Momentum Dependence of Diffraction Slopes in Meson-Nucleon Scattering

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The slopes $B(k)$ of the exponential forward peak for $\pi^{\pm}p$ and $K^{\pm}p$ elastic scattering have been surveyed in the region $0.3 \lesssim P_{\text{lab}} \lesssim 20.0$ GeV/c. Plots of the momentum dependence of $B(k)$ reveal striking enhancements at momenta corresponding to the location of known high-spin, high-elasticity resonances. There are indications that the background behavior of $B(k)$, when considered over such a wide momentum interval, might be smoothly rising in all cases; thus, the forward peak associated with *background* diffraction effects would appear to shrink not only in $K^+p \to K^+p$ interactions, where resonance formation is weak (or absent), but also in $\pi^{\pm}p \to \pi^{\pm}p$ and $K^-p \to K^-p$ interactions, where many prominent resonant states are formed. An explanation for the behavior of $B(k)$ is presented, based on the properties of the logarithmic derivative of the differential cross section in the forward direction. As a quantitative illustration, $B(k)_{K^-}$ was derived in terms of a simple phenomenological model, which involves a linear superposition of diffractive-like and resonant amplitudes. In this way a fit to the $B(k)_{K^-}$ and K^-p total-cross-section data could be obtained throughout the entire momentum region. The information which can be derived from the experimental study of $B(k)$ is discussed; this has bearing on the determination of resonance properties and on the behavior of the spin-flip amplitude.

I. INTRODUCTION

IN a recent phenomenological study[1] of elastic meson-nucleon interactions, we presented a plot of the momentum dependence of the slope of the diffraction-like peak, $B(k)$, observed for the process $K^-p \to K^-p$. It was, in particular, pointed out there that diffraction phenomena play an important role in the scattering process even at relatively low energy ($\sim$1 GeV) where the formation of hyperon resonances is prominent. The interpretation of the differential cross sections for $K^-p \to K^-p$ in the 1-GeV region, in terms of the superposition of diffractive and resonant amplitudes, provided quantitative evidence for the above speculation.

As remarked in Ref. 1, $B(k)$ showed striking enhancements at K^- laboratory momenta corresponding to the formation of known resonant states. The first indication of a possible structure in $B(k)$ for $\pi^{\pm}p \to \pi^{\pm}p$ scattering was pointed out by Damouth *et al.*[2] In Sec. II of this paper we present a comprehensive study of $B(k)$ for the processes $\pi^-p \to \pi^-p$, $\pi^+p \to \pi^+p$, $K^+p \to K^+p$, together with more recent data for $K^-p \to K^-p$. In Sec. III we discuss, on the basis of the model of Ref. 1, a quantitative explanation for the behavior of $B(k)$ for $K^-p \to K^-p$, $B(k)_{K^-}$, together with a fit to the total K^-p cross sections from 0.3 to $\sim$16 GeV/c. In principle, a similar description should also be valid for $B(k)_{\pi^{\pm}}$ and $B(k)_{K^+}$, though we do not attempt a fit to the actual data at this time. The content of information in $B(k)$ is emphasized in Sec. IV.

II. MOMENTUM DEPENDENCE OF $B(k)$

A. Determination of $B(k)$

$B(k)$ was determined by fitting the forward differential cross sections with the form

$$d\sigma/dt \underset{t\to 0}{\approx} A(k)e^{B(k)t} \tag{1}$$

through the method of least squares. In this expression, t is the invariant four-momentum transfer squared and $A(k)$ and $B(k)$ are constant parameters which were determined by the fit at each laboratory momentum considered. For all scattering processes, the determination of $B(k)$ was performed over the region in $|t|$ where $d\sigma/dt$ could be approximated by Eq. (1). After a preliminary selection of the t intervals on the basis of visual inspection of the angular distributions, it was required that an acceptable χ^2 be obtained in the fit. Whenever this condition was not met, the interval was reduced until the χ^2 became acceptable (including a minimum of three data points). This procedure defined intervals $0 \leq |t| \leq |t|_{\text{cutoff}}$ with $|t|_{\text{cutoff}}$ smoothly increasing from a typical value of $\sim$0.1 (GeV/c)^2 at a laboratory momentum of 0.3 GeV/c to a constant value of $\sim$0.45 (GeV/c)^2 above 1.5 GeV/c. The discontinuities in $|t|_{\text{cutoff}}$, reflecting mostly the finite bin-size of the experimental angular distributions, were $\sim\pm 0.05$ (GeV/c)^2. As we see in Sec. B, there is considerable structure in the plots of $B(k)$. We shall assume that the slow variation in $|t|_{\text{cutoff}}$ is not responsible for the more pronounced structure in these plots.

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† Work supported in part by USAF-EOAR 68-0015.
[1] T. Lasinski, R. Levi Setti, and E. Predazzi, Phys. Rev. **163**, 1792 (1967).
[2] D. E. Damouth, L. W. Jones, and M. L. Perl, Phys. Rev. Letters **11**, 287 (1963).

Plots of $B(k)$ for $\pi^-p\to\pi^-p$ and $\pi^+p\to\pi^+p$ are shown in Figs. 1 and 2, respectively. Similar plots of $B(k)$ for $K^-p\to K^-p$ and $K^+p\to K^+p$ as a function of laboratory momentum are presented in Fig. 3. We refer to the Appendix for a tabulation of the values of $B(k)$ and for references to the original experimental data from which these values were obtained. For completeness, the determination of $B(k)$ was extended to low momenta, in a region most likely below the onset of diffraction phenomena.

We point out, first, some of the most noticeable features in the above plots, which emerge in spite of an excessive scatter of the points, intrinsic perhaps in a survey of unrelated experiments. We then attempt to give a quantitative interpretation of these observations in terms of phenomena which may or may not be common to all processes considered.

