a general effective interaction Lagrangian, will be invariant under $U(6)_W$ if some of the external and internal lines in the associated Feynman diagram can be brought to rest by a single Lorentz transformation under which the three-momenta of the remaining lines simultaneously become parallel or collinear. For our quark-quark example, (3.2) the first-order diagram, Fig. 1(a), for scattering *in the forward direction* will be $U(6)_W$-invariant, but second, Fig. 1(b), and higher orders will not be, since internal lines in the second and higher orders must be integrated over noncollinear momenta. For general two-body scattering in the forward direction, diagrams with no more than one internal line (i.e., for which the momentum is fixed by momentum conservation), will contribute $U(6)_W$-invariant S-matrix elements. Furthermore all three-point functions constructed from "broken $U(6,6)$" fields will be $U(6)_W$-invariant, since momentum conservation allows only two independent momenta. This means that the coupling constant relations obtained from the *formally* $U(6,6)$-invariant three-point functions will be the same as the $U(6)_W$ coupling-constant relations, when compared in the same Lorentz frame.

Finally, we consider the kinematical configuration for which there are *three independent* four-momenta, p, k, and q. The elements of the invariance group for such an S-matrix element will satisfy

$$M^{-1}(q_\mu\gamma_\mu\otimes I)M=(q_\mu\gamma_\mu\otimes I)\,, \tag{3.5}$$

as well as conditions (2.1), (3.1), and (3.4) above. This defines a $U(3)\otimes U(3)$ group. If we choose a frame in which one of the momenta p, k, or q is at rest and the other two three-momenta define the X-Z plane, that particular $U(3)\otimes U(3)$ will be generated by $\{I,i\gamma_2\gamma_5\}\otimes\{\lambda\}$ (note that in our representation $i\gamma_r\gamma_5=-\gamma_4\sigma_r$, $i\sigma_{rs}=-\epsilon_{rst}\sigma_t$). Under this *coplanar* $U(3)\otimes U(3)$ the quarks moving in the X-Z plane with spin quantized up (down) along the Y axis, the normal to the plane, transform according to the [3,1], ([1,3]) representation, whereas the antiquarks with spin up (down) transform under the [3*,1] ([1,3*]) representation. Then the "**143**" mesons transform according to the reducible representation

$$([3,1]\oplus[1,3])\otimes([3^*,1]\oplus[1,3^*])=[9,1]\oplus[1,9]\oplus[3,3^*]\oplus[3^*,3]\,, \tag{3.6}$$

where [9,1] ([1,9]) is a nonet of vector mesons with spin-projections $+1$ (-1), $[3,3^*]\oplus[3^*,3]$ contains a nonet of vectors with spin-projection 0, and a nonet of pseudoscalar mesons. The "**364**" baryons transform according to the totally symmetrized direct product of three-quark representations:

$$\{([3,1]\oplus[1,3])\otimes([3,1]\oplus[1,3])\otimes([3,1]\oplus[1,3])\}_{\text{sym}}=[10,1]\oplus[1,10]\oplus[6,3]\oplus[3,6]\,, \tag{3.7}$$

where [10,1] ([1,10]) is a decuplet of spin-$\frac{3}{2}^+$ baryons with projection $+\frac{3}{2}$ $(-\frac{3}{2})$, and [6,3] ([3,6]) is a decuplet of spin-$\frac{3}{2}^+$ baryons with projection $+\frac{1}{2}$ $(-\frac{1}{2})$, along with an octet of spin-$\frac{1}{2}^+$ baryons with projection $+\frac{1}{2}$ $(-\frac{1}{2})$. [We have derived the expressions (3.6) and (3.7) so that we will be able to determine below the number of *coplanar* $U(3)\otimes U(3)$-invariant amplitudes involved in meson-baryon scattering.] From the preceding paragraphs, it should now be clear that a diagram for which some of the external and internal lines can be brought to rest by a Lorentz transformation which simultaneously leaves all the other lines in the same plane (coplanar), will be invariant under *coplanar* $U(3)\otimes U(3)$. In particular, for two-body scattering, all diagrams with no more than one internal line will be coplanar, since momentum conservation allows only three independent external momenta and conservation also fixes the single internal momentum. All such diagrams will thus be *coplanar* $U(3)\otimes U(3)$ symmetric.

In summary, then, given an effective Lagrangian fully contracted on "broken $U(6,6)$" fields, all diagrams calculated from this Lagrangian will be (i) invariant under $U(6)\otimes U(6)$ if all lines in the diagram can be simultaneously brought to rest; (ii) invariant under $U(6)_W$ if some lines can be brought to rest while the remaining lines are collinear in three-momenta; (iii) invariant under *coplanar* $U(3)\otimes U(3)$ if some lines can be brought to rest while those remaining are coplanar. Then for scattering of *two* degenerate "broken $U(6,6)$" multiplets, all diagrams containing, at most, one internal line, will be rest $U(6)\otimes U(6)$-symmetric at threshold, $U(6)_W$-symmetric in the forward direction, and *coplanar* $U(3)\otimes U(3)$-symmetric for arbitrary kinematical configurations [the "tree graphs" are always *coplanar* $U(3)\otimes U(3)$-symmetric]. We see then, that in the limit of mass degeneracy, using trilinear interactions only to second order, and quadrilinear couplings only to first order, results in *coplanar* $U(3)\otimes U(3)$ symmetry for the S-matrix elements in two-body scattering.

IV. EFFECTIVE LAGRANGIAN

We next write our effective interaction Lagrangian for meson-baryon scattering,[19] containing formally $U(6,6)$-invariant trilinear and quadrilinear couplings. The trilinear terms lead to manifest $U(6,6)$ symmetry breaking in second order, due to the imposition of free-field conditions on the external particles, as well as the propagator for the exchanged particle. The discussion of Sec. III will facilitate an analysis of the invariances of the resulting scattering amplitude.

With $\phi_{i\alpha}{}^{j\beta}(k)$ the "**143**" mass-degenerate mesons and $\psi_{i\alpha,j\beta,k\gamma}(p)$ the "**364**" degenerate baryons, the effective interaction Lagrangian is, as in I,

$$\begin{aligned}\mathcal{L}_I=\alpha\bar{\psi}^{ABC}(p')\psi_{ABC}(p)\phi_E{}^D(-k')\phi_D{}^E(k)+4\beta\bar{\psi}^{ABC}(p')\psi_{ABD}(p)[\phi_C{}^E(-k')\phi_E{}^D(k)+\phi_C{}^E(k)\phi_E{}^D(-k')]\\+4\tilde{\beta}\bar{\psi}^{ABC}(p')\psi_{ABD}(p)[\phi_C{}^E(-k')\phi_E{}^D(k)-\phi_C{}^E(k)\phi_E{}^D(-k')]+\gamma\bar{\psi}^{ABC}(p')\psi_{ADE}(p)\phi_B{}^D(-k')\phi_C{}^E(k)\\+ig\tfrac{1}{4}m_0Tr(\phi\phi\phi)+iG\bar{\psi}^{ABC}\psi_{ABD}\phi_C{}^D\,,\end{aligned} \tag{4.1}$$

where A, B, $\cdots$ are pairs (i,α), (j,β), $\cdots$, and G and g are coupling constants which will be fixed by the known π-nucleon coupling and by the decay width of the ρ into two pions, respectively. The parameters α, β, $\tilde{\beta}$, and γ are undetermined functions of the scalar invariants s, t, and u, and must satisfy the following restrictions due to crossing symmetry:

$$\begin{aligned}\alpha(s,t,u)=\alpha(u,t,s),\quad \beta(s,t,u)=\beta(u,t,s)\,,\\ \tilde{\beta}(s,t,u)=-\tilde{\beta}(u,t,s),\quad \gamma(s,t,u)=\gamma(u,t,s)\,.\end{aligned} \tag{4.2}$$

Since s, t, and u are not independent, we can choose to write the four parameters as functions of the two independent variables t and $(s-u)$. Then the above restrictions imply that α, β, and γ must be even functions of $(s-u)$, whereas $\tilde{\beta}$ must be an odd function of $(s-u)$. Later we will assume for simplicity that the parameters are constants, so that $\tilde{\beta}$ will have to be set equal to zero.

When free-field solutions (2.5) and (2.6) are substituted into (4.1), an effective interaction Lagrangian will result containing trilinear couplings among the vector and pseudoscalar mesons, the spin-$\frac{1}{2}$ baryons and and spin-$\frac{3}{2}$ baryons and because of the momentum dependences in the free-field solutions, there will be form factors multiplying those interaction terms.[16] In particular, the familiar couplings for the nucleons, pions, and ρ mesons, $\bar{u}\boldsymbol{\tau}\gamma_5 u\cdot\boldsymbol{\phi}$, $\bar{u}\boldsymbol{\tau}\gamma_5\gamma_\mu u\cdot\partial_\mu\boldsymbol{\phi}$, $\bar{u}\boldsymbol{\tau}\gamma_\mu u\cdot\mathbf{V}_\mu$, $\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi}\cdot\mathbf{V}_\mu$, will be contained in the expansion, with momentum-dependent form factors. Furthermore, the quadrilinear terms will result in contact terms among the mesons and baryons, again with momentum-dependent form factors. Among these terms will be the pion-nucleon contact interactions $\bar{u}u\boldsymbol{\phi}\cdot\boldsymbol{\phi}$ and $\bar{u}\boldsymbol{\tau}\gamma_\mu u\cdot\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi}$, and with our final choice of parameters, in Sec. V, only the latter contact term will contribute near threshold. Our effective Langrangian, then, is similar in form to the phenomenological Lagrangians,[11] except that we have maintained invariance under the nonchiral *coplanar* $U(3)\otimes U(3)$, and have made no attempt to relate the couplings here to the constants of weak interactions.[26]

Now, given the effective Lagrangian (4.1), we can derive the amplitude for scattering of the meson and baryon multiplets.

The trilinear couplings will lead to three matrix elements in second order; meson exchanges, baryon exchanges, and direct-channel baryon poles. The quadrilinear terms are only used to first order, so the resulting amplitude will be

$$\begin{aligned}M(p',k';p,k)=&\alpha\bar{\psi}^{ABC}(p')\psi_{ABC}(p)\phi_E{}^D(-k')\phi_D{}^E(k)\\&+4\beta\bar{\psi}^{ABC}(p')\psi_{ABD}(p)[\phi_C{}^E(-k')\phi_E{}^D(k)+\phi_C{}^E(k)\phi_E{}^D(-k')]\\&+4\tilde{\beta}\bar{\psi}^{ABC}(p')\psi_{ABD}(p)[\phi_C{}^E(-k')\phi_E{}^D(k)-\phi_C{}^E(k)\phi_E{}^D(-k')]+\gamma\bar{\psi}^{ABC}(p')\psi_{ADE}(p)\phi_B{}^D(-k')\phi_C{}^E(k)\\&-\tfrac{1}{4}m_0Gg\bar{\psi}^{ABC}(p')\psi_{ABD}(p)\langle\phi_C{}^D(p'-p),\,\bar{\phi}_E{}^F(p'-p)\rangle_+[\phi_F{}^G(-k')\phi_G{}^E(k)+\phi_F{}^G(-k')\phi_G{}^E(k)]\\&+G^2\bar{\psi}^{ABC}(p')\phi_C{}^D(-k')\langle\psi_{ABD}(p+k),\,\bar{\psi}^{EFG}(p+k)\rangle_+\phi_E{}^H(k)\psi_{HFG}(p)\\&+G^2\bar{\psi}^{ABC}(p')\phi_C{}^D(k)\langle\psi_{ABD}(p-k'),\,\bar{\psi}^{EFG}(p-k')\rangle_+\phi_E{}^H(-k')\psi_{HFG}(p)\,,\end{aligned}\tag{4.3}$$

where the brackets $\langle\ \rangle_+$ represent the momentum-space propagators for the multiplet specified, which were defined in (2.11) and (2.12). When the free-field solutions and the propagators are substituted into the amplitude $M(p',k';p,k)$, the α, β, $\tilde{\beta}$, and γ terms will give rise to amplitudes containing polynominals in the invariants s, t, and u, and no pole terms. These contact terms are then of the same form as the Cini-Fubini polynomianl approximations to the double spectral functions in the Mandelstam representation.[4] Hence we interpret them as approximations to singularities distant from the threshold region of the physical cut in the Mandelstam plane,[2] i.e., as short-range forces. The terms containing the propagators, when expanded, will consist of contact terms and pole terms, and thus represent single-particle intermediate states—the single spectral functions or long-range forces—with the additional contact terms to ensure that these are purely $U(6,6)$-breaking terms (as specified in Sec. II). Because of the additional contact terms, the symmetry-breaking single-particle exchanges will all vanish at the degenerate-mass threshold. Thus the dynamical scheme we have chosen is one in which long-range forces are generated by the exchange of the same particles that are scattered—the low-lying hadron states—and the short-range forces, approximated by *formally* $U(6,6)$-invariant contact terms, dominate the scattering at and near threshold.

Next we investigate the number of independent amplitudes that should enter from symmetry considerations in the scattering of "**143**" mesons on "**364**" baryons. If $U(6,6)$ were a good symmetry of the scattering amplitude (which it is not) we would have only four independent invariant amplitudes, since in $U(6,6)$, $\mathbf{364}\otimes\mathbf{143}=\mathbf{364}\oplus\mathbf{572}\oplus\mathbf{16016}\oplus\mathbf{35100}$ couples in four invariant ways to another $\mathbf{364}\otimes\mathbf{143}$ [note that we have ignored the $U(6,6)$ scalar part of the meson representation]. If free-field conditions were not imposed on the external particles, one choice of four independent invariants would be the α, β, $\tilde{\beta}$, and γ terms of (4.3). By then decomposing the fields according to the $SU(2)_S\otimes SU(3)$ subgroup, where $SU(2)_S$ is the group of ordinary spin transformations, relations would be obtained among the many reactions described by the four $U(6,6)$ invariants. Specifically, all two-body scatterings of the vector or pseudoscalar nonets by the baryon octet or decuplet would be specified by fixing the four functions, α, β, $\tilde{\beta}$, and γ.[27] Then the well-known difficulties with the implementation of unitarity would arise.[28] However, because of the introduction of free-field conditions and

[26] A similar Lagrangian has also been used for πN scattering at low energies, requiring PCAC; H. S. Mani, Y. Tomozawa, and Y. P. Yao, Phys. Rev. Letters **18**, 1084 (1967).

