Electron-Impact Broadening of Overlapping He I Lines Plasmas

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The general impact theory of electron broadening, developed previously by Griem, Baranger, Kolb, and Oertel (GBKO), is extended in order to take into account the off-diagonal matrix elements necessary to treat the case of partially overlapping lines.

The new time integrals arising in the theory have rigorously been evaluated. It follows a generalized and consistent treatment of overlapping lines for any degree of degeneracy. The "complete degeneracy case" (hydrogen) and the "isolated-line case" appear as limiting situations in our theoretical description. In order to describe the four possible kinds of second-order atomic transitions, we are led to generalize the A and B functions of GBKO. Properties of these new A (z_1, z_2) and B (z_1, z_2) functions are considered in detail.

1. INTRODUCTION

The generalized impact theory for electron broadening of neutral helium lines, developed by Griem, Baranger, Kolb, and Oertel[1] (GBKO), is restricted to isolated lines. Nevertheless, electron-density measurements at $T_e \geqslant 2\times10^4$ °K and $N_e \geqslant 10^{14}$ cm^{-3} render very desirable the consideration of the most intense He I lines, arising from the $2P$-nQ (where $3 \leqslant n \leqslant 5$ and $Q = P, D, F, G$) hydrogenic transitions. Previous treatment by Griem[2] of the partially overlapping lines $2^1S - 4^1Q$ was based on a full degeneracy assumption which is acceptable only in the presence of a high electron density ($N_e \geqslant 10^{17}$ cm^{-3}). The nonlinear static Stark effect of the He I excited levels, resulting in a weakening of the ionic broadening, makes necessary a careful evaluation of the electron-impact contribution to the line broadening. Thus there appears the need to generalize the GBKO electron-collision operator, in order to be able to consider not only diagonal matrix elements (isolated lines) but also off-diagonal ones, which are usually neglected. Hydrogenic He I lines offer a very sensitive test of this complete impact theory of electron broadening, because their static Stark patterns can be determined with very high accuracy.[3-4]

The present paper is organized as follows:

In Sec. 2 the electron-impact-broadening theory is adapted to the case of partial degeneracy. The off-diagonal matrix elements are obtained in Sec. 3 with the aid of a rigorous treatment of the time integrals, the details of which are given in the Appendix. Symmetry considerations and numerical results are discussed in Sec. 4. Also in Sec. 4 the particular cases of completely degenerate levels (hydrogen) and of inelastic transitions for partially degenerate levels are emphasized. An expression for the quadrupole correction is given in Sec. 5. The theory developed here is very well suited to describe the Stark broadening of overlapping He I lines in the presence of an external magnetic field (Sec. 6).

The applicability of the present generalization to partially degenerate ionic lines does not appear self-evident because the use of the classical path approximation for the perturber motion is questionable in that case.[5] On the other hand, it could be of interest to apply the analytical methods developed in this work to atomic impact broadening.[6]

2. ELECTRON BROADENING OF PARTIALLY DEGENERATED LINES

A. Initial Formulation

Our starting point will be the expression (2.10) of Ref. 1 for the electronic part of the line shape of the emitted light polarized along direction $\vec{e}$:

$$I_{nn'}(\omega,\vec{e}) = -\,\mathrm{Re}\sum_{i,j,k,l}\langle ni\,|\,\vec{e}\cdot\vec{r}\,|\,n'j\rangle\langle n'k\,|\,\vec{e}\cdot\vec{r}\,|\,nl\rangle \times\langle ni\,|\langle n'j\,|\,[i\omega - i\hbar^{-1}(\mathcal{H}_n - \mathcal{H}_{n'}) + \phi_{nn'}]^{-1}\,|\,nl\rangle\,|\,n'k\rangle\,, \qquad (1)$$

where ω is the angular frequency separation from the unperturbed line $2P$-nD. (In the following we shall fix $n' = 2$ for the lower level.) The summation runs over the sublevels i and l of the upper level n and over the sublevels j and k of the lower level n'. Also, $\mathcal{H}_n$ and $\mathcal{H}_{n'}$ are the Hamiltonians[3-4] describing the various sublevels, in subspaces of principal quantum numbers n and n', respectively, as functions of the static electric field $\vec{F}$. The kets $|ni\rangle$ and $|nl\rangle$ (or $|n'j\rangle$ and $|n'k\rangle$) are eigenvectors of $\mathcal{H}_n$ (or $\mathcal{H}_{n'}$) and are given by the following expressions:

$$|ni\rangle = \sum_{p=1}^{n^2} a_i^{\,p}\,|np\rangle\,, \quad |nl\rangle = \sum_{s=1}^{n^2} a_l^{\,s}\,|ns\rangle\,, \quad |n'j\rangle \sum_{q=1}^{n'^2} a_j^{\,q}\,|n'q\rangle\,, \quad |n'k\rangle = \sum_{r=1}^{n'^2} a_k^{\,r}\,|n'r\rangle\,, \tag{2}$$

where the static perturbation is wholly contained in the eigenvector components $a_i^{\,p}$, etc. The kets $|nq\rangle$, etc., on the right-hand side represent the eigenfunctions $|nlm\rangle$ of the unperturbed hydrogen atom. $\phi_{nn'}$ is the impact-broadening operator to be discussed in the next section.

