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s-Wave Scattering Lengths, $\pi\pi$ Dynamics, and a Universal Partial Width for a Regge Trajectory

SHARASHCHANDRA H. PATIL
Department of Physics, University of Delhi, Delhi, India
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Logunov-Tavkhelidze equations are solved for $\pi\pi$ scattering partial-wave amplitudes. The forces due to exchanges of the experimental ρ and f^0 mesons and a g meson with a somewhat larger width are found to reproduce ρ, f^0, and g reasonably well. The same forces, however, very strongly imply that the $I=0$, 2 scattering lengths are both negative. Analysis of positive- and negative-signature trajectories leads to the conclusion that the particles on a given trajectory all have approximately the same width. Implications of this for high-energy scattering are discussed, and are found to be in good agreement with experiment.

I. INTRODUCTION

It is indeed an unhappy situation that we have so slight a knowledge about the $\pi\pi$ interaction, which perhaps is the most fundamental of strong interactions.

Our ignorance and confusion is nowhere as deep as it is in the s-wave part of the $\pi\pi$ interaction. The theorists have predicted almost every possibility from one and two s-wave resonances[1] all the way to negative scattering lengths.[2] Not to be outdone, experimentalists have found a narrow resonance,[3] a broad resonance,[4] and a negative scattering length.[5] Yet s-wave interaction is useful in many descriptions—the $K_2^0 \to 3\pi$ and $\eta \to 3\pi$ decays, the K_1^0–K_2^0 mass difference,[6] and the Roper-resonance decay into $N\pi\pi$, to mention a few obvious cases.

The situation about partial waves with $l>0$ is somewhat better. We know that in the $I=l=1$ state we have a ρ-meson resonance, and in the $I=0$, $l=2$ state the f^0 resonance. It is less certain, but likely, that the g meson[7] is an $I=1$, $l=3$ resonance. Correspondingly, the theorists have been reasonably successful[8] in analyzing the forces which might give rise to these resonances.

The $\pi\pi$ interaction has profound implications in another domain of strong interactions—namely, high-energy phenomena. For example, the Pomeranchuk trajectory, which may pass through the f^0, governs all the high-energy total cross sections; the ρ-meson trajectory plays a dominant role in π-p charge-exchange scattering. It is understood that these high-energy effects can be extrapolated from the low-energy $\pi\pi$ interaction. Yet the relation between these two domains of strong interaction has not been fully exploited.[9]

We will discuss $\pi\pi$ dynamics in the framework of the Logunov-Tavkhalidze (L-T) equation,[10] which has the advantages that it contains unitarity, it is relativistic and reasonably simple to solve, and the prescription for the forces allows the calculations to be made without a cutoff. With the help of solutions to this equation we elucidate some points raised before, extend the applications, and predict certain results.

The model consists of solving the L-T equation for continuous-l partial waves of the $\pi\pi$ interaction. The input forces are due to exchanges of the ρ meson with $m_\rho \approx 750$ MeV and $\Gamma_\rho \approx 110$ MeV, the f^0 meson with $m_{f^0} \approx 1250$ MeV and $\Gamma_{f^0} \approx 120$ MeV, and the g meson with[11] $m_g \approx 1650$ MeV and $\Gamma_g \approx 240$ MeV. In particular, we solve[12] for $l=0$, 1, 2, 3, 4, and 5; some of the relevant results are as follows:

(i) The ρ, f^0, and g are reasonably well reproduced. The widths are, in general, somewhat larger, and this may be due to not taking the higher-spin exchanges into account.

(ii) The model predicts infinitely rising trajectories,[13] for both even and odd signatures. In particular, spin-4 and -5 particles are predicted, one of which lies on the Pomeranchuk trajectory through the f^0 and the other on the odd-signature ρ trajectory through the ρ and g mesons. The trajectories are approximately straight lines, though they become quadratic in l for higher values of l. All these statements, however, have to be qualified by the shortcoming that the forces we have taken are inadequate for the description of large-l partial waves.

