The last terms cancel as another application of Eq. (A14) for $N=1$ and these equations establish Eqs. (A13) and (A14) in the case $N=2$. Finally, suppose that Eqs. (A13) and (A14) hold for $N=R$ and $N=R-1$, where $R=2, 3, 4, \cdots$. Multiply the $N=R$ equations from the left by $s_j$ and $s_k$, eliminate the $sS^\dagger$ products using Eq. (A10), and discard terms as permitted by Eqs. (A13) and (A14) for $N=R$ and $N=R-1$. The result is Eqs. (A13) and (A14) for the $N=R+1$ case. This establishes them in general and completes the proof that the $\mp$ helicity functions (A12) are the solutions of the wave equation (14).

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# Mass-Shell Evaluation of the Amplitude, Partial Conservation of Axial-Vector Current, and Equal-Time Commutators in Low-Energy $K^{\pm}p$ Scattering*

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For the on-mass-shell calculation of the scattering amplitude, we have used the following method. On the basis of the hypothesis of partially conserved axial-vector current (PCAC), we introduce the axial-vector current $J=A+c\partial\phi$, where $A$ is the current appearing in the PCAC relation, and $\phi$ is the pion field. Using this current in the Lehmann-Symanzik-Zimmermann formalism, the amplitude is decomposed into two terms $W^{00}$ and $W^0$ due to the equal-time commutators (ETC) of the divergence of the current $J$, respectively; a term $W^1$ due to an ETC leading to a vector current; and a term $W$ giving contributions of the discrete intermediate states. The ETC's of the current octet $J$ are assumed to be similar in form, though not equivalent, to those normally used for the current $A$. For kaon-proton ($K^{\pm}p$) scattering, we have used this formula in conjunction with dispersion relations and the $SU(3)\otimes SU(3)$ scheme, and derived sum rules for $W^0$ and $W^1$. Using the scattering length, the sum $W^{00}+W^0$ is evaluated. It is found that in order to obtain the correct signs and magnitudes of the $K^{\pm}p$ scattering lengths, with a single coefficient $c$ in the PCAC relation, we must take into account the sum $W^{00}+W^0$, which seemed to be negligible in $\pi N$ scattering. The difference between the on- and off-mass-shell amplitudes is derived, and seen to depend on the type of particles involved. Assuming that only the sum $W^{00}+W^0$ is a smooth function of the squares of the kaon four-momenta, this difference is found to be negligible in the $K^-p$ case, while it is 40% of the $K^+p$ scattering amplitude at threshold.

## I. INTRODUCTION

FOR the study of low-energy meson scattering, the hypothesis of partially conserved axial-vector current (PCAC)[1,2] has been used at times in conjunction with the current algebras. Among other authors,[3] Weinberg,[4] in the study of the pion scattering lengths, and Raman and Sudarshan,[5] in case of pion-nucleon ($\pi N$) scattering, have applied this combination to the off-mass-shell amplitude. Subsequently, for the calculation of the scattering lengths, other authors[6–8] have used the same technique, which consists of making use of the off-mass-shell amplitude and extrapolating the results to the physical threshold. In these calculations there are essentially two known sources of error. First, there is an error due to the extrapolation which may be considered in two parts: one arising from the use of PCAC in the current-algebra expression of the amplitude, while shifting $k^2$ and $k'^2$ from zero to $m^2$ ($k$ and $k'$ are the 4-momenta of the incoming and outgoing mesons, and $m$ is the meson mass); the other due to the difference between the on- and off-mass-shell terms in the amplitude. The effect of these approximations which seems to be negligible in the $\pi N$ scattering[4] has not been investigated in the kaon-nucleon ($KN$) case. The second source of error is, except for the ($\pi\pi$) scattering,[4,7] the omission of the $\sigma$ term which is due to the time derivative and the divergence of the axial-vector currents. Weinberg's reason for this approximation, in the $\pi N$ case, is that the above term is in the order of $m_\pi^2/M^2$ as compared with the terms linear in pion 4-momenta ($M$ and $m_\pi$ being the nucleon and pion masses). The fact that such an argument is not applicable to the $KN$ case, however, leads one to believe that the calculation of the

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[1] Y. Nambu, Phys. Rev. Letters 4, 380 (1959).
[2] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).
[3] See, for example, Y. Tomozawa, Nuovo Cimento **46**, 707 (1966); A. P. Balachandran, M. G. Gundzik and F. Nicodemi, *ibid.* **44**, 1257 (1966); see the footnote 5 of Ref. 4.
[4] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).
[5] K. Raman and E. C. G. Sudarshan, Phys. Rev. **154**, 1499 (1967).
[6] N. N. Khuri, Phys. Rev. **153**, 1477 (1967).
[7] H. J. Schnitzer, Phys. Rev. **158**, 1471 (1967).
[8] P. Roy, Phys. Rev. **162**, 1644 (1967); **172**, 1849(E) (1968).

$K^+p$ scattering length made in Ref. 8 may contain some error, and the use of the PCAC relation in the $KN$ case needs further study. We shall, therefore, study the above points in the case of $K^{\pm}p$ low-energy elastic scattering by direct evaluation of the on-mass-shell amplitude with the following method.

In Sec. II, we introduce the axial-vector current octet $J=A+c\partial\phi$, where $A$ is the weak current appearing in the PCAC relation, and $\phi$ is the interaction field. Using this current in the Lehmann-Symanzik-Zimmermann (LSZ) formalism,[9] we decompose the on-mass-shell amplitude into four terms: one called $W^{00}$, due to the equal-time commutator (ETC) of the divergence of the current $J$ with the time derivative of the interpolating field; a second term $W^0$, consisting of the ETC of the divergence and the time-component of the current $J$; a third one $W'$ due to an ETC leading to a vector current; and finally a term $W$ due, to all possible intermediate states. We note that the contribution of $W^{00}$ to the scattering amplitude is known[10] to be a polynomial in $(k+k')^2$. We also observe that the sum $W^{00}+W^0$, for $k^2=k'^2=0$, is the same as the $\sigma$ term which was negligible in the $\pi N$ case[4] and ignored in the work of Ref. 8 for $K^+p$ scattering. To evaluate $W^0$ and $W^1$, we assume that the commutation rules of the currents $J$ are similar in form though not equivalent, with those normally assumed for the current $A$.[11] In applying this treatment to the $K^{\pm}p$ scattering, we approximate the term $W$ by a term $W^P$ due to one-particle intermediate states, taking into account all quadratic terms in $k$. In Sec. III, the sum rules for $W^0$ and $W^1$ are derived by combining this treatment with the dispersion relations. In Sec. IV, we evaluate the sum $W^{00}+W^0$ in terms of $W^1$ and $W^P$ and the observed scattering lengths. Also, we evaluate the difference of the on- and off-mass-shell amplitudes for $K^{\pm}p$ scattering, which is a part of the error in the extrapolation used in the previous method.[4-8] Our results may be summarized as follows.