[27] The quadrilinear terms alone were used to predict relations among many cross sections by J. M. Cornwall, P. G. O. Freund, and K. T. Mahanthappa, Phys. Rev. Letters **14**, 515 (1965); R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. Treiman, *ibid.* **14**, 518 (1965). Several of the predictions of this simple model were in violent disagreement with experiment, such as no polarization in $K^-p\rightarrow K^+\Xi^-$.

[28] This was already noticed by R. Blankenbecler *et al.* (Ref. 27).

second-order matrix elements in (4.3), $U(6,6)$ will not be an invariance group of the scattering processes. The $ip_\mu\gamma_\mu$'s that are contained in the free-field projection operators and propagators break the symmetry as explained in the preceding sections. This is equivalent to introducing symmetry-breaking "spurions" or "kinetons"[18] but in a particular, physical context; namely, as particle exchanges. The coefficients that multiply the symmetry-breaking terms are consequently related to physical coupling constants, and are not arbitrary, as they would be in the "spurion" scheme. However, it is important to note, that if all possible "spurions" are introduced into the amplitude, the remaining invariance will be only $U(3)$. By restricting our matrix element to no more than second-order terms in the coupling constants, we are considering only a subset of "spurions" such that, in general, *coplanar* $U(3)\otimes U(3)$ symmetry is maintained in the absence of mass splittings, and for particular kinematical configurations, larger subgroups of $U(6,6)$ are invariance groups.

At the threshold for the reaction, all the degenerate particles are at rest and we have seen in Sec. III that the scattering amplitude must be *rest* $U(6)\otimes U(6)$ symmetric. Then at threshold there can be only two invariant amplitudes, since in $U(6)\otimes U(6)$,

$$[56,1]\otimes[6,6^*]=[126,6^*]\oplus[210,6^*]$$

and this can couple in only two invariant ways to another $[56,1]\otimes[6,6^*]$.

Thus, of the seven terms in the amplitude M, (4.3), there can be only two independent terms at threshold. Only the α, β, and $\tilde{\beta}$ terms will be nonzero at the degenerate-mass threshold and the $\tilde{\beta}$ term will equal the β term. This has very strong implications for the scattering lengths of the reactions involved—they are all determined by the two functions α and $(\beta+\tilde{\beta})$ evaluated at threshold (i.e., two constants).

Then (for degenerate masses) the following relations are obtained for s-wave scattering lengths:

$$a(\pi^+p\to\pi^+p)=a(K^+p\to K^+p)=-[16M/(M+m)][\alpha+\tfrac{4}{3}(\beta+\tilde{\beta})],\quad(4.4)$$

$$a(\pi^-p\to\pi^-p)=a(K^+n\to K^+n)=-[16M/(M+m)][\alpha+\tfrac{2}{3}(\beta+\tilde{\beta})];$$
$$2a(K^+n\to K^+n)-a(K^+p\to K^+p)=a(K^-p\to K^-p)=a(K^-n\to K^-n);\quad(4.5)$$

$$\begin{aligned}a(\pi^+p\to\pi^+p)-a(\pi^-p\to\pi^-p)&=(\sqrt{6})a(\pi^-p\to\eta^0n)\\&=\sqrt{3}a(\pi^-p\to\eta'n)=-(2/\sqrt{3})a(K^-p\to\pi^0\Lambda^0)\\&=-2a(K^-p\to\eta^0\Lambda^0)=-\sqrt{2}a(K^-p\to\eta'\Lambda^0)\\&=-2a(K^-p\to\pi^0\Sigma^0)=-a(K^-p\to\pi^-\Sigma^+);\end{aligned}\quad(4.6)$$

$$\begin{aligned}0&=a(\pi^+p\to K^+\Sigma^+)=a(\pi^-p\to K^+\Sigma^-)\\&=a(\pi^-p\to K^0\Lambda^0)=a(K^-p\to K^+\Xi^-)\\&=a(K^-n\to K^0\Xi^-).\end{aligned}\quad(4.7)$$

When mass breaking is included, the $U(6)\otimes U(6)$ symmetry is broken and the relations among the inelastic amplitudes at threshold (4.6) and (4.7) will no longer hold since other terms in the matrix elements (4.3) will be nonzero. However, the elastic processes will still be determined by the two parameters α and $\beta+\tilde{\beta}$, although expressions (4.4) and (4.5) become modified in the following way:

$$\begin{aligned}a(\pi^+p\to\pi^+p)&=[-8M_N/(M_N+m_\pi)]\{[1+(m_\pi^2/m_0^2)](\alpha+[\beta+\tilde{\beta}])+\tfrac{2}{3}(m_\pi/m_0)[\beta+\tilde{\beta}]\},\\a(\pi^-p\to\pi^-p)&=[-8M_N/(M_N+m_\pi)]\{[1+(m_\pi^2/m_0^2)](\alpha+[\beta+\tilde{\beta}])-\tfrac{2}{3}(m_\pi/m_0)[\beta+\tilde{\beta}]\},\\a(K^\pm p\to K^\pm p)&=[-8M_N/(M_N+m_K)]\{[1+(m_K^2/m_0^2)](\alpha+\tfrac{2}{3}[\beta+\tilde{\beta}])\pm\tfrac{4}{3}(m_K/m_0)[\beta+\tilde{\beta}]\},\\a(K^\pm n\to K^\pm n)&=[-8M_N/(M_N+m_K)]\{[1+(m_K^2/m_0^2)](\alpha+\tfrac{1}{3}[\beta+\tilde{\beta}])\pm\tfrac{2}{3}(m_K/m_0)[\beta+\tilde{\beta}]\}.\end{aligned}\quad(4.8)$$

For the K-meson scattering lengths, a simple relation holds even with mass breaking:

$$a(K^+p)-a(K^-p)=2[a(K^+n)-a(K^-n)].\quad(4.9)$$

This relation is part of the Johnson-Treiman relations, which were derived from degenerate $SU(6)$.[29] Taking the experimentally determined KN and $\bar{K}N$, $T=1$ scattering lengths, we can obtain the $\bar{K}N$, $T=0$ scattering length[30,31]:

$$a(KN,\,T=0)\simeq0.0\text{ F},$$
$$a(KN,\,T=1)\simeq0.28\text{ F},$$
$$a(\bar{K}N,\,T=1)\simeq0.0+i0.7\text{ F}.$$

Then $a(\bar{K}N,\,T=0)\simeq-0.2-i1.2$, which is nonsensical since the imaginary part must be positive. This just confirms the importance of multichannel effects for the $\bar{K}N$ system. We found in I that choosing $\tilde{\beta}=0$ and $\alpha=-\beta$ gave good scattering lengths for πN and the $T=1$ KN. We will consider this again when we calculate cross sections in Sec. V. It is also important to note that although relations (4.7) cannot hold when mass breaking is included, the threshold amplitudes for these associated-production reactions will be small relative to the elastic amplitudes (by more than an order of magnitude). We take this to be a very encouraging qualitative prediction of the model, since all of the reactions involved in (4.7) have small cross sections at low energies, experimentally.

Relations similar to (4.4)–(4.7) can be written for degenerate-mass threshold amplitudes in vector-meson

[29] K. Johnson and S. Treiman, Phys. Rev. Letters **14**, 189 (1965).
[30] V. J. Stenger *et al.*, Phys. Rev. **134**, B1111 (1964).
[31] S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1962).
[32] J. K. Kim, Phys. Rev. Letters **14**, 29 (1965).

production ($P+B\rightarrow V+B$), baryon-decuplet production ($P+B\rightarrow P+D$), and double-resonance production ($P+B\rightarrow V+D$). All of these relations will be altered drastically by mass breaking.

The next simplest kinematical configuration will be the forward direction, for which all external and internal lines in the matrix elements that we have considered in (4.3) will have parallel three momenta, and hence the amplitude will be $U(6)_W$-invariant. The number of independent $U(6)_W$-invariant amplitudes for the scattering of the reducible **36** representation by the **56** representation will be *seven*, since

$$\mathbf{56}\otimes\mathbf{36}=\mathbf{56}\otimes(\mathbf{35}\oplus\mathbf{1})=\mathbf{56}\oplus\mathbf{70}\oplus\mathbf{700}\oplus\mathbf{1134}\oplus\mathbf{56}'$$

contains two **56**'s which couple invariantly in four ways to the two **56**'s contained in the outgoing **56**⊗**36**. Since our amplitude (4.3) contains only seven independent terms, in the forward direction we obtain the same predictions as in $U(6)_W$, with the further restriction that our three pole terms involve only two parameters—the coupling constants g and G (although the functional forms of the three terms are all different).

If we separate out the $SU(3)$ singlet pseudoscalar part of the meson fields by writing

$$\phi_{i\alpha}{}^{j\beta}(k)=\left[\phi_{i\alpha}{}^{j\beta}(k)-\tfrac{1}{3}\delta_\alpha{}^\beta\left(\gamma_5-\frac{ik}{m}\gamma_5\right)_i^j(\gamma_5)_k{}^l\phi_{l\gamma}{}^{k\gamma}(k)\right]+\tfrac{1}{3}\delta_\alpha{}^\beta\left(\gamma_5-\frac{ik}{m}\gamma_5\right)_i^j(\gamma_5)_k{}^l\phi_{l\gamma}{}^{k\gamma}(k)\,,\quad(4.10)$$

the bracketed term will transform according to the irreducible **35** representation of $U(6)_W$. Then substituting (4.10) into the scattering matrix (4.3), we can separate out those terms corresponding to pure **35** scattering off of **56**. From the direct product decomposition, above, we know that there will be only *four* independent $U(6)_W$-invariant amplitudes for **35**⊗**56** scattering. Then in the forward direction the three second-order terms in (4.3), corresponding to particle exchanges, will be linear combinations of the four first-order terms in (4.3) (the α, β, $\tilde{\beta}$, and γ term). Again, with our choice of propagators, the meson-exchange term will vanish for forward scattering, so the baryon-exchange terms will be linear combinations of the four first-order terms. (These remarks, as previously specified, only apply in the degenerate-mass limits.) It then follows that all of the relations among scattering amplitudes in $U(6)_W$ will hold for forward scattering of degenerate-mass particles in our model. In particular, the relations derived by Johnson and Treiman[29] and by Carter *et al.*[33] will be obtained for the forward amplitudes. Many of these relations have been shown to disagree markedly with experiment.[34] By including mass breakings, the relations will be altered and simple ratios of amplitudes will no longer be obtained.

Finally, for an arbitrary kinematical configuration in our meson-baryon scattering matrix, the amplitude will be *coplanar* $U(3)\otimes U(3)$ symmetric. Since the pseudoscalar and spin-projection zero (quantized normal to the scattering plane) vector mesons are in the $(3,3^*)_0$ and $(3^*,3)_0$ irreducible representations, and the spin-projection $+\frac{1}{2}$ baryon octet and decuplet are in the $(6,3)_{+1/2}$ representation, and the direct products

$$\begin{aligned}(3,3^*)_0\otimes(6,3)_{+1/2}&=[(10,8)\oplus(8,8)\oplus(10,1)\oplus(8,1)]_{+1/2}\,,\\(3^*,3)_0\otimes(6,3)_{+1/2}&=[(15,6)\oplus(3,6)\oplus(15,3^*)\oplus(3,3^*)]_{+1/2}\,,\end{aligned}\quad(4.11)$$

then, there will be eight *coplanar* $U(3)\otimes U(3)$-invariant amplitudes for $P+B_{+1/2}\rightarrow P+B_{+1/2}$ ($P+B_{-1/2}$ amplitudes are related to these by parity). Note that there will be no $P+B_{+1/2}\rightarrow P+B_{-1/2}$ terms since the total spin projection and parity are combined. The same eight amplitudes will also describe reactions involving the spin-projection zero vector mesons and the spin-projection $\pm\frac{1}{2}$ decuplet baryons. To completely determine production of vector mesons ($P+B\rightarrow V+B$) the initial states, represented by the decomposition in (4.11), must also be coupled to the spin-projection $+1$ (-1) vector mesons in the final state, which belong to the $(8,1)_{+1}\oplus(1,1)_{+1}((1,8)_{-1}\oplus(1,1)_{-1})$ reducible representation. When this is done there will be a total of 24-invariant amplitudes for vector-meson nonet production. These 24 amplitudes completely determine the six independent helicity amplitudes for the production of each member of the nonet. Because there are, generally, more independent amplitudes in *coplanar* $U(3)\otimes U(3)$ than the seven we have chosen in our broken $U(6,6)$ scheme, there is less arbitrariness in our prediction (and less freedom). For example, of the eight undetermined functions of energy and angle that would describe $P+B\rightarrow P+B$ scattering in *coplanar* $U(3)\otimes U(3)$, only four of those functions will be independent in our model. Furthermore, part of the functional dependence will be fixed by the kinematical factors that arise in the decomposition of the supermultiplets "**143**" and "**364**" into constituent mesons and baryons, part will arise from the dynamical assumption of single-particle intermediate states, and the remainder will be specified by fixing α, β, $\tilde{\beta}$, and γ. In specifying the functions we were guided by the Cini-Fubini representation, i.e., distant singularities in the scattering amplitude, at low energies, can be approximated by polynomials in the Mandelstam variables s and t. Polynomial terms appear naturally in the four-point functions and in the nonpole terms in the single-particle exchanges, so we make the simplest assumption, that all of the distant singularities are accounted for by these polynomial terms and the α, β, $\tilde{\beta}$, and γ merely fix the over-all strength of the four-point interactions. Then α, β, $\tilde{\beta}$, and γ are taken to be con-

[33] J. C. Carter *et al.*, Phys. Rev. Letters **15**, 373 (1965).
[34] J. D. Jackson, Phys. Rev. Letters **15**, 990 (1965).

stants. Condition (4.2) then requires $\tilde{\beta}=0$, so we have only three constants to choose in order to determine the low-energy scattering processes.