(iii) The s-wave scattering lengths for both $I=0$ and 2 are definitely predicted to be negative. In this model, it is very hard to see how they could be otherwise. The $I=0$ s-wave scattering length comes out to be -1.2;

[1] L. M. Brown, Phys. Rev. Letters **14**, 836 (1965).
[2] G. F. Chew, Phys. Rev. Letters **16**, 60 (1966).
[3] M. Feldman *et al.*, Phys. Rev. Letters **14**, 869 (1965); V. Hagopian *et al.*, *ibid.* **14**, 1077 (1965).
[4] W. D. Walker *et al.*, Phys. Rev. Letters **18**, 630 (1967); E. Malamud, and P. Schlein, *ibid.* **19**, 1056 (1967).
[5] L. W. Jones *et al.*, Phys. Letters **21**, 590 (1966).
[6] K. Nishijima, Phys. Rev. Letters **12**, 39 (1964); S. H. Patil, *ibid.* **13**, 454 (1964).
[7] A. H. Rosenfeld *et al.* University of California Radiation Laboratory Report No. UCRL 8030, 1968 (unpublished); T. F. Johnson *et al.*, Phys. Rev. Letters **20**, 1414 (1969).
[8] To mention just one case, L. A. P. Balázs and S. M. Vaidya, Phys. Rev. **140**, B1025 (1965).
[9] An investigation was initiated by L. Durand, III, and Y. T. Chiu, reported by L. Durand at the Stony Brook Conference on High-Energy Two Body Reactions, 1966 (unpublished).
[10] A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963).
[11] The experimental width varies from 80–200 MeV; see Ref. 7. We have taken a larger input width which gives us the right ρ position as an output. The larger width may be regarded as being due to the higher-spin exchanges which we have neglected.
[12] L. A. P. Balázs has investigated the width of the ρ meson due to similar forces, using the Bethe-Salpeter equation. His output width for the ρ meson is about 180 MeV (private communication).
[13] S. Mandelstam, in *Tokyo Summer Lectures in Theoretical Physics*, edited by G. Takeda and T. Fuji (W. A. Benjamin, Inc. New York, 1967).

the magnitude is in excellent agreement with that given by Jones *et al.*,[5] who also suggest that the sign is likely to be negative. We point out why this situation may be the correct one, and how the other experimental results can be reinterpreted to allow for this possibility.

In the latter part of this work, the continuation of position and the width of the resonances is analyzed as a function of l. The approximate constancy of the width of the resonances leads us to the statement of a "universal partial width" for components of a Regge trajectory. This result and the approximate linearity of Regge trajectories together imply the following:

(a) As s, the square of the total energy in the center-of-mass system, tends to infinity:

$$\sigma(\pi^- p)-\sigma(\pi^+ p)\approx 60 s^{-1/2}\ \text{mb},$$

where s is expressed in units of m_π^2. This agrees with the experimental data for lab energies greater than (say) 10 BeV, to within 10%.

(b) The asymptotic $\pi\pi$ scattering total cross section is predicted to be about 11 mb. Though there is no direct experimental information on this, the number is in reasonable agreement with the prediction of 15 mb by Gell-Mann and by Gribov and Pomeranchuk,[14] who use the factorization of Regge couplings and πN and NN scattering data to obtain this value.

II. MODEL FOR $\pi\pi$ SCATTERING

In discussing the $\pi\pi$ dynamics, we will be interested more in the qualitative features of the interaction than in the detailed quantitative features. For this purpose the Logunov-Tavkhelidze equation is ideally suited. It has been shown[15] that this equation is an approximation to the Bethe-Salpeter equation. It contains many desirable features such as unitarity and relativistic kinematics, and what is more, it is reasonably simple to solve. For the $\pi\pi$-scattering partial waves, the equation has the form

