β- and γ-Vibrational Bands of ^{152}Sm and ^{154}Gd

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Employing radioactive sources of ^{152}Eu (12.4 yr) and ^{154}Eu (16 yr) in both γ-ray singles and γ-γ coincidence experiments, we have determined the intensities of most of the weak γ-ray transitions depopulating the 0+, 2+, and 4+ members of the β-vibrational bands and the 2+, 3+, and 4+ members of the γ-vibrational bands in ^{152}Sm and ^{154}Gd. From these γ-ray intensities, ratios of reduced $E2$ transition probabilities have been determined and a detailed analysis of band mixing in these nuclei has been accomplished. Mixing of the β and γ bands into the ground-state rotational band and into each other has been considered but is found to be grossly insufficient in explaining the $B(E2)$ ratios from members of the β band. However, this treatment does seem to be adequate in explaining the ratios from members of the γ band in each nucleus. From our γ-ray intensities and literature values for the internal-conversion electron intensities and the reduced $E2$ transition probabilities, it has been possible to evaluate the reduced nuclear matrix element ρ for the electric-monopole transitions between the β and ground-state bands in each nucleus. For ^{152}Sm, ρ is determined to be 0.28 ± 0.02, while only an estimate of 0.44 is possible for ^{154}Gd. Values of X, the dimensionless ratio of the squares of the $E0$ to $E2$ reduced matrix elements, are determined and compared to the model predictions. The experimental values are generally 2.5 to 5 times smaller than predicted. Furthermore, the effects of nonadiabatic perturbations on the ground-state rotational band are considered. The contributions of centrifugal stretching to the observed energy shifts and to changes in radius within the rotational spectra can be estimated from our experimental determination of the band-mixing amplitudes. The results indicate that centrifugal stretching of the nucleus cannot explain the energy shifts in ^{152}Sm and ^{154}Gd but probably can account for the observed changes in radius for ^{152}Sm and ^{154}Gd.

I. INTRODUCTION

THE even-even nuclei ^{152}Sm and ^{154}Gd are at the very beginning of the deformed rare-earth region. Each of these nuclei displays three low-lying bands of energy levels, which are thought to result from β and γ vibrations in the nuclear surface and from rotations of the nucleus in excited and unexcited intrinsic states. A detailed study of the properties of these collective excitations is important in order to find if collective effects of nuclei in a transition region from spherical to deformed shapes can be described in the same manner as such effects in strongly deformed nuclei. Due to the "softness" of the ^{152}Sm and ^{154}Gd nuclei to vibrations in the nuclear surface, the β- and γ-vibrational bands lie at lower excitation energies than the corresponding levels of the more rigid nuclei in the middle of the deformed rare-earth region. Thus, these levels are more accessible by radioactive decay and Coulomb excitation techniques than those of the heavier nuclei. This permits critical tests of models attempting to describe collective effects.

Studies of the collective properties of ^{152}Sm and ^{154}Gd are divided into three parts. In the first, $E2$ branching ratios from members of the β- and γ-vibrational bands are measured experimentally. A comparison of these ratios to predictions of the symmetric-rotor model of Bohr and Mottelson[1] demonstrates that nonadiabatic corrections must be made to this model. This is done by changing the wave function of the β, γ, or ground-state band to include small contributions from the other two in the usual perturbation approach. The amplitudes of these admixtures are then found by fitting the new predictions to the experimental branching ratios. If this fitting process yields a consistent set of amplitudes, the basic ideas of rotational motion, which are essential to this treatment, are verified without the use of a detailed theory.

The second approach involves a study of the electric monopole ($E0$) transitions from members of the β-vibrational bands to members of the ground-state rotational bands in ^{152}Sm and ^{154}Gd. The interactions leading to $E0$ transitions occur only while the atomic electron is within the nuclear volume where it can sense any change in the proton charge distribution. Consequently, such electric monopole processes should be sensitive to subtle details of nuclear structure and should serve as good tests of particular nuclear models.

The form of the nuclear $E0$ matrix element M is

$$M=\rho R^2, \tag{1}$$

where ρ, the reduced nuclear $E0$ matrix element or "nuclear strength parameter," is given by

$$\rho\simeq\sum_p\int\psi_f^*(r_p/R)^2\psi_i d\tau. \tag{2}$$

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[1] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **27**, 16 (1953).

Here r_p is the position vector of the pth proton and R is the nuclear radius. There are higher-order terms of r_p but they are usually of minor importance. Church and Weneser[2] first pointed out that, in addition to $E0$ transitions between two 0+ states, there should occur $E0$ transitions of observable intensities between any two states of the same angular momentum and parity, since there are no angular-momentum variables in the $E0$ transition operator.

It has been suggested by Rasmussen[3] that, since the monopole matrix element is of the form $\langle r^2 \rangle$, one might expect the oscillations of a deformed nucleus about its equilibrium shape to provide a collective contribution to the $E0$ process. Indeed, several $E0$ transitions have now been observed between members of β-vibrational and ground-state rotational bands in deformed nuclei (see, for example, Refs. 4–6). We have used our data in conjunction with literature values of the K-shell internal-conversion electron intensities[7–9] and of the reduced $E2$ transition probabilities from Coulomb excitation[10–12] of the β bands to compute values of the reduced $E0$ matrix element ρ for each nucleus. These values are then compared with the predictions[3,13–16] of several theoretical treatments.

In the third part, the effects of nonadiabatic perturbations on the ground-state rotational bands are considered. The shifts in the energies of these levels resulting from band mixing are determined. Also, the monopole matrix element found for each nucleus is used to estimate the change in radius between the 0+ and 2+ levels of the ground-state band, as expected from band mixing. This change in radius is compared to the values observed through muonic atom and isomer shift experiments.[17–19]

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For the accumulation of the experimental data in the present studies, we have utilized Ge(Li) and NaI detectors to perform singles and coincidence measurements on radioactive sources of long-lived ^{152}Eu and ^{154}Eu. Although we are concerned with the general decay properties of these two nuclei, particular emphasis was placed on a determination of the γ-ray intensities for the transitions depopulating the 0+, 2+, and 4+ members of the β-vibrational bands and the 2+, 3+, and 4+ members of the γ-vibrational bands in the daughter nuclei ^{152}Sm and ^{154}Gd. It is true that previous γ-ray decay studies[20–22] have been made on these two nuclei, but it is only with the recent advent of high-resolution Ge(Li) detectors that an extensive study of these transitions from the β and γ bands has become feasible. Results of some of our earlier experiments are given in Refs. 23–28.

II. EXPERIMENTAL PROCEDURE

The 12.4-yr ^{152}Eu source was prepared by neutron irradiation of Eu_2O_3 enriched to 92% in ^{151}Eu. Our γ-ray spectra indicate that there was about 1.3% ^{154}Eu source contamination. Similarly, the 16-yr ^{154}Eu source was obtained from neutron irradiation of enriched ^{153}Eu. The ^{152}Eu contamination in the ^{154}Eu source was less than 1%.

Both γ-ray singles and γ-γ coincidence measurements were performed on the nuclei. Lithium-drifted germanium detectors Ge(Li) of 6-, 20-, and 35-cm^3 active volume were used in the singles experiments. The data reported previously[27] were obtained through the use of a spectrometer consisting of the 6-cm^3 Ge(Li) detector, a Tennelec TC-130 preamplifier, TC-200 amplifier, and TC-250 post-biased amplifier, and a 1600-channel Victoreen SCIPP pulse-height analyzer. Recent experiments have been performed using the 20-cm^3 Ge(Li) detector and a TC-135 preamplifier coupled to the other components mentioned above, and also the 35-cm^3 detector with Tennelec electronics and a Nuclear Data 4096-channel analyzer. The resolution obtained with the last arrangement was approximately

[20] *Nuclear Data Sheets*, compiled by K. Way *et al.*, (U.S. Government Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington, D.C., 1964).
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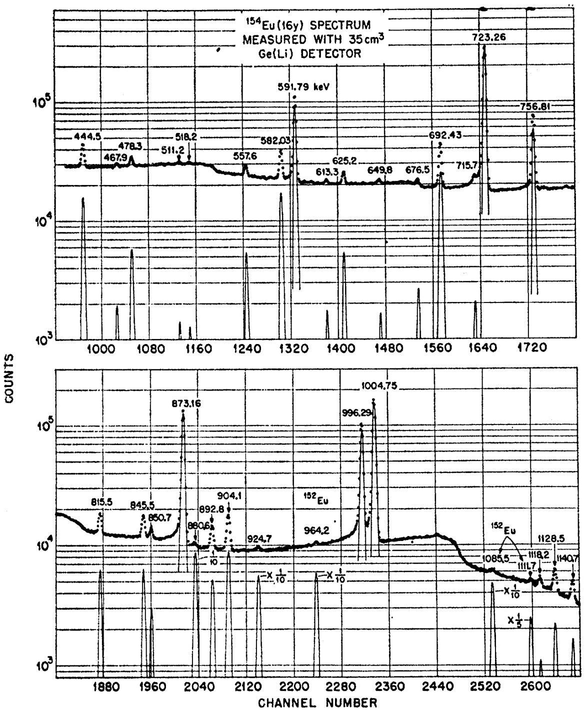

FIG. 1. Portion of ^{154}Eu γ-ray spectrum containing transitions from the β- and γ-vibrational bands in the daughter nucleus ^{154}Gd.

2.7-keV full width at half-maximum (FWHM) for 1332 keV.

The efficiencies of the detectors were measured over a range of 0.06–2.75 MeV through the use of standard sources with known disintegration rates and of sources with transitions of well-known relative intensities. Absolute intensities of the transitions of some of these sources were measured with a 7.5-cm×7.5-cm NaI detector, while the strengths of others were specified upon purchase. The errors in the efficiency calibrations of the Ge(Li) detectors are estimated to be 5% for a determination of relative intensities. The detectors were energy calibrated using various standard sources over the range of interest. Accurate energies of the intense transitions in ^{152}Sm and ^{154}Gd were determined by counting ^{152}Eu or ^{154}Eu and sources with transitions of well-known energies (as given by Marion[29]) simultaneously on the spectrometer involving the 35-cm^3 Ge(Li) detector and the 4096-channel analyzer. This system was checked for linearity of pulse-height response with a precision pulser and was found to deviate by no more than 0.6 channels from the best straight line over 90% of the total region. Energies of the weaker γ rays in ^{152}Sm and ^{154}Gd were then determined using the energies of these strong γ rays.