V. UNITARITY, ANGULAR DISTRIBUTIONS, AND CROSS SECTIONS

To proceed with the calculation it is necessary to specify the four as yet undetermined functions that multiply the four-point functions in the Lagrangian, and the two coupling constants g and G for the trilinear interactions. The coupling constants are fixed, as in I, by relating G to the π-N coupling and g to the ρ-$\pi\pi$ coupling to obtain $G^2/4\pi=2.05$ and $g^2/4\pi=0.05$ (having taken $g_{\rho\rho\pi^0}{}^2/4\pi=15$ and $g_{\rho\pi\pi}{}^2/4\pi=\frac{1}{2}$). This fixes the couplings of the "**364**" baryons to the "**143**" mesons and the couplings of the "**143**" mesons to the "**143**" mesons. The four functions α, β, $\tilde{\beta}$, and γ are chosen to be constants for simplicity. Crossing symmetry for the four-point functions demands that α, β, and γ be even under the interchange of the Mandelstam variables s and u, whereas $\tilde{\beta}$ must be odd. Hence $\tilde{\beta}$ must vanish when it is taken to be a constant, leaving three constants, α, β, and γ. Next, requiring that the π-N s-wave scattering lengths satisfy the relation $2a_3+a_1=0$, which is obtained by assuming pure isovector exhcange in the t channel at the s-channel threshold[35] and is fairly well satisfied experimentally,[36] imposes the restriction that $\alpha+\beta=0$. Finally, the calculated s-wave phase shifts for πN, $T=\frac{1}{2}$ at $P_{\text{lab}}=100$ MeV/c and KN, $T=1$ at $P_{\text{lab}}=140$ MeV/c are required to agree with the experimentally determined phase shifts,[37] which fixes the two remaining independent parameters, as in I,

$$\alpha=-\beta=-0.0957\mu_\pi{}^{-1},$$
$$\gamma=3.00\mu_\pi{}^{-1}.$$

Having made these choices for the parameters, it is now possible to use the Lagrangian to second order as specified previously, to calculate the low-energy scattering of any member of the "**143**" meson representation on any member of the "**364**" baryon representation. However, it should be clear that such a calculation can only produce real amplitudes, and may violate single-channel and multichannel unitarity, even in the low-energy region considered.

To compare our results with experimental cross sections, angular distributions, and excitation functions it is necessary to implement unitarity. We would like to do this in a way that introduces few new parameters into the model yet, is, in some sense, a better approximation to a more complete theory. One general method often used to generate an amplitude satisfying unitarity, from a Born-term input, is the N/D method.[38] In our case, however, the lowest-order terms that would be used as input in an N/D iteration procedure were chosen to be a good approximation to threshold phenomena. Those terms contain polynomials in s, t, and u which approximate the distant singularities only for low energies. In calculating the D function the polynomial terms would have to be cutoff to give nondivergent results, and since the polynomials become very large in the medium-energy region the results would be strongly dependent on the cutoff parameters chosen for each partial-wave amplitude. Thus we feel that the N/D method is not a natural procedure for unitarizing our amplitudes.

A simpler assumption, and a more natural one from our point of view, is to assume that our partial-wave amplitudes are an effective-range approximation to the real K matrix.[39] Such an ansatz is implemented without introducting any new parameters, and does not involve the behavior of the amplitudes at higher energies. The resulting T matrix then satisfies unitarity automatically, since $T^{-1}=K^{-1}-i\rho$, where ρ is the appropriate phase space matrix, and $\text{Im}T^{-1}=-\rho$. It is easy to justify this procedure when the calculated real partial-wave amplitudes are small, as in s-wave πN scattering, where $(\cos\delta\,\sin\delta/q)\simeq\delta/q\simeq 1/q\,\cot\delta=K$. In the discussion of the $N_{33}{}^*$ resonance, below, we will also see that this method provides a simple way to unitarize resonant amplitudes, so that the correct threshold behavior is guaranteed. In the case of large, nonresonant partial waves it is not clear that this approach is completely justified, but we will attempt to make it plausible. Consider a perturbative scheme in which the T and K matrix are expanded in powers of some coupling constant λ^2, i.e.,

$$T=\sum_{n=1}^{\infty}\lambda^{2n}T_n$$

and

$$K=\sum_{n=1}^{\infty}\lambda^{2n}K_n.$$

Then the relation $T^{-1}+i\rho=K^{-1}$ yields an iterative

[35] This prediction is a result of several different theoretical models. S. Weinberg [Phys. Rev. Letters **17**, 616 (1966)] and A. P. Balachandran, M. G. Gundzik, and F. Nicodemi [Nuovo Cimento **44A**, 1257 (1966)] use the current algebra to obtain this result. J. J. Sakurai [Phys. Rev. Letters **17**, 552 (1966)] uses vector-isovector meson dominance. J. Schwinger (Ref. 11) and Wess and Zumino (Ref. 11), among others, use chiral symmetric phenomenological Lagrangians. Finally, the PCAC assumption gives this result as a consistency condition at threshold as shown by S. L. Adler, Phys. Rev. **137**, B1022 (1965).

[36] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963); L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965); S. W. Barnes *et al.*, *ibid.* **117**, 226 (1960), all agree that $2a_3+a_1$ is small. C. Lovelace, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), has obtained $2a_3+a_1=0.069$, which is in strong disagreement with the other determinations. However, if we choose to fit the Lovelace values, our parameters α and β change by only 10%, and our final conclusions will be effected only very slightly.

[37] S. W. Barnes *et al.*, Phys. Rev. **117**, 226 (1960); S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1962).

[38] See, for example, A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963), where earlier references are also given.

[39] R. H. Dalitz, Rev. Mod. Phys. **33**, 471 (1961).

solution, the first few terms of which are symbolically

$$K_1 = T_1,$$
$$K_2 = T_2 - T_1 i\rho T_1,$$
$$K_3 = T_3 - T_2 i\rho T_1 - T_1 i\rho T_2 - T_1 i\rho T_1 i\rho T_1.$$

Hence, in such an iteration scheme, the lowest-order K matrix is the lowest-order T-matrix element, i.e., the Born term. In our case we can identify the four-point interaction terms with a low-energy approximation to the higher-order terms in K.[40] It should be emphasized that the preceding discussion is intended only as a plausibility argument. We do not calculate any higher-order T-matrix elements.

To illustrate some general consequences of this assumption, consider a partial-wave amplitude for pseudoscalar-meson–spin-$\frac{1}{2}$-baryon scattering with total angular momentum $J = l \pm \frac{1}{2}$, satisfying elastic unitarity,[41]

$$f_{l\pm}(q) = [e^{i\delta_{l\pm}(q)} \sin\delta_{l\pm}(q)]/q = 1/\{[K_{l\pm}(q)]^{-1} - iq\},$$

where $K_{l\pm}(q) = [\tan\delta_{l\pm}(q)]/q$. Now, if the partial-wave projections of the matrix elements obtained from the Lagrangian (4.1) are set equal to the K matrix, the elastic unitarity condition, $\mathrm{Im}\{[f_{l\pm}(q)]^{-1}\} = -q$, and the threshold conditions, $\mathrm{Re}f_{l\pm}(q) \sim q^{2l}$ and $\mathrm{Im}f_{l\pm}(q) \sim q^{4l+1}$, are automatically satisfied since the $K_{l\pm}$ will be real and satisfy $K_{l\pm}(q) \sim q^{2l}$ near threshold. Furthermore, the $f_{l\pm}(q)$ will have the same singularities as the input matrix elements—no new poles can be generated by this procedure. This is in keeping with the spirit of the model; low-lying resonant states are explicitly accounted for by pole diagrams (it is essentially an N/D procedure in which the N function is approximated by the aforementioned matrix elements and the real part of the D function is set equal to 1). Chosen as stated, the $K_{l\pm}$ will have pole terms and polynomial terms and consequently its magnitude will become a monotonically increasing function beyond some finite energy, so that the $f_{l\pm}$ will all tend to zero at high energies. These $K_{l\pm}$'s having no poles in the physical region will produce phase shifts that approach $\pm\frac{1}{2}\pi$ asymptotically. These asymptotic properties will be of no concern, however, since the model is only used up to energies slightly above inelastic thresholds. For those $K_{l\pm}$'s that become large in magnitude at low energies, the corresponding $\delta_{l\pm}$ will approach $\pm\frac{1}{2}\pi$ more rapidly. In a few cases we will consider the implications of multichannel unitarity for the K matrix, but these general results will still apply.

[40] Recently Remiddi, Pusterla, and Mignaco have used a Padé approximant technique to sum T-matrix elements to fourth order to determine πN phase shifts. The $P[2,2]$ approximant has extra effective-range parameters included to account for short-range forces, so that except for the fourth-order terms, the partial-wave amplitudes have a form similar to our K-matrix approximation. [E. Remiddi, M. Pusterla, and J. A. Mignaco, CERN Report No. 895, 1968 (unpublished).]

[41] S. C. Frautschi and J. D. Walecka, Phys. Rev., **120**, 1486 (1960).

Having specified the calculational procedure, and its qualitative features, we will now proceed to discuss the quantitative results of the model for particular reactions.

A. $\pi N \to \pi N$

In I we obtained the lowest-order terms from the effective Lagrangian for πN scattering, projected out partial waves, and identified them with the real parts of the partial-wave amplitudes $f_{l\pm}^{(T)}$. In the low-energy region ($P_{\mathrm{lab}} = 0$–300 MeV/c) the calculated real partial waves were found to be in good agreement with the experimentally determined set of Roper, Wright, and Feld,[42] with the exception of the resonant amplitude ($T = \frac{3}{2}$, $J = \frac{3}{2}$). In the latter case, the model did not take the nonzero width of the N_{33}^* into account, and consequently this amplitude was infinite at the resonance position. In order to compare more directly with experimental data, namely, differential and total cross sections, it is necessary to include a width for the N_{33}^*. We have done this without introducing any new parameters, by using the K-matrix approximation, discussed above, which is equivalent to a narrow-width approximation[43] with elastic unitarity and correct threshold behavior, as we now demonstrate.

Consider the pure-pole contribution to the P_{33} amplitude (which is identical to the narrow-width approximation to a dispersion relation for a kinematical-singularity-free amplitude)

$$\tilde{f}_{1+}^{(3/2)}(q) = \frac{E+M}{W}\frac{1}{6}\frac{q^2}{M^2}\frac{g_{N^*N\pi}^2/4\pi}{M_R - W}, \qquad (5.1)$$

where $f_{1+}^{(3/2)}$ is the usual $T = \frac{3}{2}$, $l = 1$, $J = \frac{3}{2}$ partial-wave amplitude (defined in I), W is the total energy in the c.m. system, E is the nucleon energy, q is the magnitude of the c.m. momentum, M_R is the resonance mass, and $g_{N^*N\pi}^2/4\pi$ is a dimensionless coupling constant. This amplitude is pure real and has a pole at $W = M_R$. In order to produce the usual Breit-Wigner resonance form, near the resonance, replace M_R by $M_R - i(q/q_R)\frac{1}{2}\Gamma$, where Γ is identified with the full width at half-maximum. Then the new amplitude will satisfy unitarity *at* the resonance position, providing the following identification is made:

$$\Gamma = (E_R + M/W)\tfrac{1}{3} q_R^3 (M^{-2}) g_{N^*N\pi}^2/4\pi, \qquad (5.2)$$

where the subscript R refers to the value of the subscripted quantity at the resonance energy. This identification gives a width of 100 MeV in our model, where $g_{N^*N\pi} = \frac{2}{3}[1 + (M + M_R)/m_0]G$. If unitarity is to be satisfied for all energies, Γ must be a function of energy

[42] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965); also A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968), who obtain phase shifts in agreement with the above at the low energies considered here.

[43] A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964).

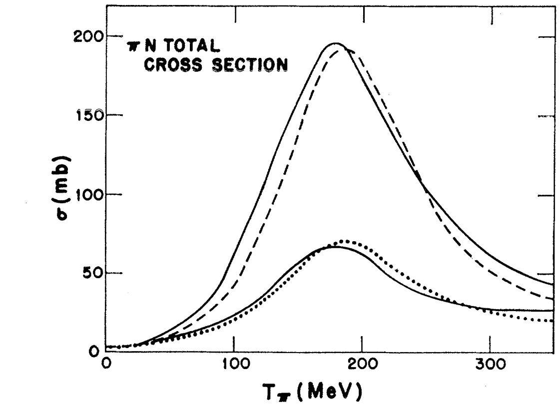

FIG. 2. $\pi^{\pm}p$ total sections. The experimental curves (Ref. 45) are given by solid lines for $\pi^{\pm}p$. The cross sections calculated from our model are given by dashed and dotted lines for $\pi^{+}p$ and $\pi^{-}p$, respectively.

satisfying relation (5.2) at $W=M_R$. If, furthermore, we require that the form (5.1) be preserved, except for the substitution of a complex mass for the resonance,[44] then the complete amplitude must be

$$f_{1+}{}^{(3/2)}(W)=\frac{\Gamma(W)/2}{q_R(M_R-W)-iq\Gamma(W)/2},$$

where

$$\Gamma(W)=(E+M/W)\tfrac{1}{3}q_Rq^2/(M^2)g_{N^*N\pi}{}^2/4\pi. \qquad (5.3)$$

This amplitude also satisfies the threshold conditions

$$\mathrm{Re}f_{1+}\to q^2,\quad \mathrm{Im}f_{1+}\to q^5.$$

We can rewrite (5.3) in the form

$$f_{1+}{}^{(3/2)}(W)=\{[1/\tilde{f}_{1+}{}^{(3/2)}(W)]-iq\}^{-1}, \qquad (5.4)$$

where $\tilde{f}_{1+}{}^{(3/2)}$ is the narrow-resonance approximation in (5.1), and this is just the K-matrix approximation

$$\tilde{f}_{1+}{}^{(3/2)}(W)=K_{1+}{}^{(3/2)}(W).$$

The full P_{33} amplitude, calculated from the model, contains nonpole contributions arising from the baryon propagators and the other terms in the Lagrangian, so that when this amplitude is identified with the K matrix there arises a nonresonant background which will be important at the very low energies.