The two matrix elements $W^0$ and $W^1$ may be expressed in terms of polynomials of $k^2$ similar to that known for $W^{00}$.[10] the Goldberger-Treiman coefficient,[12] $c$ in the PCAC relation, andthe scattering length which may be obtained experimentally. For a given $c$ the sum $W^{00}+W^0$ is found to be appreciable at threshold and independent of the kaon energy. It is noted that if the sum $W^{00}+W^0$ has been omitted from the scattering amplitude, as was done in Ref. 8, then it would have been impossible to obtain the correct signs and magnitudes of both $K^{\pm}p$ scattering lengths with a single coefficient $c$ given by the PCAC relations.

[9] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **1**, 205 (1965).

[10] See the work of D. Amati, Nuovo Cimento **2**, 190 (1958); see also S. S. Schweber, *Relativistic Quantum Field Theory* (Row, Peterson and Co., 1961), pp. 789 and 790.

[11] The commutation rules of the weak current $A$ are those suggested by M. Gell-Mann, Physics **1**, 63 (1964).

[12] M. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

The difference between the on- and off-mass-shell amplitudes is seen to depend on the type of the particles involved in the scattering, and an unknown constant, namely the off-mass-shell value of the matrix element of a scalar field. Assuming that the sum $W^{00}+W^0$ is a slowly varying function of $k^2$, this difference is found to be negiligble for $K^-p$ scattering, and 40% of the threshold amplitude for $K^+p$ scattering.

## II. ON-THE-MASS-SHELL EVALUATION OF THE AMPLITUDE

### A. Analysis and Decomposition of the Amplitude

The two-particle scattering amplitude derived from LSZ treatment,[9] as a function of the time-ordered products of the interpolating field $\phi$, may be expressed as

$$T_{ba}(\omega,k^2,k'^2)=\Gamma_0(\omega,k^2,k'^2)+\Gamma_1(\omega,k^2,k'^2)\,, \qquad (1a)$$

with

$$\Gamma_0(\omega,k^2,k'^2)=m^4\int d^4x d^4y \times e^{-ik'\cdot x+ik\cdot y}\langle p'|T\{\phi_b(x)\phi_a^\dagger(y)\}|p\rangle\,, \qquad (1b)$$

and

$$\Gamma_1(\omega,k^2,k'^2)=\int d^4x d^4y \times e^{-ik'\cdot x+ik\cdot y}[m^2(\partial_{x_\mu}{}^2+\partial_{y_\nu}{}^2)+\partial_{x_\mu}{}^2\partial_{y_\nu}{}^2] \times\langle p'|T\{\phi_b(x)\phi_a^\dagger(y)\}|p\rangle\,. \qquad (1c)$$

Here $k$ and $k'$ are the initial and final momenta and $a$ and $b$ the initial and final $SU(3)$ indices for the mesons, while $p$ and $p'$ are the initial and final momenta, and $|p\rangle$ and $|p'\rangle$ are the initial and final state of the target. The functions $\Gamma_0$ and $\Gamma_1$ have the following properties: For $k^2=k'^2=m^2$ we have

$$\Gamma_0(\omega,\, k^2=k'^2=m^2)=\Gamma_1(\omega,\, k^2=k'^2=m^2)\,, \qquad (2a)$$

and for $k^2=k'^2=0$,

$$\Gamma_1(\omega,\, k^2=k'^2=0)=0\,, \qquad (2b)$$

where we have made use of some partial integrations and of the fact that the spatial terms in these integrations vanish at spatial infinity.

Introducing the PCAC relation[1,2,13]

$$\partial_\mu A^\mu=cm^2\phi\,, \qquad (3)$$

where $A$ is the axial-vector current, $m$ is the meson mass, and $c$ is the coefficient which is evaluated at $k^2=0$ by Ref. 1 or 12, Eqs. (2b) and (1a) give

$$T_{ba}(\omega,\, k^2=k'^2=0)=c^{-2}\int d^4x d^4y\; e^{-ik'\cdot x-ik\cdot y} \times\langle p'|T\{\partial_\mu A_b{}^\mu(x),\partial_\nu A_a{}^{\nu\dagger}(y)\}|p\rangle_{k^2=k'^2=0}\,. \qquad (4)$$

[13] S. L. Adler, Phys. Rev. **137**, B1022 (1965).

It is clear from (1b) that Eq. (1a) in its present form cannot be evaluated directly. The method used by previous authors,[4–8] hereafter referred to as the "previous method," is to calculate the amplitude (4) and extrapolate the results to the physical mass shell. The error involved in this treatemnt may be analyzed into two parts: one is due to the use of (2) while varying $k^2$ from zero to $m^2$, and its evaluation requires a knowledge of the "residual current" involved in the definition of PCAC as given by Adler[13]; the other is

$$\epsilon = T_{ba}(\omega,\, k^2=k'^2=m^2) - T_{ba}(\omega,\, k^2=k'^2=0)\,, \quad (5)$$

which may or may not be important, depending on the particles involved in the scattering, as can be seen from the work in Refs. 4–8 and our final results.

To avoid extrapolation, therefore, we deal directly with the amplitude (1a) by the following method.