Both NaI-NaI and Ge(Li)-NaI γ-γ coincidence experiments were performed. The NaI detectors were 7.5 cm×7.5 cm and the Ge(Li) detector was 6 cm^3, since the 20- and 35-cm^3 detectors had not been yet obtained. Antiscattering baffles were used in all coincidence measurements. The data were accumulated in a 100×200-channel matrix with a Victoreen multiparameter analyzer. Two sets of Ge(Li)-NaI measurements were performed. In the first, "crossover-pickoff" timing was used and the coincidence resolving time was $2\tau=178$ nsec; in the second, we were able to reduce the resolving time to $2\tau=69$ nsec by employing "leading-edge" timing. The results in both cases were essentially the same. Corrections for random coincidences were made by a computer program.

III. EXPERIMENTAL RESULTS

Portions of the γ-ray spectra resulting from the decays of ^{154}Eu and ^{152}Eu are given in Figs. 1 and 2, respectively. The 35-cm^3 Ge(Li) detector was used to obtain these data. The transitions from the β- and γ-vibrational bands to the ground band are of prime interest in this paper. We therefore show only the regions of the spectra containing these transitions.

γ-ray intensities I_γ are found from the γ-ray spectra generally through the fitting of standard Gaussian curves to the peaks after background had been subtracted. The intensities for the pertinent transitions are given in column 5 of Table I and in columns 4 and 6 of Table II. The errors on the intensities result from the uncertainty in the efficiency calibration, from statis-

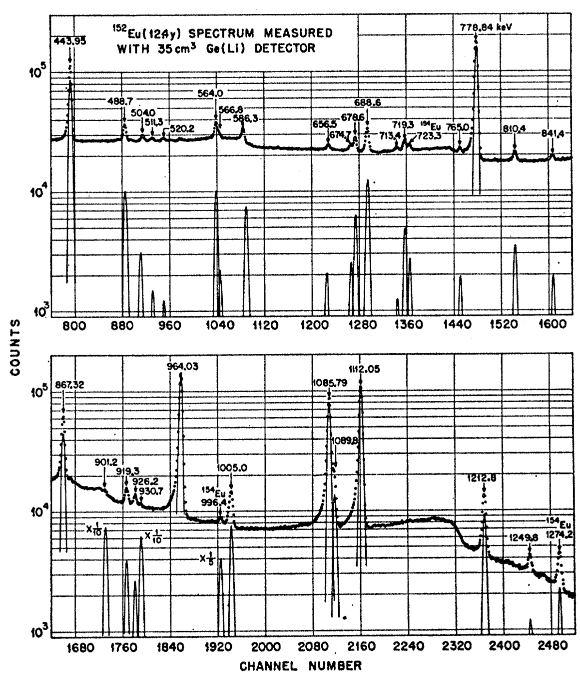

FIG. 2. Portion of ^{152}Eu γ-ray spectrum containing transitions from β- and γ-vibrational bands in the daughter nucleus ^{152}Sm.

[29] J. B. Marion, Nucl. Data 4, 301 (1968).

TABLE I. Relative γ-ray and electron intensities between members of the β-vibrational and ground-state bands in ^{152}Sm and ^{154}Gd.

Nucleus	$(I\pi)_i$ [a]	$(I\pi)_f$	Transition energy (keV)	I_γ [b]	$10^3\times I_{ek}$ [c]	$10^3\times I_{ek}$ ($E0$)
^{152}Sm	4+″	4+	656.5	0.42±0.10	25±5	23±5
	4+″	2+	901.2	0.13±0.05		
	2+″	4+	444.0	0.9±0.3		
	2+″	2+	688.6	2.91±0.23	108±8	94±8
	2+″	0+	810.4	1.00±0.13		
^{154}Gd	4+″	4+	676.5	0.43±0.11	27±4	25±3
	4+″	2+	924.7	0.19±0.10		
	2+″	4+	444.6	1.69±0.15		
	2+″	2+	692.43	4.97±0.30	233±13	207±13
	2+″	0+	815.5	1.38±0.18		
	0+″	2+	557.6	0.74±0.10		
	0+″	0+	681.0		21±5	21±5

[a] Double primes refer to members of the β band.
[b] γ-ray intensities normalized to 100 for the 344-keV transition in ^{152}Gd and for the 1274-keV transition in ^{154}Gd.
[c] Electron intensities normalized to 3.0 for the 344-keV transition of ^{152}Gd and to 0.068 for the 1274-keV transition in ^{154}Gd. Intensities for ^{152}Sm are from Ref. 7; those for ^{154}Gd from Ref. 9.

tical considerations, and from possible variations in the choice of backgrounds under the peaks. The first of these errors is estimated to be approximately 5%, the second is almost always negligible, and the third becomes dominant only for low-intensity γ rays. Partial level schemes of ^{154}Gd and ^{152}Sm containing the transitions from the β and γ bands to the ground-state band are shown in Figs. 3 and 4, respectively.

A. ^{154}Gd

The γ-ray intensities which are listed in Tables I and II for the transitions from the β and γ bands in ^{154}Gd are those obtained from singles experiments involving the 20- and 35-cm^3 Ge(Li) detectors. These values are in good agreement with the results[27] of earlier experiments with the 6-cm^3 detector, although the new values have substantially lower error limits due to the improved quality of the spectra resulting from the larger volume and better resolution of the detectors used.

As discussed in earlier reports[27,30] and as will be further discussed in Sec. IV A of this paper, the singles γ-ray intensities of the three transitions from the 2″ state in ^{154}Gd yield values of the band mixing param-

TABLE II. Relative γ-ray intensities between members of the γ vibrational and ground-state bands in ^{152}Sm and ^{154}Gd.

		^{152}Sm		^{154}Gd	
$(I\pi)_i$ [a]	$(I\pi)_f$	Transition energy (keV)	I_γ [b]	Transition energy (keV)	I_γ [b]
4+′	4+	1005.0	2.40±0.24	892.8	1.31±0.10
4+′	2+	1249.8	0.64±0.09	1140.7	0.69±0.10
3+′	4+	867.32	14.1±0.7	756.81	12.9±0.6
3+′	2+	1112.05	47.9±2.4	1004.75	50.6±2.5
2+′	4+	719.3	1.11±0.16	625.2	0.89±0.12
2+′	2+	964.03	51.2±2.6	873.16	34.8±1.7
2+′	0+	1085.79	36.3±2.2	996.29	29.4±1.5

[a] Primes refer to members of the γ band.
[b] γ-ray intensities normalized to 100 for the 344-keV transition in ^{152}Gd and for the 1274-keV transition in ^{154}Gd.

[30] Y-t Liu, O. B. Nielsen, P. Salling, and O. Skilbreid, Bull. Acad. Sci. USSR, Phys. Ser. **31**, 69 (1967).

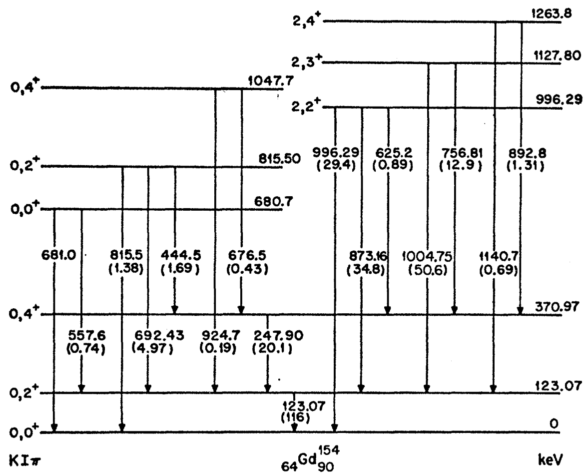

FIG. 3. Partial level scheme of ^{154}Gd showing the transitions from the β- and γ-vibrational bands. The 681.0-keV transition is based on the electron measurements of Brantley *et al.* (Ref. 9).

eter Z which are not internally consistent. (Note that double primes refer to members of the β band, single primes to members of the γ band; spins with no primes correspond to levels in the ground-state band). The singles intensity of the $2''\rightarrow 2$ transition is approximately twice the $E2$ intensity needed to provide internal consistency in the Z_β values. To check this point, we have made extensive coincidence measurements on the 692.43-keV transition in ^{154}Gd. These experiments indicate that this peak is not composite and that it is appropriately placed as a transition between the 815.50- and 123.07-keV levels. Furthermore, the placement of all, except two, of the γ-ray transitions from the members of the β and γ bands has been verified in our coincidence experiments. The 924.7- and 1140.7-keV γ rays are assigned to transitions on the basis of energy fits.

All of the γ-ray transitions from the β and γ bands in ^{154}Gd are considered to be $E2$ in character. This is certainly a valid assumption for the $I\rightarrow I\pm 2$ transitions from the β and γ bands down to the ground-state band. The $2''\rightarrow 2$ transition has been shown by Hamilton *et al.*[31] to be essentially pure $E2$, while Rasera *et al.*[32] have found the $E2$ component of the $2'\rightarrow 2$ transition to be 100 times larger than any possible $M1$ component. Debrunner and Kündig[33] have measured the absolute value of $\delta=(E2/M1)^{1/2}$ to be greater than 11 for the $3'\rightarrow 2$ transition.

