Production of Particles by Gravitational Fields

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The production of particles by gravitational fields is calculated within the framework of quantum field theory. It turns out that the expansion of the universe—giving rise to a time-dependent gravitational field—does not lead to an appreciable production at present, since the frequencies involved are too low. Particle production was, however, of importance during the first 10^{-20} sec after a "big bang." This mechanism of particle creation leads to equal numbers of particles and antiparticles.

I. INTRODUCTION

SEVERAL explanations have been proposed to the questions concerning the origin of the particles populating the universe. The following models are of current interest:

1. "Big bang" theories.[1,2] According to these the universe originated from a pointlike singular state about 10 billion years ago. During this event particles must have been created. In a very early and dense phase of the universe, when thermonuclear reactions were still intense, the hydrogen gas created originally was burned to helium, which could explain the high (10%) helium content of the universe in a natural way.[3,4] The cosmic black-body radiation observed by Penzias and Wilson,[5] Roll and Wilkinson,[6] and many others supports the assumption of such a dense state of the universe. This cosmic model agrees more or less with the predictions[1] of general relativity. It does not offer an explanation of the origin of elementary particles. Also, the particle-antiparticle asymmetry we observe at least in our neighborhood remains unexplained. A global asymmetry—the whole universe consisting of protons rather than antiprotons—seems plausible. Otherwise, complete annihilation would have taken place in the early dense phase. However, one can imagine a model of the early universe where this total asymmetry is not required.

2. Oscillating models. They differ only slightly from "big bang" theories; the difference concerns the singular initial state. Matter and universe are considered eternal here; therefore problems of creation and asymmetry do not arise.

3. Theory of Klein and Alfvén.[7,8] It assumes a charge-symmetric initial state which is infinitely dilute. Charge asymmetry is caused by local condensations, which first contract and afterwards re-expand, giving rise to the observed red shifts. Black-body radiation and high helium content remain unexplained in this theory, whose main attractive feature is its charge-symmetric initial state.

4. Steady-state theories.[9–12] They postulate continuous creation of matter so that the density of matter in the universe remains constant despite the expansion.

If one tries to treat these questions of cosmic particle creation from the point of view of quantum field theory (which is up to now the only reasonable framework for this purpose), there are in principle two ways in which cosmologically significant numbers of particles can be created. Firstly, one can introduce a field with negative energy into the theory, so that the vacuum becomes unstable and particles are created spontaneously together with quanta of the negative energetic field. On the other hand—and this is the creation mechanism we shall concentrate upon here—the interaction with the gravitational field automatically leads to charge-symmetric particle creation.

II. PARTICLE PRODUCTION BY GRAVITATIONAL FIELDS

One need not alter general relativity (by introducing new fields) at all if one wants charge-symmetric particle creation. An attractive idea seems to be the use of the time-dependent gravitational field stemming from the expansion of the universe to explain a continuous creation of particles. We assume that a number of particles is already present and causes a time-dependent gravitational field. This field will give rise to pair production. If the number of particles created is small compared to the number of particles already present, the change in the gravitational field caused by the creation of particles can be neglected. This is the external-field approximation.

We shall consider the first-order process in this approximation. Particle production is possible when-

[1] H. Bondi, *Cosmology* (Cambridge University Press, London, 1961).
[2] R. H. Dicke, in *Relativity, Groups and Topology*, edited by C. deWitt and B. S. deWitt (Gordon and Breach, Science Publishers, Inc., New York, 1964), pp. 195–317.
[3] F. Hoyle and R. J. Tayler, Nature **203**, 1108 (1964).
[4] J. E. Peebles, Phys. Letters **16**, 410 (1966).
[5] A. A. Penzias and R. W. Wilson, Astrophys. J. **142**, 420 (1965).
[6] P. G. Roll and D. T. Wilkinson, Phys. Rev. Letters **16**, 405 (1966).
[7] H. Alfvén and O. Klein, Arkiv Fysik **23**, 187 (1962).
[8] H. Alfvén, Rev. Mod. Phys. **37**, 652 (1965).
[9] F. Hoyle, in *Evidence for Gravitational Theories*, edited by C. Moller (Academic Press Inc., New York, 1962), p. 141.
[10] F. Hoyle and T. V. Narlikar, Proc. Roy. Soc. (London), **290**, 143 (1966).
[11] F. Hoyle and T. V. Narlikar, Proc. Roy. Soc. (London), **290**, 162 (1966).
[12] F. Hoyle and T. V. Narlikar, Proc. Roy. Soc. (London), **290**, 177 (1966).