On the basis of the PCAC relation (3) we introduce the axial-vector current octet

$$J_a{}^{\mu}(x) = A_a{}^{\mu}(x) + c\partial^{\mu}\phi_a(x) \quad (6a)$$

as a source of the interacting field, so that

$$(\partial^2+m^2)\phi_a(x) = c^{-1}\partial_{\mu}J_a{}^{\mu}(x)\,. \quad (6b)$$

Using this current and (2), the amplitude (1a) can be expressed as

$$T_{ba}(\omega,k^2,k'^2) = c^{-2}\int d^4x d^4y \times e^{-ik'\cdot x+ik\cdot y}\langle p'|\, T\{\partial_{\mu}J_b{}^{\mu}(x),\partial_{\nu}J_a{}^{\nu\dagger}(y)\} - m^{-2}\delta(x_0-y_0)\partial_{y0}[\partial_{\mu}J_b{}^{\mu}(x),\partial_{\nu}A_a{}^{\nu\dagger}(y)]\,|p\rangle\,. \quad (7a)$$

The second term on the right-hand side of (7a) is known[10] to contribute a polynomial in $(k'+k)^2$ with a finite number of terms and with coefficients depending on $t=(k'-k)^2$. We may, therefore, write

$$\int d^4x d^4y\; e^{-ik'\cdot x+ik\cdot y}\delta(x_0-y_0) \times \langle p'|\,\partial_{y0}[\partial_{\mu}J_b{}^{\mu}(x),\partial_{\nu}A_a{}^{\nu\dagger}(y)]\,|p\rangle = \frac{i(2\pi)^4\delta(p'+k'-p+k)}{(2\pi)^6(4k_0k_0')^{1/2}}\sum_{n=0}^{N}\alpha_n(t)(k+k')^{2n}, \quad (7b)$$

where the $\alpha$'s are coefficients depending on the momentum transfer, and $N$ is the number of terms in the polynomial.[14] Also, the currents are normalized such that

$$\langle 0|\,A_a{}^{\mu}(0)\,|k,b\rangle = [2(2\pi)^3k_0]^{-1/2}ck^{\mu}\delta_{ab}\,. \quad (7c)$$

The first matrix element in (7a) can be decomposed into three terms, either by using some partial integrations combined with the commutator properties of the field $\phi$ or by using the generalized Ward-Takahashi identies, as done by Raman and Sudarshan.[5] Carrying out this decomposition and using (7b) in (7a) we find

$$T_{ba}(\omega,k^2,k'^2) = -\frac{i(2\pi)^4\delta(p'+k'-p-k)}{(2\pi)^6(4k_0k_0')^{1/2}c^2} \times (W^{00}+W^0+W^1-W)_{ba}\,. \quad (7d)$$

Here,

$$W^0(\omega,k^2,k'^2) = -i\int d^4z\, e^{-ik\cdot z}\langle p'|\,\delta(-z_0)[J_b{}^0(0),\partial_{\nu}J_a{}^{\nu\dagger}(z)]\,|p\rangle\,, \quad (7e)$$

$$W^1(\omega,k^2,k'^2) = k_{\mu}\int d^4z\, e^{-ik\cdot z}\langle p'|\,\delta(-z_0)[J_a{}^{0\dagger}(z),J_b{}^{\mu}(0)]\,|p\rangle\,, \quad (7f)$$

$$W(\omega,k^2,k'^2) = -ik_{\mu}'k_{\nu}\int d^4z\; e^{-ik\cdot z}\langle p'|\, T\{J_b{}^{\mu}(0),J_a{}^{\nu\dagger}(z)\}\,|p\rangle\,, \quad (7g)$$

and

$$W^{00}(t,k^2,k'^2) = \frac{1}{m^2}\sum_{n=0}^{N}\alpha_n(t)(k+k')^{2n}\,. \quad (7h)$$

Now consider the decomposition of the amplitude (4), which can be found in Ref. 4:

$$T_{ba}(k^2=0) \propto [R^0(A)+R^1(A)+R(A)]\,, \quad (8a)$$

where $R$, $R^1$, and $R$ represent the $\sigma$ term, the term due to a vector current operator, and the terms due to the discrete intermediate states, respectively. In the present context, $A$ indicates that the amplitude (8a) depends on the commutators of the current $A$ only. Noticing from (2) that at $k^2=0$ the two amplitudes (7a) and (8a) are completely equivalent, we find the connections between our treatment and the previous method as

$$R^0(A) = \alpha_0 + W^0(J,\, k^2=0)\,, \quad (8b)$$

$$R^1(A) = W^1(J,\, k^2=0)\,, \quad (8c)$$

$$R(A) = W(J,\, k^2=0)\,. \quad (8d)$$

Here, $\alpha_0$ the zero-order term in (7h) is a constant, and $J$ only indicates that the amplitudes $W$'s are originated by the commutators of the current $J$.

To evaluate the amplitudes (7e) and (7f) we need the commutation rules of the current $J$. A simple choice which guarantees the relations (8b) to (8d) is to express the commutation rules of the current $J$ in a form similar to those[11] for the current $A$, i.e.,

$$\delta(z_0)[J_b{}^0(0),\partial_{\nu}J_a{}^{\nu\dagger}(z)] = id_{abc}[\sigma_c{}^J(z)]\delta^4(z) + \text{S.T.}\,, \quad (9a)$$

$$\delta(z_0)[J_a{}^{0\dagger}(z),J_b{}^{\mu}(0)] = 2if_{abc}[V_c{}^{J\mu}(z)]\delta^4(z) + \text{S.T.} \quad (9b)$$

It is understood, however, that $\sigma^J$ and $V^J$, the scalar and vector current operators associated with the cur-

[14] The polynomial expression for the equal-time commutator, given by (7b), appears both in the dispersive part of the amplitude and in its relevant $S$-matrix element, described by H. J. Bremermann, R. Oehme, and J. G. Taylor, Phys. Rev. **109**, 2178 (1958).

rent $J$, are not equivalent to those $\sigma$ and $V$ normally used in the relations (9) as applied to the current $A$. Also, in (9) we have the usual $SU(3)$ structure constants, $d_{abc}$ and $f_{abc}$, and possibly the Schwinger terms (S.T.), which we shall neglect hereafter.

### B. Application to $K^{\pm}p$ Scattering

For further development of Eqs. (7) we shall consider the case of $K^{\pm}p$ scattering for which the initial currents

$$J_{K^{\pm}}=(1/\sqrt{2})(J_4\mp iJ_5)$$

are given by the $SU(3)\otimes SU(3)$ scheme which we follow. Using these currents, Eqs. (6a), (9a), and (7e), and setting for convenience $k^2=k'^2$, we find

$$W_{\pm}{}^0(\omega,k^2)=\langle p'|\sigma_3{}^J(0)-(1/\sqrt{3})\sigma_8{}^J(0)|p\rangle\,, \quad (10)$$

where the subscripts $\pm$ refer to $K^{\pm}p$, and $\omega$ is the meson lab energy. Similarly, using the above currents (6a), (9b), and (7f), for $k^2=k'^2$, we may write[15]