In column 6 of Table I are shown electron intensities taken from Brantley *et al.*[9] for ^{154}Gd. Electron intensities are given only for those transitions ($I''\rightarrow I$), where the $E0$ mode is expected to occur. We can determine the amount of the $E0$ mode present by using the theoretical $E2$ K-conversion coefficients of Sliv and Band[34] to find the $E2$ K-electron intensity. (It is now known[35] that the K-conversion coefficients for the nuclei in this region are in good agreement with theory.) The $E0$ intensity, given in column 7 of Table I, is the difference between the total K-electron intensity and the theoretical $E2$ K-electron intensity. A comparison of columns 6 and 7 shows that in each case most of the observed electron intensity for the transition, $I''\rightarrow I$, is due to the $E0$ mode of deexcitation.

B. ^{152}Sm

The intensities listed in Tables I and II for ^{152}Sm are those obtained from singles experiments in all cases except three. In the first case, the intensity of the 1005.0-keV γ ray has been reduced by 25% to account for contamination from the intense 1004.75-keV γ ray of ^{154}Eu. Second, 0.4 unit of the peak seen at 964.03 keV in the singles spectrum of Fig. 2 is attributed to a transition from a 1− state at 963.2 keV to the ground state. This level is strongly populated in the decay of the 0− isomeric state (9.3 h) of ^{152}Eu but only weakly in the decay of the 3− ground state of ^{152}Eu. The 841.4-keV γ ray of the $1-\rightarrow 2+$ transition is observed in the spectrum of Fig. 2. From our intensity of this γ ray and from the value of $I_\gamma(963.2)/I_\gamma(841.4)=0.85$ as measured by Dzhelepov *et al.*[36] in the decay of ^{152m}Eu, we arrive at our estimate of the contribution of the $1-\rightarrow 0+$ transition to the 964.03-keV peak. The

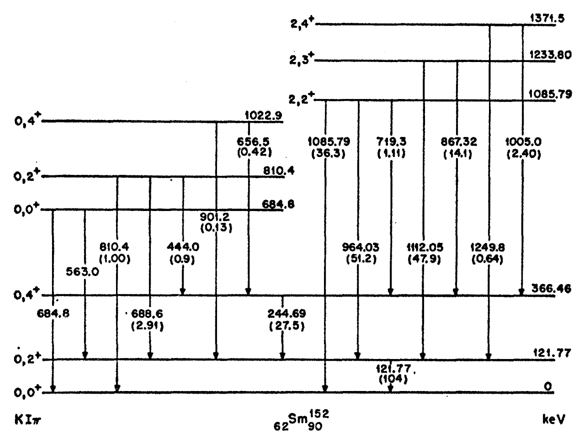

FIG. 4. Partial level scheme of ^{152}Sm showing the transitions from the β- and γ-vibrational bands. The 684.8- and 563.0-keV transitions were observed by Andersson and Ewan (Ref. 39).

[31] J. H. Hamilton, A. V. Ramayya, and L. C. Whitlock, Phys. Rev. Letters **22**, 65 (1969).

[32] R. L. Rasera, J. Lange, W. Schäffner, W. Kesternich, and E. Bodenstedt, Bull. Am. Phys. Soc. **13**, 671 (1968).

[33] P. Debrunner and W. Kündig, Helv. Phys. Acta **33**, 395 (1960).

[34] L. A. Sliv and I. M. Band in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Co., Amsterdam, 1965), p. 1639.

[35] R. S. Dingus, W. L. Talbert, and M. G. Stewart, Nucl. Phys. **83**, 545 (1966).

[36] B. S. Dzhelepov, N. M. Zhukovskii, and A. S. Maloyan, Yadern Fiz. **1**, 941 (1965), [English transl.: Soviet J. Nucl. Phys. **1**, 671 (1965)].

remainder of this intensity is assigned to the $2'\rightarrow2$ transition.

In the third case, the coincidence data indicate that the intensity of the $2''\rightarrow4$ transition is 0.9 ± 0.3, which is less than 10% of the total intensity seen at 443.95 keV in the singles spectrum of ^{152}Eu. This conclusion is reached mainly from the coincidence spectrum shown in Fig. 5, which results from gating on the 244.69-keV transition with a Ge(Li) detector. The 1213-keV peak in this spectrum results from a transition of that energy to the 366.46-keV level. The remainder of the 443.95-keV singles intensity is observed to be in coincidence with the 964.03-keV γ ray, indicating that a 444-keV transition also feeds the 2+ member of the γ band. This transition is only very weakly in coincidence with the 244.69-keV γ ray, since the $2'\rightarrow4$ feeding cascade is very weak.

There is poor agreement in the Z_β values determined from the three transitions from the 2+ member of the β band in ^{152}Sm, as is the case in ^{154}Gd. Therefore, we have performed coincidence measurements on the 688.6-keV γ ray. The data are not quite of the same statistical quality as in the ^{154}Gd experiment but they show no indications that this γ ray results from other than the $2''\rightarrow2$ transition. There is, however, additional evidence that supports this conclusion. Sayer[37] has observed, after Coulomb excitation of the ^{152}Sm nucleus,

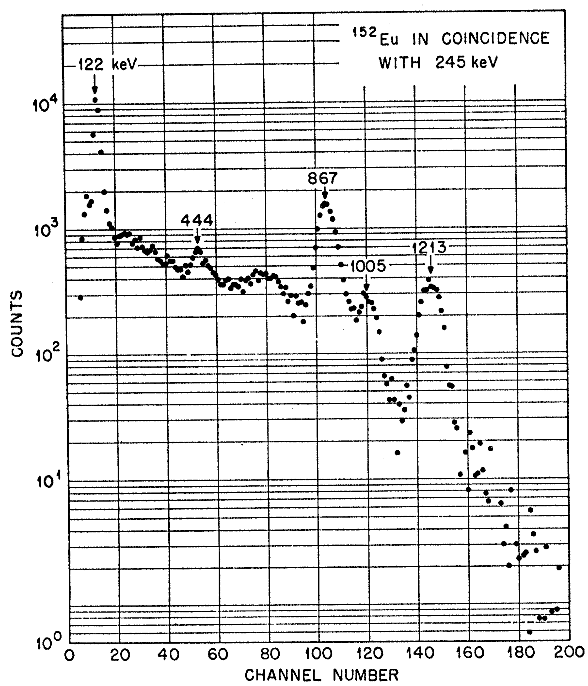

FIG. 5. γ-ray spectrum (NaI) of ^{152}Eu in coincidence with the 245-keV transition. The gating detector was Ge(Li).

[37] R. O. Sayer, Oak Ridge National Laboratory Report No. ORNL-TM-2211, 1968 (unpublished).

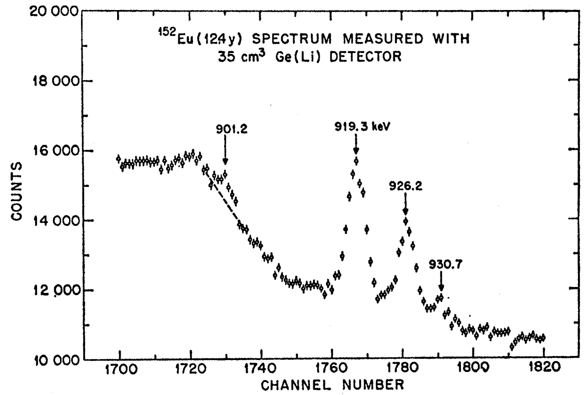

FIG. 6. Expanded section of ^{152}Eu γ-ray spectrum shown in Fig. 2.

γ rays corresponding to the three transitions from the $2''$ level and gives relative γ-ray intensities which are very similar to ours. Since only some of the levels in ^{152}Sm populated by radioactive decay of ^{152}Eu are populated also by Coulomb excitation, it is unlikely that two transitions at approximately 688.6 keV would occur with the same relative intensities in each process.

Of the transitions from the γ band, only the one at 1249.8 keV has been placed solely as a result of energy considerations. The placements of the other γ-band transitions have been verified by our coincidence measurements. Details of the remaining parts of the ^{152}Sm and ^{154}Gd level schemes will be reported at a later date.

As in ^{154}Gd, all of the γ-ray transitions from the β band in ^{152}Sm are assumed to result from the $E2$ mode. Results of angular distribution experiments by McGowan *et al.*[11] on the 688.6-keV $2''\rightarrow2$ transition are compatible with the assumptions of pure $E2$ radiation. However, not all of the transitions from the γ band can be considered entirely $E2$ in character. The experiments of McGowan *et al.*[11] indicate that the $2'\rightarrow2$ transition contains a 7% $M1$ contribution. Bisgaard *et al.*[38] found the $3'\rightarrow4$ transition to be only $(97.5_{-0.7}^{+0.5})$% $E2$, while Debrunner and Kundig[33] obtained $\delta>7$ for the $3'\rightarrow2$ transition.

The very weak peak in the singles spectrum at 901.2 keV is assigned to the $4''\rightarrow2$ transition on the basis of energy fit. It was not observed in the earlier work[27] involving the 6-cm^3 Ge(Li) detector, since it was obscured by the compton edge of the 1112.05-keV γ ray. The region around 900 keV in the spectrum of Fig. 2 is enlarged and shown in Fig. 6. Error bars on the points are given in the latter figure.