ever the Fourier decomposition of the external field contains frequencies $k_0 > \sum m_i \approx 10^{23}$ Hz, where the sum has to be extended over all particles created in one elementary process. At present the expansion of the universe is a phenomenon becoming pronounced only at distances of the order of 10^8 light years, which corresponds to frequencies of 10^{-25} Hz and is therefore a negligible source of particle creation. An appreciable number of particles was created, however, during the first 10^{-20} sec of the lifetime of the universe. At that time cosmic expansion was a sufficiently rapid process.

In the first-order process the external field has to contain frequencies $\sum m_i$ in order to create particles. In a second-order process, however, two interactions with the field can work together to create particles, each of them thus requiring frequencies half as large. Continuing this to all orders one ends up with the well-known Klein paradox, namely, that even a static field can create particles. The creation rate due to this process (infinitely many quanta of zero frequency acting together) is proportional to e^{-mR}, where R is the distance in which a particle, being accelerated by the external field, starting from rest acquires a kinetic energy equal to its rest mass. Such situations where a particle is accelerated almost up to the speed of light within distances of 10^{-13} cm do not exist, neither on a cosmic scale [here $R \simeq$ radius of the visible universe, so $e^{-mR} \simeq \exp(-10^{40})$] nor in quasars (typical radii have the order of magnitude of light years).

To calculate the production probability for pairs to lowest order in $\kappa = f^2$, we expand the Lagrangian of general relativity in powers of f:

$$L = \tfrac{1}{2}(\Psi_{ik,m}\Psi^{ik,m} - 2\Psi_{ik,m}\Psi^{im,k} + 2\Psi_{ik}{}^{,k}\Psi_m{}^{m,i} - \Psi_{ik}{}^{,k}\Psi^i{}_m{}^{,m}) + \Lambda + f\Psi^{ik}T_{ik},$$

where $g_{ik} = \eta_{ik} - 2f\Psi_{ik}$, η_{ik} being the flat space-time metric, T_{ik} the flat space-time energy-momentum tensor.

The pair production probability summed over all 2-particle states is

$$w_2 = \sum_2 |(2|S^{(1)}|0)|^2$$

with the first-order S matrix given by

$$S^{(1)} = if\int d^4x\, \Psi_{ik} : T^{ik} : .$$

The sum over the 2-particle states can be extended over a complete set of states, since only the matrix elements wanted in w_2 will contribute. Thus

$$w_2 = f^2 \int d^4x d^4x'\, \Psi_{ik}(x)\Psi_{jm}(x')K^{ikjm}(x-x'),$$

where the tensor $K_{ikjm}(x-x') = (0|:T_{ik}(x)::T_{jm}(x'):|0)$ depends only on $x-x'$ by the usual arguments. Further invariance arguments imply that it can be built up from some invariant functions with the help of η_{ik} and ∂_i. The general form is limited by the symmetry and conservation properties of $T_{ik}(x)$ to be ($D^{ik} = \eta^{ik} - \partial^i\partial^k$, ( ) and [ ] indicate symmetrization and antisymmetrization as usual)

$$K^{ikjm}(x) = D^{ik}D^{jm}K(x) + D^{i(j}D^{m)k}I(x).$$

K and I can be determined from the two independent scalars $K_i{}^i{}_j{}^j$ and $K_{ik}{}^{ik}$ (essentially the two quadratic invariants $T_i{}^iT_j{}^j$ and $T_{ik}T^{ik}$ of the energy-momentum tensor; we shall write $T_i{}^i = T$) by

$$\Box^2 K = (1/15)(2K_i{}^i{}_j{}^j - K_{ik}{}^{ik}),$$
$$\Box^2 I = (1/15)(3K_{ik}{}^{ik} - K_i{}^i{}_j{}^j).$$