B. Qualitative Discussion of the Behavior of $B(k)$

There are three prominent features of Fig. 1 which we would like to point out. In the first place, there is a considerable amount of structure in $B(k)$ associated with some of the more pronounced resonant effects, in both $I=\frac{1}{2}$ and $\frac{3}{2}$ isospin states. The most significant peak in $B(k)$ occurs at $P_{\pi^-}\approx 1$ GeV/c, probably due to the presence of two states of $J^P=\frac{5}{2}^-$ and $\frac{5}{2}^+$, viz. $N(1670)$ and $N(1688)$, respectively, both of sizable elasticity. Other enhancements can be noticed at $P_{\pi^-}\approx 0.7$ GeV/c, corresponding to the location of $N(1525)$ ($J^P=\frac{3}{2}^-$), at $\sim$1.65 GeV/c, where $\Delta(1920)$ is formed, and finally, at $\sim$2.1 GeV/c, where $N(2190)$ ($J^P=\frac{7}{2}^-$) is known. As can be noticed, these four enhancements occur in conjunction with the formation of the known nucleon resonances of higher spin and

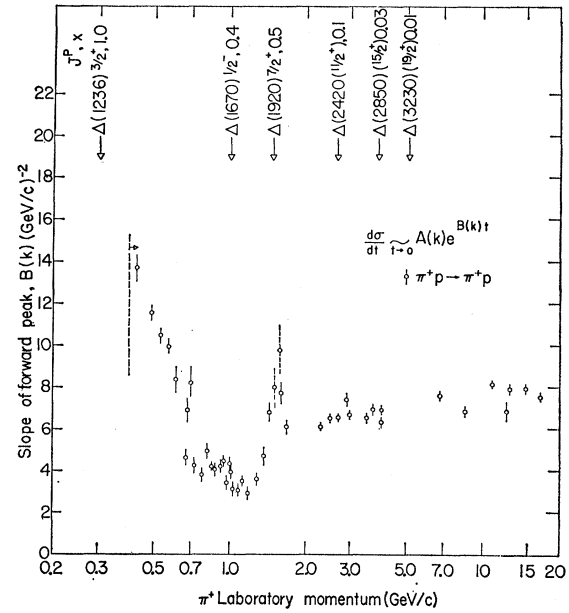

FIG. 2. $B(k)$ versus laboratory momentum for $\pi^+p\to\pi^+p$. (See the Appendix for references.)

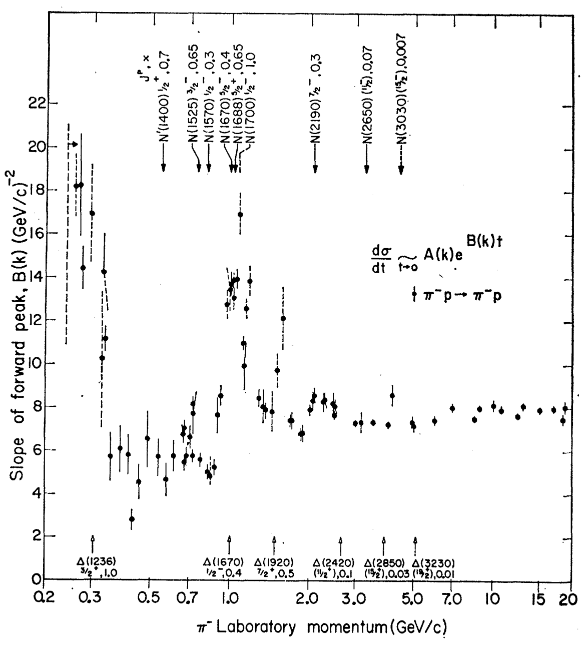

FIG. 1. Momentum dependence of the slope $B(k)$ of the forward peak in $d\sigma/dt$ for $\pi^-p\to\pi^-p$. The $B(k)$ values have been obtained from least-squares fits to data collected from the literature (see the Appendix). Points with dashed error bars resulted from fits to only three experimental $d\sigma/dt$ data points.

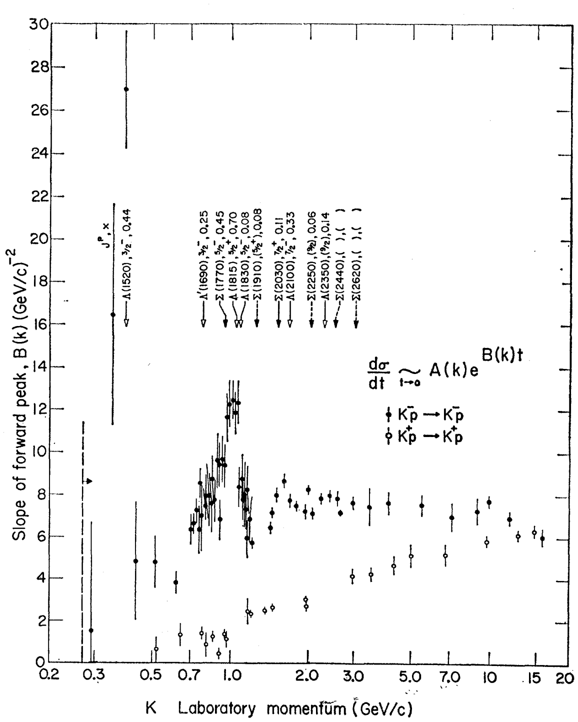

FIG. 3. $B(k)$ versus laboratory momentum for $K^-p\to K^-p$ and $K^+p\to K^+p$. (See the Appendix for references.)

elasticity beyond $P_{\pi^-}\approx 0.5$ GeV/c. $B(k)$ shows, in addition, a rapid rise at lower momenta, toward the region of $\Delta(1236)$ ($J^P=\frac{3}{2}^+$); we are inclined to regard this effect as due primarily to that resonance, rather than to other phenomena.

Another feature in Fig. 1 which might be of significance is the possible presence of at least two dips, one at ~ 0.85 GeV/c, the other at ~ 1.9 GeV/c. The presence of dips of this nature in $B(k)$ is given a simple interpretation in the following.

The third point to make about Fig. 1 concerns the background behavior of $B(k)$ as a function of π^- laboratory momentum. Aside from the structure noted above, $B(k)$ gradually increases fron a value of ~ 5 (GeV/c)$^{-2}$ at $P_{\pi^-}\approx 0.5$ GeV/c toward an asymptotic value of ~ 8 (GeV/c)$^{-2}$, reached in the region of ~ 7 GeV/c.

Figure 2, representing the behavior of $B(k)$ for $\pi^+p\rightarrow\pi^+p$, shows essentially similar features to Fig. 1, although with considerably less structure. This is clearly to be attributed to the smaller number of resonant states contributing to this pure $I=\frac{3}{2}$ channel. Once again a clear peak is associated with $\Delta(1920)$ and a small enhancement is present at $P_{\pi^-}\approx 0.95$ GeV/c. As will be shown later, $l=0$ resonances are not expected to contribute enhancements to $B(k)$. Thus, we do not associate this effect with $\Delta(1670)$ ($J^P=\frac{1}{2}^-$). It is not inconceivable that this small bump may be due to the P and D states in this region recently proposed by Donnachie *et al.*[3] There might be a dip at ~ 1.2 GeV/c. The background $B(k)_{\pi^+}$ increases from ~ 4 (GeV/c)$^{-2}$ at ~ 0.7 GeV/c to an asymptotic value of ~ 8 (GeV/c)$^{-2}$ as for $B(k)_{\pi^-}$.