The electron intensities from Malmsten *et al.*[7] are given in Table I for the $4''\rightarrow4$ and $2''\rightarrow2$ transitions in ^{152}Sm. These intensities are in excellent agreement with the earlier values of Katoh and Spejewski.[8] The values of the former group are used, because the resolution of

[38] K. Maack Bisgaard, K. Bonde Nielsen, and J. Sodemann, Phys. Letters 7, 57 (1963).

their spectrometer allows lower error limits than those of the latter group. The $E0$ components found in the $2''\to2$ and $4''\to4$ transitions are observed to be large, as is the case for ^{154}Gd. The $0''\to0$ transition was not observed either by Malmsten *et al.*[7] or by Katoh and Spejewski,[8] probably due to the proximity of the strong 688.6-keV conversion-electron line. Andersson and Ewan[39] have observed the $0''\to0$ transition in the decay of 9.3-h ^{152m}Eu and have measured the $E0$ K-electron intensity of this 684.8-keV transition relative to the photon intensity of the $0''\to2$, 563.0-keV transition as 0.014 ± 0.002. This result is quite similar to the earlier value of 0.013 ± 0.001 observed by Marklund *et al.*[40] There has existed some question about this latter value since they had failed to see a close-lying line only 4 keV away. However, the agreement between the two results probably does imply that one should not halve the value of Marklund *et al.*[40] as suggested[3] previously. Our coincidence experiments indicate that 1.4 ± 0.4 units of the 564.0-keV γ ray observed in the singles spectrum of Fig. 2 feed the 1085.79-keV state, while the total singles intensity is 1.91 ± 0.19. This leaves 0.5 ± 0.4 units for the $0''\to2$ transition. If the energy of this transition from the β band is actually 563.0 keV, its contribution to the 564.0-keV peak is very small, since the latter peak is not noticeably widened on the low side in Fig. 2. The intensities of the $0''\to0$ and $0''\to2$ transitions are not entered in Table I, since we will merely use the ratio of these intensities measured by Andersson and Ewan.[39]

IV. DISCUSSION

A. Band Mixing

One test required for any nuclear model which attempts to describe collective phenomena is to find if it can predict the branching ratios of levels resulting from these collective effects. The simple Bohr-Mottleson approach[1] is to assume that the nucleus is an axially-symmetric rotor, the rotational and intrinsic motions of which do not disturb each other. This allows the $E2$ transition matrix element between members of two collective bands to be expressed as a product of a Clebsch-Gordan coefficient and a reduced matrix element which is independent of the spins of the levels involved. A ratio of reduced $E2$ transition probabilities between members of such bands is then merely a ratio of the appropriate Clebsch-Gordan coefficients squared, as was pointed out by Alaga *et al.*[41] However, it has been known for some time that for the γ-vibrational band these simple predictions do not hold and that the adiabatic assumption for the axially-symmetric nucleus is the source of the problem. Nonadiabatic coupling of the intrinsic and rotational motions of the nucleus mixes the wave functions of the vibrational and rotational bands and leads to corrections in the predicted reduced $E2$ transition probability $B(E2)$. If this problem is treated phenomenologically, the exact form of the coupling need not concern us. We wish to find if a simple mixing of β, γ, and ground-state bands can account for all the deviations of the observed $B(E2)$ ratios from the adiabatic predictions.

The state function of the adiabatic nucleus can be written as $|nKI\rangle$, which can be broken down as usual into the rotation matrix times a function describing the intrinsic state of the nucleus $|nK\rangle$. The quantum number n refers to the order of the nuclear vibration, while K is the projection of the total angular momentum I on the nuclear symmetry axis. Members of the β, γ, and ground-state bands can be written as $|10I\rangle$, $|12I\rangle$, and $|00I\rangle$, respectively. Each of these functions is perturbed by the coupling between intrinsic and rotational motion so that the correct state functions become linear combinations of the zero-order functions

$$|00I\rangle=|00I\rangle_0-\epsilon_\beta f_\beta(I)|10I\rangle_0-\epsilon_\gamma f_\gamma(I)|12I\rangle_0, \tag{3a}$$

$$|10I\rangle=|10I\rangle_0+\epsilon_\beta f_\beta(I)|00I\rangle_0+\epsilon_{\beta\gamma} f_\gamma(I)|12I\rangle_0, \tag{3b}$$

$$|12I\rangle=|12I\rangle_0+\tfrac{1}{2}[1+(-)^I]\epsilon_\gamma f_\gamma(I)|00I\rangle_0 -\tfrac{1}{2}[1+(-)^I]\epsilon_{\beta\gamma} f_\gamma(I)|10I\rangle_0. \tag{3c}$$

In these equations, ϵ_β, ϵ_γ, and $\epsilon_{\beta\gamma}$ are those parts of the perturbation amplitudes depending on the intrinsic variables only. The spin dependencies of the perturbation amplitudes are contained entirely in $f_\beta(I)$ and $f_\gamma(I)$. From the form of the perturbation matrix element, one can find that $f_\beta(I)=I(I+1)$ and $f_\gamma(I)=[2(I-1)I(I+1)(I+2)]^{1/2}$. As is clear from Eq. (3) we have included not only mixing between the β or γ band and the ground-state band but also mixing between the β and γ bands.

Corrections for band mixing can be made to the reduced $E2$ transition probability by using the modified state functions in the transition matrix element. In deriving the new form of $B(E2)$, one assumes that the intrinsic quadrupole moments for the bands are equal

$$\langle 12|Q(E2)|12\rangle=\langle 10|Q(E2)|10\rangle =\langle 00|Q(E2)|00\rangle =Q_{00}, \tag{4}$$

where $Q(E2)$ is the electric quadrupole operator. Also, one neglects terms quadratic in ϵ and terms proportional

[39] I. Andersson and G. T. Ewan (private communication from G. T. Ewan); Atomic Energy of Canada Limited, Chalk River Ontario, Physics Division Progress Report No. PR-P-73, 1967 (unpublished).

[40] I. Marklund, O. Nathan, and O. B. Nielsen, Nucl. Phys. **15**, 199 (1960).

[41] G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, 9 (1955).

TABLE III. Correction factors for the reduced $E2$ transition probability between members of the β and γ bands and members of the ground-state band.[a]

I_i $K=0$ or 2	I_f	$\dfrac{B(E2;12I_i\to 00I_f)}{B_0(E2;12I_i\to 00I_f)}$	$\dfrac{B(E2;10I_i\to 00I_f)}{B_0(E2;10I_i\to 00I_f)}$
$I-2$	I	$[1+(2I+1)Z_\gamma+I(I-1)Z_{\beta\gamma}]^2$	$[1+2(2I-1)Z_\beta-(I-2)(I-3)\zeta_{\beta\gamma}]^2$
$I-1$	I	$[1+(I+2)Z_\gamma]^2$	
I	I	$[1+2Z_\gamma-\frac{1}{3}I(I+1)Z_{\beta\gamma}]^2$	$[1+3(I-1)(I+2)\zeta_{\beta\gamma}]^2$
$I+1$	I	$[1-(I-1)Z_\gamma]^2$	
$I+2$	I	$[1-(2I+1)Z_\gamma+(I+1)(I+2)Z_{\beta\gamma}]^2$	$[1-2(2I+3)Z_\beta-(I+3)(I+4)\zeta_{\beta\gamma}]^2$

[a] Previously given in Ref. 42 in a slightly different convention.

to $\langle 12\,|\,Q(E2)\,|\,10\rangle$. Because of the poor overlap of the wave functions of the β- and γ-vibrational states,[42] the latter terms are negligible compared to terms in $\langle 12\,|\,Q(E2)\,|\,00\rangle=Q_\gamma$, $\langle 10\,|\,Q(E2)\,|\,00\rangle=Q_\beta$, and Q_{00}. For a transition from a state of spin I_i of the γ band to a state of I_f in the ground band, the $B(E2)$ value becomes

$$B(E2;12I_i\to 00I_f)=B_0(E2;12I_i\to 00I_f)\times[1+Z_\gamma F_\gamma(I_i,I_f)+Z_{\beta\gamma}F_{\beta\gamma}(I_i,I_f)]^2, \quad (5)$$

where

$$Z_\gamma=-(\sqrt{24})\epsilon_\gamma(Q_{00}/Q_\gamma), \quad (6)$$

$$Z_{\beta\gamma}=-(\sqrt{6})\epsilon_{\beta\gamma}(Q_\beta/Q_\gamma), \quad (7)$$

$$F_\gamma(I_i,I_f)=(\sqrt{24})^{-1}\left[f_\gamma(I_f)\frac{\langle I_i220\,|\,I_f2\rangle}{\langle I_i22-2\,|\,I_f0\rangle}-\tfrac{1}{2}[1+(-)^{I_i}]f_\gamma(I_i)\frac{\langle I_i200\,|\,I_f0\rangle}{\langle I_i22-2\,|\,I_f0\rangle}\right], \quad (8)$$

and

$$F_{\beta\gamma}(I_i,I_f)=\tfrac{1}{2}[1+(-)^{I_i}]\frac{f_\gamma(I_i)}{\sqrt{6}}\frac{\langle I_i200\,|\,I_f0\rangle}{\langle I_i22-2\,|\,I_f0\rangle}. \quad (9)$$

For a transition from the β band, one gets for the reduced transition probability

$$B(E2;10I_i\to 00I_f)=B_0(E2;10I_i\to 00I_f)\times[1+Z_\beta F_\beta(I_i,I_f)+\zeta_{\beta\gamma}F_{\beta\gamma}'(I_i,I_f)]^2, \quad (10)$$

where

$$Z_\beta=-\epsilon_\beta(Q_{00}/Q_\beta), \quad (11)$$

$$\zeta_{\beta\gamma}=\frac{1}{6}\frac{Q_\gamma^2}{Q_\beta^2}Z_{\beta\gamma}=\frac{1}{6}\frac{B_0(E2;000\to 122)}{B_0(E2;000\to 102)}Z_{\beta\gamma}, \quad (12)$$

$$F_\beta(I_i,I_f)=f_\beta(I_f)-f_\beta(I_i), \quad (13)$$

and

$$F_{\beta\gamma}'(I_i,I_f)=-(\sqrt{6})f_\gamma(I_i)\frac{\langle I_i22-2\,|\,I_f0\rangle}{\langle I_i200\,|\,I_f0\rangle}. \quad (14)$$

[42] P. O. Lipas, Nucl. Phys. **39**, 468 (1962).

In each case, $B_0(E2)$ is the unmixed reduced transition probability and the various Z factors are the mixing parameters. The F factors are evaluated for various combinations of I_i and I_f and are given in Table III. This table was first presented by Lipas.[42] It is reproduced here for convenience to the reader and because there is a slight change in his correction factors due to the way in which Z_β is defined in the paper. The definitions of Z_β, Z_γ, and $Z_{\beta\gamma}$ and the sign conventions correspond to those in recent works.[43–45] This notation represents a shift from that used in our previous work.[27]

Using the experimental $B(E2)$ ratios obtained from relative γ-ray intensities, one is able to calculate the various mixing parameters, since a $B_0(E2)$ ratio is merely a ratio of Clebsch-Gordan coefficients squared. If the above perturbational treatment of the nonadiabatic mixing is legitimate, then the values of Z_β, Z_γ, and $Z_{\beta\gamma}$ calculated from the various branching ratios of members of the β and γ bands should be consistent, since these parameters represent the spin-independent parts of the mixing amplitudes.

