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†Based on part of the thesis of Kenneth A. Pickar submitted in partial fulfillment of the requirements of the Physics Department for the degree of Doctor of Philosophy, University of Pennsylvania, 1966 (unpublished).

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# Surface Waves on Bulk Liquid Helium*†

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The velocity of a surface wave on bulk liquid helium is measured as a function of frequency at temperatures above and below the λ point. The results do not reproduce those of Everitt, Atkins, and Denenstein. Below the λ point our results follow the classical theory, and no two-fluid effects are seen. There is some deviation from simple theory above the λ point. The variation in wave amplitude as a function of temperature is qualitatively consistent with the amplitude variation expected from consideration of the specific-heat curve.

## INTRODUCTION

The velocity of a surface wave on bulk liquid helium was first measured by Everitt, Atkins, and Denenstein.[1] They filled the inner chamber of their third-sound apparatus above the level of the stainless-steel mirror. A surface wave on the resulting helium "puddle," which was ~0.1 cm deep, was excited and detected by the same techniques used in the film work. The data at 1.25°K are shown in Fig. 1. The phase velocity $V_p$ of a surface wave of a wavelength λ on an ideal classical liquid is given by[2]

$$V_p^{\,2}=\left(\frac{(\rho-\rho')g\lambda}{(\rho+\rho')2\pi}+\frac{2\pi\sigma}{(\rho+\rho')\lambda}\right)\tanh\left(\frac{2\pi h}{\lambda}\right), \qquad (1)$$

where $\rho$ is the liquid density, $\rho'$ is the vapor density, $\sigma$ is the surface tension, $g$ is the acceleration due to gravity, and $h$ is the depth of the liquid.

Plotting this function in Fig. 1 for different values of $h$, we see that it is in poor agreement with the observed data. Even if the wavelengths measured correspond to the third harmonic of the driving frequency, as sometimes happens in this kind of experiment, the agreement is not good. Everitt, Atkins, and Denenstein suggested that the thermomechanical effect and other two-fluid effects might influence the velocity of a surface wave on liquid helium II.

Khalatnikov[3] has shown that a classical surface-tension wave can exist on liquid helium. However, since he ignores evaporation effects at the surface, his treatment does not exclude the possibility that other surface modes might exist. We therefore decided to pursue the matter further experimentally.

## APPARATUS

To check the previous experimental results and to extend them by varying the temperature and liquid depth, some modifications of our third-sound apparatus were made. In order to eliminate the effects of wave scattering from the prism support and mitigate the problem of end reflections, a new geometry was devised as shown in Fig. 2. A trough open at the ends, of interior dimensions $1\times0.3\times3.6$ cm$^3$, was constructed by cutting and milling a section of $X$-band wave guide. The helium level in the trough was the same as that in the inner vessel and could be raised by opening the needle valve connecting the inner vessel to the helium Dewar and lowered by pumping on the inner chamber. The trough was level to 0.02 cm along its length. The liquid depth was read with a cathetometer to $\pm0.01$ cm.

The 45° mirror for reflecting the chopper light remained in place from the third-sound experiment. The length of the strip of mercury arc across the mirror was less than in the third-sound experiment, because of the shadow cast by the walls of the trough. Obviously, the mechanism for light modulation by the plane wave no longer involved polarization as in the film, but rather scattering from the oscillating liquid surface. A polished stainless-steel slab, 0.1 cm thick, was placed along the bottom of the trough to reflect the detecting light. The slab was polished so that it might be correctly oriented for maximum reflection.

The signal obtained was very weak; so changes were made to increase the intensity of chopping radiation. A 250-W Sylvania quartz-iodide lamp was used instead of the 40-W G. E. lamp. The stainless-steel slab was coated with a thin opaque layer of lamp black by holding it above a kerosene flame. This assured that ~95% of the radiation would be absorbed. These measures, together with the use of a larger aperture and a lower chopper frequency, increased the amount of radiation absorbed at the mirror by a factor of 200 over the previous third-sound apparatus. A thin transverse strip of polished metal was left unblackened to permit the reflection of the detector light.