$$W_{\pm}{}^1(\omega,k^2)=\pm 2\bar{u}(p')\times\left\{k'F_1(t)+\mu_pF_2(t)\left[k'-\frac{k'\cdot(p'+p)}{2M}\right]\right\}u(p)\,, \quad (11\text{a})$$

where the $u$'s are the spinors obeying the Dirac equation and $\mu_p$ is the anomalous magnetic moment of the proton. In writing (11a) we have expressed the matrix element of the current $V^J$ in the usual way, in terms of the $\gamma_\mu$ matrices $(p'-p)_\mu$ and $\sigma_{\mu\nu}(p'-p)^\nu$, and have used the vector current conservation condition. In (11a) we have also $F_1$ and $F_2$, the unknown form factors corresponding to the mass shell $k^2=m^2$, which should satisfy the relation (8c), that is,

$$F_1(t=0)=F_2(t=0)=1 \quad \text{for} \quad k^2=0\,.$$

Hence, (11a) yields

$$W^1(\omega,\,t=0,\,k^2=0)=\pm 2|\omega|\,, \quad (11\text{b})$$

in agreement with the previous work.[8] From the properties of the scalar and vector $\sigma^J$ and $V^J$ and the currents $J_{K^{\pm}}$, we find

$$\begin{aligned} W_+{}^0(\omega,k^2)&=W_-{}^0(\omega,k^2)\,,\\ W_+{}^1(\omega,k^2)&=-W_-{}^1(\omega,k^2)\,. \end{aligned} \quad (11\text{c})$$

We shall use these conditions with the dispersion relations for deriving the sum rules for $W^0$ and $W^1$ in the next section. Also, we shall compute $W^0$ plus $W^{00}$ given by (7h), and $W^1$ in Sec. IV, using the observed data for the scattering lengths.

To evaluate the matrix element $W$, Eq. (7g), we choose only the one-particle intermediate states. These states for $K^{\pm}p$ scattering in the $s$ and $u$ channels are $\Lambda$, $\Sigma$, $Y_1^*$, and $Y_0$. We neglect the contribution of the $t$ channel. Then, denoting $T_{ba}=T_{K^{\pm}p}=T_+$, $T_{ab}=T_{K^-p}=T_-$, and using (A15) of the Appendix, we may express the terms due to one-particle intermediate states, denoted by $W^P$, as

$$W_{\pm}{}^P(\omega,k^2)=W_{1\pm}{}^P(\omega)+k^2W_{2\pm}{}^P(\omega)\,, \quad (12\text{a})$$

where

$$\mathrm{Re}W_{1\pm}{}^P=\omega^2\left[-\sum_{J=\Lambda,\Sigma}\frac{G(J)}{\omega_J\pm\omega}-\frac{G(Y_0^*)}{\omega_{Y_0^*}\pm\omega}-\frac{G(Y_1^*)}{3}\frac{M}{M_{Y_1^*}}\frac{M_{Y_1^*}+M\pm\omega}{\omega_{Y_1^*}\pm\omega}\right] \quad (12\text{b})$$

and

$$\mathrm{Re}W_{2\pm}{}^P=\frac{1}{2M}\left[\sum_{J=\Lambda,\Sigma}G(J)\frac{M_J+M\pm\omega}{\omega_J\pm\omega}+G(Y_0^*)\frac{M_{Y_0^*}-M\pm\omega}{\omega_{Y_0^*}\pm\omega}+\frac{G(Y_1^*)}{3}\frac{M_{Y_1^*}+M\pm\omega}{\omega_{Y_1^*}\pm\omega}\right]. \quad (12\text{c})$$

In (11c) we have from (A14), for $\omega^2\ll M_i^2+m^2$ and $p=(0,0,0,\mathrm{M})$,

$$\begin{aligned} \omega_i&\simeq M_i-M+(\omega^2-m^2)/2M_i\,,\\ G(i)&=(M/M_i)g_A{}^2(i)\,, \end{aligned} \quad (12\text{d})$$

where $g_A(i)$ refers to the axial-vector coupling constant for a given intermediate state $i$. The absorptive part of $W^P(\omega,k^2)$ will not be needed in this work, since we shall use the threshold values of $W^P$ in our computations. For $k^2=0$, the contribution of the term $W^P$ reduces to

$$R(\omega,\,k^2=0)=W_1{}^P(\omega)\,, \quad (12\text{e})$$

making use of Eqs. (8d) and (12a).

Making use of Eqs. (7h) and (10)–(12) in (7d), the $K^{\pm}p$ scattering amplitudes for the forward direction are

$$\begin{aligned} T_{\pm}(\omega,k^2)=&-(i8\pi^2c^2|\omega|)^{-1}\\ &\times[W^{00}(k^2)+W^0(\omega,k^2)+W_{\pm}{}^1(\omega,k^2)\\ &-W_{1\pm}{}^P(\omega)-k^2W_{2\pm}{}^P(\omega)-W_{\pm}{}^H(\omega,k^2)]\,, \end{aligned} \quad (13)$$

where we have left out the $\delta$ function from (7d). Here, $W^H(\omega,k^2)$, which is extracted from (7g), represents the terms due to more than one-particle intermediate states.

## III. USE OF DISPERSION RELATIONS

To obtain further information on the matrix elements due to the equal-time commutators, we use the dispersion relation given by (A12) of the Appendix, viz.,

$$\begin{aligned} T_{\pm}(\omega,1)=&-(i8\pi^2c^2|\omega|)^{-1}[W^{00}(1)+W^0(m,1)\\ &+(\omega/m)W_{\pm}{}^1(m,1)-W_S{}^H(m,1)\pm W_A{}^H(m,1)\\ &-W_{\pm}{}^P(\omega,1)]+|\omega|^{-1}D_{\pm}(\omega,1)\,. \end{aligned} \quad (14\text{a})$$

Here $(\omega,1)$ denotes $(\omega,\,k^2=m^2)$ and (1) means $(k^2=m^2)$; $W_A{}^H$ and $W_A{}^H$ represent the terms $W^H$ in (A12)

[15] The $SU(3)\otimes SU(3)$ scheme gives $f_{abc}V_c(z)=V_3{}^\mu(z)+\sqrt{3}V_8{}^\mu(z)$ and for this relation we find (11a), following the procedure seen, e.g., in S. Gasiorowicz, *Elementary Particle Physics* (John Wiley & Sons, Inc., New York, 1966), p. 435.