Figure 3 contains a plot of $B(k)$ for $K^-p\rightarrow K^-p$ and $K^+p\rightarrow K^+p$. The plot for $K^-p\rightarrow K^-p$, already presented and discussed in Ref. 1, is based here on data of better statistical significance in the region[4] 0.8–1.2 GeV/c and is extended to lower momenta. In addition, all values of $B(k)$ have been redetermined more critically. As previously pointed out, $B(k)_{K^-}$ peaks very prominently at ~ 1 GeV/c where the two $J^P=\frac{5}{2}^-$ and $\frac{5}{2}^+$ states, $\Sigma(1760)$ and $\Lambda(1820)$, are formed. In addition, there is a peak at ~ 1.6 GeV/c, where $\Sigma(2030)$ and $\Lambda(2100)$, both of $J=\frac{7}{2}$ and opposite parity, are known and possibly some other structure at higher momenta. There is a peak corresponding to $Y_0^*(1520)$ ($J^P=\frac{3}{2}^-$), related as for $\Delta(1236)$, to the resonance angular distribution. Minima in $B(k)_{K^-}$ which could be interpreted as dips may exist at ~ 0.6 and ~ 1.3 GeV/c. If one regards the plot of $B(k)_{K^-}$ as the superposition of peaks to a smooth background [as in the case of $B(k)_{\pi^\pm}$], then the background increases from a value of ~ 4 (BeV/c)$^{-2}$ at ~ 0.5 GeV/c to an asymptotic value of ~ 8 (GeV/c)$^{-2}$, which seems to be reached already in the region of ~ 3 GeV/c or even earlier.

In great contrast with the structure observed in $B(k)_{\pi^\pm}$ and $B(k)_{K^-}$, $B(k)_{K^+}$ in Fig. 3 exhibits a smooth behavior, increasing slowly from zero at ~ 0.5 GeV/c toward a value in the region 6–8 (GeV/c)$^{-2}$ at ~ 20 GeV/c. It is tempting to associate this different behavior with the known absence of high-spin high-elasticity resonances in the K^+p system. $B(k)_{K^+}$ is then most likely representative of a purely diffractive contribution. It should be remarked that, even so, the existence of K^+p resonances cannot be ruled out on the basis of this observation. Since, in fact, known resonances of low spin and elasticity do not reveal themselves as peaks in $B(k)_{\pi^-}$, $B(k)_{\pi^+}$, $B(k)_{K^-}$, it cannot be excluded that states of similar properties indeed exist also in K^+p.[5]

III. MODEL FOR THE BEHAVIOR OF $B(k)$

A. Relation between $B(k)$ and the Elastic Scattering Amplitudes

We wish to elaborate now on a quantitative interpretation for the presence of peaks and dips in $B(k)$ corresponding to the formation of resonances on the basis of the model of Ref. 1. Within the context of the latter, the differential cross section is given by

$$d\sigma/dt=(\pi/k^2)(d\sigma/d\Omega)=(\pi/k^2)[|g|^2+|h|^2], \quad (2)$$

where

$$g=g^D+g^R,$$
$$h=(h^D+h^R)\sin\theta_{\mathrm{c.m.}}. \quad (3)$$

Here the superscripts D and R refer to diffractive and resonant contributions, respectively. More explicitly, it was shown in Ref. 1 that the background contribution could indeed be attributed to diffraction scattering on the basis of several observations. Among these is the fact that the forward scattering amplitude is predominantly imaginary and that the forward elastic peak falls off exponentially even at relatively low momenta. Accordingly, the following parametrizations were used:

$$g^D=G(k)e^{bt}, \quad h^D=H(k)e^{b't}, \quad (4)$$

b,b' const; $G(k)$, $H(k)$ functions of k only

$$g^R=\frac{1}{k}\sum_{l=l_R}\{(l+1)a_{l^+}{}^R+la_{l^-}{}^R\}P_l(x),$$
$$h^R=\frac{1}{k}\sum_{l=l_R}(a_{l^+}{}^R-a_{l^-}{}^R)\frac{dP_l(x)}{dx}, \quad (5)$$

[3] A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[4] N. M. Gelfand, D. Harmsen, R. Levi Setti, E. Predazzi, M. Raymund, J. Doede, and W. Männer, Phys. Rev. Letters **17**, 1224 (1966), and improved data from the same group.

[5] For evidence concerning K^+p resonances, see R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters **17**, 102 (1966); R. J. Abrams, R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, and D. N. Michael, *ibid.* **19**, 259 (1967).

where the sums are over resonating partial waves only and $x=\cos\theta_{\mathrm{c.m.}}$. The resonant amplitudes are, as usual, expressed in terms of a Breit-Wigner formulation,

$$a^R=\frac{1}{2}\frac{\Gamma_{\mathrm{el}}(k)/2}{(E_R-E)^2+\Gamma_{\mathrm{tot}}^2(k)/4}[(E_R-E)+\tfrac{1}{2}i\Gamma_{\mathrm{tot}}(k)], \quad (6)$$

where $\Gamma(k)$ includes centrifugal barrier effects.[6]

By comparing linear expansions of Eqs. (1) and (2) we make the identification

$$B(k)\approx\left[\frac{d}{dt}\left(\frac{d\sigma}{dt}\right)\Big/\left(\frac{d\sigma}{dt}\right)\right]_{t=0}. \quad (7)$$

This relation, expressing $B(k)$ as the logarithmic derivative of $d\sigma/dt$ at $t=0$, rests on the assumption that linear approximations of Eqs. (1) and (2) are valid. As a result, Eq. (7) is expected to hold only over an interval in $|t|$ such that $|t|\lesssim 1/B(k)$ [from Eq. (1)] and $|t|\lesssim 4k^2/l(l+1)$ [from Eq. (2)], where l refers to the highest-order partial wave of appreciable amplitude. It is to be noted that Eq. (7) does not depend upon a specific parametrization for the amplitudes in Eq. (2). If we now evaluate Eq. (7) from Eqs. (2)–(5), we obtain

$$\begin{aligned}B(k)=&\frac{1}{[k^2(d\sigma/d\Omega)_{t=0}]}\\&\times[(\mathrm{Re}g^D+\mathrm{Re}g^R)(2bk^2\,\mathrm{Re}g^D+\mathrm{Re}dg^R/dx)\\&+(\mathrm{Im}g^D+\mathrm{Im}g^R)(2bk^2\,\mathrm{Im}g^D+\mathrm{Im}dg^R/dx)\\&-(\mathrm{Re}h^D+\mathrm{Re}h^R)^2-(\mathrm{Im}h^D+\mathrm{Im}h^R)^2], \quad (8)\end{aligned}$$

where g^D, h^D, g^R, h^R, and dg^R/dx are computed at $t=0$ $(x=1)$.[7]

