Effect of Weak Magnetism in the Semileptonic Decay of Hyperons

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For the baryon semileptonic decay $B \to B'+e+\bar{\nu}$, the correlation function among the spin of the final baryon and the momenta of the leptons is computed, assuming only first-class hadronic weak currents. The weak-magnetism term is found to enter the correlation function in the first order in $(m_B-m_{B'})/m_B$ via interference with the vector and the axial-vector terms. The expression may be useful in the future experimental determination of the magnetic form factor for various hadronic weak currents.

ASSUMING the $V-A$-type current-current form of the semileptonic weak interaction of hadrons, and the most general form of the matrix element of the hadron current, expressions for the decay rates, the final-particle energy spectra, and the final baryon-lepton momentum correlation have been obtained in previous literature.[1] The average final baryon polarization, as well as averages of its components along various combinations of the final momenta, have been obtained by Alles[2] for the case of $\Sigma \to \Lambda^0 e\nu$, but with only the vector and axial-vector (γ_μ and $\gamma_\mu\gamma_5$) terms in the $\Sigma\Lambda$ matrix element. Barash *et al.*[3] have obtained the correlation among the spin of the Λ and the momenta of the leptons, with the same form $(f\gamma_\mu+g\gamma_\mu\gamma_5)$ of the $\Sigma\Lambda$ matrix element.

For an experimental determination of the magnetic form factor for the decay $B \to B'e\bar{\nu}$, its effects which are of the first order in $\beta \equiv (m_B-m_{B'})/m_B$ are desired. One such is in the electron energy spectrum,[1] while another would be in the expression for the final baryon polarization as a function of the energies of the final baryon and one of the leptons. The latter has been obtained by Willis and Thompson,[4] with the most general form of the hadronic current. Cabibbo and Franzini[5] have derived the expression for the decay rate for given polarizations of the two baryons and for given momentum of the final baryon, which, too, contains the magnetic form factor in terms that are of the first order in β. Finally, the extension of the correlation obtained in Ref. 3, with the magnetic term included, is given below. The expression is valid to the first order in β. Although time-reversal invariance implies reality of the form factors,[1] they are retained as complex numbers in order to display explicitly the effects of any T violation. The momentum-transfer dependence of the form factors is ignored. As in Ref. 2, the mass of the electron is completely neglected, so that the various expressions are valid only for hyperon decays with $m_B-m_{B'} \gg m_e$.

Let us take the hadronic matrix element to be of the form[6]

$$\langle B'|J_\mu|B\rangle = \left(\frac{m_B m_{B'}}{E_B E_{B'}}\right)^{1/2} \bar{u}(p_{B'}) \times \left(g_V\gamma_\mu + g_A\gamma_\mu\gamma_5 + g_T\frac{\sigma_{\mu\nu}q_\nu}{2m_B}\right)u(p_B), \quad (1)$$

where $q \equiv p_B - p_{B'}$, and let us take the relevant Hamiltonian to be

$$H_{\text{weak}} = (G/\sqrt{2})[J_\mu(x)j_\mu^\dagger(x) + \text{H.c.}], \quad (2)$$

where $G \equiv G_{\text{Fermi}}$ and j_μ is the leptonic current.

The most general hadronic matrix element contains three additional terms[1]; two of these are associated with the so-called second-class currents[7] and have been dropped. A third term (the pseudoscalar term) has been dropped too, since its contribution is proportional to $(m_e/m_B)^2$.

Then the decay rate for given directions of the lepton momenta and for given polarization of the final baryon [correct to $O(\beta)$] is found to be