With expressions (2), Eq. (1) takes the form

$$I_{nn'}(\omega,\vec{e}) = -\,\mathrm{Re}\sum_{i,j,k,l}\left(\sum_{p,q,r,s} a_i^{\,p} a_j^{\,q} a_k^{\,r} a_l^{\,s}\langle np\,|\,\vec{e}\cdot\vec{r}\,|\,n'q\rangle\langle n'r\,|\,\vec{e}\cdot\vec{r}\,|\,ns\rangle\right)$$
$$\times\langle ni\,|\langle n'j\,|\,[i\omega - i\hbar^{-1}(\mathcal{H}_n - \mathcal{H}_{n'}) + \phi_{nn'}]^{-1}\,|nl\rangle\,|n'k\rangle\,. \tag{3}$$

The sums over p, q, r, s, and the algebraic quantity

$$\langle ni\,|\langle n'j\,|\,\omega - \hbar^{-1}(\mathcal{H}_n - \mathcal{H}_n\;)\,|nl\rangle\,|n'k\rangle = i[\omega - \hbar^{-1}(\xi_i - \xi_j)]\delta_{il}\,\delta_{jk}$$

are evaluated directly in terms of the eigenquantities given in Refs. 3 and 4.

B. Impact-Broadening Operator

The electron-broadening problem is then reduced to the evaluation of the matrix elements of the impact-broadening operator given by Eq. (2.17) of Ref. 1.

$$\phi_{nn'} = N_e\int_0^\infty vf(v)\,dv\int_0^\infty 2\pi\rho\left\{S_n(0)S_{n'}^{\,*}(0) - 1\right\}_{\substack{\text{angular}\\ \text{average}}} d\rho\,, \tag{4}$$

where N_e is the electron density, * means the complex conjugate of the given quantity, $f(v)$ denotes the Maxwellian distribution function of the electron velocity, ρ is the impact parameter, and $S(0)$ the collision S matrix evaluated at time $u = 0$. Using the linear relation $r_\nu(u) = \rho_\nu + v_\nu u$ for the ν component of the electron trajectory, Eq. (4) can be solved with exactly the same procedure as used by GBKO.[1] The well-known second-order development of the S matrices allows us to write

$$\left\{\langle ni\,|\langle n'j\,|\,S_n(0)S_{n'}(0) - 1\,|nl\rangle|n'k\rangle\right\}_{\substack{\text{angular}\\ \text{average}}}$$
$$= -\frac{e^2}{\hbar^2}\Bigg\{\delta_{jk}\sum_{\sigma,l',\nu}\langle ni\,|\,r_\sigma\,|nl'\rangle\langle nl'\,|\,r_\nu\,|nl\rangle\int_{-\infty}^{\infty}du_1\int_{-\infty}^{u_1}du_2\,\exp[i(\omega_{il'}u_1 + \omega_{l'l}u_2)]E_{1\sigma}(u_1)E_{1\nu}(u_2)$$
$$+\,\delta_{il}\sum_{\sigma,k',\nu}\langle n'j\,|\,r_\sigma\,|n'k'\rangle^*\langle n'k'\,|\,r_\nu\,|n'k\rangle^*\int_{-\infty}^{\infty}du_1\int_{-\infty}^{\infty}du_2\,\exp[-i(\omega_{jk'}u_1 + \omega_{k'k}u_2)]\,E_{1\sigma}(u_1)E_{1\nu}(u_2)$$
$$-\left[\sum_\sigma\langle ni\,|\,r_\sigma\,|nl\rangle\int_{-\infty}^{\infty}\exp(i\omega_{il}u_1)E_{1\sigma}(u_1)\,du_1\right]\left[\sum_\nu\langle n'j\,|\,r_\nu\,|n'k\rangle^*\int_{-\infty}^{\infty}\exp(-i\omega_{jk}u_2)E_{1\nu}(u_2)\,du_2\right]\Bigg\}_{\substack{\text{angular}\\ \text{average}}}\cdots\,, \tag{5}$$