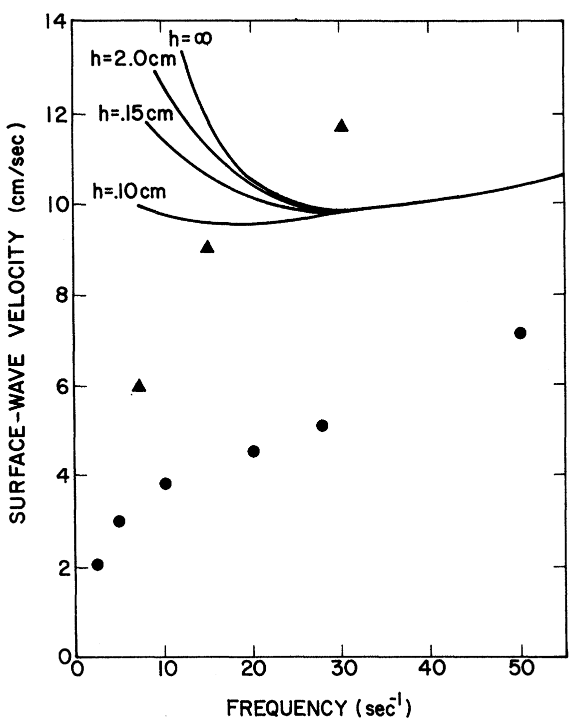


FIG. 1. Velocity of surface wave versus frequency. Circles are raw data from Everitt *et al.* ($T=1.25$° K). Triangles are from the same data assuming that the actual frequency of the wave was the third harmonic of the driving frequency. The solid lines are from the classical theory of surface waves.

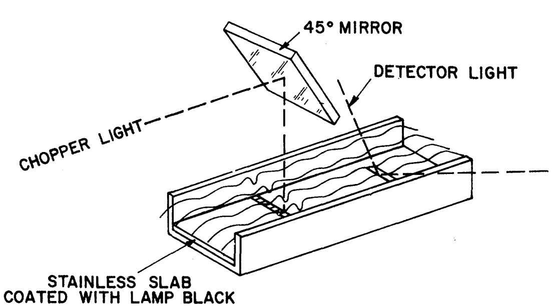


FIG. 2. Diagram of trough containing bulk liquid helium (depth variable to 3 mm). On the surface a wave is generated and detected.

The heated strip of lamp black probably acted as a second-sound source. The second-sound wavelength for a characteristic chopper frequency of 10 Hz is 200 cm, which is much larger than the dimensions of the apparatus. One might expect, then, that the heated strip acted as a thermal radiator which served to evaporate the liquid at the surface above the heated area.

In the same manner as in the third-sound calculation,[4] we may determine what wave amplitude may be generated by evaporation for the energy available. The result is $7\times 10^{-4}$ cm, which is only a 0.7% ripple on a 0.1-cm-deep liquid channel.

## RESULTS AND DISCUSSION

Figure 3 shows a plot of velocity $V_p$ against frequency at 1.25°K for the present experiment. The theoretical curves are taken from Eq. (1) with $\rho' \ll \rho$.

$$V_p^{\,2} = (g\lambda/2\pi + 2\pi\sigma/\rho\lambda)\tanh(2\pi h/\lambda). \qquad (2)$$

Experimentally, $h$, the depth of the liquid, was $0.15 \pm 0.05$ cm, the uncertainty being due to changes in the level during the experiment and fluctuations. The theoretical curves are for $h = 0.10$, 0.15, 0.20 cm and $\infty$. They are seen to fit the data reasonably well. For larger depths the signal-to-noise ratio

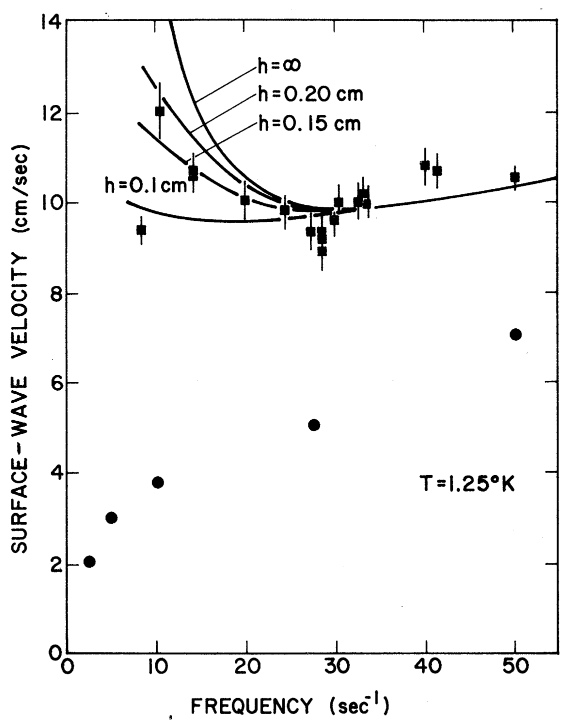


FIG. 3. Velocity of surface wave versus frequency. Squares are present results. Circles are again results from Everitt *et al.*