The presence of peaks in $B(k)$ corresponding to the formation of higher spin resonances is readily understood on the basis of Eq. (8). We see, in fact, that $B(k)$ is proportional to the real and imaginary part of $dg^R/dx|_{x=1}$, which in turn contains terms in $dP_l(x)/dx|_{x=1}=\frac{1}{2}l(l+1)$. It is precisely this latter factor, rapidly increasing with l, which is primarily responsible for the enhancements in $B(k)$ when a^R goes through resonance. As a consequence, $l=0$ resonances will not contribute enhancements in $B(k)$. In fact, at lower momenta $l=0$ resonances could give significant dips, although calculations with a reasonable background indicate that they cause only moderate depressions in $B(k)$.

An unusual, and possibly informative, feature of Eq. (8) is the presence of terms in h^D and h^R at $t=0$. Thus, information on the spin-flip amplitude is contained in $B(k)$ due to the fact that dh/dx is actually a maximum at $t=0$. Dips in $B(k)$ are likely to occur, as can be seen from Eq. (8), when these terms become important relative to the spin-nonflip contributions. This situation may arise (a) in a momentum interval devoid of resonances, or (b) when the interference term

$$\mathrm{Re}h^R\propto\sum_{l=l_R}(\mathrm{Re}a_{l^+}{}^R-\mathrm{Re}a_{l^-}{}^R)\frac{dP_l}{dx}\bigg|_{x=1}$$

adds a significant contribution to $\mathrm{Re}h^D$. Clearly, the latter situation depends, for example, on the relative parity of two adjacent resonances, and in principle, if the sign of h^D were known, the observation of dips in $B(k)$ could be related to the parity of these resonances.

B. Fit to $B(k)_{K^-}$ and Total K^-p Cross Sections

A quantitative illustration of the above qualitative discussion is provided by a calculation based on Eq. (8), to apply to $K^-p\rightarrow K^-p$. It must be remarked at this point that the values of $|t|_{\mathrm{cutoff}}$ used in Sec. II A were generally larger than the limits required for Eq. (7) to be valid. In most cases the available data simply did not permit us to use a value of $|t|_{\mathrm{cutoff}}$ consistent with these limits. Despite this difficulty, however, we would like to indicate a possible, though most likely incomplete, explanation for the properties of $B(k)$.

For this limited purpose, we have chosen a simplified momentum-dependent parametrization for the diffractive amplitudes g^D and h^D:

$$\begin{aligned}&\mathrm{Im}g^D=(k/4\pi)(\sigma_\infty+G/k^\alpha)e^{b_\infty t}, \quad \mathrm{Re}g^D=0,\\&\mathrm{Re}h^D=Hk^\beta e^{b't}, \quad \mathrm{Im}h^D=0.\end{aligned} \quad (9)$$

Here σ_∞ is a constant corresponding to the asymptotic value of the total K^-p cross section, b_∞ is $\frac{1}{2}B(k)$ as $k\rightarrow\infty$, and G, H, α and β are free parameters. Note that b' does not appear in Eq. (8) and cannot, therefore,

TABLE I. Resonant parameters adopted in fitting $B(k)_{K^-}$ and $\sigma_{\mathrm{tot}}(K^-p)$.

Mass (GeV/c^2)	Total width (GeV)	Elasticity	J^P, I	Reference
1.519	0.020	0.450	$\frac{3}{2}^-$, 0	a
1.665	0.030	0.025	$\frac{3}{2}^-$, 1	b
1.695	0.040	0.230	$\frac{3}{2}^-$, 0	b
1.768	0.110	0.380	$\frac{5}{2}^-$, 1	b
1.819	0.075	0.700	$\frac{5}{2}^+$, 0	b
1.827	0.076	0.080	$\frac{5}{2}^-$, 0	a
1.870	0.040	0.100	$\frac{7}{2}^+$, 0	b
1.905	0.060	0.060	$\frac{5}{2}^+$, 1	b
2.020	0.130	0.140	$\frac{7}{2}^+$, 1	b
2.100	0.140	0.300	$\frac{7}{2}^-$, 0	b
2.250	0.230	0.094	$(\frac{9}{2}^-)$[c], 1	b
2.340	0.140	0.120	$(\frac{9}{2}^+)$[c], 0	b
2.450	0.140	0.042	$(\frac{11}{2}^+)$[c], 1	a
2.595	0.140	0.042	$(\frac{11}{2}^-)$[c], 1	a

[a] A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. **40**, 77 (1968). J^P, I information was also taken from this reference.
[b] See Ref. 9, Bugg *et al.*
[c] These are tentative J^P assignments.

[6] See, for example, R. D. Tripp, in *Proceedings of the International School of Physics "Enrico Fermi"* (Academic Press Inc., New York, 1966), Course 33, p. 83.

[7] From the fact that f^R and h^R decrease with increasing k, faster than $1/k$, and under the assumption that $\mathrm{Re}g^D$ and h^D fall off faster than $\mathrm{Im}g^D$, we find from Eq. (8) that $B(k)\rightarrow_{k\rightarrow\infty}2b$.

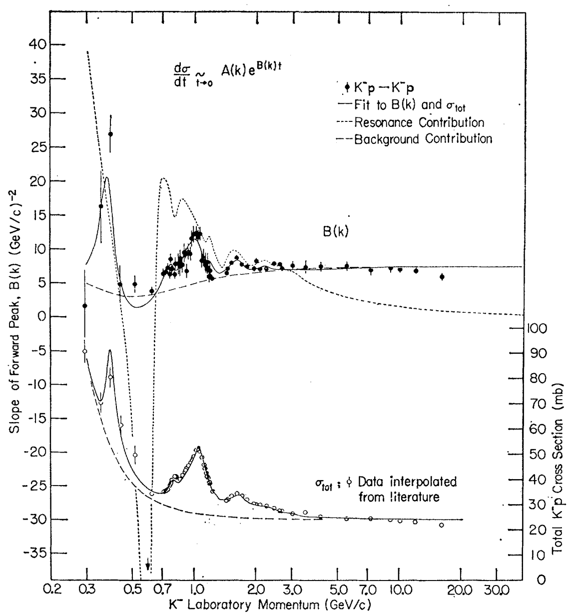

FIG. 4. $B(k)$ and σ_{tot} for $K^-p \to K^-p$ compared with a fit to the data (full curve) in terms of the interference between a diffractive background and resonant amplitudes (see Sec. III). The fit to σ_{tot} included only data points (Ref. 9) at momenta corresponding to those where $B(k)$ was determined.

be determined from a fit to $B(k)$. At $t=0$, a parametrization similar to that for $(4\pi/k)\mathrm{Im}g^D$ was used by Foley *et al.*[8] to explain the high-energy behavior of the total cross sections. The term G/k^α $(\alpha>0)$ describes the increase observed in $\sigma_{tot}(K^-p)$ at low k, and may be regarded as a crude description of below-threshold effects.