$$T_l^I(\nu',s)=V_l^I(\nu',\nu,s)+\frac{1}{\pi}\int_0^\infty \frac{d\nu''}{\nu''-\nu}\left(\frac{\nu''}{\nu''+1}\right)^{1/2} V_l^I(\nu',\nu'',s)T_l^I(\nu'',s)\,, \quad (1)$$

where

$$T_l^I(\nu,s)=[(\nu+1)/\nu]^{1/2}e^{i\delta_l}\sin\delta_l\,,$$

I stands for the isotopic spin, and $\nu=\frac{1}{4}s-1$. The potentials are defined in terms of projections of resonant amplitude in the t and u channels. The t-channel amplitude is

$$T^{I'}(t,\cos\theta_t)=\left(\frac{t}{t-4}\right)^{1/2}\sum_l (2l+1)e^{i\delta_l}\sin\delta_l \times P_l(\cos\theta_t)\,. \quad (2)$$

[14] M. Gell-Mann, Phys. Rev. Letters **8**, 263 (1962); V. N. Gribov and I. Ya. Pomeranchuk, *ibid.* **8**, 343 (1962).

[15] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966).

Then

$$V_l^I=\frac{1}{2}\int_{-1}^{1} C_{I'}{}^I T^{I'}(l,\cos\theta_t)P_l(\cos\theta_s)d\cos\kappa_s\,, \quad (3)$$

and $C_{I'}{}^I$, including the u-channel contribution, is

$$C_{I'}{}^I=\begin{bmatrix} \frac{2}{3} & 2 & 10/3\\ \frac{2}{3} & 1 & -5/3\\ \frac{2}{3} & -1 & \frac{1}{3}\end{bmatrix}. \quad (4)$$

The t-channel partial-wave amplitudes are saturated by the ρ, f^0, and g mesons. The g-meson width is still very uncertain; we will fix its value so as to give the right ρ-meson position as an output of Eq. (1). For a ρ meson of width 110 MeV, we have

$$[t/(t-4)]^{1/2}e^{i\delta_1}\sin\delta_1\approx\gamma_1(\tfrac{1}{4}t-1)/(m_\rho^2-t)\,, \quad (5)$$

where $\gamma_1\approx0.7$. Similarly, for the f^0 width of 120 MeV, we have

$$[t/(t-4)]^{1/2}e^{i\delta_2}\sin\delta_2\approx\gamma_2(\tfrac{1}{4}t-1)^2/(m_{f^0}{}^2-t)\,, \quad (6)$$

with $\gamma_2\approx0.022$; and for the g-meson width of 240 MeV,

$$[t/(t-4)]^{1/2}e^{i\delta_3}\sin\delta_3\approx\gamma_3(\tfrac{1}{4}t-1)^3/(m_g^2-t)\,, \quad (7)$$

with $\gamma_3\approx4.8\times10^{-3}$. The prescription for defining potentials which are asymptotically well behaved is to evaluate the t-dependent numerators at the respective pole positions. Then the off-shell potentials in (3) are of the form

$$V_l^I(\nu',\nu'',s)=\sum_i f_i(s)\frac{1}{2q'q''}Q_l\left(\frac{m_i^2+\nu'+\nu''}{2q'q''}\right), \quad (8)$$

where $q'^2=\nu'$ and $q''^2=\nu''$, and the summation is over the ρ, f^0, and g exchange terms; the f_i are

$$f_\rho(s)=3C_1{}^I\gamma_1(\tfrac{1}{4}m_\rho^2-1)P_1[1+2s/(m_\rho^2-4)]\,,$$
$$f_{f^0}(s)=5C_0{}^I\gamma_2(\tfrac{1}{4}m_{f^0}{}^2-1)^2P_2[i+2s/(m_{f^0}{}^2-4)]$$
$$f_g(s)=7C_1{}^I\gamma_3(\tfrac{1}{4}m_g^2-1)^3P_3[1+2s/(m_g^2-4)]\,.$$