which are, respectively, even and odd functions of $\omega$. Also,

$$D_{\pm}(\omega,1)=\mathbf{k}^2P\int_m^\infty\left(\frac{\sigma_+(\omega')+\sigma_-(\omega')}{k'(\omega'^2-\omega^2)}\omega' \pm\omega\frac{\sigma_-(\omega')-\sigma(\omega')}{k'(\omega'^2-\omega^2)}\right)d\omega' \quad (14b)$$

is (A8) of the Appendix, in which the optical theorem for the amplitude (A10) and the total cross sections $\sigma_{\pm}(\omega)$ are used. Comparing (14a) with (13), in which the $W^H$ term is split into even the odd functions of $\omega$, and noticing from (7h) and (11c) that $W^{00}$ and $W^0$ are even functions of $\omega$ while $W_1$ is odd, we find

$$W^0(\omega,1)+W_S{}^H(\omega,1)=W^0(m,1)+W_S{}^H(m,1) +8\pi^2c^2\mathbf{k}^2P\int_m^\infty\frac{\sigma_+(\omega')+\sigma_-(\omega')}{k'(\omega'^2-\omega^2)}\omega'd\omega', \quad (15a)$$

and

$$W_{\pm}{}^1(\omega,1)\pm W_A{}^H(\omega,1) =\pm(|\omega|/m)[W^1(m,1)+W_A{}^H(m,1)] \mp 8^2c^2\mathbf{k}^2|\omega|P\int_m^\infty\frac{\sigma_-(\omega')-\sigma_+(\omega')}{k'(\omega'^2-\omega^2)}d\omega'. \quad (15b)$$

The dispersion relation for the off-mass-shell amplitudes $T_{\pm}(\omega,\, k^2=0)$ may be expressed by assuming that all conditions leading to (A12) of the Appendix hold as $k^2\to 0$. Hence,

$$T_{\pm}(\omega,0)=-(8\pi^2c^2|\omega|)^{-1}[W^{00}(m,0)+W^0(m,0) +(\omega/m)W_{\pm}{}^1(m,0)-W_S{}^H(m,0)\pm W_A{}^H(m,0) -W_{\pm}{}^P(\omega,0)]+|\omega|^{-1}D_{\pm}(\omega,0). \quad (16)$$

Here $(\omega,0)$ denotes $(\omega,\, k^2=0)$ and $D_{\pm}(\omega,0)$ is given by (A8) of the Appendix. Comparing (16) with (13) in which the $k^2=0$ is used, and with the same procedure for obtaining Eqs. (15) from (14) and (13), we find

$$W^0(\omega,0)+W_S{}^H(\omega,0)=\beta^0(m,0)+W_S{}^H(m,0) +16\pi c^2\mathbf{k}^2P\int_m^\infty\frac{\mathrm{Im}[A^+(\omega')-\omega'B^+(\omega')]}{(\omega'^2-\omega^2)(\omega'^2-m^2)}\omega'd\omega' \quad (17a)$$

and

$$W_{\pm}{}^1(\omega,0)\pm W_A{}^H(\omega,0)=\pm(|\omega|/m)W^1(m,0)\pm W_A{}^H(m,0) \mp 16\pi c^2\mathbf{k}^2|\omega|P\int_m^\infty\frac{\mathrm{Im}[A^-(\omega')-\omega'B^-(\omega')]}{(\omega'^2-\omega^2)(\omega'^2-m^2)}d\omega'. \quad (17b)$$

We now note that in the $K^{\pm}p$ case the total cross sections $\sigma_{\pm}(\omega)$ can be considered as slowly varying functions[16] of $\omega$ (except for a small interval in the low-energy range). With this approximation extended to the off-mass-shell $k^2=0$, the integrals in the right-hand sides of Eqs. (15) to (17b) vanish. In any case, the small contributions of these integrals may be neglected in compensation with all the terms $W_{\pm}{}^H$ which correspond to more-than-one-particle intermediate states. Hence, Eqs. (15) and (17) give

$$W^0(\omega,1)\simeq W^0(m,1),\quad W^0(\omega,0)\simeq W^0(m,0) \quad (18a)$$

and

$$W_{\pm}{}^1(\omega,1)=\pm(|\omega|/m)W^1(m,1),\qquad W_{\pm}{}^1(\omega,0)=\pm(|\omega|/m)W^1(m,0). \quad (18b)$$

From (18a) we learn that the function $W^0(\omega,k^2)$ is independent of $\omega$; hence, considering (18a), (10), and (8b), we may write

$$W^0(\omega,1)=\sum_{i=0}^{I}\beta_i(t=0)k^{2i}, \quad (19a)$$

a polynomial similar to (7h), with coefficients $\beta$ depending on $t=(k'-k)^2$. Using (19a), for $k^2=0$, in (8b),

$$\beta_0(t=0)+\alpha_0(t=0)=R^0(t=0,\, k^2=0), \quad (19b)$$

where $\beta_0$ is the zero-order terms of the polynomials in (19a) and $\alpha_0$ and $R^0$ are described in (8b).

On the other hand, Eqs. (18b), (11b) and (11c) allow us to rewrite (11a), for $t=0$, as

$$W_{\pm}{}^1(\omega,1)=\pm[2+R_1(t=0,\, k^2)]\omega, \quad (20a)$$

in which

$$R_1(t=0,\, k^2)=\sum_{i=1}^{I}\gamma_n(t=0)k^{2i}. \quad (20b)$$

We shall evaluate $R_1$ for the threshold, in the next section, using the observed values of the $K^{\pm}p$ scattering lengths.

The difference between the on- and off-mass-shell amplitudes, Eq. (5), is

$$\epsilon_{\pm}=-(i8\pi^2c^2\omega)^{-1}[W^{00}(m,1)+W^0(m,1) -R^0(0)\pm\omega R_1(1)+m^2W_2{}^P(\omega)_{\pm}], \quad (21)$$

making use of Eqs. (20), (19), (14a), (16), (13), and (12).