In the above treatment, we have included the first-order effects of mixing of the β or γ band into the ground-state band and the second-order effect of mixing between the β and γ bands. Additional second-order correction terms to $B_0(E2)$ would arise by allowing the intrinsic quadrupole moments of the β, γ, and ground-state bands to be different, i.e., by not setting the requirements expressed in Eq. (4). However, including such terms in Eqs. (5) and (10) would result in yet another mixing parameter to be found from the experimental $B(E2)$ ratios and would thus complicate a phenomenological analysis of band mixing. Certainly one could neglect β-γ band mixing and take into account variations of the quadrupole moments for the three collective bands, as is described by Mottelson.[46] We

[43] E. R. Marshalek, Phys. Rev. **158**, 993 (1967).
[44] C. Gunther and D. R. Parsignault, Phys. Rev. **153**, 1296 (1967).
[45] O. Nathan and S. G. Nilsson in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), p. 601.
[46] B. R. Mottelson, J. Phys. Soc. Japan Suppl. **24**, 87 (1968).

TABLE IV. Experimental and theoretical ratios of reduced $E2$ transition probabilities from members of the γ band in ^{152}Sm and ^{154}Gd.

Nucleus	$\frac{I'\to I_1{}^a}{I'\to I_2}$	$\frac{B(E2;I'\to I_1)}{B(E2;I'\to I_2)}$ Expt	Theory [b]	$10^2\times Z_\gamma$ [c]	$10^2\times Z_{\beta\gamma}$
^{152}Sm	$\frac{4'\to4}{4'\to2}$	11.2±1.9	2.94	8.1±0.8	−(0.2±0.4)
	$\frac{3'\to4}{3'\to2}$	1.00±0.05	0.40	7.7±0.5	
	$\frac{2'\to2}{2'\to4}$	10.0±1.4	20.0	6.7±1.8	−(0.4±0.8)
	$\frac{2'\to2}{2'\to0}$	2.38±0.18	1.43	8.8±1.4	−(0.8±1.0)
	$\frac{2'\to0}{2'\to4}$	4.19±0.61	14.0	7.6±1.1	−(0.1±1.6)
^{154}Gd	$\frac{4'\to4}{4'\to2}$	6.41±0.97	2.94	5.1±0.9	1.2±0.4
	$\frac{3'\to4}{3'\to2}$	1.05±0.06	0.40	8.2±0.5	
	$\frac{2'\to2}{2'\to4}$	7.34±0.95	20.0	11.4±2.3	1.2±0.8
	$\frac{2'\to2}{2'\to0}$	2.30±0.12	1.43	8.3±0.9	0±0.8
	$\frac{2'\to0}{2'\to4}$	3.21±0.42	14.0	9.8±1.1	2.3±1.8

[a] States with primes are in the γ-vibrational band and those without primes are in the ground-state rotational band.

[b] Predictions from adiabatic symmetric-rotor model.

[c] Calculated assuming that β-γ band mixing is negligible.

choose to include the former rather than the latter due to the fact that the β and γ vibrations occur at similar excitation energies in ^{152}Sm and ^{154}Gd.

We have determined the $B(E2)$ ratios from members of the β and γ bands in ^{152}Sm and ^{154}Gd using the γ-ray intensities given in Tables I and II. For ^{152}Sm, the intensities of the $3'\to4$ and $2'\to2$ transitions have been reduced by 2.5 and 7%, respectively, to account for $M1$ components,[33,11] as discussed in Sec. III B. The ratios from the γ bands are displayed in column 3 of Table IV. From these experimental ratios, the mixing parameters can be determined through the use of Table III. These Z_γ parameters describe the amount of mixing which must be invoked in order to bring agreement between the experimental $B(E2)$ ratios and the symmetric-rotor predictions of column 4. If mixing of the β and γ bands is neglected, the terms in $Z_{\beta\gamma}$ do not enter and thus a value for Z_γ can be determined from each experimental ratio. These values are listed in column 5 of Table IV.

For ^{152}Sm, one finds that the error limits on the various Z_γ values at least overlap each other. However, if the singles intensity of the $2'\to2$ transition is not reduced to account for a 7% $M1$ admixture[11] (by way of contrast, Rasera *et al.*[32] found less than a 1% $M1$ component in this transition), the mixing parameters from the $2'\to2/2'\to4$ and $2'\to2/2'\to0$ ratios become $(5.8\pm1.7)\times10^{-2}$ and $(10.1\pm1.2)\times10^{-2}$, respectively. Assuming that the $2'\to2$ transition of ^{152}Sm is pure $E2$ would thus lead to an inconsistency in the Z_γ values for the nucleus. In ^{154}Gd, the $2'\to2$ transition has been considered entirely $E2$ in character. The mixing parameters determined from the $2'\to2/2'\to4$ and $2'\to2/2'\to0$ ratios are seen to differ significantly in magnitude, although there is slight overlap of their error limits. Including any amount of $M1$ admixture in the $2'\to2$ transition of ^{154}Gd would result in clear disagreement between the Z_γ values from the $2'$ state. Also, Z_γ from the $4'\to4/4'\to2$ ratio in ^{154}Gd deviates greatly from the other parameters. Although the error limits on the Z_γ

values of Table IV do overlap for ^{152}Sm, they certainly do not for ^{154}Gd. One could conclude that there is not exact agreement between the Z_γ values for each nucleus, especially in view of the discrepancy between the two measurements[11,32] of the $M1$ component in the $2'\rightarrow 2$ transition of ^{152}Sm. This conclusion differs from our earlier statement[27] that the Z_γ values were in excellent agreement and is based on further experiments with large-volume high-resolution Ge(Li) detectors, which have yielded more reliable γ-ray intensities. The larger uncertainties in the previous intensities[27] led to mixing parameters which were consistent within experimental error.

It is interesting then to find if mixing of the β and γ bands can allow the experimental $B(E2)$ ratios to be fit to a consistent set of mixing parameters. The 3+ level of the γ band contains no admixture from the β band, since the latter does not have a member with a spin of 3. Thus, there will be no $Z_{\beta\gamma}$ corrections to the $3'\rightarrow 4$ and $3'\rightarrow 2$ transitions, as is shown in Table III. Therefore, Z_γ calculated from this ratio can be assumed to be characteristic of the band and used with the other experimental $B(E2)$ ratios to extract $Z_{\beta\gamma}$ values. These values are listed in column 6 of Table IV. For each nucleus, there appears to be consistency in the $Z_{\beta\gamma}$ values.

Prior to this study, investigators (e.g., Refs. 42, 44, and 47) have found that, within the large uncertainties in the determined Z_γ values, one could explain the observed $B(E2)$ ratios for these and other deformed nuclei using only a Z_γ correction term. However, the present measurements in ^{152}Sm and ^{154}Gd appear to give the first indication that there are small, but significant, deviations of the experimental γ-band $B(E2)$ ratios from those expected through a treatment which accounts only for mixing between the γ and ground-state bands. More importantly, these deviations can be explained by including mixing of the β and γ bands. The weighted average of the various $Z_{\beta\gamma}$ values is $-(0.29\pm0.33)\times 10^{-2}$ for ^{152}Sm and $(1.0\pm0.3)\times10^{-2}$ for ^{154}Gd, yielding $Z_{\beta\gamma}/Z_\gamma=-(0.04\pm0.05)$ for ^{152}Sm and 0.12 ± 0.04 for ^{154}Gd. These ratios of mixing parameter are considerably larger than the value $-(0.01\pm0.03)$ found in ^{166}Er by Gunther and Parsignault.[44] This difference in ratios may arise because the β and γ band heads are much closer in ^{152}Sm and ^{154}Gd than in ^{166}Er. According to a tentative assignment of a β-vibrational band by Burson *et al.*,[48] the 2+ members of the β and γ bands in ^{166}Er are separated by approximately 744 keV, while this separation is 275 keV for ^{152}Sm and 181 keV for ^{154}Gd. Certainly for each nucleus in which γ-vibrational bands are populated, there is a need for refined measurements which reduce the error limits on Z_γ. Then it can be assessed if mixing of the β and γ bands is appreciable only at the beginning of the deformed region.

Experimental $B(E2)$ ratios from members of the β bands are shown in Table V. Neglecting β-γ band mixing, we are able to calculate Z_β from each ratio; these values are given in column 5. A wide variation in the Z_β values determined from the decays of the $2''$ state is evident for each nucleus. It is interesting to find if these discrepancies also can be removed by including a correction for mixing of the β and γ bands. The magnitude of this mixing has been determined from branching ratios of members of the γ band in each nucleus. After including this correction, we can recalculate the Z_β needed to bring each experimental ratio into agreement with the adiabatic-model prediction shown in column 4. The correction terms given in Table III depend on the parameter $\zeta_{\beta\gamma}$, which is found from the average $Z_{\beta\gamma}$ through the use of Eq. (12). This parameter is also dependent on the unmixed $B(E2)$ values to the 2+ members of the β and γ bands, which can be deduced from Coulomb excitation measurements.[10–12] A discussion of the reduced $E2$ transition probabilities used will be given in Sec. IV B. The redetermined Z_β values are then given in column 6 of Table V. For each nucleus, correction for mixing of the β and γ bands has not allowed a consistent set of parameters to be fit to the experimental $B(E2)$ ratios from the β band, as was done in the case of the γ band.