$$\begin{aligned} W(\mathbf{S},\hat{p},\hat{p}') = (G^2\Delta^5/1920\pi^5)\{&[|g_A|^2(3-5\beta)+|g_V|^2(1-\beta)]+[|g_V|^2(1-\tfrac{5}{2}\beta)-|g_A|^2(1+\tfrac{3}{2}\beta)](\hat{p}\cdot\hat{p}') \\ &+\tfrac{3}{2}\beta(|g_A|^2-|g_V|^2)(\hat{p}\cdot\hat{p}')^2+[|g_A|^2(2-\tfrac{7}{2}\beta)-\tfrac{1}{2}\beta|g_V|^2+2\,\mathrm{Re}g_Ag_V^*(1-\beta)+\tfrac{1}{2}\beta\,\mathrm{Re}g_Ag_T^*-\tfrac{1}{2}\beta\,\mathrm{Re}g_Vg_T^*]\mathbf{S}\cdot\hat{p} \\ &+[|g_A|^2(\tfrac{7}{2}\beta-2)+\tfrac{1}{2}\beta|g_V|^2+2\,\mathrm{Re}g_Ag_V^*(1-\beta)+\tfrac{1}{2}\beta\,\mathrm{Re}g_Ag_T^*+\tfrac{1}{2}\beta\,\mathrm{Re}g_Vg_T^*]\mathbf{S}\cdot\hat{p}'+\beta\,\mathrm{Im}g_Ag_T^*\mathbf{S}\cdot\hat{p}\times\hat{p}' \\ &+2\,\mathrm{Im}g_Vg_A^*(1-3\beta)\mathbf{S}\cdot\hat{p}\times\hat{p}'-3\beta\,\mathrm{Im}g_Vg_A^*(\mathbf{S}\cdot\hat{p}\times\hat{p}')(\hat{p}\cdot\hat{p}') \\ &-[\tfrac{5}{2}\beta|g_A|^2+\tfrac{1}{2}\beta|g_V|^2+3\beta\,\mathrm{Re}g_Ag_V^*+\tfrac{1}{2}\beta\,\mathrm{Re}g_Ag_T^*+\tfrac{1}{2}\beta\,\mathrm{Re}g_Vg_T^*](\mathbf{S}\cdot\hat{p})(\hat{p}\cdot\hat{p}') \\ &+[\tfrac{5}{2}\beta|g_A|^2+\tfrac{1}{2}\beta|g_V|^2-3\beta\,\mathrm{Re}g_Ag_V^*-\tfrac{1}{2}\beta\,\mathrm{Re}g_Ag_T^*+\tfrac{1}{2}\beta\,\mathrm{Re}g_Vg_T^*](\mathbf{S}\cdot\hat{p}')(\hat{p}\cdot\hat{p}')\}, \quad (3) \end{aligned}$$

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[1] M. M. Nieto, Rev. Mod. Phys. **40**, 140 (1968); references to previous literatures can be found there. Also see I. Bender, V. Linke, and H. J. Rothe, Z. Physik **212**, 190 (1968).

[2] W. Alles, Nuovo Cimento **26**, 1429 (1962).

[3] N. Barash, T. B. Day, R. G. Glasser, B. Kehoe, R. E. Knop, B. Sechi-Zorn, and G. A. Snow, Phys. Rev. Letters **19**, 181 (1967).

[4] W. Willis and J. Thompson, in *Advances in Particle Physics*, edited by R. L. Cool and R. E. Marshak (Interscience Publishers, Inc., New York, 1968), Vol. I.

[5] N. Cabibbo and P. Franzini, Phys. Letters **3**, 217 (1963).

[6] The convention about γ matrices and 4-vector products adopted here is that used in Ref. 1. The coupling constants g of course include the Cabibbo angle.

[7] S. Weinberg, Phys. Rev. **112**, 1375 (1958); N. Cabibbo, in *Particle Symmetries and Axiomatic Field Theory*, edited by M. Chrétien and S. Deser (Gordon and Breach, Science Publishers, Inc., New York, 1965), Vol. II. It should be remembered, however, that experimentally the existence of second-class currents has been neither established nor ruled out: P. Hertel, Z. Physik **202**, 383 (1967); also see Refs. 3 and 9.

where $\Delta \equiv m_B - m_{B'}$ $\beta \equiv \Delta/m_B$ **S** is the polarization of B', $\hat{p} \equiv \hat{p}_e$, and $\hat{p}' \equiv \hat{p}_{\bar{\nu}}$. [A sketch of the derivation of Eq. (3) is given in Appendix A.]

Here g_T enters in the first order in β, while in the total rate[1] [which is quoted below correct to $O(\beta^2)$] it enters in the second order:

$$W_{\text{total}}(B \to B'e\bar{\nu}) = (G^2\Delta^5/60\pi^3)\{3|g_A|^2[1-\tfrac{3}{2}\beta+(4/7)\beta^2]+|g_V|^2[1-\tfrac{3}{2}\beta+(6/7)\beta^2] + |g_T|^2\beta^2/7+\text{Re}\, g_V g_T^* \times 3\beta^2/7\}. \quad (4)$$

Thus, for example, for the $\Sigma^- \to \Lambda e\nu$ decay, the contribution of g_T in the total rate is only about $\frac{1}{2}\%$.[1,10] (See Appendix B for similar estimates for other decays.) The $\text{Re}(g_A g_T^*)$ and $\text{Re}(g_A g_V^*)$ interference terms appear [to $O(\beta)$] in the electron energy spectrum[1] but not in W_{total}. For experimental comparison the likelihood function (involving experimental data) may be constructed.[3] Thus, for example, for the case of $\Sigma^- \to \Lambda e\nu$, if g_V is neglected,[8] one obtains