where the first term represents the second-order perturbation of sublevels in the upper state n, the second term is the same quantity attached to the lower state n', and the third term is a product of two first-order terms contributing to the narrowing of the line. Also, in Eq. (5), it appeared to be useful to introduce the usual angular frequencies

$$\omega_{il} = \hbar^{-1}(\xi_i - \xi_l), \quad \text{etc.} \tag{6}$$

The angular average $\{\circ\cdots\}_{\substack{\text{angular}\\ \text{average}}}$ is performed with the help of

$$\left\{E_{1\sigma}(u_1)E_{1\nu}(u_2)\right\}_{\substack{\text{angular}\\ \text{average}}} = e^2\frac{\delta_{\mu\nu}}{3}\,\frac{(\rho^2 + v^2u_1u_2)}{(\rho^2 + v^2u_1^{\,2})^{3/2}(\rho^2 + v^2u_2^{\,2})^{3/2}} \tag{7}$$

and of $\vec{\rho}\cdot\vec{v}=0$. So with the dimensionless quantities

$$z_1=\omega_{il'}\rho/v, \quad z_2=\omega_{ll'}\rho/v, \quad z_3=\omega_{il}\rho/v,$$
$$z_1'=\omega_{jk'}\rho/v, \quad z_2'=\omega_{kk'}\rho/v, \quad z_4=\omega_{jk}\rho/v, \quad x=vu/\rho, \tag{8}$$

one obtains

$$\left\{\langle ni|\langle n'j|S_n(0)S_{n'}^*(0)-1|n'k\rangle|nl\rangle\right\}_{\text{angular average}}$$
$$=-\frac{1}{3}\left(\frac{e^2}{\hbar\rho v}\right)^2\left(\delta_{jk}\sum_{\sigma,l'}\langle ni|r_\sigma|nl'\rangle\langle nl'|r_\sigma|nl\rangle\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{x_1}dx_2\frac{(1+x_1x_2)e^{i(z_1x_1-z_2x_2)}}{(1+x_1^2)^{3/2}(1+x_2^2)^{3/2}}\right.$$
$$+\delta_{il}\sum_{\sigma,k'}\langle n'j|r_\sigma|n'k'\rangle^*\langle n'k'|r_\sigma|n'k\rangle^*\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{x_1}dx_2e^{-i(z_1'x_1-z_2'x_2)}\frac{(1+x_1x_2)}{(1+x_1^2)^{3/2}(1+x_2^2)^{3/2}}$$
$$\left.-\sum_\sigma\langle ni|r_\sigma|nl\rangle\langle n'j|r_\sigma|n'k\rangle^*\int_{-\infty}^{\infty}dx_1e^{iz_3x_1}\int_{-\infty}^{\infty}dx_2\frac{e^{iz_4x_2}(1+x_1x_2)}{(1+x_1^2)^{3/2}(1+x_2^2)^{3/2}}\right)+\cdots. \tag{9}$$

The essential part of our work is now the rigorous evaluation of Eq. (9).

3. OFF-DIAGONAL MATRIX ELEMENTS

The expression (9) has already been considered in the following two complementary situations:

(a) *Isolated Lines*

These are produced by allowed transitions of well-separated levels. In their vicinity the intensity of forbidden components is weak. Nevertheless, the levels of the forbidden components contribute to the broadening of the isolated lines through electronic collisions involving the different levels. This case is obtained by putting $z_1=z_2$ and $z_1'=z_2'$ in Eq. (9). It was studied at length by GBKO.[1]

(b) *Strongly Overlapping Lines*

Here, in view of their quasilinear Stark effect, forbidden components can have an intensity of the same order of magnitude as those arising from allowed transitions. Electron broadening is then well described by the approximation $\exp(\pm izx)=1$ of Eq. (9). This procedure is the classical one used for hydrogen broadening.[7] It was also applied by Griem[2] to the hydrogenic transitions 2^1S-4^1D and 2^1S-4^1F of neutral helium. Experimental results[8] at $T_e=2\times10^4\,°\text{K}$ and $N_e<10^{17}\text{ cm}^{-3}$ strongly suggest that the profile of $2P$-$4Q$ transitions cannot be computed within the framework of this approximation. It appears necessary to consider separately in the upper level n the contribution of each second-order transition $i\to l'\to l$, and to investigate matrix elements with $z_1\neq z_2$.