fell off considerably. We have no sure explanation for the discrepancy between our results and those of Everitt *et al.* The geometries were different but the velocity of a plane wave in a trough is the same as that of a circular wave. In the earlier experiment, the wave was probably a distorted circular wave complicated by the fact that it was generated by a plane source and was subjected to obstacle scattering. The intensity of the chopper light was small compared to ours, but one might expect nonclassical thermomechanical effects to be enhanced by the increased chopper intensity. Also it should be mentioned that the earlier data were the results of only one run.

As we raised the temperature the signal became weaker, disappearing completely at about 1.7°K. The signal reappeared very strongly above the $\lambda$-point at about 2.5°K. This is not surprising if one examines the variation of specific heat with temperature. Above 1.2°K the specific heat curve rises very sharply, increasing by a factor of 5 between 1.2°K and 2.0°K and going to infinity at the $\lambda$ point. Above the $\lambda$ point, it falls to a minimum at 2.5°K and then rises slowly. In our technique for exciting the surface wave, the temperature swing $\Delta T$ is inversely proportional to the specific heat, and $\Delta T$ in turn determines the amplitude of the surface wave because the evaporated mass per unit time is proportional to $\Delta T$.

Figures 4 and 5 give experimental and theoretical data for the velocity of surface waves at 2.50 and 4.26°K. The signal at 2.5°K was stronger than that seen at any other temperature, in fact about 100 times stronger than the signal seen at 1.2°K. At these higher temperatures, the helium should behave as a normal liquid insofar as any thermal property is concerned. However, vapor effects are important because the density of the vapor is one-tenth the density of the liquid at the boiling point, and so Eq. (1) was used to calculate the velocity. The signal strength decreased markedly

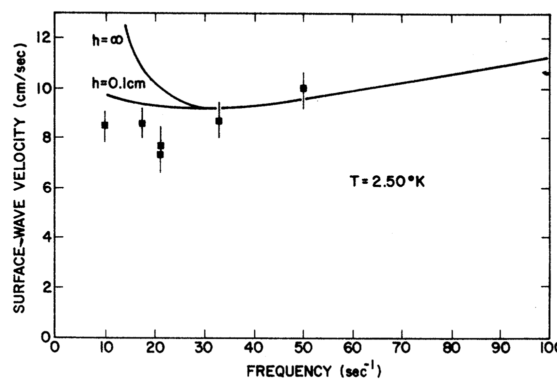


FIG. 4. Velocity of surface wave versus frequency at 2.50°K.

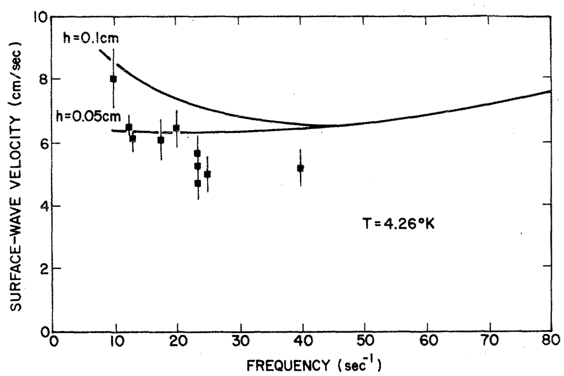


Fig. 5. Velocity of surface wave versus frequency at 4.20°K.

as the frequency was increased becoming very poor past 40 Hz. This was because the wavelength became comparable with the width of the detecting light (0.1 cm). The results were not very sensitive to the depth of the liquid. Any variations in velocity as $h$ was varied from 0.05 to 0.20 cm were obscured in the data scatter. The points are seen to lie somewhat lower than the theoretical curves.

It is difficult to draw any quantitative conclusions about the behavior of the attenuation. The attenuation in all cases was very small (~0.1 per wavelength). It appeared to increase with temperature and frequency.

## CONCLUSION

Thus, we have examined the surface wave for possible two-fluid effects and have not seen any. Within the accuracy of our measurements, the results can be described in terms of purely classical surface waves. Discrepancies from the simple classical theory (which are greatest above the $\lambda$ point) might be explained by a consideration of vapor effects on the velocity of the wave.

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