Finally, the well-known existence of spectral representations for expressions like $K_i{}^i{}_j{}^j(x)$ and $K_{ik}{}^{ik}(x)$ leads to the form

$$K^{ikjm}(x) = \frac{-i}{16\pi^2}\int_{4m^2}^{\infty} ds \left(1 - \frac{4m^2}{s}\right)^{1/2} \times\{[4f_1(s) + f_2(s)]D^{ik}D^{jm} + f_2(s)D^{i(j}D^{m)k}\}\Delta^+(x, \sqrt{s}).$$

[For later convenience we have extracted a common factor $-(i/16\pi^2)(1-4m^2/s)^{1/2}$ out of the spectral functions $f_2(s)$ for $\Box^2 I$ and $4f_1(s)+f_2(s)$ for $\Box^2 K$; see Ref. 13 for normalizations.]

In the case of our derivative coupling, the use of the Lagrangian instead of the Hamiltonian for constructing the S matrix fixes a certain gauge of Ψ, which depends on the special type of matter field we use. Therefore, we prefer not to use any gauge explicitly to derive our final result. By the convolution and Parseval theorem, w_2 becomes in momentum space

$$w_2 = f^2 \int \frac{d^4k}{(2\pi)^4} \Psi_{ik}{}^*(k) K^{ikjm}(k) \Psi_{jm}(k).$$

On partial integration, the differential operators become tensors constructed from k which, apart from a common factor k^4, take the form $P^{ik}P^{jm}$ and $P^{i(j}P^{m)k}$ with $P^{ik} = \eta^{ik} - k^ik^k/k^2$. In w_2 we can replace the second one simply by $P^{ij}P^{mk} = \Pi^{ikjm}$. Whereas P acts as a projection operator in the space of Lorentz vectors, $\Pi = P \otimes P$ is a projection operator in the space of second-rank tensors. Therefore, in matrix notation, we can write $\Psi^*\Pi\Pi\Psi$ instead of $\Psi^*\Pi\Psi$. Next we note that—as is easily verified—the gravitational field equations in momentum space can be written in the form

$$f\Upsilon_{ik}/k^2 = 2P_{i[k}P_{j]m}\Psi^{jm} = P_{ik}P_{jm}\Psi^{jm} - \Pi_{ikjm}\Psi^{jm},$$

with the contraction

$$f\Upsilon/k^2 = 2P_{jm}\Psi^{jm}.$$

[13] W. E. Thirring, *Principles of Quantum Electrodynamics* (Academic Press Inc., New York, 1958).

Here Υ_{ik} is the energy-momentum tensor of the particles that are already present and produce the gravitational field which in turn creates new particles (of course, Υ_{ik} and T_{ik} have strictly to be kept apart from one another); and $\Upsilon \equiv \Upsilon_i{}^i$. Hence we can eliminate the potentials Ψ_{ik} completely from w_2 to arrive at the manifestly gauge-invariant expression

$$w_2=\frac{\kappa^2}{8\pi}\int_{k^2>4m^2;\,k_0=0}\frac{d^4k}{(2\pi)^4}\left(1-\frac{4m^2}{k^2}\right)^{1/2}(k^2)^{-2}\times[\Upsilon(k)\Upsilon^*(k)f_1(k^2)+\Upsilon^{ij}(k)\Upsilon_{ij}{}^*(k)f_2(k^2)].$$

It is interesting to compare this result with the corresponding pair creation rate by an external electromagnetic field, which is given by

$$w_2=\frac{1}{8\pi}\int_{k^2>4m^2;\,k_0=0}\frac{d^4k}{(2\pi)^4}\left(1-\frac{4m^2}{k^2}\right)^{1/2}\times(k^2)^{-2}J^i(k)J_i{}^*(k)f(k^2).$$