Analytically, our problem will be to evaluate, without any simplifications, the three double integrals appearing in Eq. (9). The third integral immediately gives

$$\int_{-\infty}^{\infty}\frac{e^{iz_3x_1}dx_1}{(1+x_1^2)^{3/2}}\int_{-\infty}^{\infty}\frac{e^{iz_4x_2}(1+x_1x_2)}{(1+x_2^2)^{3/2}}dx_2=2[|z_3||z_4|K_1(|z_3|)K_1(|z_4|)+z_3z_4K_0(|z_3|)K_0(|z_4|)], \tag{10}$$

where K_0 and K_1 are the modified Bessel functions of the second kind.

The second integral reduces to the first when $z_1'=-z_1$ and $z_2'=-z_2$. So we are left with

$$2^{-1}\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{x_1}dx_2e^{i(z_1x_1-z_2x_2)}\frac{(1+x_1x_2)}{(1+x_1^2)^{3/2}(1+x_2^2)^{3/2}}\equiv A(z_1,z_2)+iB(z_1,z_2). \tag{11}$$

As shown in the Appendix, the integrations over x_1 and x_2 can be expressed in terms of Bessel functions,

$$A(z_1,z_2)=|z_1||z_2|K_1(|z_1|)K_1(|z_2|)+z_1z_2K_0(|z_1|)K_0(|z_2|), \tag{12}$$

hypergeometric functions, and integrals of Bessel functions,

$$B(z_1,z_2)=z_1[{}_1F_2(1;\tfrac{1}{2},\tfrac{3}{2};\tfrac{1}{4}z_1^2)-\tfrac{1}{4}\pi|z_1|{}_1F_2(\tfrac{3}{2};2,\tfrac{3}{2};\tfrac{1}{4}z_1^2)]|z_2|K_1(|z_2|)$$

$$-[{}_1F_2(1;\tfrac{1}{2},\tfrac{1}{2};\tfrac{1}{4}z_1^2)-\tfrac{1}{2}\pi|z_1|\,{}_1F_2(\tfrac{3}{2};1,\tfrac{3}{2};\tfrac{1}{4}z_1^2)]z_2K_0(|z_2|)$$

$$+\sum_{n=0}^{1}\left(z_2\int_0^\infty\frac{dt\,tJ_n(t)}{t^2+z_2^2}\int_0^\infty\frac{dt'\,t'(t+t')J_n(t')}{(t+t')^2+(z_1-z_2)^2}+(z_2-z_1)\int_0^\infty\frac{dt\,t^2J_n(t)}{t^2+z_2^2}\int_0^\infty\frac{dt'\,t'J_n(t')}{(t+t')^2+(z_1-z_2)^2}\right),\quad(13)$$

where $J_n(t)$ are Bessel functions of the first kind.

Expressions (12) and (13) are very well suited for numerical evaluation. The quantities involving the K functions vanish exponentially for large z_2 values, and the double integrals with fixed limits are easily handled by a Legendre-Gauss quadrature. The particular case $z_1=z_2$ gives again the values of $A(z)$ and $B(z)$ used for the isolated-line case $(z=z_1=z_2)$. More precisely, the real part

$$A(z)=z^2[K_1^2(|z|)+K_0^2(|z|)]\quad(12')$$

is the one already found by GBKO.[1] The relations[9]

$$\int_0^\infty dt'\,t'J_1(t')/(t+t')=(\pi t/2\cos\pi)[H_{-1}(t)-N_{-1}(t)],$$

$$\int_0^\infty dt'\,t'J_0(t')/(t+t')=1-\tfrac{1}{2}\pi t[H_0(t)-N_0(t)],\quad\int_0^\infty dt'\,t'J_0(t')/(t'^2+z^2)=K_0(|z|),\quad(14)$$

where H_0 and H_{-1} are the Struve functions, and N_0 and N_{-1} the Neuman functions, give the imaginary part

$$B(z)=z\Big(|z|K_1(|z|)[{}_1F_2(1;\tfrac{1}{2},\tfrac{3}{2};\tfrac{1}{4}z^2)-\tfrac{1}{4}\pi|z|\,{}_1F_2(\tfrac{3}{2};2,\tfrac{3}{2};\tfrac{1}{4}z^2)]$$

$$-K_0(|z|)[-1+{}_1F_2(1;\tfrac{1}{2},\tfrac{1}{2};\tfrac{1}{4}z^2)-\tfrac{1}{2}\pi|z|\,{}_1F_2(\tfrac{3}{2};1,\tfrac{3}{2};\tfrac{1}{4}z^2)]$$

$$-\frac{\pi}{2}\int_0^\infty dt\,t^2J_1(t)\frac{H_{-1}(t)-N_{-1}(t)}{t^2+z^2}-\frac{\pi}{2}\int_0^\infty dt\,t^2J_0(t)\frac{H_0(t)-N_0(t)}{t^2+z^2}\Big),\quad(13')$$