$$\mathcal{L}_y(\hat{q}_\pi|\hat{p},\hat{p}') = \prod_{i=1}^{n} \frac{W(\vec{\mathbf{S}}_i,\hat{p}_i,\hat{p}_i')}{2W(\vec{\mathbf{0}},\hat{p}_i,\hat{p}_i')} = \prod_{i=1}^{n} \frac{1}{2}\left\{1+\frac{2\alpha\hat{q}_{\pi i}\cdot\{(\hat{p}_i-\hat{p}_i')+\frac{1}{4}\beta[(y-7)\hat{p}_i+(y+7)\hat{p}_i'-((y+5)\hat{p}_i+(y-5)\hat{p}_i')(\hat{p}_i\cdot\hat{p}_i')]\}}{(3-\hat{p}_i\cdot\hat{p}_i')-\beta[5+\frac{3}{2}(\hat{p}_i\cdot\hat{p}_i')-\frac{3}{2}(\hat{p}_i\cdot\hat{p}_i')^2]}\right\}, \quad (5)$$

where $\alpha = \Lambda$ decay asymmetry parameter $\simeq -0.647$,[9] $\hat{q}_{\pi i}$ is the direction of π flight in the subsequent decay $\Lambda \to p+\pi$, and $y = g_T/g_A$ [taken to be real in Eq. (5)]. If the total number of measured $\Sigma \to \Lambda e\nu$ events n is sufficiently large, $\mathcal{L}$ can be expected to be a Gaussian function of y centered at the actual value.

The $SU(3)$ estimate[10] of $g_T|_{\Sigma^-\Lambda}$ is[11]

$$g_T \simeq 2.98 \cos\theta_V, \quad (6a)$$

while for $g_A|_{\Sigma^-\Lambda}$, Matsuda, Oneda, and Desai[12] have used broken-$SU(3)$ sum rules and some latest experimental data[13] to estimate

$$g_A|_{\Sigma^-\Lambda} \simeq 0.618 \cos\theta_A. \quad (6b)$$

If the Cabibbo angles θ_V and θ_A are taken to be equal,[12,13] one has

$$g_T/g_A|_{\Sigma^-\Lambda} \simeq 4.83. \quad (6c)$$

A preliminary fit to 45 $\Sigma^- \to \Lambda e\bar{\nu}$ events by Kehoe, Sechi-Zorn, and Snow[14] indicated that the said number of events is not sufficient to yield an accurate value of g_T/g_A. In fact, for values of g_T/g_A from about -5 to about $+5$, the variation in $\mathcal{L}_y$ was less than a standard deviation. With more statistics, this method may yield a reliable value for g_T/g_A.

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APPENDIX A: SKETCH OF THE DERIVATION OF EQUATION (3)

The 2×2 nonrelativistic reduction of (1), correct to $O(\beta)$, is

$$(m_B m_{B'}/E_B E_{B'})^{1/2}(\chi_{B'}{}^{\text{Pauli}})^\dagger h_\mu(\chi_B{}^{\text{Pauli}}), \quad (\text{A1})$$

where

$$h_4 = g_V + (g_A/2m_{B'})\boldsymbol{\sigma}\cdot\mathbf{q},$$

$$h_k = ig_A\sigma_k + i(g_V+g_T)(\boldsymbol{\sigma}\cdot\mathbf{q}/2m_{B'})\sigma_k - (ig_T/2m_B)q_k, \quad (k=1,2,3).$$

(Here $\mathbf{p}_B = \mathbf{0}$ was explicitly used.) From the Hamiltonian

[8] Even if the electromagnetic Σ^0-Λ mixing is taken into account, $g_V/g_A|_{\Sigma^-\Lambda}$ may be estimated to be only about -0.07, which may be neglected here. See Ref. 12; also R. H. Dalitz and F. Von Hippel, Phys. Letters **10**, 153 (1964).

[9] A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. Willis, Rev. Mod. Phys. **40**, 77 (1968).

[10] N. Brene, B. Hellesen, and M. Roos, Phys. Letters **11**, 344 (1964); N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. **149**, 1288 (1966). H. T. Nieh and M. M. Nieto [*ibid*.**172**, 1694 (1968)] also give the relevant expressions. A sample evaluation of the magnetic form factor is given in Ref. 1.

[11] This value of $g_T|_{\Sigma\Lambda}$ corresponds to $\mu_{\Sigma^0\Lambda}$ (the electromagnetic $\Sigma^0\Lambda$ transition moment) $\simeq 2$ or T_{Σ^0} (lifetime of $\Sigma^0 \to \Lambda+\gamma$) $\simeq 7 \times 10^{-20}$ sec: N. Cabibbo (Ref. 7); N. Cabibbo and R. Gatto, Nuovo Cimento **15**, 159 (1960).

[12] S. Matsuda, S. Oneda, and P. Desai, Phys. Rev. **178**, 2129 (1969). The notation there differs slightly from the present one.