The problem of solving the integral equation (1) is greatly simplified by approximating the potential (8) with[15,16]

$$V_l^I(\nu',\nu'',s)\approx[V_l^I(\nu',\nu',s)V_l^I(\nu'',\nu'',s)]^{1/2}. \quad (9)$$

For our qualitative analysis, this is not unreasonable, since for $\nu'=\nu''$ our potential is exact; and even for $\nu'\neq\nu''$, the approximation is not bad for smooth potentials. With this approximation, the solution to (1) can be directly written down:

$$T_l^I(\nu,s)=V_l^I(\nu,s)\Big/\left[1-\frac{1}{\pi}\int_0^\infty\frac{d\nu''}{\nu''-\nu}\times\left(\frac{\nu''}{\nu''+1}\right)^{1/2}V_l^I(\nu'',\nu'',s)\right]. \quad (10)$$

[16] Such an approximation was first proposed by A. N. Mitra (private communication). The approximation works very well; for example, for the πN static Bethe-Salpeter equation, the results are within 10% of the other calculations by S. N. Biswas and L. A. P. Balázs, Phys. Rev. **156**, 1511 (1967) and by D. Bandyopadhyay, S. N. Biswas, and R. P. Saxena, *ibid.* **160**, 1272 (1967).

To obtain a simple expression for (10), we use the extreme relativistic approximation

$$[\nu''/(\nu''+1)]^{1/2}\approx 1 \tag{11}$$

and the asymptotic expression

$$Q_l\left(\frac{m_i^2+2\nu''}{2\nu''}\right)\approx\frac{\pi^{1/2}\Gamma(l+1)}{2^{l+1}\Gamma(l+\frac{3}{2})}\left(\frac{2\nu''}{m_i^2+2\nu''}\right)^{l+1}. \tag{12}$$

Then the integral in (10) can be explicitly evaluated by using

$$I_l\equiv\int_0^\infty\frac{d\nu''}{(\nu''-\nu-i\epsilon)}\frac{\nu''^l}{(\nu''+m^2)^{l+1}}$$
$$=\frac{1}{\nu}\left[\frac{\nu^{l+1}}{(m^2+\nu)^{l+1}}(i\pi-\pi\cot\pi l)-\frac{\nu}{lm^2}F(1,1|1-l|-\nu/m^2)\right]. \tag{13a}$$

For positive integer values of l, this integral reduces to

$$I_l=\frac{1}{(\nu+m^2)}\sum_{n=0}^{l-1}\frac{1}{l-n}\left(\frac{\nu}{\nu+m^2}\right)^n+\frac{\nu^l}{(\nu+m^2)^{l+1}}\ln\left|\frac{m^2}{\nu}\right|+\frac{i\pi\nu^l}{(\nu+m^2)^{l+1}}, \tag{13b}$$

where the first term is zero for $l=0$. We can now study the amplitude (10) with the help of (8), (11), (12), and (13).

III. CHARACTERISTICS OF T_l

The input forces due to the ρ and f^0 exchanges [Eqs. (5) and (6)] correspond to their experimental mass and decay widths. The g-meson input width is as yet rather uncertain. Therefore, we fix its value so that the ρ meson appears at 750 MeV in $T_1^{I=1}$. This gives a value of $\gamma_3\approx 4.8\times 10^{-3}$, corresponding to a g width of about 240 MeV. This is larger than expected, but may be looked upon as due to higher-spin exchanges which we have ignored.

With the above inputs, the ρ-meson width comes out to be $\Gamma_\rho\approx 1.5m_\pi$, which is somewhat large but not unreasonable. The calculations for $T_3^{I=1}$ yield a resonance at about 1800 MeV and a width of about $\Gamma_g\approx 1.7m_\pi$. The forces are actually strong enough to produce resonances for all $T_{l=1,3\ldots}^{I=1}$, and the corresponding Regge trajectory is infinitely rising. Thus, for example, for $l=5$, there is a resonance at about 2800 MeV, with a width of about $2.5m_\pi$. The trajectory is approximately linear for small valves of l, and has a value of $l=0.4$ at $s=0$. For larger l it acquires a quadratic term. However, one must remember that our forces are inadequate for the treatment of large-l partial waves.