[Here $f(s)$ is defined by

$$\tfrac{1}{3}(0|:j_k(x)::j^k(x'):|0)=\frac{-i}{16\pi^2}\int_{4m^2}^{\infty}ds\left(1-\frac{4m^2}{s}\right)^{1/2}\Delta^+(x,\sqrt{s})f(s);$$

$j_k(x)$ is the current of particles to be created in contrast to the current J_k which is the source of the external field.] From this we have the following conclusions: (i) In both cases only the high-frequency parts $k_0\geqq 2m$ are effective, which is just the adiabatic theorem; a slowly varying perturbation does not cause transitions between states separated by a finite energy gap and thus not particle production. (ii) Such high-frequency parts will in general be due to individual motions of the particles generating the external field rather than to collective motions of these which tend to be slower. Electromagnetic production is then much more important than the gravitational one, and both interactions do not solve the problem of asymmetric creation. The only thing really created by these processes is, therefore, radiation. Thus, gravitational production of particles may be neglected, with one possible exception to which we shall turn below.

For the sake of explicitness, we shall briefly consider the production of (massive) neutral spin-zero and (massive) spin-$\frac{1}{2}$ particles. The energy-momentum tensor reads

$$T_{ik}=\tfrac{1}{2}[\Phi_{,i}\Phi_{,k}+\Phi_{,k}\Phi_{,i}-\eta_{ik}(\Phi_{,m}\Phi^{,m}-m^2\Phi^2)],$$
$$T_{ik}=\tfrac{1}{4}i(\bar\psi\gamma_k\psi_{,i}-\bar\psi_{,i}\gamma_k\psi+\bar\psi\gamma_i\psi_{,k}-\bar\psi_{,k}\gamma_i\psi),$$

respectively; for the latter expression we obtain $T=m\bar\psi\psi$, using Dirac's equation. From the commutation relations (for details and normalizations see Ref. 13) we obtain

$$(0|:T(x)::T(x'):|0)=-8m^4\Delta^+\Delta^++8m^2\Delta^+{}_{,i}\Delta^{+,i}-2\Delta^+{}_{,ik}\Delta^{+,ik},$$
$$(0|:T_{ik}(x)::T^{ik}(x'):|0)=-3m^4\Delta^+\Delta^++2m^2\Delta^+{}_{,i}\Delta^{+,i}-\Delta^+{}_{,ik}\Delta^{+,ik},$$

and

$$(0|:T(x)::T(x'):|0)=-m^2\,\mathrm{Tr}S^+(x-x')S^-(x'-x),$$
$$(0|:T_{ik}(x)::T^{ik}(x'):|0)=2m^2\,\mathrm{Tr}S^+\gamma^kS^-\gamma_k-2\,\mathrm{Tr}S^+{}_{,i}\gamma^kS^{-,i}\gamma_k-\mathrm{Tr}S^+{}_{,i}\gamma^iS^-{}_{,k}\gamma^k-\mathrm{Tr}\gamma^kS^+{}_{,k}\gamma^iS^-{}_{,i}-\mathrm{Tr}\gamma^kS^+{}_{,ki}\gamma^iS^--\mathrm{Tr}S^+\gamma^iS^-{}_{,ik}\gamma^k,$$

respectively, where the comma always indicates derivatives with respect to the arguments, and the omitted arguments are $x-x'$ for Δ^+ and S^+, and $x'-x$ for S^-. The spectral representations of these expressions can be found from Table I which might also be helpful elsewhere.