[13] B. Sechi-Zorn, Bull. Am. Phys. Soc. **13**, 555 (1968); E. Bierman, S. Kounosu, U. Nauenberg, L. Seidlitz, and A. P. Colleraine, Phys. Rev. Letters **20**, 1459 (1968). References to other experiments may be found in Ref. 12.

[14] B. Kehoe, B. Sechi-Zorn, and G. A. Snow (private communication). The events were measured at the University of Maryland; details may be found in Ref. 3.

TABLE I. Estimated effects of weak magnetism on some hyperon semileptonic decay rates.

Decay	g_A	g_T	g_V	$R_T/(R_A+R_V)$ (%) If g_V has sign as shown	If g_V has sign opposite to that shown
$\Sigma^- \to \Lambda e\nu$	$0.618\cos\theta$	$2.98\cos\theta$	0	0.518	0.518
$\Sigma^+ \to \Lambda e\nu$	$0.618\cos\theta$	$2.98\cos\theta$	0	0.427	0.427
$\Lambda \to pe\nu$ [a]	$-0.945\sin\theta$	$-2.61\sin\theta$	$-(\sqrt{\frac{3}{2}})\sin\theta$	1.49	-0.31
$n \to pe\nu$ [a]	$1.27\cos\theta$	$3.71\cos\theta$	$\cos\theta$	~ 0	~ 0
$\Sigma^- \to ne\nu$	$0.244\sin\theta$	$2.59\sin\theta$	$-\sin\theta$	-1.10	8.62
$\Xi^- \to \Xi^0 e\nu$	$0.244\cos\theta$	$2.86\cos\theta$	$-\cos\theta$	~ 0	~ 0
$\Xi^- \to \Lambda e\nu$	$0.319\sin\theta$	$-0.2\sin\theta$	$(\sqrt{\frac{3}{2}})\sin\theta$	-0.145	$+0.160$
$\Xi^0 \to \Sigma^+ e\nu$	$1.27\sin\theta$	$5.19\sin\theta$	$\sin\theta$	0.962	0.236

[a] For these two decays, the signatures of g_V have been experimentally determined to be as listed (Ref. 9).

(2), summing its matrix element (squared) over all variables except $\mathbf{S}$, $\hat{p}$ and $\hat{p}'$, one finds

$$W(\mathbf{S},\hat{p},\hat{p}')=\frac{G^2 m_{B'}}{64\pi^5}\int_0^{a_0} dE\, E\int_{a_-}^{a_+} dE'\, E' \times\delta(m_B-E_{B'}-E-E')\,\mathrm{Tr}\left(h_\mu h_{\mu'}{}^\dagger\frac{1+\mathbf{S}\cdot\boldsymbol{\sigma}}{2}\right)\frac{T_{\mu\mu'}}{E_{B'}},\quad (A2)$$

where E and E' are the electron and neutrino energies, and

$$a_0=m_B(\beta-\tfrac{1}{2}\beta^2),\quad a_-=a_0-E,\quad a_+=a_-/(1-2E/m_B),$$

$$T_{\mu\mu'}=\pm(p_{\mu'}'p_\mu+p_\mu'p_{\mu'}-\delta_{\mu\mu'}p\cdot p'+\epsilon^{\mu\mu'\lambda\rho}p_\lambda'p_\rho).$$

(The $-$ sign enters for $\mu'=4$; the $+$ sign enters for $\mu'=1, 2, 3$.) Inserting (A1) into (A2), and taking proper account of the recoil of B', one arrives at Eq. (3) of the text. [The above rate is related to the total rate by

$$W_{\text{total}}=2\times(4\pi)^2\langle W(0,\hat{p},\hat{p}')\rangle,\quad (A3)$$

where $\langle\ \rangle$ stands for angular averaging: e.g., $\langle(\hat{p}\cdot\hat{p}')^2\rangle=\frac{1}{3}$.]

APPENDIX B: ESTIMATES OF THE EFFECT OF WEAK MAGNETISM IN THE RATES OF VARIOUS SEMILEPTONIC DECAYS

In Table I, g_T has been estimated using the expressions of Ref. 10, while the numbers for g_A are as in Ref. 12. (The convention adopted here corresponds to taking $g_A/g_V|_{np}>0$.) In the following, the results corresponding to both the signatures of g_V are displayed. The first column corresponds to the signatures as in the Cabibbo theory.[10,15]

Here R_A (R_V) denotes the contribution to the total rate due to g_A (g_V). R_T includes the contributions from the g_T^2 as well as the $g_T g_V$ terms. These have been obtained from Ref. 1. As mentioned in the text, the q^2 dependence of form factors as well as the electromagnetic corrections are ignored, and the Cabibbo angles θ_V and θ_A are taken to be equal.

[15] The sign convention used here is as listed by S. Matsuda, University of Maryland Department of Physics and Astronomy Technical Report No. 768, 1967 (unpublished).