In both ^{152}Sm and ^{154}Gd, the differing Z_β values would be brought into agreement if the $E2$ intensity of the $2''\rightarrow 2$ transition were only about one-half of the total singles intensity. However, our coincidence experiments show that all of the singles intensity of the 692.43-keV γ ray of ^{154}Gd results from the $2''\rightarrow 2$ transition and Hamilton *et al.*[31] have performed directional correlation experiments on this γ ray to find that it is pure $E2$ radiation. In ^{152}Sm our coincidence experiments on the 688.6-keV γ ray and the fact that this γ ray is observed in Coulomb excitation[37] and in our radioactive-decay experiments with the same intensity relative to the other γ rays from the $2''$ level indicate that this singles intensity belongs exclusively to the $2''\rightarrow 2$ transition. Also, the angular distribution experiments of McGowan *et al.*[11] have shown that the γ ray of the $2''\rightarrow 2$ transition in ^{152}Sm is either greater than 98.5% $E2$ or 77% $M1$ in character. Neither possibility is able to bring internal consistency to the Z_β values.

One thus concludes that variation in Z_β is real and unexplainable by mixing of any known excitation into the β-vibrational band. This is the first case in which such widely varying mixing parameters were found and was reported earlier by us[27] and by Liu *et al.*[30] Thi discrepancy seems too large to be explainable by a nonequality of the intrinsic quadrupole moments of the β and ground-state bands.

Contrary to the evidence from the γ-band $B(E2)$

[47] O. B. Nielsen, in *Proceedings of the Rutherford Jubilee Conference on Nuclear Physics*, edited by J. B. Birks (Heywood and Co. Ltd., London, 1961), p. 317.

[48] S. B. Burson, P. F. A. Goudsmit, and J. Konijn, Phys. Rev. **158**, 1161 (1967).

TABLE V. Experimental and theoretical ratios of reduced $E2$ transition probabilities from members of the β band in ^{152}Sm and ^{154}Gd.

Nucleus	$\frac{I''\to I_1}{I''\to I_2}$ [a]	$\frac{B(E2;I''\to I_1)}{B(E2;I''\to I_2)}$ Expt	Theory [b]	$10^2\times Z_\beta$ without β-γ mixing	$10^2\times Z_\beta$ with β-γ mixing
^{152}Sm	$\frac{2''\to 4}{2''\to 2}$	2.80±0.87	1.8	1.8±1.4	1.7±1.4
	$\frac{2''\to 0}{2''\to 2}$	0.15±0.02	0.7	8.8±0.6	9.1±0.6
	$\frac{2''\to 4}{2''\to 0}$	18.3±6.0	2.6	5.5±1.0	5.6±1.0
	$\frac{4''\to 2}{4''\to 4}$	0.06±0.03	1.1	5.4±0.4	5.7±0.5
^{154}Gd	$\frac{2''\to 4}{2''\to 2}$	3.15±0.30	1.8	2.3±0.5	2.6±0.5
	$\frac{2''\to 0}{2''\to 2}$	0.12±0.02	0.7	9.7±0.5	9.0±0.6
	$\frac{2''\to 4}{2''\to 0}$	25.2±3.7	2.6	6.5±0.4	6.3±0.4
	$\frac{4''\to 2}{4''\to 4}$	0.09±0.06	1.1	5.1±0.6	4.4±0.7

[a] States with double primes are in the β-vibrational band and those without primes are in the ground-state rotational band.
[b] Predictions from adiabatic symmetric-rotor model.

ratios, these results for the β band may indicate that the perturbational treatment of an intrinsic-rotational coupling is invalid for ^{152}Sm and ^{154}Gd. This would possibly be connected to the fact that these nuclei are in the transition region from spherical to highly deformed shapes and are not good symmetric rotors to first order. However, it was shown earlier[27] that the asymmetric-rotor model of Davidson[49] is also unable to predict the peculiar $E2$ branching ratios from members of the β bands. A better possibility is that these so-called β-vibrational states contain significant admixtures of other states which are at this time unknown. It is obvious that, just as for transitions from the γ band, detailed branching ratios from members of β bands in more strongly deformed nuclei are much needed at this time.

B. Monopole Matrix Elements

It is evident from Table I that the electric monopole process is an important mode of deexcitation of the members of the β-vibrational band, since it accounts for approximately 90% of the conversion electrons resulting from the $2''\to 2$ and $4''\to 4$ transitions and for, of course, 100% of the $0''\to 0$ electrons in ^{152}Sm and ^{154}Gd. Within the framework of the Bohr-Mottelson collective model the $E0$ process is forbidden between members of the γ and ground-state bands, due to the selection rule $L\geq |K_i-K_f|$, where L is the multipole order of the radiation emitted. Deviations from this rule are to be expected in cases where there is an appreciable mixing of the wave functions of the β and γ-vibrational bands. For the nuclei presently considered, there is evidence for some mixing of the β and γ bands, but this has no apparent effect on the model predictions in that the K-shell conversion coefficients for the $2'\to 2$ transitions agree with the theoretical $E2$ values. We now proceed to examine if various nuclear models are capable of predicting the amount of $E0$ radiation observed in the transitions from the β-vibrational levels.

The absolute $E0$ transition probability between two states of the same spins and parities, $T(E0;\, I''\to I)$, has been defined by Church and Weneser[2] as

$$T_K(E0;\, I''\to I)=\Omega_K\rho^2, \quad (15)$$

where ρ is the nuclear strength parameter or reduced

[49] J. P. Davidson (private communication); details of calculations given in J. P. Davidson and M. G. Davidson, Phys. Rev. **138**, 316 (1965).

TABLE VI. ^{152}Sm and ^{154}Gd values of X, the ratio of the squares of the reduced $E0$ to the reduced $E2$ matrix elements.

			10×X (Theor)				
Nucleus	I''	10×X (Expt.)	Rasmussen[b]	Reiner[c]	Bes[d]	Davydov and Rostovsky[e]	Davidson[f]
^{152}Sm	0	0.7±0.1 [a]	3.7	1.6	6.5	3.6	3.0
	2	4.5±0.5	12.9	10.7		13.0	16.4
	4	6.6±2.1	14.2				21.1
^{154}Gd	0	1.1±0.3	3.7	1.3	7.3	3.4	2.8
	2	4.5±0.4	12.9			12.3	15.4
	4	5.8±1.8	14.2				20.1

[a] Calculated using K-electron/photon intensity ratio of 0.014±0.002 obtained from Ref. 39.
[b] Reference 3.
[c] Reference 13.
[d] Reference 14.
[e] Reference 15.
[f] Reference 16.

nuclear monopole matrix element, Ω_K is a factor completely determined from the electron wave functions and given in graphic form in Ref. 2, and the K subscript refers to conversion of K-shell electrons. Listengarten and Band[50] have more recently included effects of screening in computing Ω_K. Their values differ from those of Church and Weneser[2] by as much as 6–7% for low transition energy and high atomic number. For cases under consideration in this report, the corrections are negligible. The transition probability $T_K(E0; I''\rightarrow I)$ can be found from the relation

$$\frac{T_K(E0; I''\rightarrow I)}{T(E2; I''\rightarrow I_f)}=\frac{N_K(E0; I''\rightarrow I)}{N(E2; I''\rightarrow I_f)}, \qquad (16)$$

where $N_K(E0)$ and $N(E2)$ refer to the number of K-shell $E0$ transitions and total $E2$ transitions, respectively, from the I'' state in the β band to the I and I_f states of the ground state band, where I and I_f need not be the same. The $E2$ transition probability $T(E2; I''\rightarrow I_f)$ is given by

$$T(E2; I''\rightarrow I_f)=1.23\times10^{13}E_\gamma{}^5 B(E2; I''\rightarrow I_f). \qquad (17)$$

Here E_γ is the γ-ray energy of the $E2$ transition in MeV, and $B(E2; I''\rightarrow I_f)$ is the reduced $E2$ transition probability in units of $e^2\times10^{-48}$ cm^4. Using the values of $N_K(E0)$ and $N(E2)$ from Table I and the known $B(E2)$ values from Coulomb excitation measurements, we are able to compute the nuclear strength parameter ρ.

Knowledge can be gained about the $E0$ process by considering the ratio of the $E0$ intensity to $E2$ intensity for a given transition. This quantity can be determined strictly from radioactive decay experiments without information on the $E2$ transition probability to the β band. Consideration of the $E0/E2$ intensity ratios before determining ρ for each nucleus is worthwhile, since, as will be discussed later, there are problems involved in extracting $B(E2)$ values from Coulomb excitation data. The quantity X is defined by Rasmussen[3] to be the dimensionless ratio of the $E0$ to $E2$ reduced matrix elements squared and is written as

$$X(I''\rightarrow I)=\frac{\rho^2e^2R_0{}^4}{B(E2; I''\rightarrow I)}, \qquad (18)$$

where $R_0=1.2\ A^{1/3}$ fm is the nuclear radius. In the determination of X for the $0''\rightarrow0$ transition, the $B(E2)$ for the $0''\rightarrow2$ transition is used.

Values of X are found from an equivalent form of Eq. (18), given by Davidson[16] as

$$X(I''\rightarrow I)=2.53(\mu_K A^{4/3}E_\gamma{}^5/\Omega_K)\times10^9, \qquad (19)$$

where $\mu_K=N_K(E0; I''\rightarrow I)/N(E2; I''\rightarrow I)$.

We have determined X for the $0''$, $2''$, and $4''$ levels in ^{152}Sm and ^{154}Gd and listed them in Table VI. It is interesting to note that the experimental value of X for each of the observed levels in the β band of ^{154}Gd is nearly equal to the value for the corresponding level in ^{152}Sm, which implies that these states in the two nuclei are very similar in character. This similarity was also evident in the previous section, where the mixing parameters determined from the $E2$ branching ratios of the β band displayed the same pattern in ^{152}Sm and ^{154}Gd.

These ratios of $E0/E2$ matrix elements are also compared to various model predictions in Table VI. In the predictions of column 4, Rasmussen[3] employs a model of a vibrating axially-symmetric rotor, where the surface oscillations are assumed to be volume conserving. Reiner[13] uses an axially-symmetric rotor as did Rasmussen, but in addition includes a correction for the rotation-vibration interaction, which alters considerably his values of $B(E2)$ and hence his values of X. The improved agreement between experiment and theory indicates that this correction is very impor-

[50] M. A. Listengarten and I. M. Band, Bull. Acad. Sci. USSR, Phys. Ser. **23**, 225 (1959).