Included in the resonant amplitudes g^R and h^R were the resonances indicated in Fig. 3 with the parameters listed in Table I. The full curves in Fig. 4 represent a fit to $B(k)$ and σ_{tot},[9] obtained for the following diffraction parameters: $\sigma_\infty=23.94$ mb, $G=1.47$,[10] $H=1.76$ [10]; $b_\infty=3.809$ $(\mathrm{GeV}/c)^{-2}$, $\alpha=3.151$, and $\beta=0.836$. The values of σ_∞ and b_∞ (determined by the fit) are close to those one would reasonably expect in the asymptotic limit on the basis of what is known up to $\sim$20 GeV/c. As can be seen, the gross features in the structure of $B(k)$ are quite well reproduced. In spite of this over-all visual agreement with the data, the χ^2 of the fit remains unacceptably high, 343 for 113 degrees of freedom. The partial χ^2's are 107 from σ_{tot}, 236 from $B(k)$; 173 points of the latter are contributed by 10 data points scattered throughout the entire momentum range. We are inclined to attribute this fact to inconsistencies in the error assignments within the data, collected from many different experiments, rather than to a failure of the theoretical model in fitting the data.

The dotted curve in Fig. 4 corresponds to the contribution of the resonances (without background) to $B(k)$. The pure background contribution is also indicated as a dashed line. It can be noticed that, if the background were absent, $B(k)$ would exhibit very pronounced peaks and dips, of much greater amplitude than observed. It is the interference with the background which dampens these sharp oscillations to yield what is observed. A comment is in order regarding the large negative dip in $B(k)$ for the pure resonance contribution, occurring at $P_{K^-}\approx 0.55$ GeV/c. It corresponds, as we have verified, to an actual dipping of $d\sigma/d\Omega$, calculated only with resonance contributions, in the forward direction.

Also to be noticed is the fact that the chosen parametrization for g^D and h^D yields in a natural way a smoothly rising background in $B(k)$, above $\sim$0.5 GeV/c, as speculated in Sec. II B. Furthermore, we have found that the above diffraction parameters [with $b'\sim 11$ $(\mathrm{GeV}/c)^{-2}$] and set of resonances predict differential cross sections and polarizations, near $t=0$, in reasonable agreement with the data[4,11] for K^- momenta from 0.8–2.4 GeV/c. Clearly, however, the background parametrization of Eq. (9) is most likely inadequate to describe the behavior of $d\sigma/dt$ and $P(\theta)$ for all t values; Eq. (9) is, on the other hand, to be regarded as a limiting form of a more accurate parametrization.

Perhaps the most unsatisfactory representation of the data in Fig. 4 is in the region 1.150–1.45 GeV/c. The fitted curve, based on the existence of a $\Sigma(1910)$ of $J^P=\frac{5}{2}^+$, $\Gamma=0.06$ GeV, and $x=0.06$, clearly does not account for a pronounced dip indicated by the data despite the gap between 1.22 and 1.43 GeV/c. The fit

[8] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love' S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys' Rev. Letters **19**, 330 (1967).

[9] The fit included values of σ_{tot} at the same K^- laboratory momenta where the values of $B(k)$ were obtained. Such σ_{tot} values were either taken directly or interpolated from the following references: R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters **16**, 1228 (1966); J. D. Davies, J. D. Dowell, P. M. Hattersley, R. J. Homer, A. A. Carter, K. F. Riley, R. J. Tapper, D. V. Bugg, R. S. Gilmore, K. M. Knight, D. C. Salter, G. H. Stafford, and E. J. N. Wilson, *ibid.* **18**, 62 (1967). The numerical values for the total cross sections of the above references were taken from a compliliation by G. Lynch, Lawrence Radiation Laboratory, Memo 600(1966) (unpublished); M. Watson, M. Ferro-Luzzi, and R. D. Tripp, Phys. Rev. **131**, 2248 (1963); W. F. Baker, R. L. Cool, E. W. Jenkins, T. F. Kycia, R. H. Phillips, and A. L. Read, *ibid.* **129**, 2285 (1963); V. Cook, B. Cork, T. F. Hoang, D. Keefe, L. T. Kerth, W. A. Wenzel, and T. F. Zipf, *ibid.* **123**, 320 (1961); G. von Dardel, D. H. Frisch, R. Mermod, R. H. Milburn, P. A. Piroué, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters **5**, 333 (1960); A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, Phys. Rev. **132**, 2721 (1963); D. V. Bugg, R. S. Gilmore, K. M. Knight, D. C. Slater, G. H. Stafford, E. J. N. Wilson, J. D. Davies, J. D. Dowell, P. M. Hattersley, R. J. Homer, A. W. O'Dell, A. A. Carter, P. J. Tapper, and K. F. Riley, *ibid.* **168**, 1466 (1968); R. J. Abrams, R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, and D. N. Michael, Phys. Rev. Letters **19**, 678 (1967).

[10] In our calculations, the c.m. momentum k was in units of $mb^{-1/2}$. Thus, G and H are in units of $\mathrm{mb}^{1+\alpha}$ and $\mathrm{mb}^{1/2-\beta}$, respectively.

[11] C. Daum, F. Erné, J. P. Lagnaux, J. C. Sens, M. Steuer, F. Udo, G. Plaut and S. Andersen, in *Proceedings of the Heidelberg International Conference on Elementary Particles* (North-Holland Publishing Co., Amsterdam, 1967), p. 117.

would improve if $\Sigma(1910)$ were assigned $J^P=\frac{1}{2}^-$; it is not inconceivable that, if more complete data on $B(k)$ in this region were available, one could by this method determine J^P for this resonance. As seen in Fig. 4, the total K^-p cross sections are quite well described over a vast momentum region. Within the context of the present determination of $B(k)$, however, the fit to σ_{tot} was included just to provide a constraint on Img. A better description of σ_{tot} could clearly be obtained by including in the fit all available data[9] and by varying the resonance parameters.