## IV. USE OF THE SCATTERING LENGTHS

Here, we compute the sum $W^{00}+W^0$ and the polynomial $R_1(1)$, Eq. (20b), using the $K^{\pm}p$ scattering lengths $a_{\pm}$. The $s$-wave scattering length may be obtained as $2\pi i$ times the reduced mass times the coefficient of the $\delta$ function in (7d), evaluated for forward scattering at threshold, and for the $s$-wave contributions

[16] For $K^+p$ scattering data see C. Cook *et al.*, Phys. Rev. **129**, 2743 (1963); for $K^-p$ data see R. H. Dalitz, Ann. Rev. Nucl. Sci. **13** (1963). The $K^{\pm}p$ cross sections $\sigma_{\pm}(\omega)$ have also been considered as slowly varying functions of $\omega$ by D. Amati, Phys. Rev. **113**, 1692 (1959).

of the amplitudes,[17]

$$a_{\pm}=\frac{1}{4\pi c^2}\left(\frac{Mm}{M+m}\right)[W^{00}(1)+W^0(m,1)+W_{\pm}{}^1(m,1)-W_{\pm}{}^P(m,1)]_{l=0}, \quad (22a)$$

where $l$ is the orbital angular momentum, and we choose[13]

$$c\simeq -i[\sqrt{2}g_A(\Lambda)/g_{\Lambda NK}]M\equiv -ic_0M, \quad (22b)$$

having used the vertex form factor $F_{\Lambda NK}(k^2=m^2)=1$.

To obtain $W^0$ from (22a) we evaluate first $W_{\pm}{}^P(m,1)$ from Eqs. (13) in which the contributions of the $t$ channel are neglected. Using the coupling constants[18] $g_A(\Lambda)=0.70$, $g_A(\Sigma)=0.23$, $g_A(Y_0{}^*)=0.15$, and $g_A(Y_1{}^*)=0.67$ in Eqs. (13), we find the $s$-wave contributions of the term $W_{\pm}{}^P$ as

$$W_2{}^P(m,1)_+=0.9m^{-1}, \quad (23a)$$

$$W_2{}^P(m,1)_-=-0.149m^{-1}, \quad (23b)$$

$$W_+{}^P(m,1)=0.436m, \quad (23c)$$

$$W_-{}^P(m,1)=-0.302m. \quad (23d)$$

In writing Eqs. (22), we note that in the $K^+p$ case all four intermediate states $\Lambda$, $\Sigma$, $Y_1{}^*$, and $Y_0{}^*$ contribute in the $u$ channel. However, in the $K^-p$ case the $s$-wave contribution to the amplitude is due to $Y_0{}^*$ only ($\Lambda$, $\Sigma$, and $Y_0{}^*$ contribute to the $p$-wave amplitude, as can be verified by their spins and parities).

Using Eqs. (10) and (22b), and Eqs. (23) in (22a), we have

$$m^{-1}[W^{00}(1)+W^0(m,1)]=12\pi c^2Ma_+-[1.56+R_1(1)]=12\pi c^2Ma_-+[1.70+R_1(1)]. \quad (24a)$$

Here, the observed $K^{\pm}p$ scattering lengths $a_{\pm}$ are[19]

$$a_+=a_1=(-0.29\pm0.01)\ \mathrm{F},\quad a_-=\tfrac{1}{2}[a_1+a_0]=(-0.83\pm0.09)\ \mathrm{F}, \quad (24b)$$

in which $a_1$ and $a_0$ are the scattering lengths corresponding to isospin $I=1$ and $I=0$, respectively.

For $c_0=0.155$, obtained from the Cabibbo theory and used in Ref. 8, Eqs. (24a) and (24b) give

$$R_1(1)=0.41, \quad (25a)$$

$$W^{00}(1)+W^0(1)=-3.3m, \quad (25b)$$

$$R^0(0)=-3.3m-\sum_{n=1}^{N}\alpha_nm^{2(n-1)}-\sum_{i=1}^{I}\beta_im^{2i}. \quad (25c)$$

Note that $R_1(1)$, which is the difference $W^1(m,1)-W^1(m,0)$, is 20% of $W^1(m,0)$, the off-mass-shell ($k^2=0$) value of $W^1$. If we had neglected the terms $W^{00}+W^0$ and $R_1$ in our calculation, we would have gotten $a_+=0.258$ F, about 11% less than the observed $a_+$ given by (24b). This value is reasonable, considering the approximation involved in data for $g_A$, $c$, etc.; however, it has the wrong sign.[20] For the $K^-p$ case, on the other hand, the above approximation would have given $a_-=-0.28$ F, which has the correct sign, but is three times smaller than the observed $a_-$ given by (24b).

The difference between the on- and off-mass-shell amplitudes depends on the constant $R^0(0)=W^{00}(m,0)+W^0(m,0)$, given by (19b) and (25c). Here, we assume that both amplitudes $W^{00}$ and $W^0$ are smooth functions of $k^2$; then, we may write

$$W^{00}(1)+W^0(m,1)-R^0(0)\approx 0. \quad (26)$$

In this case, Eqs. (23) to (26) and (21) give

$$\epsilon_+\simeq 0.40T_+(m,1),\quad \epsilon_-\simeq 0.04T_-(m,1). \quad (27)$$

## V. CONCLUSION AND COMMENTS

Considering Eqs. (19) to (27) and the remark, in the preceding section on the consequence of neglecting the terms $W^{00}$, $W^0$, and $R_1$ from the expression of the $K^{\pm}p$ scattering lengths, we conclude the following.

(1) Each of the matrix elements $W^0$ and $W^1$ is essentially a polynomial of $k^2$, similar to the matrix element $W^{00}$ whose general form was already known to us. The terms $W^{00}$ and $W^0$, as well as their respective values $\alpha_0$ and $\beta_0$ corresponding to the off-mass-shell $k^2=0$, are not known at the present time. We have only been able to compute the $\sigma$ term $W^{00}+W^0$ given by Eq. (25b), and found it to be appreciable as compared to the terms $W^1$ and $W^P$. (2) With a single coefficient $c$ in the PCAC relation, the correct signs of the $K^{\pm}p$ scattering lengths can only be obtained if the sum $W^{00}+W^0$, or at least one of these terms, is taken into account. For the $K^{\pm}p$ cases, in contrast to the $\pi N$ scattering, the magnitude of $W^{00}+W^0$ exceeds that of the term $W^1$. In both cases, the difference of the on-mass-shell values of $W^1$, given by (25a), as well as the contribution of the pole term $W^P$, are appreciable and should not be ignored. (3) The difference between the on- and off-mass-shell amplitudes at threshold depends on the type of incident

[17] This method of computation of $s$-wave scattering length is given in Ref. 4 and others mentioned in Ref. 4. The same result, of course, would be obtained from the standard method as given by J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[18] The values of $g_A(\Lambda)$, $g(\Sigma)$, and $g_A(Y_1{}^*)$ are those gathered by Ref. 8. The coefficient $g_A(Y_0{}^*)$ is calculated by (19b), in which we have used $g_{pY_0{}^*K}=\alpha g_{Y_0{}^*\Sigma\pi^0}$ and $(g_{Y_0{}^*\sigma\pi^0}{}^2/4\pi)=0.045$, and $\alpha=10$ according to C. Weil's work [C. Weil, University of Minnesota report (unpublished)], as recorded in Ref. 8.