Among the various terms in the Lagrangian, the α and β terms dominate the lowest-energy region, contributing to the s-wave scattering. Near threshold, all other terms contribute to p waves and higher partial waves. As stated previously, this situation arises because the matrix elements calculated from the γ term, the ρ-meson-exchange term, and the direct and crossed N and N^* terms all vanish at threshold in this model. At energies in the N_{33}^* region, the direct N^*-pole term, of course, dominates, with all other terms contributing to a coherent background.

Now, using the K-matrix approximation for all the partial waves important in the low-energy region, namely, s and p waves, we calculate the total π^+p and π^-p cross sections, plotted in Fig. 2, and compare with the experimental data compiled by Bareyre, Bricman, and Villet.[45] The agreement is very good. In Figs. 3 and 4 we plot the elastic π^+p and π^-p differential cross sections at several energies and compare with the data of Barnes *et al.*[37] We have not included Coulomb corrections to our amplitudes, so there will be some systematic differences with the data, especially at very low energies and small angles. Furthermore, the partial-wave analysis of Barnes *et al.* agrees with the more recent and more complete analysis of Roper, Wright, and Feld,[42] except for the $T=\frac{1}{2}$, $J=\frac{3}{2}^+$ partial wave which is positive in the former analysis, negative in the latter. This discrepancy casts some doubt on the data near the backward direction in π^-p .With these qualifications, we consider the

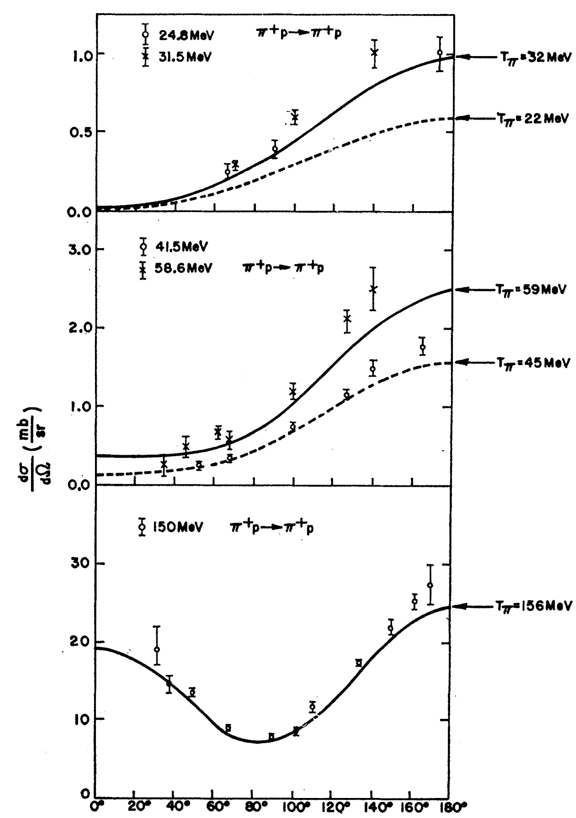

FIG. 3. π^+p differential cross sections. The plotted points are from Ref. 37. The solid curves are the predictions of the model.

[44] Without this requirement $\Gamma(W)$ could have an arbitrary extra energy-dependent factor, say, $\gamma(W)$, only satisfying the conditions $\gamma(M_R)=1$ and $\gamma(W)$ at threshold not zero. For example, a factor $(M_R-E_R)/(W-E)$ would give the Chew-Low linear extrapolation or $(q^3\cot\delta_{33})/\omega$ [G. Chew and F. Low, Phys. Rev. **101**, 1570 (1956)] in the static limit.

[45] P. Bareyere, C. Bricman, and G. Villet, Phys. Rev. **165**, 1730 (1968).

calculated angular distributions to be in fairly good agreement with experiment.

In order to carry these calculations to higher energies we must consider the inelastic processes, the multipion production and associated production. We do not attempt to account for the uncorrelated multipion production amplitudes, but we will consider some of the quasi-two-body processes later.

B. $KN \rightarrow KN$

KN elastic scattering is particularly amenable to study in our model, since there are no direct-channel resonances in the hypercharge-2 system (at least in the low-energy region that we are considering here). The matrix elements that contribute to this process are the α and β terms, ρ and ω exchange, and Λ, Σ, and Y_1^* exchange. There is no contribution from the γ term since it corresponds to the **56** representation of $SU(6)_W$ in the direct channel, and thus does not contain a $Y=2$ member. Of all these matrix elements, the α and β terms will dominate the low-energy region; the exchange terms contribute primarily to the p-wave scattering, in this model, which is suppressed by the threshold factor q^2 relative to the s waves. Hence, we expect the KN scattering to proceed primarily through s wave as experiment indicates. It is important to realize that in a more conventional model, with only pure exchange terms, the s-wave scattering would be significantly smaller than our s waves, and, as Warnock and Frye[46] have shown, it becomes necessary to include polynomial terms in the amplitude, representing short-range forces

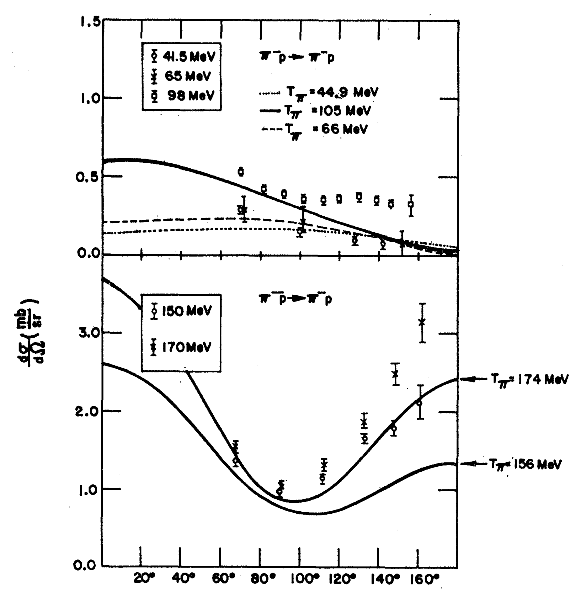

FIG. 4. π^-p differential cross sections. The plotted points are data from Ref. 37. The solid curves are the predictions of the model.

[46] R. L. Warnock and G. Frye, Phys. Rev. **138**, B947 (1965).

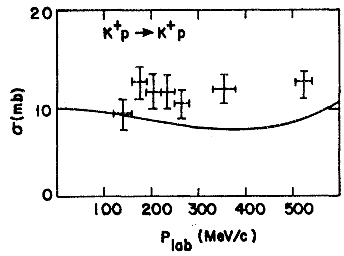

FIG. 5. K^+p elastic cross section. The plotted points are data from Ref. 47. The solid curve is the prediction of the model.

in the Cini-Fubini[4] approximation. In our model these polynomial terms appear naturally from the α- and β-matrix elements.

The calculated K^+p elastic-scattering cross section is plotted in Fig. 5 along with the data of Goldhaber *et al.*[47] The agreement is good—the total cross section is essentially flat from threshold to $P_{\rm lab}\sim 600$ MeV/c, and in Fig. 6, the differential cross sections indicate the predominantly s-wave character of the interaction, as we predict. We have used the K-matrix approximation to perform the calculation, but in K^+p there is only a very small unitarity correction, since all the phase shifts are small in this case. Hence, we predict very small polarization in this region, below the inelastic threshold.

For the K^+n elastic scattering, the α and β terms produce a very large s wave for $T=0$, that exceeds the unitarity bound if these matrix elements are taken to be the real part of the partial-wave amplitude. However, by equating the partial-wave projections with the K-matrix elements, unitarity is preserved, as we have outlined above. The resulting real part of the s-wave amplitude increases rapidly from threshold, reaches a maximum, and then gradually decreases. The other partial waves are hardly effected by the unitarity correction, since they are relatively small.

Experimentally, the K^+n interaction must be extracted from K^+d scattering using some form of impulse approximation and the known values for the K^+p scattering. There are difficulties with this procedure at very low energies where multiple-scattering effects will be important. Stenger *et al.*[30] have measured K^+d scattering in the range $P_{\rm lab}=350$–812 MeV/c and have extracted the $T=0$ phase shifts. There is an unresolved Fermi-Yang ambiguity and they have determined both sets, with large uncertainties at the lower momenta (recently, a K^+d experiment at ~ 600 MeV/c[47] measured polarization and favors the Yang set). We compare K^+n angular distributions and cross sections with the model, and these results are presented in Figs. 7 and 8. The agreement is not very good, although the calculated cross

[47] S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1962). There is a possibility of an over-all normalization error in the total cross sections obtained, but the relative angular distributions definitely support pure s-wave scattering. For a thorough discussion of the data, and a phase-shift analysis, see A. T. Lea, B. R. Martin, and G. C. Oades, Phys. Rev. **165**, 1770 (1968); A. K. Ray *et al.*, Bull. Am. Phys. Soc. **13**, 703 (1968).

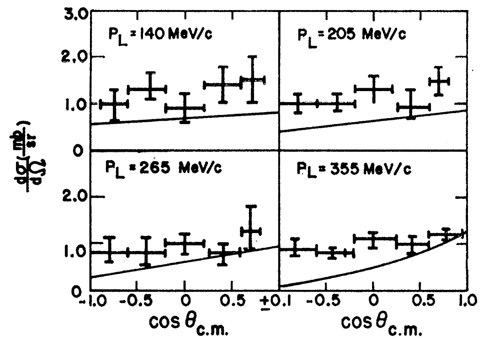

FIG. 6. K^+p differential cross sections. The plotted points are data from Ref. 47. The solid curve is the prediction of the model.

sections (Fig. 7) are not inconsistent with the data at the two low-energy points, $P_{\text{lab}}=350$ and 530 MeV/c. From the differential cross-section data (Fig. 8), it is seen that the model does not adqeuately reproduce the large p wave of Stenger's phase-shift analysis. More accurate measurements must be made before definite conclusions can be drawn from the data.

C. $\bar{K}N \to \bar{K}N$

Low-energy $\bar{K}N$ scattering is the most complicated of the experimentally accessible elastic reactions. A theoretical understanding of this system depends on a multichannel analysis involving the as yet unobserved $\pi\Sigma$ and $\pi\Lambda$ channels, and the $\bar{K}N$ bound states—the $Y_1^*(1385)$, $J^P=\frac{3}{2}^+$ and the $Y_0^*(1405)$, $J^P=\frac{1}{2}^-$.[48] The $Y_1^*(1385)$ is included in the **56** representation of $SU(6)_W$ that we are considering, but the $Y_0^*(1405)$ is not. It is not clear to which representation of $SU(6)_W$ the latter resonance should be assigned—it seems to be a singlet in $SU(3)$.[49] Without an assignment for the Y_0^*, we could simply ignore it and perform a multichannel calculation, but this would not be in the spirit of the model, which requires that all important states be included.

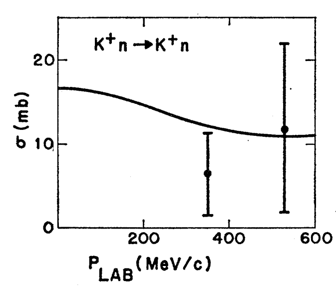

FIG. 7. K^+n elastic cross section. The two plotted points are data extracted from Ref. 30. The solid curve is the prediction of the model.

[48] J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967). This paper gives the most extensive phenomenological analysis of the data.

[49] R. H. Dalitz, T. C. Wong, and G. Rajasekaran, Phys. Rev. **153**, 1617 (1967). These authors generate a primarily singlet Y_0^* by vector-meson forces. They include references to earlier theoretical models for the $\bar{K}N$ system.

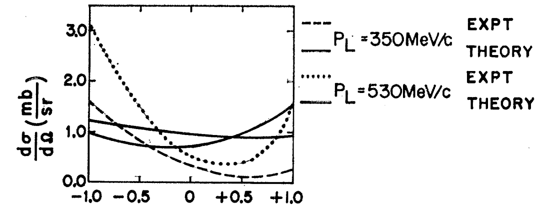

FIG. 8. K^+n differential cross section. The predicted curves for the two low energies are compared with the data of Ref. 30.

D. $\pi N \to \eta^0 N$

Of all the pseudoscalar-meson–nucleon reactions, η production has the lowest inelastic threshold at $W=1487$ MeV. If other energetically allowed multipion final states are ignored, then only πN elastic scattering competes with η production, and through multichannel unitarity the πN and ηN systems are coupled together. Since the ηN system is pure $T=\frac{1}{2}$, only the $T=\frac{1}{2}$ part of the πN is coupled. Now at this threshold, our matrix elements are no longer adequate to quantitatively describe the πN partial waves, although the calculated πN cross sections are still of the right order of magnitude. However, without further refinements in the model, we only want to get a rough estimate of the effects of multichannel unitarity on η production by again setting the partial-wave projections, obtained from the model, equal to the K-matrix elements.