IV. DISCUSSION AND CONCLUSIONS

The present study has emphasized several known features and brought to light new aspects of the momentum dependence of $B(k)$ which seem to be common to all meson-nucleon elastic scattering processes. These can be summarized as follows:

(a) At the highest momenta investigated, in the region 15–20 GeV/c, $B(k)$ for $\pi^\pm p \to \pi^\pm p$, $K^-p \to K^-p$, and very likely also $K^+p \to K^+p$ is consistent with a value of $\sim$7–8 (GeV/c)$^{-2}$.

(b) Aside from the observed structure, $B(k)$ appears in all cases to increase toward the asymptotic value from a much lower value at low momenta. This feature, very prominent throughout the entire momentum region in $K^+p \to K^+p$, seems present, at least below $\sim$3 GeV/c, also in $K^-p \to K^-p$ and $\pi^\pm p \to \pi^\pm p$.

(c) Whenever the meson-nucleon collision process involves the formation of elastic resonant states (with $l\neq 0$), an associated structure of enhancements, and occasionally dips, appears in $B(k)$. This is spectacularly evident in $B(k)_{K^-}$, $B(k)_{\pi^-}$, and consistently also in $B(k)_{\pi^+}$. The contrast of these examples with the absence of structure in $B(k)_{K^+}$ is particularly striking.

In fact, the similarities pointed out in (c) become even more striking when the detailed structure of $B(k)_{K^-}$ and $B(k)_{\pi^-}$ are compared. In this case, such similarities are suggestive of a connection between the two processes $K^-p \to K^-p$ and $\pi^-p \to \pi^-p$, naturally understood within the context of SU^3. Thus, the most prominent peaks in both plots are found to correspond to the formation of well-known members of the same SU_3 multiplets. The most outstanding example of this correspondence is certainly given by the peaks at $\sim$1 GeV/c, where $\Lambda(1815)$ and $N(1688)$ are members of a $J^P=\frac{5}{2}^+$ octet and $\Sigma(1770)$ and $N(1670)$ belong in a $J^P=\frac{5}{2}^-$ octet.[12]

The observations (a), (b), and (c) above can be consistently interpreted by assuming that $B(k)$ results from the interference of two main contributions to the scattering amplitude. Of these, one, the "background," would yield by itself a smoothly rising $B(k)$, whereas the other is represented by the resonant amplitudes. This viewpoint was indeed shown to lead to a fair quantitative representation of the $B(k)_{K^-}$ data. The description of the scattering amplitude as a superposition of a diffractive-like background and resonant effects, as described in Sec. III, is the essence of the model of Ref. 1. The parametrization given in Ref. 1 for the diffractive amplitude in $K^-p \to K^-p$, over the limited range 850–1130 MeV/c, was modified here in a first attempt (valid only near $t=0$) to cover a much wider momentum range.

We wish at this point to elaborate on the relationship between the present approach (diffraction interference model, DIM) and that of the "interference model"[13] (Regge interference model, RIM), with regard, in particular, to the problem of "double counting."[14] The two approaches have in common the description of the scattering amplitudes as the linear superposition of a background and a resonance contribution. The difference rests in the prescription adopted to represent the background term and in its interpretation. In RIM the background is described in terms of Regge amplitudes determined on the basis of the t-channel exchange of permitted trajectories. When *all* permitted trajectories are taken into account, as pointed out in Ref. 14, however, one already includes within the background the averaged contribution of the direct channel resonances; therefore, the superposition of resonant amplitudes to this particular form of background would lead to a double counting of the resonant effects. In DIM, on the other hand, a purely phenomenological background is employed; since its parameters are determined by a fit to the data, the background thus contributes only that part of the amplitude which is required to complement the resonant amplitude, insufficient by itself to satisfy the observations. In this spirit DIM is unlikely to be affected by double counting. This becomes apparent from the present fit to the total cross section [$\sigma_{\text{tot}}=(4\pi/k)\text{Im}g(k,t=0)$], where the background and resonant contribution are simply additive and the former is seen to lie well under the resonant peaks (see Fig. 4). We do realize, however, that the correct determination of the background in DIM is affected by our incomplete knowledge about the existence of resonant states at high energy. An inspection of the resonant contribution to $B(k)_{K^-}$ in Fig. 4, in fact, shows that this contribution drops in a perhaps unnatural way beyond $\sim$3 GeV/c. Were many additional states present beyond this region, the resonant contribution would be enhanced at the expense of a lower background. This brings to bear a speculation[15]

[12] R. D. Tripp, D. W. G. Leith, A. Minten, R. Armenteros, M. Ferro-Luzzi, R. Levi Setti, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granet, J. Meyer, and J. P. Porte, Nucl. Phys. **B3**, 10 (1967).

[13] See, for example, V. Barger and D. Cline, Phys. Rev. Letters **16**, 913 (1966); **16**, 1135 (E) (1966).

[14] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).

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that the background at high energy might be entirely due to the superposition of the effects of a spectrum of infinite resonances. If this were the case, we should consider as "background" only part of the background as presently determined: this would dominate a low energy, as demanded by the behavior of the total cross section, and would rapidly decrease with increasing energy.

A modification of RIM which would be exempt from double counting was implicit in a comment of Freund[16] and was discussed in detail by Harari.[17] On the basis of finite-energy sum rules,[14] the latter author argues that, for elastic processes, only the Pomeranchuk trajectory should be combined with s-channel resonances.

We have investigated this speculation by replacing our background amplitude by that corresponding to the Pomeranchuk trajectory. The parametrization used was that of solution 1 of Phillips and Rarita.[18] In addition we included the contribution of an s wave, $I=0$ scattering length, the real and imaginary parts of which are related to the mass and width of the $Y_0^*(1405)$. These contributions, together with the resonances used in Sec. III B, essentially reproduced the curves in Fig. 4 below ~ 1.5 GeV/c. Above this momentum, however, the total cross section was too low by ~ 4 mb. We do not believe that this difficulty is due to the use of an incorrect value for the Pomeranchuck residue function[18] since it is consistent with the K^+p total cross sections above ~ 2 GeV/c. We suspect, rather, that this discrepancy might be due to the presence of many as yet undetected resonances above 1.5 GeV/c.