TABLE I. Invariant function.
$$\frac{-i}{16\pi^2}\int_{4m^2}^{\infty}ds\left(1-\frac{4m^2}{s}\right)^{1/2}\Delta^+(x,\sqrt{s})\rho(s).$$

Invariant function[a]	$\rho(s)$
$\Delta^+(x)\Delta^+(x)$	-1 [b]
$\Delta^+{}_{,i}(x)\Delta^{+,i}(x)$	$-(m^2-\frac{1}{2}s)$ [c]
$\Delta^+{}_{,ik}(x)\Delta^{+,ik}(x)$	$-(m^2-\frac{1}{2}s)^2$ [c]
$\mathrm{Tr}S^+(x)\gamma^kS^-(-x)\gamma_k$	$4(s+2m^2)$ [b]
$\mathrm{Tr}S^+(x)S^-(-x)$	$2(4m^2-s)$
$\mathrm{Tr}S^+{}_{,i}(x)\gamma^kS^{-,i}(-x)\gamma_k$	$2(s+2m^2)(s-2m^2)$
$\mathrm{Tr}S^+{}_{,i}(x)\gamma^iS^{-,k}(-x)\gamma_k$	$2m^2(s-4m^2)$
$\mathrm{Tr}\gamma^kS^+{}_{,k}(x)\gamma^iS^-{}_{,i}(-x)$	$2m^2(s-4m^2)$
$\mathrm{Tr}\gamma^kS^{+,}{}_{ki}(x)\gamma^iS^-(-x)$	$2m^2(s-4m^2)$
$\mathrm{Tr}S^+(x)\gamma^iS^-{}_{,ik}(-x)\gamma^k$	$2m^2(s-4m^2)$

[a] Comma = derivative with respect to the argument.
[b] See Ref. 13.
[c] H. V. R. Pietschmann, Phys. Rev. 130, B446 (1965).

Collecting terms, we get for the scalar field

$$f_1(s)=(1/60)(s-4m^2)^2,$$
$$f_2(s)=(1/60)[\tfrac{1}{2}(s-4m^2)^2+10m^2(s-m^2)],$$

and for the spinor field

$$f_1(s)=-(1/30)(s-4m^2)(s+m^2),$$
$$f_2(s)=(1/30)(s-4m^2)[3(s+m^2)+5m^2].$$

It is not immediately obvious that these expressions give rise to positive values of w_2, as is required. An analogous question arises when one calculates electromagnetic pair creation (see, e.g., Ref. 13 for Dirac particles). There one easily proves the property $\theta(k^2)J_i(k)J^{*i}(k)\leqq 0$ for the Fourier transform of a conserved current, and this establishes $w_2\geqq 0$. By the same method (i.e., putting the 1 axis into the direction of $\mathbf{k}$ and using the conservation property) one obtains for $\Upsilon_{ij}(k)$, $k^2 \geqq 0$:

$$0 \leqq \tfrac{1}{3}\Upsilon\Upsilon^* = \tfrac{1}{3}\left| -(1-k_1^2/k_0^2)\Upsilon_{11} - \Upsilon_{22} - \Upsilon_{33}\right|^2 \leqq (1-k_1^2/k_0^2)^2|\Upsilon_{11}|^2 + |\Upsilon_{22}|^2 + |\Upsilon_{33}|^2 \leqq (1-k_1^2/k_0^2)^2|\Upsilon_{11}|^2 + 2(1-k_1^2/k_0^2) \times (|\Upsilon_{12}|^2 + |\Upsilon_{13}|^2) + |\Upsilon_{22}|^2 + |\Upsilon_{33}|^2 + 2|\Upsilon_{23}|^2 = \Upsilon_{ij}\Upsilon^{*ij}.$$

(One of these inequalities is derived by observing the fact that the squared mean value is less than or equal to the mean square, which is also valid for complex numbers, if the square is replaced by the absolute square.) Thus we have the inequality

$$\theta(k^2)[3\Upsilon_{ij}(k)\Upsilon^{*ij}(k) - \Upsilon(k)\Upsilon^*(k)] \geqq 0$$

for the Fourier transform of a conserved energy-momentum tensor. This in turn implies the non-negativity of w_2 in the two cases discussed above.