Explicitly, we calculate the amplitudes for $\pi N \to \pi N$ ($T=\frac{1}{2}$), $\pi N \to \eta N$, and $\eta N \to \eta N$, at the ηN threshold and above. Calling these $\alpha_{l\pm}(W)$, $\beta_{l\pm}(W)$, and $\gamma_{l\pm}(W)$, respectively, we equate

$$K_{l\pm}(W)=\begin{pmatrix}\alpha_{l\pm}(W) & \beta_{l\pm}(W)\\ \beta_{l\pm}(W) & \gamma_{l\pm}(W)\end{pmatrix}. \tag{5.5}$$

Then the phase-space matrix ρ is defined as

$$\rho(W)=\begin{pmatrix}q_1(W) & 0\\ 0 & q_2(W)\end{pmatrix}, \tag{5.6}$$

where $q_1(W)$ and $q_2(W)$ are the c.m. momenta in the πN and ηN systems at the c.m. energy W. The T matrix is then given by

$$T_{l\pm}(W)=[(K_{l\pm}(W))^{-1}-i\rho(W)]^{-1}, \tag{5.7}$$

so that[50] $\text{Im}\{[T_{l\pm}(W)]^{-1}\}=-\rho(W)$. For a *given partial wave* the T-matrix elements may be parametrized either by an inelasticity parameter η, and two phase shifts δ_1

[50] There is no need to extract threshold factors and branch points for each partial wave, since the matrix elements that we use for the $K_{l\pm}$ matrix already have the correct analytic properties, having been obtained from a relativistically invariant Lagrangian.

and δ_2, or by the K-matrix elements α, β, and γ:

$$T=\begin{bmatrix} \dfrac{\eta e^{2i\delta_1}-1}{2iq_1} & \dfrac{1}{2}\left(\dfrac{1-\eta^2}{q_1q_2}\right)^{1/2} e^{i(\delta_1+\delta_2)} \\ \dfrac{1}{2}\left(\dfrac{1-\eta^2}{q_1q_2}\right)^{1/2} e^{i(\delta_1+\delta_2)} & \dfrac{\eta e^{2i\delta_2}-1}{2iq_2} \end{bmatrix}$$

$$=[1-q_1q_2(\alpha\gamma-\beta^2)-i(q_1\alpha+q_2\gamma)]^{-1} \times\begin{pmatrix} \alpha-iq_2(\alpha\gamma-\beta^2) & \beta \\ \beta & \gamma-iq_1(\alpha\gamma-\beta^2) \end{pmatrix}. \quad (5.8)$$

The two sets of parameters are then related as follows:

$$\delta_1+\delta_2=\arctan\left(\frac{q_1\alpha+q_2\gamma}{1-q_1q_2\delta}\right),$$

$$\delta_1-\delta_2=\arctan\left(\frac{q_1\alpha-q_2\gamma}{1+q_1q_2\delta}\right), \quad (5.9)$$

$$\eta=\left[\frac{(1+q_1q_2\delta)^2+(q_1\alpha-q_2\gamma)^2}{(1-q_1q_2\delta)^2+(q_1\alpha+q_2\gamma)^2}\right]^{1/2},$$

where $\delta=\det K=\alpha\gamma-\beta^2$. The parameters η and δ completely determine the πN elastic partial-wave amplitudes. In a complete calculation, the η parameters in (5.8) would be determined not only by transitions to the ηN inelastic channels, but by transitions to all other open channels [$\pi\pi N$, $\pi\pi\pi N$, $\pi N^*(1238)$]. Hence, if the other inelastic channels are strongly coupled to the πN channel, the η parameter that we calculate from the two-channel approximation will be closer to unity ($0\leq\eta\leq 1$) than the experimentally determined η. Even if this is true, the η-meson channels may still be coupled most strongly to the πN system, so that the other open channels have a smaller effect on ηN than they have on πN. We assume that this is the case, so that two-channel unitarity will be a good approximation to full unitarity corrections for η production, although not necessarily for πN elastic scattering.

We have calculated α, β, and γ in (5.5) for s and p waves near the η-production threshold. Higher partial waves are small in this energy region for the η-N system. Upon unitarizing the partial-wave amplitudes, as specified, we have determined the angular distributions and cross sections for $\pi N\to\eta N$, and have plotted them in Figs. 9 and 10. They are compared with the experimental data on[51,52] $\pi^-p\to\eta^0 n$, (where the η^0 decays into two photons) by dividing the cross-section data by the

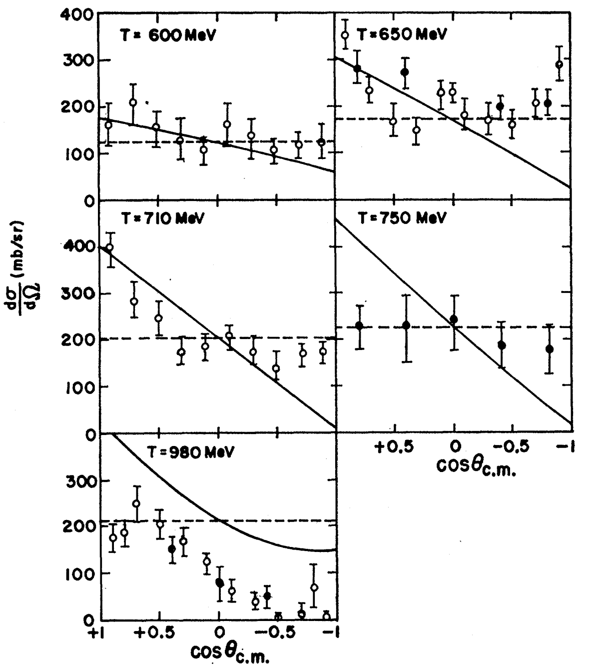

FIG. 9. $\pi^-p\to\eta^0 n$ differential cross sections. The predicted curves are compared with the data of Bulos *et al.* (Ref. 51) and Richards *et al.* (Ref. 52).

$\eta^0\to 2\gamma$ branching ratio of 0.34.[53] The two sets of data differ in that the earlier experimental angular distribution of Bulos *et al.*[51] is compatible with pure s-wave production up to $T_\pi=1$ BeV, whereas the data of Richards *et al.*[52] require p waves at $T_\pi=655$ MeV. The model predicts relatively large p-wave contributions at the lower energy in agreement with Richards *et al.*, but these become too large beyond $T_\pi=750$–800 MeV, and in the vicinity of 1 BeV the predicted differential cross section is too large over all angles. The total production cross section is also in good agreement with experiment up to

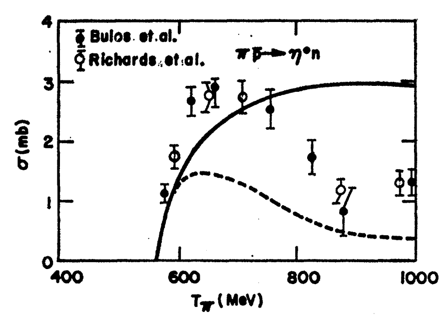

FIG. 10. $\pi^-p\to\eta^0 n$ production cross section. The predicted curve is compared with the data of Bulos *et al.* (Ref. 51) and Richards *et al.* (Ref. 52.) The dashed curve is the predicted s-wave contribution.

[51] F. Bulos *et al.*, Phys. Rev. Letters **13**, 486 (1964).

[52] W. B. Richards *et al.*, Phys. Rev. Letters **16**, 1221 (1966). More recent data agree with the work of Richards *et al.*; cf. W. G. Jones *et al.*, Phys. Letters **23**, 597 (1966); E. Hyman *et al.*, Phys. Rev. **165**, 1437 (1968).

[53] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968). The branching ratio, for $\eta\to 2\gamma$ decay, may be ~30% higher than the value we have quoted. There are some discrepancies in the experimental determinations. This situation introduces a large uncertainty in the *over-all* normalization of the cross sections.

TABLE I. Phase shifts and inelasticities for S-wave πN elastic scattering above the threshold for η-meson production. The calculated parameters for the two-channel approximation are given at the left. The parameters are defined in the text. The phase shift and inelasticity parameter on the right are taken from Ref. 54.

Partial wave	W (BeV)	α (BeV^{-1})	β (BeV^{-1})	γ (BeV^{-1})	δ_1 (deg)	δ_2 (deg)	η	δ_1 (deg)	η
$S_{1/2}$	1.49	1.71	−1.40	3.52	36.5	0.0	1.00	34	0.54
	1.51	1.82	−1.43	3.61	35.7	19.8	0.88	33	0.52
	1.54	1.94	−1.46	3.68	39.1	30.2	0.86	38	0.46
	1.56	2.07	−1.49	3.77	41.8	36.7	0.85	43	0.43
	1.58	2.21	−1.52	3.84	44.0	40.9	0.86	51	0.40
	1.62	2.45	−1.57	3.96	47.8	46.2	0.86	65	0.41
	1.64	2.62	−1.60	4.02	50.5	49.0	0.87	75	0.45
	1.66	2.82	−1.63	4.10	53.3	51.6	0.89	82	0.51
	1.72	3.46	−1.73	4.30	60.5	57.5	0.92	100	0.72

$T_\pi \sim 800$ MeV, but beyond that energy the measured cross section decreases, whereas the calculated values remain flat. This indicates that above $T_\pi \sim 800$ MeV the calculated p waves should be reduced further by unitarity than we are able to accomplish in the two-channel approximation, i.e., other channels are important for p waves in the higher-energy region. The s-wave contribution to the total cross section is also shown in Fig. 10; it has the correct behavior—a sharp rise at threshold and a gradual fall-off due to unitary corrections.

Now it is important to realize that there are no resonances included in the model in the region of η productions, although relevant resonances in πN have been reported at $T_\pi = 660$ MeV $[N_{1/2}^*(1550), \frac{1}{2}^-]$, at $T_\pi = 880$ MeV $[N_{1/2}^*(1680), \frac{5}{2}^-]$, at $T_\pi = 900$ MeV $[N_{1/2}^*(1688), \frac{5}{2}^+]$, and at $T_\pi = 940$ MeV $[N_{1/2}^*(1710), \frac{1}{2}^-]$.[53] The two $\frac{1}{2}^-$ resonances should have noticeable effects on the η-production amplitudes as well as the πN elastic channel, but our nonresonant calculation satisfactorily accounts for the s-wave production amplitude near threshold. If we compare the extrapolated threshold production cross section $(q_{\text{initial}}/q_{\text{final}}) \times \sigma(q_{\text{final}} \to 0)$ with the prediction of the model, we find good agreement:

experimental extrapolation -6.22 ± 0.80 mb;
prediction of model -6.04 mb.

This indicates that the $N_{1/2}^*(1550)$ may not be necessary in explaining η production near threshold. In fact in Table I we compare the resulting πN elastic S_{11} phase shift with the phase shifts determined by Lovelace *et al.*,[54] where the 1550-MeV resonance is *not* required, and we find the phase shifts in good agreement up to $T_\pi \sim 700$ MeV ($W \sim 1580$ MeV). This is easily understood, since the analysis[54] indicates a rapidly decreasing η from the threshold for $\pi N \to \pi N^*(1238)$, so that the effect of the η-meson production is superimposed on the already small η parameter, and a more complete calculation would require at least three-channel unitarity. Beyond $T_\pi \sim 700$ MeV the phase-shift analysis gives a monotonically increasing phase shift passing through $\frac{1}{2}\pi$ at $W \sim 1710$ MeV, and here the model is unable to give a resonating amplitude since the calculated K matrix has no singularity here.

Hence, we see that the model provides the proper s-wave threshold enhancement for η production without an $N_{1/2}^*(1550)$, but cannot account for the decreasing p wave at higher energies above threshold, nor for an $N_{1/2}^*(1710)$ which may have only a small branching ratio into the ηN channel. The terms in the effective Lagrangian that contribute to this reaction are the β and γ contact interactions and the nucleon pole in both direct and crossed channels. The β term provides the largest part of the threshold behavior; the direct-nucleon term dominates at energies above the maximum in the cross section, causing a large p-wave contribution. At lower energies, the latter is essentially cancelled by the γ term and the nucleon-exchange term. Thus, in this model the behavior of the amplitude for η production near threshold is determined primarily by short-range forces (the β term) and coupling to elastic πN scattering through unitarity. This is in marked contrast to more detailed fits to the data, without unitarity, that require the nucleon-pole term and a $D_{13}(1512)$ resonance[55]; or an $S_{11}(1560)$, a $P_{11}(1503)$, and a $D_{13}(1631)$[56]; or a nucleon pole, and $S_{11}(1567)$, a $P_{11}(1430)$, and a $D_{13}(1512)$ with finite widths[57]; or these and an additional F_{15}-(1688).[58] We emphasize that in our less satisfactory fit to the data, there were no free parameters and no $T = \frac{1}{2}$ resonances—short-range forces and multichannel unitarity play the major role in the near-threshold behavior.

The $\pi N \to \eta N$ calculation is typical of the inelastic processes we consider. The model provides an adequate description of the scattering very near threshold, but even with multichannel unitarity approximated, we are unable to account for the marked decrease in the cross section at somewhat higher energies. Hence, in the following reactions we will calculate only the extrapolated

[54] A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[55] G. Altarelli, F. Buccella, and R. Gatto, Nuovo Cimento **35**, 331 (1965).

[56] F. Uchiyama-Campbell and R. K. Logan, Phys. Rev. **149**, 1220 (1966).

[57] T. A. Moss, Phys. Rev. **163**, 1785 (1967).

[58] S. R. Deans and W. G. Holladay, Phys. Rev. **165**, 1886 (1968). These authors obtain the best fit to the data up to $T_\pi = 975$ MeV by varying seven parameters for the partial widths of the resonances.

threshold cross sections. Unitarity is more difficult to approximate adequately for the higher threshold processes (where there are many competing channels) and, it is hoped, will not play a significant role very near threshold.