For the $I=0$ states, we find that there is a resonance in the $l=2$ state at about 1180 MeV having a width of about $1.2m_\pi$, which is in good agreement with the f^0. The Regge-trajectory has approximately the same slope as the negative-signature ρ trajectory. In particular, it implies a resonance with $l=4$ at about 2300 MeV with a width of about $2m_\pi$. The trajectory takes on a value of about $l=0.7$ at $s=0$. While this does not exactly agree with the Pomeranchuk trajectory, it is not far from it, and it may be that a more elaborate calculation will bring the trajectory into agreement with the Pomeranchuk one.

The s-wave results are of particular interest, and will be discussed separately.

IV. NEGATIVE SCATTERING LENGTHS

The forces due to the ρ, f^0, and g exchanges do provide a qualitatively correct picture for the description of the ρ, f^0, and g. They also describe the general characteristics of the two negative- and positive-signature trajectories, namely, the slopes and the $s=0$ intercepts. We therefore expect that they may provide a qualitatively correct description for the low-energy s-wave scattering amplitudes with $I=0$, 2. We find that in this model, the scattering lengths for both the $I=0$ and the $I=2$ amplitude are compellingly negative.

For $I=0$, the scattering length is about $a_0\approx -1.2$. The phase shift starts from 180°, changes rapidly, and at 550 MeV is about 120°. The scattering length for the $I=2$ state is also negative and is -0.4. These results[17] more than bear out Chew's conjectures[2] based on the Pomeranchuk trajectory.

While at first thought these results might appear disastrous, a closer analysis indicates that the real situation may not be very different from this.

First consider the $I=2$ scattering length. The negative $I=2$ scattering length is implied by many experiments—especially the small asymmetry observed[4] in the process

$$p+\pi^+\rightarrow\pi^++\pi^0+p. \tag{14}$$

As regards the $I=0$ scattering, the change in the asymmetry[5] for

$$p+\pi^-\rightarrow\pi^++\pi^-+n \tag{15}$$

at about 500 MeV provides a strong argument that the phase shift starts at 180°. Furthermore, Jones *et al.* give other arguments, based on interference between $I=0$ and $I=2$ for

$$p+\pi^-\rightarrow\pi^0+\pi^0+n, \tag{16}$$

which indicate that the $I=0$ scattering length is also

[17] The negative scattering lengths here are due to a zero in the denominator function for negative s. On physical grounds this ghost should be accompanied by a zero in the numerator function. Mathematically, this is not necessary in general, and it does not happen in our case, though one expects a complete theory to have this property.

negative. There is another analysis[18] of the K_1^0-K_2^0 system which indicates that the $I=0$ phase shift is negative at the K-meson mass.

The interference around the ρ-meson position tells us that the $I=0$ phase shift is large, i.e., near 90°, at the ρ position. In our model, this phase shift is about 115°. We do not claim that we have a quantitative picture of s-wave scattering. It may be that the inelastic effects do change the phase shifts to some extent, above the threshold. But it is highly unlikely that any of these effects could significantly affect the low-energy s-wave scattering, particularly the negative sign of the scattering length.

Our scattering phase shifts give the correct K_1^0-K_2^0 mass difference. This may be seen from the fact that in the dispersion relations for the K_1^0-K_2^0 mass difference, it is $\sin^2\delta$ which is relevant. These values are approximately the same as those used by Dutta-Roy and Lapidus,[19] who get a very good result for the mass difference.

It has been pointed out[20] that the Dalitz-plot distribution is well described by the interference between the s-wave state and the $\rho^\pm$ pole in the $\pi^\pm\pi^0$ states. If the s-wave phase shifts are negative, this interference is opposite. However, we note that in some sense there is double counting when the poles in various channels are added up. We find that an alternative explanation is provided by the rapidly changing s-wave phase shift, without including crossed-channel poles. The slope from our model is about 1.6, as compared to the experimental value of about 2.5. Considering all the approximations involved, the agreement is not unreasonable.