As pointed out by Damouth *et al.*,[2] the structure observed in $B(k)$ could be profitably used as a tool for the detection of resonant states, in particular, at high energies where the interaction is mostly inelastic. In this connection, we wish to emphasize here the relevance of the experimental information which can be obtained from a detailed study of the momentum dependence of $B(k)$. This information can be grouped into two main categories: one relevant to the determination of resonance parameters and the other concerning the momentum dependence of the spin-flip amplitude.

Although the study of total cross sections provides some information on the resonance parameters, it cannot reveal which partial wave is resonating. Thus, at resonance the total cross section behaves as

$$\text{background}+(J+\tfrac{1}{2})xP_l(\cos\theta=1)\,,$$

where $x=\Gamma_{\text{el}}/\Gamma_{\text{tot}}$. Since all $P_l=1$ in the forward direction, it is not possible to determine J and x simultaneously from the total cross section. However, at resonance $B(k)$ is due to the interference of the above behavior with

$$\text{background}+(J+\tfrac{1}{2})x\left(\frac{dP_l(\cos\theta)}{d(\cos\theta)}\right)_{\cos\theta=1},$$

where the $[dP_l(\cos\theta)/d(\cos\theta)]_{\cos\theta=1}$ $[=\frac{1}{2}l(l+1)]$ increases rapidly with l. The most dramatic example of this effect is provided by an s-wave resonance. It will reveal itself as an enhancement in σ_{tot}, but since $[dP_0(\cos\theta)/d(\cos\theta)]_{\cos\theta=1}=0$, it will not appear as an enhancement in $B(k)$, though it may yield a slight dip.

As observed in Sec. III A, the spin-flip amplitude (divided by $\sin\theta$) appears in the expression for $B(k)$. The fit of Sec. III B showed how this could yield a determination of the sign and momentum dependence of $\text{Re}h^D$ in reasonable agreement with polarization data. It might be of interest to illustrate the behavior of $B(k)$ when the scattering occurs on a polarized proton target.[19] For totally polarized protons, the differential cross sections are given by

$$(d\sigma/d\Omega)_\pm=|g|^2+|h^2|(1-x^2)\pm 2\,\text{Im}(gh^*)(1-x^2)^{1/2},\quad(10)$$

where we redefine here h to be the spin-flip amplitude divided by $\sin\theta_{\text{c.m.}}$. Very near $x=1$,

$$\frac{d}{dx}\left(\frac{d\sigma}{d\Omega}\right)_\pm\approx 2\,\text{Re}g\frac{d\,\text{Re}g}{dx}+2\,\text{Im}g\frac{d\,\text{Im}g}{dx}-2|h|^2x\mp 2\,\text{Im}(gh^*)\frac{x}{(1-x^2)^{1/2}}.\quad(11)$$

The last term of Eq. (11) diverges for $x\to 1$, implying that the linear expansion required to derive Eq. (7) is no longer permissible. This in turn means that the first derivative of the differential cross section, for scattering on a polarized target will have a strong t dependence as $t\to 0$ $(x\to 1)$. Equation (11) will yield Eq. (8) if one averages over the polarization states. Thus, even though scattering from an unpolarized target would show an exponential forward peak, scattering from a polarized target (when $h\neq 0$), will, in general, be nonexponential for sufficiently small t. Whether this effect might be detectable or not will depend on the relative magnitude of g and h.

In conclusion, we feel that the accurate determination of the momentum dependence of $B(k)$ would be a worthwhile experimental undertaking in itself, in particular, in those regions where the fragmentary nature of the data collected here has left gaps and discrepancies. A similar study for $pp\to pp$ would be, of course, desirable, as well as a study of $B(k)$ for $\bar{p}p\to\bar{p}p$. Here the formation of heavy, high-spin boson states might be detectable, and if so, several of their quantum numbers could be determined by this approach.

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[19] This possibility was suggested to us by V. L. Telegdi (private communication).

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APPENDIX

The following tables summarize values of $B(k)$ obtained in least square fits to

$$d\sigma/dt = A(k)e^{B(k)t}$$

TABLE II. $\pi^- p \to \pi^- p$. $B(k)$ in $d\sigma/dt = A(k)e^{B(k)t}$.

$p_{\rm lab}$ (GeV/c)	B (GeV/c)$^{-2}$	Reference
0.231	23.16±1.58	a
0.253	21.63±1.74	b
0.257	23.73±0.86	a
0.257	18.20±1.39	c
0.268	18.24±2.22	d
0.276	14.40±0.99	b
0.295	16.99±2.29	e
0.328	10.22±3.13	f
0.331	14.23±1.81	g
0.338	11.18±0.59	c
0.355	5.71±1.16	h
0.385	6.07±1.07	h
0.425	5.81±0.93	h
0.428	2.82±0.44	i
0.452	4.57±0.79	h
0.490	6.53±1.30	j
0.532	5.71±0.79	j
0.573	4.68±0.73	j
0.614	5.73±0.72	j
0.675	5.48±0.33	j
0.675	6.77±0.37	k
0.678	7.05±0.33	l
0.685	5.71±0.45	m
0.707	6.62±0.55	l
0.727	5.77±0.28	j
0.727	7.72±1.02	n
0.777	5.58±0.30	j
0.826	5.01±0.34	l
0.848	4.86±0.82	j
0.875	5.15±0.25	o
0.900	7.63±0.82	n
0.925	8.54±0.44	o
0.975	12.79±0.68	o
1.000	13.42±0.93	o
1.003	13.71±0.53	l
1.030	13.08±0.50	o
1.030	13.91±0.25	k
1.055	13.97±0.44	o
1.080	16.98±0.92	o
1.121	11.02±0.30	l
1.132	9.88±1.10	p
1.151	12.54±0.32	j
1.180	13.86±0.67	o
1.280	8.43±0.38	o
1.335	8.05±0.79	q
1.360	8.00±0.36	o
1.440	7.81±0.86	o
1.505	9.65±0.76	o
1.579	12.19±1.44	o
1.700	7.41±0.26	r
1.720	7.40±0.37	s
1.880	6.80±0.28	r
1.890	6.83±0.33	s
2.010	7.94±0.23	t
2.070	8.59±0.33	r
2.070	8.32±0.41	s
2.270	8.19±0.30	r
2.270	8.29±0.41	s
2.460	8.21±0.47	s
2.500	8.07±0.26	r
2.500	7.69±0.10	u
3.000	7.31±0.10	u
3.150	7.32±0.49	v
3.500	7.35±0.10	u
4.000	7.28±0.12	u
4.130	8.62±0.50	v
4.950	7.39±0.26	v
5.000	7.15±0.17	u
6.000	7.45±0.18	u
7.000	8.06±0.13	w
8.500	7.52±0.09	x
8.900	8.00±0.11	w
10.000	8.19±0.28	y
10.800	8.00±0.11	w
12.400	7.68±0.09	x
13.000	8.10±0.11	w
15.000	8.04±0.12	w
17.000	8.00±0.15	w
18.400	7.53±0.20	x
18.900	8.08±0.29	w

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TABLE III. $\pi^+p \rightarrow \pi^+p$. $B(k)$ in $d\sigma/dt = A(k)e^{B(k)t}$.