E. $\pi N \to K\Lambda$

The amplitude for $\pi^- p \to K^0\Lambda^0$ is pure $T=\frac{1}{2}$ and the production threshold is at $T_\pi = 766$ MeV (where the η-production amplitude is still large). There are known to be large polarization effects in this reaction fairly near threshold,[59] and unitarity becomes important in the region of the rapidly decreasing cross section above $T_\pi \sim 900$ MeV, where resonant amplitudes are also important.[60] At threshold the extrapolated cross section $(q_{\text{initial}}/q_{\text{final}}) \times \sigma(q_{\text{final}} \to 0)$ is 1.75 mb in the model, and 1.29 ± 0.35 mb averaged over the four lowest-energy measurements.[59] The total production cross section for low energies is given in Fig. 11. It is clear that the model gives an adequate representation of the data below the maximum in the cross section. The fact that the calculated cross section is close to the experimental values is a good test of the method we have adopted to split the particle masses from their degenerate $U(6)\otimes U(6)$ multiplet values; in the absence of mass breaking, $U(6)\otimes U(6)$ symmetry predicts that the cross section would be exactly zero at threshold [see relation (4.7)] and slowly rising above. Hence, the nonzero cross section is due entirely to mass-broken contact and exchange terms, primarily contact terms at threshold. This is true for the other associated-production reactions, also; i.e., in $U(6)\otimes U(6)$ symmetry, at rest, all associated-production reactions have zero amplitudes, and any small finite cross sections are due to symmetry-breaking effects. We consider this to be an important qualitative result of our broken-symmetry model.

F. $\pi N \to K\Sigma$

The associated production of Σ hyperons involve both $T=\frac{1}{2}$ and $T=\frac{3}{2}$ amplitudes, and, as in Λ production, there are many competing channels that contribute to unitarity corrections. Here we reach the first significant failure of the model. The total cross section for $\pi^- p \to K^+\Sigma^-$ and $\pi^+ p \to K^+\Sigma^+$ are predicted to be an order of magnitude too large, e.g., at $P_{\text{lab}} = 1170$ MeV/c, $\sigma(\pi^- p \to K^+\Sigma^-) = 0.231 \pm 0.006$ mb whereas the model predicts 3.50 mb, and $\sigma(\pi^+ p \to K^+\Sigma^+) = 0.205 \pm 0.014$ mb[61] whereas the model predicts 9.13 mb. Furthermore, the model predicts large backward peaks and forward minima at $P_L = 1170$ MeV/c for both Σ^-- and Σ^+-production angular distributions whereas experiment

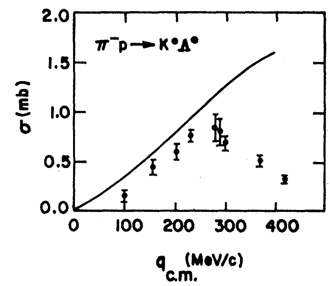

FIG. 11. $\pi^- p \to K^0\Lambda^0$ production cross section. The plotted points are data taken from Ref. 59. The solid curve is the prediction of the model.

indicates a large forward as well as backward peak for Σ^- production[62] and only a slightly backward peaked distribution for Σ^+ production.[62] The reason for the large discrepancy in this case can be traced to the direct-channel pole terms—the nucleon and the N_{33}^*. Because of our choice of propagator for the baryons, Eq. (2.18), these terms have nonpole contributions that increase with c.m. energy as W^4. Very near elastic threshold those nonpole terms were useful in canceling the pole contribution exactly, so that only the four-point functions or contact terms determined the near-threshold elastic scattering. Beyond the $K\Sigma$-production threshold, however, these terms dominate over all other contributions—the contact terms β and γ, the K^* exchange, and the Λ, Σ, and Y_1^* exchanges—and produce very large cross sections. Only in those cases where these terms cancel against one another can we expect to be able to continue the model to higher energies as in the $K\Lambda$ production. On the other hand, where it is feasible to apply multichannel unitarity, as in η production above, these high-energy divergences are suppressed.

G. $K^- p \to \eta^0 \Lambda$

Of all the inelastic reactions we have considered, the $K^- p \to \eta^0\Lambda$ is known to have the most striking behavior near threshold. Without applying any unitarity corrections, the model predicts a very large s-wave interaction near threshold, due primarily to contact terms, and p waves that are smaller by an order of magnitude. The extrapolated cross section $(q_i/q_f)\sigma(q_f \to 0)$ is predicted to be 9.8 mb, whereas the extrapolation from the data[63] ranges between 5 and 10 mb. Hence, the rapid increase from threshold that we calculate is consistent with the data, see Fig. 12, and the predicted enhancement is predominantly s wave, as verified by the angular distribution collected by Berley *et al.*[63]

It is clear that rather extreme behavior will be necessary for the T matrix, to give the rapid decrease so near

[59] L. Bertanza *et al.*, Phys. Rev. Letters **8**, 332 (1962). Earlier experimental references are included here.

[60] See the model of G. T. Hoff, Phys. Rev. **131**, 1302 (1963), where a resonating $P_{1/2}$ state is necessary to explain the experimental cross sections, as well as K^* exchange.

[61] F. S. Crawford, Jr., F. Grard, and G. A. Smith, Phys. Rev. **128**, 368 (1962).

[62] J. C. Doyle, F. S. Crawford, Jr., and J. A. Anderson, Phys. Rev. **165**, 1483 (1968).

[63] D. Berley *et al.*, in *Proceedings of the Twelfth Annual Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965), p. 635; D. Berley *et al.*, Phys. Rev. Letters **15**, 641 (1965); P. L. Bastein *et al.*, *ibid.* **8**, 114 (1962).

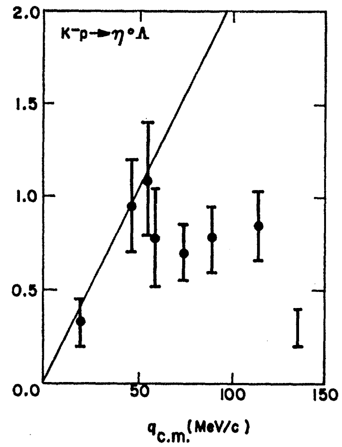

FIG. 12. $K^-p \to \eta^0\Lambda$ production cross section. The plotted points are data taken from Ref. 63. The curve is the prediction of the model.

threshold and unitarity alone may not accomplish this. Berley *et al.*[63] attempted to fit the data by using either resonances or large constant scattering lengths (in a two-channel Dalitz-Tuan model[64]), and obtained the best fit for an $S_{1/2}$ resonance at mass 1675 MeV and a very narrow width of 15 MeV approximately equal to the partial width in the $\eta\Lambda$ channel. If this interpretation is correct,[65] then our model could not account for the data, even with multichannel unitarity.

H. $P+B \to V+B$

The production of vector mesons near their thresholds is also predicted by the model. The relevant invariant amplitudes, and the contributions to these amplitudes from the various terms in the matrix element (4.3), are summarized in the Appendix. The α term in (4.3) does not contribute at all to these reactions, and at threshold only the β term and the baryon-exchange amplitude contribute; the latter only through factors that are proportional to mass differences of the initial and final particles. For degenerate masses of the $U(6)\otimes U(6)$ multiplets there would be only one independent *rest* $U(6)\otimes U(6)$-invariant amplitude at threshold, as for the $P+B \to P'+B'$ reaction in (4.6) and (4.7), and the vector-meson production reaction amplitudes would be in simple ratios,

$$\begin{aligned} A_5(\pi^-p \to \rho^-p) &= -\tfrac{1}{4}A_5(\pi^+p \to \rho^+p) = -\tfrac{1}{5}\sqrt{2}A_5(\pi^-p \to \rho^0 n) \\ &= A_5(K^+n \to K^{*+}n) = -\tfrac{1}{4}A_5(K^+p \to K^{*+}p) \\ &= -\tfrac{1}{5}\sqrt{2}A_5(\pi^-p \to \omega^0 n) = -(2/3\sqrt{3})A_5(K^-p \to \rho^0\Lambda) \\ &= -(2/3\sqrt{3})A_5(K^-p \to \omega^0\Lambda)\,, \end{aligned} \tag{5.10}$$

$$0 = A_5(\pi^-p \to \varphi^0 n) = A_5(K^-p \to \varphi^0\Lambda)\,, \tag{5.11}$$

where A_5 is the invariant amplitude defined in (A1). For this degenerate-mass case, A_5 is simply (16/9) $\times\beta(b^+-b^-)$, which leads to threshold cross sections of the order of 1 mb. The prediction that the $\pi^-p \to \varphi^0 n$ reaction is zero holds even with mass breaking, for all energies, in this model.[66] This is in agreement with experimental information,[67] for which the production cross section $\pi^-p \to \varphi^0 n$ is compatible with zero, or at most it 50 times smaller than the ω-production cross section as comparable c.m. momenta.

When mass breaking is included in the model, the other relations (5.10) and (5.11) are altered. Furthermore, there is another invariant amplitude that contributes at *inelastic* threshold—the B amplitude defined in (A6)—so that the cross sections will not be in simple ratios. There is a contribution to both A_5 and B from the baryon-exchange terms, (A13) and (A14), but the magnitudes of these are appreciably smaller than the β terms (A8), since they vanish for degenerate masses, thus they involve higher orders of the mass differences than do the β terms.

Away from thresholds the dominant terms in the model are the baryon exchanges, and, as in $\pi N \to K\Sigma$, those terms rapidly become very large. It is known that the meson-exchange terms should dominate the production in the BeV region,[68] in contradiction to the prediction of the model. This is again an indication that beyond the vector-meson-production thresholds the background terms and contact terms in the exchanges in (4.3) are no longer a good approximation to the short-range forces, i.e., the polynomial terms begin to diverge. Thus the only meaningful predictions that can be made

TABLE II. Predicted cross sections for vector-meson production processes very near thresholds. The extrapolated cross sections are given in the first column; the ratios of those cross sections with respect to $\pi^-p \to \rho^-p$ are given in the second column; the ratios that result by assuming mass degeneracy are given in the third column.

Reaction	$(q_{\text{initial}}/q_{\text{final}}) \times\sigma(q_{\text{final}} \to 0)$ (mb)	Ratios	Degenerate mass ratios
$\pi^-p \to \rho^0 n$	27.1	12.5	12.5
$\pi^+p \to \rho^+p$	34.7	16.0	16
$\pi^-p \to \rho^-p$	2.2	1.0	1
$\pi^-p \to \omega^0 n$	27.1	12.5	12.5
$K^+p \to K^{*+}p$	35.9	16.5	16
$K^+n \to K^{*+}n$	2.3	1.1	1
$K^-p \to \rho^0\Lambda$	18.3	8.4	6.7
$K^-p \to \omega^0\Lambda$	18.3	8.4	6.7
$K^-p \to \phi^0\Lambda$	0.007	0.0032	0

[64] R. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **10**, 307 (1960).

[65] An enhancement has been observed in the neutral hyperon spectrum in $K^-p \to \Sigma + m\pi$ at 3.5 geV/c; Birmingham-Glasgow-London (I.C.)-Oxford-Rutherford Collaboration, Phys. Rev. **152**, 1148 (1966). The mass, however, is somewhat lower, 1645±6 MeV; and the width larger, 40±10 MeV. With this mass the enhancement would be an $\eta\Lambda$ *bound* state, which did not give as good a fit in the analysis of Berley *et al.* (Ref. 63).

[66] The φ meson does *not* couple to strangeness = 0 mesons and baryons, so that all the diagrams that are included in (4.3) will give vanishing contributions to this process.

[67] R. I. Hess *et al.*, Phys. Rev. Letters **17**, 1109 (1966).

[68] J. D. Jackson, J. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. **139**, B428 (1965).

from the model, will be the values of the cross sections extrapolated to threshold. Unfortunately, the experimental measurements of the cross section near threshold are difficult to perform since the uncorrelated multipion states mask the vector-meson productions. Nevertheless, we will make predictions for the extrapolated cross sections, with the expectation that there will eventually be enough data to make comparisons.

The predictions of the model for $(q_{\text{initial}}/q_{\text{final}}) \times\sigma(q_{\text{final}}\to 0)$ are listed in Table II. Notice that the ratios of the cross sections are not very different from the ratios that would be derived from the threshold amplitudes for degenerate-mass particles, (5.10) and (5.11).

As we have stated, experimental data is not available near enough to the production thresholds to make comparisons with the predictions meaningful. The lowest energies for which measurements have been made are in the region where the production angular distributions show sharp forward peaking, so that the resonating meson system can be disentangled most easily from the the background of uncorrelated mesons.[69–74] Since our predictions are presumably valid in the region where s-wave production should predominate, we must await lower-energy data. There is another difficulty in $\pi^-p\to\rho N$ and $\pi^-p\to\omega n$, since the thresholds for these reactions are very near the S_{11} resonance at $W=1710$ MeV [which we were unable to account for in our discussion of the coupled $(\pi N,\eta N)$ system], which may have a large effect on the amplitudes.

We can, however, make the following qualitative comparisons with the lowest-energy data. In $\pi^-p\to\rho^-p$[70] and $K^-p\to\varphi^0\Lambda^0$,[73] the predicted cross sections are of the correct order of magnitude if we assume only slight variation from threshold to the measured values. In $\pi^-p\to\omega^0 n$,[71] $K^-p\to\omega^0\Lambda^0$,[72] $K^+p\to K^{*+}p$,[74] there must be a rapid decrease in the amplitudes (as in $K^-p\to\eta^0\Lambda^0$, discussed above) if our threshold values are to match the higher-energy measured values. This latter circumstance could occur if there were s-wave resonance effects very near threshold. In $\pi^-p\to\omega^0 n$, the $N^*(1710)$ S_{11} resonance could provide the mechanism, as mentioned, but it would have to be only weakly coupled to the $T=\frac{1}{2}$ ρN system so as not to destroy our qualitative agreement for $\pi^-p\to\rho^-p$. In the $K^-p\to\omega^0\Lambda^0$ threshold region for K^-p scattering there are no observed s-wave, $T=0$ resonances, so this reaction may be unexplained by the model. For K^+p scattering near the $K^*(890)$ production threshold, there is a definite enhancement in the total cross section[75] but a resonance interpretation for this phenomenon is still in doubt.[76]

We emphasize that it would be of great interest to have data available closer to threshold, since our predictions for these vector-meson reactions depend crucially on the ability of our symmetry model to relate spin-0^- and spin-1^- meson interactions.