TABLE VII. Comparison of experimental monopole matrix elements to model predictions. For each nucleus, ρ is determined using the $E0$ intensity of the $2''\to2$ transition, the $E2$ intensity of the $2''\to0$ transition, and $B(E2;0\to2'')$.

		ρ (Theor)			
Nucleus	ρ	Reiner[a]	Bes[b]	Davydov and Rostovsky[c]	Davidson[d]
^{152}Sm	0.28±0.02	0.53	0.82	0.75	0.84
^{154}Gd	0.44[e]	0.51	0.56	0.72	0.81

[a] Reference 13.
[b] Reference 14.
[c] Reference 15.
[d] Reference 16.
[e] Calculated under the assumption that $B(E2;0\to2'')$ for ^{154}Gd is twice the value for ^{152}Sm. No error limit is assigned to denote the fact that this value is merely an estimate.

tant. Reiner[13] feels that a basic change in the assumption of incompressibility of nuclear matter is necessary in order to achieve better agreement between experiment and theory. Bes's predictions[14] result from a microscopic model including pairing and quadrupole interactions. Davydov and Rostovsky[15] calculated ρ for nonspherical nuclei, axially symmetric in the ground state, through the use of a collective model with deformation and asymmetry vibrations. Davidson[16] considered deformation vibrations in asymmetric nuclei to find X. Except for the fairly good agreement between the experimental X value and the prediction of Reiner[13] in ^{154}Gd for $I''=0$, the model predictions are generally 2.5 to 5 times greater than our experimental values. This essentially verifies the general observation for the rare-earth nuclei by Davidson,[16] who finds agreement within a factor of 2, however, in the actinide region.

Having established that the ratios of the $E0$ to $E2$ matrix elements are very similar for members of the β bands in ^{152}Sm and ^{154}Gd, we proceed to determine if the magnitudes of the reduced monopole matrix elements ρ reflect the same behavior. Here it is necessary to use $B(E2)$ values measured in Coulomb excitation experiments. For ^{152}Sm, taking a weighted average of the very similar reduced transition probabilities from the (α,α') experiments of Veje *et al.*[10] and of McGowan *et al.*[11] yields a $B(E2;0\to2'')$ of $(2.28\pm0.16)e^2\times10^{-50}$ cm^4. The earlier values obtained from Coulomb excitation with oxygen ions by Yoshizawa *et al.*[12] and by Seaman *et al.*[51] must be corrected to account for band-mixing effects, as discussed by Veje *et al.*[10] However, making this correction is difficult, since it involves a choice of a unique mixing parameter from the various values in Table V. Thus, $B(E2)$ from the (α,α') experiments is used in our calculations.

Coulomb excitation experiments involving α particles on the ^{154}Gd nucleus have not yet been reported. The result of Yoshizawa *et al.*[12] is available, but this is not used in view of the Z_β-dependent correction which must be made. An estimate of $B(E2;0\to2'')$ for ^{154}Gd can be obtained in another way.

Bloch *et al.*[52] found that the differential cross section for inelastically scattering deuterons off a ^{154}Gd target through 125° is approximately twice that in the case of a ^{152}Sm target. As discussed by Elbek *et al.*,[53] the quantity $[d\sigma/d\Omega]/B(E2)$ is approximately independent of mass number for the rotational and for the quadrupole vibrational states. Therefore, an increase in the differential cross section by a factor of 2 implies a corresponding increase in $B(E2)$ in ^{154}Gd over the known value in ^{152}Sm. The estimate of $B(E2;0\to2'')$ for ^{154}Gd is then $4.6e^2\times10^{-50}$ cm^4.

The reduced $E0$ matrix element ρ for the $2''\to2$ transition is determined from the number of $E0$ events in the $2''\to2$ transition, the number of $E2$ events in the $2''\to0$ transition, and the reduced $E2$ transition probability for the $2''\to0$ transition by combining Eqs. (15), (16), and (17). The values for $N(E2;2''\to0)$ and $N_K(E0;2''\to2)$ are obtained from Table I, while the $B(E2)$ values discussed above are used. The resulting experimental determinations of ρ are given in Table VII.

Unfortunately, we have not been able to accomplish one of the original goals of the present experiments, *viz.*, a determination of ρ for the different states in the β bands of ^{152}Sm and ^{154}Gd. As seen in Table VII, only one value of ρ is given for each nucleus. In principle, ρ for the $0''\to0$ and $4''\to4$ transitions could be determined by comparing the $E0$ intensities of these transitions to the $E2$ intensities of the $0''\to2$, $4''\to4$, and $4''\to2$ transitions. However, each of these calculations would require finding $B(E2)$ for the appropriate transition from the experimental value of $B(E2;0\to2'')$ through the nonadiabatic symmetric rotor model, as expressed in Eq. (10). Since this process would involve the mixing parameter, an average of the various Z_β values in Table V would be used. Thus, to find the appropriate $B(E2)$ from the known $B(E2;0\to2'')$, one would be forced to use a model which really works rather poorly, as evidenced by the great variations in the Z_β values of Table V. This dependence on a pres-

[51] G. G. Seaman, J. S. Greenberg, D. A. Bromley, and F. K. McGowan, Phys. Rev. **149**, 925 (1966).

[52] R. Bloch, B. Elbek, and P. O. Tjøm, Nucl. Phys. **A91**, 576 (1967).

[53] B. Elbek, T. Grotdol, K. Nybø, P. O. Tjøm, and E. Veje, J. Phys. Soc. Japan (Suppl.) **24**, 180 (1968).

ently inadequate model makes the whole process questionable and explains our reason for extracting for each nucleus the one value of ρ which is model-independent and therefore realistic. This philosophy is contrary to that of Ng *et al.*,[54] who recently performed radioactive decay experiments to determine ρ for the $0''$ and $2''$ states of ^{154}Gd. Their results differ from ours also since they used the $B_0(E2; 0\rightarrow 2'')$ values of Yoshizawa *et al.*[12] and since their conversion electron intensities do not agree with those given in Table I (taken from Brantley *et al.*[9]). Ng *et al.*[54] do not observe the $4''\rightarrow 4$ transition and give a $0''\rightarrow 0$ electron intensity which is 35% larger than the value in Table I. The work of Hamilton and Manthuruthil,[55] however, confirm the data of Brantley *et al.*[9] for the 676.5- and 681.0-keV transitions.

From nuclear reaction studies, Lönsjo and Hagemann[56] have recently determined ρ^2 for ^{154}Gd as $(5\pm 2)\times 10^{-2}$; they find no indication that ρ^2 for ^{152}Sm is different from that of ^{154}Gd. However, a comparison between their values and ours is also difficult, since the ρ^2 which they determine for ^{154}Gd is extremely model-dependent. At this time, it was not known that the model was lacking in description of the β-vibrational band. Also, the $B(E2)$ information from (α,α') experiments was not available then and the correction of the values of Yoshizawa *et al.*[12] was not made in the manner later described by Veje *et al.*[10]

Various model predictions of ρ are given in Table VII for comparison to our experimental determinations. The value for ^{154}Gd is in fair agreement with the predictions of Reiner[13] and of Bes,[14] while ρ for ^{152}Sm is much lower than the predictions. Each theory, with the exception of Bes's predicts that ρ will be approximately a constant for these two nuclei, which is in disagreement with the present results. Perhaps our determined increase in ρ reflects the estimated increase of a factor of 2 in $B(E2; 0\rightarrow 2'')$ for ^{154}Gd over that for ^{152}Sm. To state it another way, the increase in ρ may actually indicate the degree to which $[d\sigma/d\Omega]/B(E2)$ deviates from its assumed constancy for the $2''$ levels of ^{152}Sm and ^{154}Gd. One must consider this possibility in view of the great similarity in the $E0/E2$ intensity ratios displayed in Table VI for the corresponding β-vibrational levels of the two nuclei.

C. Nonadiabatic Perturbations of Ground-State Rotational Band

As discussed in Sec. IV A, the band mixing resulting from the interaction of the intrinsic and rotational motions causes very significant adjustments in the $E2$ branching ratios in most cases. This nonadiabatic interaction also has a sizable perturbing effect on the energies of the ground-state rotational band and on the mean square radius of the rotating nucleus. If we determine the magnitude of this mixing, we can then calculate the resulting energy shifts and changes in radius. However, the problem is that we are not able to determine uniquely the magnitude of mixing of the β and ground-state bands, since the Z_β values determined from the various $B(E2)$ ratios are in no way constant. In spite of this, we proceed to estimate these nonadiabatic effects on the rotational spectra in the hope that these tenuous results will give a general idea of the contribution of the intrinsic-rotational coupling to energy shifts and changes in radius.

1. Energy Shifts

For even-even deformed nuclei, the simple $I(I+1)$ rule, resulting from the adiabatic model, is found experimentally to be incapable of describing their rotational spectra. Rather, the energies of the members of the ground-state band must be given by an expansion of the form

$$E(I)=(\hbar^2/2\mathcal{I})I(I+1)+BI^2(I+1)^2+\cdots, \quad (20)$$

where $\mathcal{I}$ is the moment of inertia of the nucleus and B is a parameter characterizing the nonadiabaticity of the rotation.

The contributions to the second term in the energy expansion due to mixing of the β and γ bands into the ground-state band are denoted by B_β and B_γ, respectively. As discussed in Ref. 45, B_β and B_γ are found from

$$B_\beta=-\epsilon_\beta^2(\hbar\omega)_\beta \quad (21)$$

and

$$B_\gamma=-2\epsilon_\gamma^2(\hbar\omega)_\gamma, \quad (22)$$

where ϵ is the spin-independent amplitude of admixture of either the β or γ band into the rotational band and $\hbar\omega$ is the energy of the β- or γ-band head. These amplitudes of admixture are related to the band mixing parameters according to Eqs. (6) and (11). In these equations, Q_{00}/Q_γ and Q_{00}/Q_β are equal to the square roots of the ratios of the appropriate unmixed $B(E2)$ values. Therefore, knowledge of Z_β and Z_γ from branching-ratio measurements and of $B_0(E2)$ to the 2, $2'$, and $2''$ levels allows us to determine ϵ_β and ϵ_γ, and thus B_β and B_γ. Mixing effects are extracted from the $B(E2)$ values established by Coulomb excitation measurements according to Eq. (10) through the use of the determined mixing parameters. The resulting values of B_β and B_γ are given in Table VIII.