Now we turn to the exceptional situation mentioned above where particle production might be important. This situation prevails during the first 10^{-20} sec of life of the universe in a "big bang" model. We assume a closed universe expanding from an almost pointlike state according to some expansion law $R(t)$ (R= radius of universe). It turns out that creation rates are determined by the quantity $m\dot{R}$, where m is the rest mass of the particles to be created. Proton production was thus possible only during the first 10^{-20} sec, electron production about 1000 times longer. A naive application of the equations developed above yields particle numbers $N_2 \simeq (T_0 m)^n$, where T_0 is the total lifetime of the expanding (closed) universe and n is of order unity. For $n=2$ one obtains $N_2 = 10^{80}$, which is approximately equal to the number of particles observed today. In this way one arrives at the following cosmological model: We consider a "big bang" that starts with an empty universe which creates the particles contained in it via the time-dependent gravitational field. The question whether the particles or the universe existed earlier is meaningless since the relevant time scale is of the order 10^{-23} sec. Both, particles and universe, originate together in one self-consistent elementary explosion. The main objection against such a model is that creation is charge symmetric. The question arises how annihilation of the particle-antiparticle pairs is prevented. A preliminary answer to this has been given by Omnès,[14] who showed that the $(\frac{3}{2}, \frac{3}{2})$ pion-nucleon resonance gives rise to thermodynamic instability of a homogeneous phase consisting of pions and nucleons against fluctuations of the n and π^- numbers.

[14] R. Omnès (unpublished).

III. EXACTLY SOLUBLE PROBLEMS

As was stressed above, our general result is gauge-invariant, i.e., invariant with respect to general coordinate transformations up to first order in f. Special gravitational fields are known, however, where the particle production can even be calculated exactly within the external field approximation.[15,16] In one of these examples (Ref. 16), the metric tensor is diagonal and depends on time only in the following way:

$$g_{ik}(t) = \theta(|t|-\tau)\eta_{ik} + \theta(\tau - |t|)\gamma_{ik},$$

where $\tau > 0$, $\gamma_{ik} \neq \eta_{ik}\eta_{ik}$ is constant. Putting ($\alpha, \beta = 1, 2, 3$)

$$w^2 = \mathbf{k}^2 + m^2, \quad w_\gamma^2 = (1/\gamma^{00})(\gamma^{\alpha\beta}k_\alpha k_\beta + m^2),$$
$$Y = \gamma^{00}(-\det\gamma_{ik})^{1/2},$$

the result is

$$\langle n^{\text{out}}(\mathbf{k})\rangle = \frac{1}{4}\left(\frac{w_\gamma Y}{w} - \frac{w}{w_\gamma Y}\right)\sin^2 2w_\gamma\tau,$$

where $n^{\text{out}}(\mathbf{k})$ is the number of particles created with momentum $\mathbf{k}$. This result is analogous to the one obtained for a pointlike source of the same time dependence, as far as the dependence on τ is concerned. In the example discussed here, the field strength is zero, apart from two "kicks" at $t=-\tau$ and $t=\tau$. An oscillator is excited at $-\tau$, oscillates freely, and its state of motion after $t=\tau$ depends on the phase, at which the oscillator suffers the second "kick."

Imamura[16] has shown that it is possible in this case to express $n^{\text{out}}(\mathbf{k})$ in terms of the Riemann tensor. It turns out that particle creation can also be calculated exactly in slightly more general models, e.g.,

$$\begin{aligned} g_{ik}(t) &= \eta_{ik}, & t<\tau_0,\quad t>\tau_3 \\ &= \gamma_{ik}^{(1)}, & \tau_0<t<\tau_1 \\ &= \gamma_{ik}^{(2)}, & \tau_1<t<\tau_2, \quad \gamma_{ik}^{(A)} = \text{const} \\ &= \gamma_{ik}^{(3)}, & \tau_2<t<\tau_3. \end{aligned}$$

The results are rather lengthy and will not be given here (for details see Ref. 17). It turns out, however, that the number of particles created can no longer be expressed in terms of the curvature tensor in this generalized model.

[15] F. L. Scarf, in *Les Théories Relativistes de la Gravitation*, (Centre National de la Recherche Scientifique, Paris, 1962).
[16] T. Imamura, Phys. Rev. **118**, 1430 (1960).
[17] H. K. Urbantke, Ph.D. thesis, University of Vienna, 1965 (unpublished).