P_{lab} (GeV/c)	B (GeV/c)$^{-2}$	Reference	P_{lab} (GeV/c)	B (GeV/c)$^{-2}$	Reference
0.428	13.71±0.59	a	1.444	6.82±0.47	b
0.490	11.54±0.34	a	1.505	7.98±0.92	c
0.532	10.48±0.34	a	1.579	9.74±0.11	c
0.573	9.96±0.33	a	1.600	7.73±0.41	d
0.614	8.36±0.63	a	1.689	6.12±0.31	b
0.675	4.63±0.36	a	2.300	6.11±0.11	f
0.678	6.84±0.51	b	2.500	6.57±0.13	f
0.707	8.22±0.66	b	2.700	6.59±0.12	f
0.727	4.26±0.35	a	2.920	7.41±0.32	g
0.777	3.81±0.31	a	3.000	6.72±0.18	f
0.825	4.98±0.40	b	3.500	6.55±0.22	f
0.875	4.07±0.38	c	3.700	6.95±0.22	f
0.925	4.03±0.23	c	4.000	6.32±0.19	f
0.950	4.48±0.22	c	4.000	6.94±0.18	h
0.975	3.44±0.32	c	6.800	7.60±0.15	i
1.000	4.37±0.23	c	8.500	6.88±0.20	j
1.003	3.99±0.29	b	8.800	7.90±0.16	i
1.030	3.16±0.33	c	10.800	8.17±0.17	i
1.080	3.09±0.24	c	12.400	6.89±0.48	j
1.121	3.56±0.19	b	12.800	7.96±0.19	i
1.180	2.94±0.25	c	14.800	7.96±0.19	i
1.280	3.63±0.26	c	16.700	7.58±0.19	i
1.360	4.74±0.43	c			

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TABLE IV. $K^-p \rightarrow K^-p$. $B(k)$ in $d\sigma/dt = A(k)e^{B(k)t}$.

P_{lab} (GeV/c)	B (GeV/c)$^{-2}$	Reference	P_{lab} (GeV/c)	B (GeV/c)$^{-2}$	Reference
0.293	1.57±5.16	a	1.102	8.76±1.16	d
0.350	16.44±5.22	a	1.117	7.76±0.90	d
0.390	27.01±2.73	a	1.134	8.01±1.54	d
0.434	4.83±2.80	a	1.150	7.32±0.92	e
0.513	4.80±1.23	a	1.153	8.24±1.13	d
0.620	3.81±0.53	b	1.174	5.93±0.82	d
0.708	6.39±0.70	c	1.183	6.86±1.05	d
0.725	6.64±0.47	c	1.220	5.73±0.21	f
0.741	7.28±0.59	c	1.430	6.45±0.26	f
0.760	6.34±1.16	b	1.450	7.16±0.21	g
0.768	8.57±0.72	c	1.510	8.00±0.33	f
0.777	7.11±1.21	d	1.610	8.76±0.33	f
0.802	6.26±0.38	c	1.700	7.76±0.31	f
0.806	7.94±1.05	d	1.800	7.50±0.21	f
0.838	7.96±1.49	d	1.950	7.22±0.33	f
0.850	7.60±1.41	b	2.000	8.27±0.18	h
0.853	8.71±1.05	d	2.080	7.14±0.26	f
0.874	7.73±0.98	d	2.240	7.21±0.25	i
0.894	9.61±1.28	d	2.440	7.96±0.26	f
0.904	9.40±1.03	d	2.600	7.81±0.31	f
0.916	6.83±0.95	d	2.660	7.15±0.10	j
0.935	9.68±1.10	d	3.000	7.72±0.28	k
0.954	9.37±1.03	d	3.460	7.44±0.89	l
0.970	11.69±1.18	d	4.100	7.62±0.51	m
0.980	12.10±2.03	d	5.500	7.52±0.50	m
0.991	12.30±1.10	d	7.200	7.00±0.65	n
1.022	12.46±0.92	d	9.000	7.28±0.61	n
1.044	11.89±0.98	d	10.000	7.11±0.23	o
1.061	12.35±1.03	d	11.880	6.93±0.32	p
1.080	8.37±0.92	d	15.91	6.05±0.40	p

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TABLE V. $K^+p \to K^+p$. $B(k)$ in $d\sigma/dt = A(k)e^{B(k)t}$.

P_{lab} (GeV/c)	B (GeV/c)$^{-2}$	Reference	P_{lab} (GeV/c)	B (GeV/c)$^{-2}$	Reference
0.520	0.63±0.58	a	1.455	2.63±0.11	g
0.642	1.34±0.51	a	1.960	3.03±0.13	h
0.780	1.42±0.26	b	1.970	2.72±0.21	f
0.810	0.88±0.59	c	2.970	4.19±0.34	i
0.860	1.22±0.24	d	3.500	4.26±0.32	j
0.910	0.42±0.24	e	4.300	4.69±0.41	k
0.960	1.35±0.17	d	5.000	5.15±0.54	j
0.970	1.18±0.45	f	6.800	5.20±0.50	l
1.170	2.49±0.61	f	9.800	5.88±0.26	l
1.200	2.34±0.09	d	12.800	6.17±0.22	l
1.360	2.51±0.15	d	14.800	6.33±0.29	l

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as a function of laboratory momentum. The fits were performed over an interval $0 \leq |t| \leq |t|_{cutoff}$, where $|t|_{cutoff}$ increased smoothly from ~ 0.1 (GeV/c)^2 at a laboratory momentum of ~ 0.3 GeV/c to a constant value of ~ 0.45 (GeV/c)^2 above 1.5 GeV/c. The processes considered were $\pi^- p \to \pi^- p$ (Table II), $\pi^+ p \to \pi^+ p$ (Table III), $K^- p \to K^- p$ (Table IV), and $K^+ p \to K^+ p$ (Table V). A list of references to the sources of experimental data used in this determination is given in the footnotes to each table.