The magnitude of B in Eq. (20) can be found from fitting the experimentally observed rotational energies to this equation. However, this process yields drastically different values of B depending on how many rotational levels are used in the fitting procedure. In a

[54] L. K. Ng, K. C. Mann, and T. G. Walton, Nucl. Phys. **A116**, 433 (1968).

[55] J. H. Hamilton and J. C. Manthuruthil, Nucl. Phys. **A118**, 686 (1968).

[56] O. Lönsjo and G. B. Hagemann, Nucl. Phys. **88**, 624 (1966).

TABLE VIII. Observed and predicted coefficients of $I^2(I+1)^2$ term in energy expansion.

Nucleus	$-B$ (eV) Expt[b]	Marshalek[a] Case I	Marshalek[a] Case II	$-B_\beta$ (eV) Expt[c]	Marshalek[a] Case I	Marshalek[a] Case II	$-B_\gamma$ (eV) Expt	Marshalek[a] Case I	Marshalek[a] Case II
^{152}Sm	260	282	221	53±9	162	133	15±2	1.7	0.9
^{154}Gd	235	240	160	66[d]	104	66	22±10	2.3	1.2

[a] Reference 43.
[b] Determined from semiempirical fit to levels up to spin of 12 by Sood (Ref. 57).
[c] Calculated using weighted averages of widely varying Z_β values in Table V. If Z_β from the $2''\to0/2''\to2$ ratio is used, B_β increases to 178 eV for ^{152}Sm and to 367 eV for ^{154}Gd.
[d] Calculated under the assumption that $B(E2: 0\to2'')$ for ^{154}Gd is twice the value for ^{152}Sm.

semiempirical treatment, Sood[57] has summed an infinite series of terms in powers of $I(I+1)$ under the assumption of a constant ratio between coefficients of successive order terms. Fitting this equation to rotational levels up to spin of 12, Sood[57] finds $B=260$ eV for ^{152}Sm and 235 eV for ^{154}Gd, as are given in Table VIII. A comparison of B_β and B_γ with the total B indicates that the mixing of the β and γ bands with the ground-state band accounts for only 26% of the observed deviation of the rotational spectrum of ^{152}Sm from the adiabatic prediction. For ^{154}Gd, 37% of the shift is accounted for in this way. However, these values were calculated using weighted averages of the Z_β values in column 6 of Table V. If Z_β obtained from the $(2''\to0/2''\to2)$ $B(E2)$ ratio is used instead of the weighted average, B_β increases. For ^{152}Sm, this increase in Z_β raises B_β to approximately 178 eV, which then accounts for most of the observed energy shift. Until one can account for the variation in Z_β and thus can determine a unique mixing parameter, it is difficult to assess the contribution of band mixing to energy shifts of the ground-state band.

Marshalek[43] has recently performed microscopic calculations of nonadiabatic effects on the rotational spectra of deformed nuclei. His predicted values of the B parameter and of the contributions of centrifugal stretching to this parameter are also given in Table VIII. The two different sets of calculations resulted from the ways in which the pairing gap parameters were obtained. Although the magnitude of B seems to be correctly predicted, the observed values of B_γ are much greater than the calculated values.

2. *Change in Mean Square Radius*

The average radius of the nucleus in a β-vibrational mode is different from that in a rotational mode, as evidenced by the existence of $E0$ transitions between the β and ground bands. Thus, the mixing of these bands resulting from centrifugal stretching causes the nucleus, excited into the 2+ state of the rotational band, to have a different average radius than it does in the unexcited state. This change in the mean square radius can be related to the amplitude of the admixture by the expression

$$\Delta\langle R^2\rangle/\langle R^2\rangle=-\tfrac{10}{3}[I(I+1)/Z]\epsilon_\beta\rho, \tag{23}$$

where Z is the number of protons. From a knowledge of ϵ_β and ρ, we are able to estimate the change in rms radius expected from centrifugal stretching. The results are given in column 3 of Table IX. Through Mössbauer experiments, Yeboah-Amankwah *et al.*[17] and Steiner *et al.*[18] have deduced the change in radius between the 0+ and 2+ states of ^{152}Sm from a measurement of the isomer shift of the γ ray resulting from the $2\to0$

TABLE IX. Observed and calculated values of change in mean-square radius between 0+ and 2+ levels of ground-state band in ^{152}Sm and in ^{154}Gd.

Nucleus	$10^4\times$ $\Delta\langle R^2\rangle/\langle R^2\rangle$ Expt	$10^4\times\Delta\langle R^2\rangle/\langle R^2\rangle$ Band-mixing estimate[c]	$10^4\times\Delta\langle R^2\rangle/\langle R^2\rangle$ Theoret Marshalek[a] Total	Marshalek[a] Stretch[d]	Bes *et al.*[b]
^{152}Sm	3.7±1.1[e]	8.0±1.0	19.7	19.2	12.2
	5.9±0.4[f]				
^{154}Gd	5.9±0.8[f]	13.5[g]	8.8	8.7	

[a] Reference 61.
[b] Reference 62.
[c] Calculated using weighted averages of widely varying Z_β values in Table V.
[d] Contribution to change in radius from centrifugal stretching of nucleus.
[e] Deduced from isomer-shift measurements using References 17, 18, and 58.
[f] Reference 19.
[g] Calculated under the assumption that $B(E2; 0\to2'')$ for ^{154}Gd is twice the value for ^{152}Sm.

[57] P. C. Sood, Nucl. Data 4, 281 (1968).

transition. According to Kienle *et al.*,[58] these values must be decreased, since recent experiments[59] indicate that the change in electron densities at the nucleus was originally underestimated. The revised change in radius for ^{152}Sm is given in column 2 of Table IX. Bernow *et al.*[19,60] have measured the increase in energy of the $2+\rightarrow 0+$ γ ray of ^{152}Sm and of ^{154}Gd in the presence of a muon in a $1s$ state. The most recent experiments[19] lead to $\Delta\langle R^2\rangle/\langle R^2\rangle$ of $(5.9\pm0.4)\times10^{-4}$ for ^{152}Sm and $(5.9\pm0.8)\times10^{-4}$ for ^{154}Gd.

These fractional changes in radius are compared to our estimates from band-mixing considerations in Table IX. Admittedly, our estimates are crude since an average Z_β was used to find ϵ_β in each case. The fact that the band-mixing estimates are greater than the experimental values may indicate that the ϵ_β values used were too large. However, decreasing ϵ_β by a factor of x in order to account for the experimental $\Delta\langle R^2\rangle/\langle R^2\rangle$ value results in a decrease in B_β by a factor of x^2, in which case band mixing explains even less the experimental shifts in the rotational spectrum of ^{152}Sm and ^{154}Gd. In view of the uncertainty in the actual mixing, one could conclude that centrifugal stretching is able to account for the observed change in radius between the $0+$ and $2+$ states of ^{152}Sm, and possibly of ^{154}Gd also, but unable to explain the deviations of the ground-state energies from the $I(I+1)$ rule. The former observation agrees with the predictions of Marshalek,[61] although the size of the change in radius is overestimated in his calculations. The prediction of Bes *et al.*,[62] who use a microscopic analog of the classical centrifugal-stretching model of Diamond *et al.*,[63] is also found to be greater than the experimental value, as observed in Table IX, but in better agreement with experiment than the predictions of Marshalek.[61]

V. CONCLUSIONS

The unexplained behavior of the $E2$ branching ratios from members of the β vibrational bands in ^{152}Sm and ^{154}Gd constitutes one of the biggest problems which hinder an understanding of the collective excitations in these transitional nuclei. The fact that a consistent set of mixing parameters could not be fit to these $B(E2)$ ratios made the determination of the monopole matrix element for the $0''\rightarrow 0$ and $4''\rightarrow 4$ transitions of little value. Also, this lack of knowledge of the actual mixing between the β and ground-state bands rendered the exact determinations of the contributions of centrifugal stretching to energy shifts and changes of radius within the ground-state band impossible.

Throughout this treatment, we have assumed that the low-lying excited band of energy levels in ^{152}Sm and ^{154}Gd result solely from vibrations in the long-range quadrupole field. However, it seems that mixing of additional excitations into the $K=0$ and $K=2$ bands will be required. For example, it has been shown[64] that fluctuations in the short-range pairing field can also give rise to a $K=0$ collective mode, namely, a pairing vibration. Nevertheless, our assumption that the observed $K=0$ bands result from β vibrations seems to have been justified by the recent calculations of Mikoshiba, Sheline, Udagawa, and Yoshida.[65] They have considered from a microscopic approach the first ten excited $0+$ states in various deformed nuclei and investigated the character of each of these states. They find that at the beginning of the deformed region most of the β-vibrational collectiveness is concentrated into the lowest $0+$, while in the middle of the deformed region it is concentrated into higher-lying $0+$ states. Thus, from their calculations, one expects the first excited $0+$ states in ^{152}Sm and ^{154}Gd to result mainly from fluctuations in the quadrupole field. Nevertheless, the treatment of Mikoshiba *et al.*[65] certainly allows for the existence of higher-lying $0+$ levels which are mainly pairing-vibrational in character. It can be assumed that the existence of such $K=0$ collective modes would have some nonadiabatic effect on the ground-state band, depending on the energy of the mode.

Investigation of excited $K=0$ bands and observations of isomer shifts in heavier rare-earth deformed nuclei, coupled with more calculations concerning the properties of pairing-vibrational states, seem essential to immediate advances in understanding collective properties of deformed nuclei.

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