VI. CONCLUSIONS

The $U(6,6)$ symmetry scheme,[14–16] as a relativistic version of $SU(6)$,[77] has provided a useful classification scheme for the low-lying hadronic states and has predicted form factors and coupling constants in fair agreement with experimental information. Shortly after its inception, however, it was realized that $U(6,6)$-invariant S-matrix elements were necessarily in conflict with unitarity,[78] and the two-body scattering amplitudes derived from such invariant S-matrix elements were seen to give some untenable relations among various reactions.[27] It was shown next that intrinsic breaking of the symmetry by kinetic-energy terms would result in a hierarchy of subgroup invariances not necessarily in conflict with unitarity,[13,18] provided all possible "spurions" were included. Alternatively, the possibility of using only $U(6,6)$-invariant three-point functions for the Born terms as input in a unitarized dynamical calculation was suggested,[14] to preserve the "good predictions" for the three-point functions. Greater impetus for preserving some vestiges of the broken symmetry has recently been provided by superconvergence relations, where saturation of the spectral functions by low-lying $SU(3)$ multiplets has resulted in $U(6,6)$ coupling-constant relations.[79]

Several calculations have been performed in the last few years using some residue of the $U(6,6)$ symmetry, while at the same time approximating unitarity. Gatto and Veneziano[80] have used the $SU(6)_W$ subgroup, which is the symmetry for the three-point functions (as discussed in Sec. III), to write input Born terms in an N/D unitarization for two-body scattering. Various authors have used mass-broken $U(6,6)$ meson exchanges and "spurion"-broken $U(6,6)$-invariant amplitudes to

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[69] W. J Fickinger, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **10**, 457 (1963). These authors measured $\pi^-p\to\rho^0 n$ at $P_{\text{lab}}=1.7$ BeV/c, but give no absolute cross section.

[70] D. D. Allen *et al.*, Phys. Rev. Letters **17**, 53 (1966); $\pi^-p\to\rho^-p$ at $P_L=1.7$ BeV/c gives $\sigma=2.1\pm0.2$ mb.

[71] R. Kraemer *et al.*, Phys. Rev. **139**, B428 (1965). $\pi^-p\to\omega^0 n$ at approximately 120 MeV/c above threshold gives $\sigma=0.4\pm0.2$ mb.

[72] P. Eberhard *et al.*, Phys. Rev. **145**, 1062 (1966). $K^-p\to\omega^0\Lambda^0$ at $P_L=1.32$ BeV/c, $\sigma=0.80\pm0.06$ mb. Threshold is at $P_L=1.2$ BeV/c.

[73] J. S. Lindsey and G. A. Smith, Phys. Rev. **147**, 913 (1966). $K^-p\to\phi^0\Lambda^0$ at $P_L=2.10$ BeV/c, $\sigma=82\pm11$ μb. Threshold is at $P_L=1.8$ BeV/c.

[74] R. W. Bland *et al.*, Phys. Rev. Letters **17**, 939 (1966). $K^+p\to K^{*+}p$ (interfering with $K^+p\to KN^*$) at $P_L=1.2$ BeV/c gives $\sigma=1.53\pm0.26$ mb. Threshold is at $P_L=1.0$ BeV/c.

[75] R. L. Cool *et al.*, Phys. Rev. Letters **17**, 102 (1966).

[76] R. W. Bland *et al.*, Phys. Rev. Letters **18**, 1077 (1967).

[77] B. Sakita, Phys. Rev. **136**, B1756 (1964); F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 175 (1964).

[78] S. Coleman, Phys. Rev. **138**, B1262 (1965).

[79] V. de Alfaro, S. Fubini, G. Furlan, and G. Rosetti, Phys. Letters **21**, 576 (1966); R. Oehme, Phys. Rev. **154**, 1358 (1967); Phys. Letters **22**, 207 (1966); B. Sakita and K. C. Wali, Phys. Rev. Letters **18**, 29 (1967); H. F. Jones and M. D. Scadron, Nuovo Cimento **48A**, 545 (1967); R. Oehme and G. Venturi, Phys. Rev. **159**, 1283 (1967).

[80] R. Gatto and G. Veneziano, Phys. Letters **19**, 512 (1965); **20**, 439 (1966).

calculate the low-energy parameters for nucleon-nucleon elastic scattering.[81] In these cases the symmetry schemes have been fairly successful in relating the many parameters that are necessary to account for the nucleon-nucleon interactions. Proton-neutron charge-exchange scattering at intermediate energies has been described in a model using meson exchanges with $U(6,6)$-invariant couplings and absorptive corrections by Migneron and Moriarty.[82] And with the same model, Migneron and Watson have fitted the proton-antiproton annihilation into hyperon-antihyperon.[83] Balachandran *et al.*[84] have imposed three-point function unitarity on broken $U(6,6)$ vertices to obtain sum rules for some two-body scattering amplitudes. Carey has taken a unitarized effective-range formalism with $SU(6)$-invariant amplitudes providing the effective ranges to calculate some of the low-lying meson-baryon resonance positions.[85] And finally, Goldstein and Wali, in I, have used $U(6,6)$ interaction terms along with symmetry-breaking single-particle exchanges to describe low-energy πN and KN elastic scattering.[19]

All of these calculations have been relatively successful in relating the many parameters that are *a priori* required for phenomenological analyses of the data, and in providing decent fits to the experimental measurements. Encouraged by these results, we have extended the model used in I, to describe low-energy inelastic meson-baryon reactions, as well as the elastic reactions, in terms of the same three constant parameters that were fixed by πN and KN s-wave scattering very near threshold. We have been able to account for a large amount of experimental information without introducing any more parameters into the model, illustrating the essential utility of the broken-symmetry scheme—namely, to relate many different processes through the largest symmetry compatible with our single-particle intermediate-state approximation.

As has been stated in the Introduction, because of the automatic appearance of contact interaction and polynomial terms in the S matrix derived from the model, the scattering amplitudes are related to a Cini-Fubini approximation for low-energy scattering, and the Lagrangian is similar in form to the phenomenological Lagrangians derived from *chiral* $U(3)\otimes U(3)$ invariance. These similarities make the model especially applicable to low-energy scattering.

In the quantitative calculations, where we were able to approximate unitarity with the K-matrix formalism, there was good agreement with experiment—at least in the region not too far from threshold. This includes πN and KN elastic scattering and η production. Where multichannel unitarity was not approximated, due to the large number of competing channels, the results are not so clear cut. The $\pi N \to K\Lambda$ and $\bar{K}N \to \eta\Lambda$ threshold enhancements were predicted well within experimental uncertainties, but the $\pi N \to K\Sigma$ reactions were far from correct. Furthermore, without unitarity corrections, the baryon-pole terms begin to dominate at energies above threshold comparable to the $K\Sigma$ threshold in πN scattering. This indicates that to carry the calculation beyond associated production thresholds, more functional dependence would have to be introduced to damp out the diverging terms.

The predictions for vector-meson productions at threshold do not suffer from the difficulty of diverging contact terms and are of the correct order of magnitude but a definitive comparison for these depends on more low-energy data.

Qualitatively, the model provides a simple explanation of the small cross sections for associated production, $\pi N \to K\Lambda$ and $\pi N \to K\Sigma$; for cascade production, $\bar{K}N \to K\Xi$; and for $\bar{K}N \to \phi\Lambda$. These reactions proceed only through terms that break the $U(6)\otimes U(6)$ threshold symmetry, and the reaction amplitudes are proportional to the mass differences. The reaction $\pi N \to \phi N$ is totally forbidden, even with mass breaking, and this is in agreement with the actual situation.

In summary, the model provides qualitative explanation for the relatively small production cross sections and provides quanitatively good fits, using only three parameters, to the πN and KN elastic reactions, and the threshold enhancements in $\pi N \to \eta N$, $\pi N \to K\Lambda$, and $\bar{K}N \to \eta\Lambda$. Where resonances are known to be of importance the model necessarily does not provide for these, and above the associated production thresholds the contact terms begin to diverge.

ACKNOWLEDGMENT

The author wishes to thank Dr. K. C. Wali for his continuing interest in the course of this work, for his guidance, and for his many valuable suggestions.

APPENDIX

We have collected together in this section, the relevant formulas for vector-meson production $P+B\to V+B$. There are 12 helicity amplitudes for this reaction, and the requirement of parity conservation leaves only six independent amplitudes. Then the general T matrix for the process can be written as

$$\begin{aligned}T_{sf,i}(p',k';p,k)=\epsilon_\mu{}^{(s)}(k')\bar{u}_f(p')\{&i\epsilon_{\mu\nu\lambda\kappa}q_\nu P_\lambda Q_\kappa A_0(s,t)\\&+iP_\mu\gamma_5A_1(s,t)+iq_\mu\gamma_5A_2(s,t)\\&+iP_\mu\gamma_5\tfrac{1}{2}iQA_3(s,t)+iq_\mu\gamma_5\tfrac{1}{2}iQA_4(s,t)\\&+\gamma_5\gamma_\mu A_5(s,t)\}u_i(p)\,,\quad\text{(A1)}\end{aligned}$$

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where p (p') is the initial (final) baryon momentum, k is the incoming pseudoscalar momentum, and k' is the outgoing vector-meson momentum; $u_i(p)$ ($\bar{u}_f(p')$) is the Dirac spinor for the initial (final) baryon, $\epsilon_\mu^{(s)}(k')$ is the vector-meson polarization vector (satisfying $k'\cdot\epsilon^{(s)}=0$), and the combinations of momenta are defined as

$$P=p+k=p'+k',$$
$$Q=k+k',$$
$$q=p'-p=k-k',$$
$$s=-P^2, \quad \text{and} \quad t=-q^2.$$

Using the Dirac equation, the condition $k'\cdot\epsilon=0$ and the relation $\epsilon_{\mu\nu\lambda\kappa}=\frac{1}{8}\gamma_5[\gamma_\mu,\{\gamma_\nu,[\gamma_\lambda,\gamma_\kappa]\}]$, it can be seen that there are no other independent amplitudes.

With the T matrix normalized by the definition

$$S_{sf,i}=I-(2\pi)^4 i\delta^4(p'+k'-p-k)\times\left[\frac{M'M}{4p_0'p_0k_0'k_0}\right]^{1/2}T_{sf,i}, \quad \text{(A2)}$$

the differential cross section will be given in the c.m. system by

$$\frac{d\sigma}{d\Omega}=\frac{|\mathbf{k}'|}{|\mathbf{k}|}\frac{4M'M}{(8\pi W)^2}\sum_{\text{spins, } s,\, f,\, i}|T_{sf,i}|^2, \quad \text{(A3)}$$

where M' and M are the baryon masses and W is the total energy in the c.m. system. The summation is over all unobserved spins, and is facilitated by use of the completeness relations

$$\sum_i u_i(p)\bar{u}_i(p)=\frac{(M-ip\!\!\!/)}{2M}$$

and

$$\sum_s \epsilon_\mu^{(s)}(k')\epsilon_\nu^{(s)\dagger}(k')=\delta_{\mu\nu}+\frac{k_\mu'k_\nu'}{m'^2}. \quad \text{(A4)}$$

We will be interested particularly in the production amplitudes at threshold. At rest for the outgoing particles (we assume that $M'+m'\geqslant M+m$) there are only two independent amplitudes, and (A1) reduces to

$$T_{sf,i}(iM',im';p,k)=\epsilon_\mu^{(s)}(im')\bar{u}_f(iM')\{ik_\mu[-A_1+A_2+(m'+\tfrac{1}{2}(M'+M))(-A_3+A_4)]+\gamma_5\gamma_\mu A_5\}u_i(p), \quad \text{(A5)}$$

where the functions $A_i(s,t)$ are evaluated at

$$s=(M'+m')^2,$$

and the fourth component of ϵ_μ vanishes. Using relations (A3) and (A4), the total cross section at threshold will be given by

$$\lim_{\mathbf{k}'\to 0}\left\{\frac{|\mathbf{k}|}{|\mathbf{k}'|}\sigma\right\}=\frac{M'}{8\pi(M'+m')^2}\{\mathbf{k}^2(p_0-M)|B|^2-2\mathbf{k}^2\,\mathrm{Re}(BA_5^*)+3(p_0+M)|A_5|^2\}, \quad \text{(A6)}$$

where

$$B=-A_1+A_2+[m'+\tfrac{1}{2}(M'+M)](-A_3+A_4).$$

We next evaluate the contributions of the various terms in the scattering matrix $M(p',k';p,k)$ [Eq. (4.3)] to the invariant amplitudes $A_i(s,t)$ $i=0, 1, \cdots 5$, in (A1). The α term in (4.3) does not contribute to any inealstic process, and the $\tilde{\beta}$ term has been chosen to be zero, so we list below the invariant amplitudes resulting from the β, γ, meson exchanges, baryon-octet exchanges, and baryon-decuplet exchanges. To simplify the expressions, we first define some common kinematical functions and $SU(3)$ coefficients:

$$r_1=\frac{M'+M+(m'^2/m_0)+m_0}{M'Mm'^2}, \quad r_2=\frac{M'+M+2m_0}{M'Mm'^2},$$
$$r_3=\frac{(M'+M)^2-t}{2M'M}, \quad r_4=\frac{3m'^2+m^2-t}{2m'^2}. \quad \text{(A7)}$$

Letting B, P, and V denote the $SU(3)$ matrices for the baryon octet, pseudoscalar octet, and vector nonet, the required coefficients are defined as follows:

$$a^\pm=\langle(B\bar{B}-\bar{B}B+\langle\bar{B}B\rangle I)(\bar{V}P\pm P\bar{V})\rangle,$$
$$b^\pm=\langle(5B\bar{B}+B\bar{B}-\langle\bar{B}B\rangle I)(\bar{V}P\pm P\bar{V})\rangle,$$
$$C=-\langle\bar{B}BP\bar{V}\rangle-\langle\bar{B}B\bar{V}P\rangle+\langle\bar{B}B\rangle\langle\bar{V}P\rangle+\langle\bar{B}VBP\rangle+\langle\bar{B}PB\bar{V}\rangle+\tfrac{3}{2}\langle\bar{B}\bar{V}PB\rangle+\tfrac{3}{2}\langle\bar{B}P\bar{V}B\rangle-\tfrac{3}{2}\langle\bar{B}\bar{V}\rangle\langle BP\rangle-\tfrac{3}{2}\langle\bar{B}P\rangle\langle B\bar{V}\rangle+\langle\bar{B}BP\rangle\langle\bar{V}\rangle-\langle\bar{B}PB\rangle\langle\bar{V}\rangle, \quad \text{(A8)}$$
$$D=4\langle\bar{B}PB\bar{V}\rangle-4\langle\bar{B}PB\rangle\langle\bar{V}\rangle-2\langle\bar{B}\bar{V}BP\rangle-\langle\bar{B}\bar{V}PB\rangle-\langle\bar{B}P\bar{V}B\rangle+\langle\bar{B}\bar{V}\rangle\langle BP\rangle+\langle\bar{B}P\rangle\langle B\bar{V}\rangle,$$
$$E=\langle\bar{B}P\bar{V}B\rangle-\langle\bar{B}\bar{V}PB\rangle+\langle\bar{B}\bar{V}\rangle\langle BP\rangle-\langle\bar{B}P\rangle\langle B\bar{V}\rangle,$$

where the angular brackets denote the trace of the $SU(3)$ matrices.