All these arguments suggest that the negative scattering lengths for both the $I=0$ and the $I=2$ state are a very good possibility.

V. HIGH-ENERGY SCATTERING

We obtain information on high-energy scattering processes by continuing the Regge trajectories and their residues to $t\leqslant 0$. For the continuation, we make use of the above analysis and some experimental information.

For small l values, it was noted that the trajectory is approximately linear. For the positive signature, we take a linear trajectory passing through $l=1$ at $t=0$, and through the f^0. For the negative signature, we take it to pass through the ρ and the g. Then we get

$$(m_l^2)_+\approx(l-1)m_{f^0}^2\,,\quad (m_l^2)_-\approx(2l-1)m_\rho^2\,. \tag{17}$$

To continue the residue functions, we note that, experimentally, the ρ, g, and f^0 all have approximately the same width, between 100 and 140 MeV. In our model, the ρ and g have nearly the same width, as do the f^0 and the spin-4 particle, if we take into account the fact that the higher-spin exchanges which we have neglected would affect the larger partial waves to a greater extent. We formulate this constancy into a universality principle that the particles lying on a meson trajectory have approximately the same partial width[21] for decay into pions provided we can neglect the mass of the pions, i.e.,

$$\Gamma_l\big|_{m_\pi=0}\approx\text{const}\,. \tag{18}$$

We are now in a position to discuss high-energy scattering. Specifically let us consider the t-channel negative-signature scattering amplitude. The partial-wave decomposition of this amplitude is

$$T=\left(\frac{t}{t-4}\right)^{1/2}\sum_l(2l+1)e^{i\delta_l}\sin\delta_l\,P_l(\cos\theta_t)\times\tfrac{1}{2}(1-(-1)^l)\,. \tag{19}$$

Then the Sommerfeld-Watson transformation gives

$$T=\frac{\pi(2\alpha+1)}{2\sin\pi\alpha}\frac{\gamma_\alpha q_t^{2\alpha}P_\alpha(-\cos\theta_t)}{2m_\rho^2}\tfrac{1}{2}(e^{-i\pi\alpha}-1)+T_B\,, \tag{20}$$

where $q_t^2=\frac{1}{4}t-1$ and $\alpha=\frac{1}{2}(1+t/m_\rho^2)$, and T_B is the background integral, which may be ignored when we are interested in the asymptotic behavior in s, or when α is close to an integer. The residue function γ_α is so normalized that at the resonance position,

$$\frac{t^{1/2}}{q_t}e^{i\delta_l}\sin\delta_l=\frac{\gamma_l q_t^{2l}}{m_l^2-t-i\gamma_l(q_t^{2l+1}/t^{1/2})}\,. \tag{21}$$

Now, the statement (18) implies that

$$\Gamma_l=\gamma_l\rho_l\approx\text{const}\,, \tag{22}$$

where

$$\rho_l\approx m_l^{2l-1}/2^{2l+1}\,. \tag{23}$$

Therefore, we get

$$\gamma_l\approx\bar\gamma(2^{2l+1})/[(2l-1)m_\rho^2]^{l-1}\,, \tag{24}$$

where $\bar\gamma$ is a constant. However, this expression is expected to be valid only for $l\geqslant 1$. For a more general form which has the nonsense factors at negative half integers and which has the asymptotic form (24) for $l\geqslant 1$, we take

$$\gamma_l=\gamma 2^l a^l/m_\rho^{2l-1}\Gamma(l+\tfrac{3}{2})\,. \tag{25}$$

The constants γ and a are determined so as to give $\Gamma_\rho=\gamma_1\rho_1$ and $\Gamma_g\approx\Gamma_\rho$. The result is

$$a\approx 0.8\quad\text{and}\quad\gamma=(3/a)\pi^{1/2}\Gamma_\rho\,. \tag{26}$$

Then for $s\to\infty$, using (20), (25), and (26), we get

$$T\approx\frac{3\pi\Gamma_\delta(2a)^{\alpha-1}(2s)^\alpha(e^{-l\pi\alpha}-1)}{4m_\rho^{2\alpha+1}\Gamma(1+\alpha)\sin\pi\alpha}\,. \tag{27}$$

This is the Regge amplitude for the ρ trajectory.