[19] V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, Phys. Rev. **134**, B1111 (1964). Also, J. K. Kim, Phys. Rev. Letters **14**, 29 (1965).

[20] Note that Roy in Ref. 8 has neglected the terms $W^{00}$, $W^0$, and $W^P$ in his calculation, and has obtained $a_+=-0.41$ F, which has the correct sign, but is 40% larger than the observed $a_+$. The reason that Roy obtained the correct sign for $a_+$ is that he chose the signs of the exponentials $e^{-ik'\cdot x-ik\cdot y}$ in the formula (1a) opposite to those appearing in the Refs. 9 and 4, which we have used here. With this choice of sign, the approximation made in Ref. 8 leads to a wrong sign for the $K^-p$ scattering length.

particle, and on the unknown quantity $R^0$, Eq. (25c). This difference could be large, as seen from (27), despite the assumption of the smoothness of $W^{00}+W^0$ with respect to $k^2$.

All these results, of course, depend mainly on the accuracy of the observed scattering data, the values of $c$ and the coupling constants $g$ in $W^P$. It is also clear that our present method of evaluation of the physical amplitude is based on the assumed commutation rules (9) for the current $J$, which we hope to study in further investigations.

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## APPENDIX

Here we shall express the $K^{\pm}p$ dispersion relations in a suitable form for use in this paper.[21] Also, we shall give a summary account of the derivation of the one-particle terms in these relations, all for forward scattering. The scattering amplitude may be expressed as

$$F_{\pm}(\omega,k^2)=F^+(\omega,k^2)\pm F^-(\omega,k^2)\,, \qquad \text{(A1)}$$

where, for forward scattering, we write

$$F^{\pm}(\omega,k^2)=A^{\pm}(\omega,k^2)-\omega B^{\pm}(\omega,k^2)\,, \qquad \text{(A2)}$$

in which

$$\begin{aligned} A^{\pm}(\omega)^* &= \pm A^{\pm}(-\omega)\,,\\ B^{\pm}(\omega)^* &= \mp B^{\pm}(-\omega)\,. \end{aligned} \qquad \text{(A3)}$$

These properties of the invariant amplitudes $A^{\pm}$ and $B^{\pm}$ lead to the dispersion relations

$$\mathrm{Re}F^-(\omega,k^2)=\omega P^-(\omega,k^2)$$
$$-\frac{2\omega}{\pi}P\int_m^\infty \frac{\mathrm{Im}[A^-(\omega',k^2)-\omega' B^-(\omega',k^2)]}{\omega'^2-\omega^2}d\omega'\,, \qquad \text{(A4)}$$

$$\mathrm{Re}F^+(\omega,k^2)=\omega P^{\pm}(\omega,k^2)-\frac{2}{\pi}P\int_m^\infty d\omega'\,\mathrm{Im}B^+(\omega',k^2)$$
$$+\frac{2}{\pi}P\int_m^\infty \frac{\omega'[\mathrm{Im}A^+(\omega',k^2)-\omega' B^+(\omega',k^2)]}{\omega'^2-\omega^2}d\omega'\,. \qquad \text{(A5)}$$

Here,

$$P_{\pm}(\omega,k^2)=P^+(\omega,k^2)\pm P^-(\omega,k^2) \qquad \text{(A6)}$$

gives the contribution of the one-particle terms. On the right-hand side of (A4) and (A5), the contributions of the integrals in the unphysical region $m_\Lambda$ to $m$ are approximated by those of the resonance poles $Y_1^*$ and $Y_0^*$ which are included in $P^{\pm}(\omega,k^2)$. To eliminate the divergent part of (A5), we use a subtraction at $\omega=m$, and for the purpose of symmetry in the final expressions we carry out the same subtraction in (A4); then we combine the results in (A1) and find

$$\begin{aligned}\mathrm{Re}F_{\pm}(\omega,k^2)=&\tfrac{1}{2}(1\pm\omega/m)[F_+(m,k^2)-mP_+(m,k^2)]\\ &+\omega P_{\pm}(\omega,k)+\tfrac{1}{2}(1\mp\omega/m)\\ &\times[F_-(m,k^2)-mP_-(m,k^2)]+D_{\pm}(\omega,k^2)\,,\end{aligned} \qquad \text{(A7)}$$

where $F_{\pm}$ denote the $K^{\pm}p$ scattering amplitudes and

$$D_{\pm}(\omega,k^2)=\frac{2(\omega^2-m^2)}{\pi}P\int_m^\infty \frac{\omega\,\mathrm{Im}[A^-(\omega')-\omega' B^-(\omega')]\pm\omega'\,\mathrm{Im}[A^+(\omega')-\omega' B^+(\omega')]}{(\omega'^2-\omega^2)(\omega'^2-m^2)}d\omega'\,. \qquad \text{(A8)}$$

The functions $P_{\pm}(\omega,k^2)$, given by (A6), are proportional to the term corresponding to the one-particle intermediate states, denoted by $W^P(\omega,k^2)$, which is included in (7g) of the text; thus we define

$$P_{\pm}(\omega,k^2)\equiv-(8\pi^2c^2|\omega|)^{-1}\,\mathrm{Re}W_{\pm}{}^P(\omega,k^2)\,. \qquad \text{(A9)}$$

Since our $T$ matrix, (1a) of the text, is defined to satisfy $S_{ba}=\delta_{ba}+T_{ba}$, then (A9) means that $T_{ba}$, given by (7d), is related to the scattering amplitude $F_{ba}$ in (A7) by

$$F_{ba}=i\omega T_{ba}\,. \qquad \text{(A10)}$$

From (A10) and Eqs. (7b) to (7g) we see that

$$\mathrm{Im}F_{\pm}(\omega,k^2)=-(8\pi^2c^2|\omega|)^{-1}\,\mathrm{Im}W_{\pm}(\omega,k^2)\,, \qquad \text{(A11)}$$

where $T_{\pm}$ denote the $T$ matrices of the $K^{\pm}p$ systems. Making use of (7a) and (10c) of the text, and combining Eqs. (A7) to (A11), we find

$$\begin{aligned}T_{\pm}(\omega,k^2)=&-(i8\pi^2c^2|\omega|)^{-1}\{W^{00}(k^2)+W^0(m,k^2)\\ &+(\omega/m)W_{\pm}{}^1(m,k^2)-W_{\pm}{}^P(\omega,k^2)\\ &-\tfrac{1}{2}[W_+{}^H(m,k^2)+W_-{}^H(m,k^2)]\mp(|\omega|/2m)\\ &\times[W_+{}^H(m,k^2)-W_-{}^H(m,k^2)]\}+\omega^{-1}D_{\pm}(\omega,k^2)\,,\end{aligned} \qquad \text{(A12)}$$

where $W^H$, given by (7g), corresponds to more than one-particle intermediate state. We shall need only the