A. β Term

$$A_0^{(\beta)}=-\tfrac{1}{3}\beta a^+r_1,$$
$$A_1^{(\beta)}=-\tfrac{1}{9}\beta b^+\{(m'^2-m^2)r_1+2[(m'^2-m_0^2)/m'^2m_0]r_3\},$$
$$A_2^{(\beta)}=-\tfrac{1}{9}\beta\{-r_1(u-s)b^++2r_3[M'Mr_1+(m'^2-m_0^2)/m'^2m_0]b^-\},$$
$$A_3^{(\beta)}=-\tfrac{1}{9}\beta b^+[2(M'+M)r_1-(4/m'^2)r_3], \quad \text{(A9)}$$
$$A_4^{(\beta)}=+\tfrac{1}{9}\beta[2(M'-M)r_1b^++(4/m'^2)r_3b^-],$$
$$A_5^{(\beta)}=+\tfrac{1}{9}\beta\{[(M'-M)(m'^2-m^2)+(M'+M)(s-u)]\times r_1b^++(2/m'^2)[u-s-(M'-M)(m_0^2-m)'^2/m_0]\times r_3b^+-4r_3r_4b^-\}.$$

B. γ Term

$$
\begin{aligned}
A_0^{(\gamma)} &= -(1/36)\gamma C r_1\,,\\
A_1^{(\gamma)} &= (1/144)\gamma r_2\{[t+m'^2-m^2+2(m'^2/m_0)(M'+M)]D+3[u-s-(M'-M)(M'+M+2m_0)]E\}\,,\\
A_2^{(\gamma)} &= (1/144)\gamma r_2\{[u-s+(M'+M)(M'+M+2(m'^2/m_0))]D-3[t+m'^2-m^2-2(M'+M)(M'+M+m_0)]E\}\,,\\
A_3^{(\gamma)} &= (1/72)\gamma(1/M'Mm'^2)\{[t+m'^2-m^2+2(m'^2/m_0)(M'+M)]D-3[u-s-(M'-M)(M'+M+2m_0)]E\}\,,\\
A_4^{(\gamma)} &= (1/72)\gamma(1/M'Mm'^2)\{[u-s+(M'-M)(M'+M+2m'^2/m_0)]D\\
&\qquad +3[t+m'^2-m^2-2(M'+M)(M'+M+m_0)]E\}\,,\\
A_5^{(\gamma)} &= -(1/24)\gamma(1/M'Mm'^2)\{(2Mm_0+M^2+m^2-s)(2M'(m'^2/m_0)+M'^2+m'^2-s)\\
&\qquad -4M'Mm'^2r_3r_4+(2M'm_0+M'^2+m^2-u)[2M(m'^2/m_0)+M^2+m'^2-u)]\}E.
\end{aligned}
\tag{A10}
$$

C. Pseudoscalar-Meson Exchange

The only nonzero amplitude for this exchange is the A_2,

$$
A_2^{(P)} = (Gg/18)b^-[r_3/m_P^2(m_P^2-t)]\times[t+2m_0(M'+M+m_P)+m_P(M'+M)]\,, \tag{A11}
$$

where m_P is the mass of the exchanged pseudoscalar meson.

D. Vector-Meson Exchange

With m_V the mass of the exchanged vector meson, the amplitudes are

$$
A_0^{(V)} = \frac{Gg}{48}a^+\frac{m_0}{M'Mm'^2m_V^2}\frac{1}{(m_V^2-t)}\times\left\{t\left(M'+M+m_V+\frac{m'^2+m_0^2}{2m_0}\right)+\frac{m_V}{2m_0}(M'+M)(m'^2+m_0^2)\right\}\,,
$$

$$
A_1^{(V)} = -\frac{Gg}{144}b^+\frac{r_5}{m_V^2-t}(m'^2-m^2)(M'+M)\,,
$$

$$
A_2^{(V)} = -\frac{Gg}{144}b^+\frac{r_5}{m_V^2-t}(s-u)(M'+M)\,,
$$

$$
A_3^{(V)} = -\frac{Gg}{72}b^+\frac{r_5}{m_V^2-t}t\,,
$$

$$
A_4^{(V)} = +\frac{Gg}{72}b^+\frac{r_5}{m_V^2-t}(M'^2-M^2)\,,
$$

$$
A_5^{(V)} = -\frac{Gg}{144}b^+\frac{r_5}{m_V^2-t}[t(u-s)+(M'^2-M^2)(m'^2-m^2)]\,,
$$

where

$$
r_5 = -\frac{m_0}{M'Mm_V^2m'^2}\left[t+\frac{m'^2+m_0^2}{2m_0}\times(M'+M+m_V)-m_V(M'+M)\right]. \tag{A12}
$$

E. Spin-$\frac{1}{2}$ Baryon-Octet Exchange

For these exchanges we will only write the contributions to the pole term for exchange in the u channel. In the calculations, however, the contact terms must be included, in order that the full amplitude vanish at the degenerate-mass threshold. To simplify the expressions for both octet and decuplet exchange, we define an additional invariant amplitude $A_6(s,t)$, which occurs in the form

$$
\epsilon_\mu(k')\bar{u}(p')\gamma_5 i\mathbf{k}'\gamma_\mu A_6(s,t)u(p)\,.
$$

A_6 is not an independent amplitude, but is related to the previously defined amplitudes in (A1) by the following identity:

$$
\begin{aligned}
&[(M'+M)^2-t]\epsilon_\mu\bar{u}\gamma_5 i\mathbf{k}'\gamma_\mu u\\
&= \epsilon_\mu\bar{u}\{-\tfrac{1}{2}i\epsilon_{\mu\nu\lambda\kappa}q_\nu P_\lambda Q_\kappa+\tfrac{1}{2}[(M'+M)^2-t+m^2-m'^2]\\
&\quad\times iP_\mu\gamma_5+\tfrac{1}{2}(u-s)iq_\mu\gamma_5-(M'+M)iP_\mu\gamma_5(\tfrac{1}{2}i\mathbf{Q})\\
&\quad+(M'-M)iq_\mu\gamma_5(\tfrac{1}{2}i\mathbf{Q})-\tfrac{1}{2}[(M'+M)(u-s)\\
&\qquad+(M'-M)((M'+M)^2-t+m^2-m'^2)]\gamma_5\gamma_\mu\}u.
\end{aligned}
$$

We also define the over-all strength of the coupling,

$$
K=(G^2/36)(M'MM_B^2m_0)^{-1}[(M'+M_B)^2-m^2]\times(M'+M_B+m_0)(M+M_B+m')\,,
$$

where M_B is the mass of the exchanged baryon; and we define the $SU(3)$ coefficients:

$$
\begin{aligned}
F_1&=\tfrac{1}{3}[5(\langle\bar{B}P\bar{V}B\rangle-\langle\bar{B}PB\bar{V}\rangle+\langle\bar{B}PB\rangle\langle\bar{V}\rangle)\\
&\qquad+\langle\bar{B}\bar{V}BP\rangle-\langle\bar{B}B\bar{V}P\rangle+\langle\bar{B}BP\rangle\langle\bar{V}\rangle]\,,\\
F_2&=(2/9)[5(\langle\bar{B}P\bar{V}P\rangle+2\langle\bar{B}PB\bar{V}\rangle-2\langle\bar{B}PB\rangle\langle\bar{V}\rangle)\\
&\quad+\langle\bar{B}\bar{V}BP\rangle+2\langle\bar{B}B\bar{V}P\rangle-2\langle\bar{B}BP\rangle\langle\bar{V}\rangle-6\langle\bar{B}P\rangle\langle B\bar{V}\rangle].
\end{aligned}
$$

Then the contributions of the pole terms of spin-$\frac{1}{2}$ octet exchange to the invariant amplitudes can be written as

$$\begin{aligned}
A_0^{(B)}&=0\,,\\
A_1^{(B)}&=-A_2^{(B)}=K/(M_B^2-u)\\
&\quad\times\{(M+M_B-m')[1-(M'+M)/2M']F_1\\
&\quad+[m'-(M'+M)(M_B+M)/2m']F_2\}\,,\\
A_3^{(B)}&=-A_4^{(B)}=-K/m'(M_B^2-u)\\
&\quad\times\{(M+M_B-m')F_1+(M_B+M)F_2\}\,,\\
A_5^{(B)}&=(M_B-M)A_6=K/(M_B^2-u)(M_B-M)/m'\\
&\quad\times\{[(M_B+M)^2-m'^2]F_1+[(M_B+M)^2+m'^2]F_2\}\,.
\end{aligned}\qquad\text{(A13)}$$

F. Spin-$\frac{3}{2}$ Baryon-Decuplet Exchange

Define the over-all strength by

$$K'=-(4G^2/9M')[1+(M_B+M')/m_0]\times[1+(m_0/m'^2)(M+M_B)]H\,,$$

where the $SU(3)$ coupling coefficient H is given by

$$\begin{aligned}
H=\tfrac{1}{6}[&3(\langle\bar{B}B\rangle\langle\bar{V}P\rangle+\langle\bar{B}\bar{V}PB\rangle-\langle\bar{B}BP\bar{V}\rangle-\langle\bar{B}\bar{V}\rangle\langle BP\rangle)\\
&-2\langle\bar{B}PB\rangle\langle\bar{V}\rangle+\langle\bar{B}PB\bar{V}\rangle-\langle\bar{B}B\bar{V}P\rangle+\langle\bar{B}\bar{V}BP\rangle\\
&+\langle\bar{B}BP\rangle\langle\bar{V}\rangle].
\end{aligned}$$

We also define various common functions of masses and momentum transfer

$$r_6=\frac{(M+M_B)^2-m'^2}{2MM_B}\,,\quad r_7=M_B+\tfrac{1}{2}(M'-M)\,,$$

$$r_8^{(\pm)}=1\pm(M'M_B-M'^2-M_B^2+m^2)/3M_B^2\,,$$

$$r_9=\tfrac{1}{2}\left[t-M^2-M'^2+\frac{1}{3M_B^2}(M'M_B-M'^2-M_B^2+m^2)\times(m'^2-m^2+M'^2-M^2)\right],$$

$$r_{10}=(1/3M_B)(M'M_B+M_B^2-m^2)\,.$$

Then the pole term of decuplet exchange contributes to the invariant amplitudes as follows:

$$\begin{aligned}
A_0^{(D)}&=0\,,\\
A_1^{(D)}&=K'/(M_B^2-u)\{\tfrac{1}{2}r_6r_7r_8^{(+)}\\
&\quad+(1/2M_BM)[r_7r_9+r_{10}((M'+M)(M-M_B)\\
&\quad+M_B^2-M^2-m'^2+\tfrac{1}{2}(M^2-M'^2))]\\
&\quad-(r_9/M)+(3M'+M-2M_B)r_{10}\}\,,\\
A_2^{(D)}&=K'/(M_B^2-u)[\tfrac{1}{2}r_6r_7r_8^{(-)}+(r_9/M)\\
&\quad-(3M'+M-2M_B)r_{10}]\,,\\
A_3^{(D)}&=K'/(M_B^2-u)\{-\tfrac{1}{2}r_6r_8^{(+)}+(1/2M_BM)\\
&\quad\times[-r_9+(M'-M)r_{10}]-(2/M)r_{10}\}\,,\\
A_4^{(D)}&=K'/(M_B^2-u)[-\tfrac{1}{2}r_6r_8^{(-)}+(2/M)r_{10}]\,,\\
A_5^{(D)}&=K'/(M_B^2-u)\\
&\quad\times\{(M-M_B)([M_B(M'+M_B)-m^2]/3M_B)r_6\\
&\quad+(1/M)[-(M_B+M)r_9-(M_B[M-M'+M_B]\\
&\quad-M'M-m'^2)r_{10}]\}\,,\\
A_6^{(D)}&=K'/(M_B^2-u)\{-([M_B(M'+M_B)-m^2]/3M_B)r_6\\
&\quad+(1/M)[r_9-(M'+M)r_{10}]\}\,.
\end{aligned}\qquad\text{(A14)}$$