[18] S. Bennett *et al.*, Phys. Rev. Letters **18**, 997 (1967).
[19] B. Dutta-Roy and I. R. Lapidus, Phys. Rev. **169**, 1357 (1968).
[20] M. A. B. Bég and P. DeCelles, Phys. Rev. Letters **8**, 46 (1962); R. N. Chaudhuri, Phys. Rev. **176**, 2066 (1968).
[21] Our discussion has ignored inelasticity. However, it is hoped that this neglect does not seriously affect the universality conjecture, at least for small l values.

Though (27) was derived for $\pi\pi$ scattering, we can apply it to πN scattering in the forward direction if we generalize the universality of ρ-meson coupling to the ρ-trajectory "coupling." Then using the optical theorem, and taking $\Gamma_\rho \approx 110$ MeV, we have

$$\sigma(\pi^- p) - \sigma(\pi^+ p) \approx 60 s^{-1/2} \text{ mb}, \tag{28}$$

where s is in units of m_π^2. This is in good agreement with experiment. Thus, for a lab energy of 15 BeV, the value of (28) is about 1.5 mb, which is within 10% of the experimental values.[22]

The above reasoning can be repeated for the $I=0$ positive-signature trajectory. The signature factor in (19) is now $1+(-1)^l$, and the expression corresponding to (20) is

$$T = \frac{\pi(2\alpha+1)}{2 \sin\pi\alpha} \frac{\gamma_\alpha q_t^{2\alpha} P_\alpha(-\cos\theta_t)}{m_{f^0}^2} \times \tfrac{1}{2}(-1-e^{-i\pi\alpha}) + T_B. \tag{29}$$

The residue function is given by

$$\gamma_l = \frac{\gamma \Gamma_{f^0} l(2^l)}{(m_{f^0}^2)^{l-\frac{1}{2}} \Gamma(l+\frac{3}{2}) a^l}, \tag{30}$$

[22] R. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).

where $a \approx 2.44$ and $\gamma \approx 19.5$; these values are obtained by requiring $\Gamma_{f^0} = \Gamma_2 \approx \Gamma_4$, according to (18). Then using the optical theorem, the asymptotic $\pi\pi$ total cross section is

$$\sigma_{\text{tot}}(\pi\pi) \approx 9.6 \times 10^3 \Gamma_{f^0}/m_{f^0}^3 \text{ mb}, \tag{31}$$

where Γ_{f^0} and m_{f^0} are in units of m_π. Then for $m_{f^0} = 1250$ MeV and $\Gamma_{f^0} \approx 120$ MeV,

$$\sigma_{\text{tot}}(\pi\pi) \approx 11 \text{ mb}. \tag{32}$$

There is no direct experimental information on the $\pi\pi$ total cross section. However, Gell-Mann as well as Gribov and Pomeranchuk[14] have predicted, using factorization and πN and NN experimental data, that $\sigma_{\text{tot}}(\pi\pi) \approx 15$ mb. The agreement of (32) with this value, while reasonable, is not as striking as in the case of the ρ trajectory. This may be due to nonlinear trajectory, or perhaps the assignment of f^0 to the Pomeranchuk trajectory is not correct.[23] Some of these questions, along with the validity of the universal-partial-width assumption, will be settled when the spin-4 particle is discovered; our model predicts it to be at about 2300 MeV.

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[23] B. R. Desai and P. Freund, Phys. Rev. Letters **16**, 622 (1966).