[21] The $Kp$ dispersion relations are given by several authors, e.g., C. Goebel, Phys. Rev. **110**, 572 (1958); D. Amati and B. Vitale, Nuovo Cimento **7**, 190 (1958). See also P. T. Matthews and A. Salam, Phys. Rev. **110**, 565 (1958); **110**, 569 (1958).

dispersive part of (A12); thus, we shall evaluate the real part of $W^P(\omega,k^2)$ using (7g). We neglect the contribution of the $t$ channel. Then we follow the standard method and assume that the currents $J$ and $J^\dagger$ in Eqs. (7) are, respectively, rising and lowering operators of unit strangeness of a state on which they operate. Considering all possible one-particle intermediate states of the $s$ and $u$ channels of the $K^\pm p$ scattering, and writing $\theta(Z)=\frac{1}{2}[\epsilon(Z)+1]$, Eq. (7g) gives

$$W_\pm{}^P=W^{P+}\pm W^{P-},$$

$$\mathrm{Re}W^{P\pm}\omega,k^2$$
$$=\tfrac{1}{2}\sum_n \frac{\langle p'|k_\mu' J_b{}^\mu(0)|n\rangle\langle n|k_\nu J_a{}^{\nu\dagger}(0)|p\rangle}{\omega_{n_+}+\omega}\delta^3(\mathbf{p}_n-\mathbf{p}-\mathbf{k})$$
$$\pm\frac{\langle p|k_\mu' J_b{}^\mu(0)|n\rangle\langle n|k_\nu J_a{}^{\nu\dagger}(0)|p\rangle}{\omega-\omega_{n_-}}\delta^3(\mathbf{p}_n-\mathbf{p}'+\mathbf{k}). \quad \text{(A13)}$$

Here, the four-momentum $p_n$, the mass $M_n$, and

$$\omega_{n_\pm}=[M_n{}^2+(\mathbf{p}\pm\mathbf{k})^2]^{1/2}-p_0 \quad \text{(A14)}$$

are associated with the intermediate states $|n\rangle$ which are $\Lambda$, $\Sigma$, $Y_0{}^*$, and $Y_1{}^*$ in the present case. For the summation over spin, in (A12), we use the projection operator

$$P=(\not{p}+\not{k}+M_n)/2M$$

for $\Lambda$, $\Sigma$, and $Y_0{}^*$ and

$$P_{\mu\nu}=\frac{\not{p}+\not{k}+M_{Y_1{}^*}}{2M_{Y_1{}^*}}$$
$$\times\left[g_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu-\frac{2}{3M_{Y_1{}^*}{}^2}p^\mu p^\nu+\frac{1}{3M_{Y_1{}^*}}(p^\mu\gamma_\nu-p^\nu\gamma_\mu)\right]$$

for $Y_1{}^*$. Considering the spins and parities of the above states, we choose

$$\langle p_i|k_\mu J^\mu(0)|p\rangle=g_A(i)\bar{u}_i k_\mu\gamma^\mu\gamma_\mu\gamma_5 u_p,$$

in which $i$ represents $\Lambda$ and $\Sigma$, and

$$\langle p_{Y_0{}^*}|k_\mu J^\mu(0)|p\rangle=g_A(Y_0{}^*)_{Y_0{}^*}k_\mu\gamma^\mu u_p,$$
$$\langle p_{Y_1{}^*}|k_\mu J^\mu(0)|p\rangle=g_A(Y_1{}^*)\bar{u}_{Y_1{}^*}{}^\mu k_\mu u_p,$$

for $Y_0{}^*$ and $Y_1{}^*$, respectively. Here, $g_A$ refers to the axial-vector coupling constants, and $u$ refers to a four-component spinor. Using the above projection operators and these current matrix elements in (A12),

$$\mathrm{Re}W_1{}^{P\pm}(\omega,k^2)=\sum_{i=\Lambda,\Sigma}\frac{g_A(i)}{2M_i}$$
$$\times\left\{\left(\frac{\Delta_i{}^++\omega}{\omega+\omega_i}\pm\frac{\Delta_i{}^+-\omega}{\omega_i-\omega}\right)k^2-2M\omega^2\left(\frac{1}{\omega_i-\omega}\pm\frac{1}{\omega+\omega_i}\right)\right\}$$
$$+\frac{g_A{}^2(Y_0{}^*)}{2M_{Y_0{}^*}}\left\{\left(\frac{\Delta_{Y_0{}^*}{}^--\omega}{\omega_{Y_0{}^*}+\omega}\pm\frac{\Delta_{Y_0{}^*}{}^-+\omega}{\omega_{Y_0{}^*}-\omega}\right)k^2+2M\omega^2\right.$$
$$\left.\times\left(\frac{1}{\omega+\omega_{Y_0{}^*}}\pm\frac{1}{\omega_{Y_0{}^*}-\omega}\right)\right\}+\frac{g_A{}^2(Y_1{}^*)}{3M_{Y_1{}^*}}$$
$$\times\left[k^2-\left(\frac{M\omega}{M_{Y_1{}^*}}\right)^2\right]\left[\frac{\Delta_{Y_1}{}^+-\omega}{\omega+\omega_{Y_1{}^*}}\pm\frac{\Delta_{Y_1{}^*}{}^++\omega}{\omega_{Y_1{}^*}-\omega}\right], \quad \text{(A15)}$$

where $\Delta_n{}^\pm=M_n\pm M$ for a state $|n\rangle$.

Note that (A15) for $K^-p$ scattering contains the contributions of not only the $s$-wave state $Y_0{}^*$, but also the $p$-wave states $\Lambda$, $\Sigma$, and $Y_1{}^*$ through the $s$ channel.