an expression much easier to handle numerically than Eqs. (3.8) of GBKO,[1] because it is free from any singularity. Before leaving this section, it is necessary to emphasize the physical meaning of the generalization $z_1\neq z_2$ studied here. The isolated line case $z=z_1=z_2$ can be described in terms of an electron elastic scattering amplitude [see Fig. 1(a)]. The latter is itself expressible in terms of the inelastic cross sections $\sigma_{il'}$ as Baranger[10] pointed out. The rigorous treatment given here introduces the three additional contributions $i\to l'\to l$ with $i\neq l$ and different values of the ratio z_2/z_1 as shown on Fig. 1(b). They correspond to inelastic electron scattering amplitudes.

4. PROPERTIES OF $A(z_1,z_2)$ AND $B(z_1,z_2)$

A. Symmetry Properties and Numerical Results

Changing the signs in expressions (12) and (13) shows us immediately that

$$A(z_1,z_2)=A(-z_1,-z_2),\quad(15a)$$

$$B(z_1,z_2)=-B(-z_1,-z_2).\quad(15b)$$

These parity properties are straightforward generalizations of the same properties already existing for isolated lines.[1]

The width function $A(z_1,z_2)$ contributes to the dissipative operator part of the profile,[10] and the shift function $B(z_1,z_2)$, influences the fluctuating part. Equations (15a) and (15b) simply say that the width (inversely proportional to a lifetime) is a positive quantity and the shift is a real-valued one.

Further relations, which are not so apparent, are

$$A(z_1,z_2)=A(z_2,z_1),\quad(16a)$$

$$B(z_1,z_2)=B(z_2,z_1).\quad(16b)$$

Equation (16a) is readily obtained from expression (12), but Eq. (16b) is not evident by simple inspection of Eq. (13). Nevertheless it has been numerically verified to five significant figures (this limitation is due only to the limited accuracy of the numerical calculation). To get deeper in-

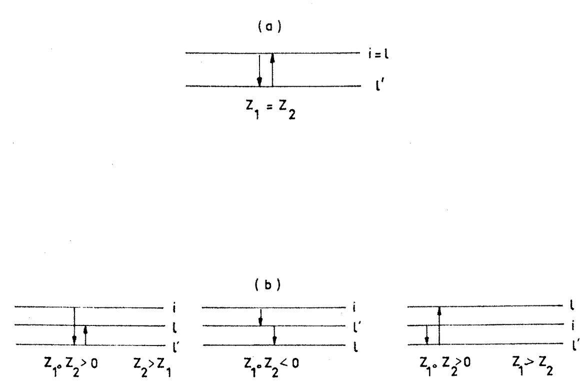

FIG. 1. The four second-order transitions that contribute to electron broadening in the dipole approximation.

sight into the physical significance of Eqs. (16a) and (16b), we have to return to expression (11) and consider

$$I(z_1, z_2) = A(z_1, z_2) + iB(z_1, z_2) \tag{17}$$

as a complex function of the complex variable z_1, parametrized with

$$z_2/z_1 = \omega_{ll'}/\omega_{il'} = a,$$

a quantity now independent of the plasma parameters ρ and v. This z_1 dependence is located in the exponential $\exp[iz_1(x_1 - ax_2)]$ of Eq. (11). First, let us choose a such that $(x_1 - ax_2) \geqslant 0$; a procedure identical to the one used in Appendix B of Ref. 1 will convince us that $I(z_1, az_1)$ is an analytical function of z_1, in the upper half plane of the complex variable z_1. So the well-known Kramers-Kronig relation[11] will appear as

$$I(z_1, az_1) = i\frac{P}{\pi}\int_{-\infty}^{\infty}\frac{I(z', az')dz'}{z_1 - z'}, \tag{18}$$

where z_1 is a point located on the real axis, and P denotes the Cauchy principal part. Separating in Eq. (18) the real and imaginary parts yields

$$A(z_1, az_1) = \frac{P}{\pi}\int_{-\infty}^{\infty}\frac{B(z', az')dz'}{z' - z_1}, \tag{19a}$$

$$B(z_1, az_1) = \frac{P}{\pi}\int_{-\infty}^{\infty}\frac{A(z', az')dz'}{z_1 - z'}$$

$$= \frac{2z_1}{\pi}P\int_0^{\infty}\frac{A(z', az')dz'}{z'^2 - z_1^2}. \tag{19b}$$

It is now clearly apparent that the symmetry of $A(z_1, z_2)$ automatically implies the symmetry of $B(z_1, z_2)$. The same arguments apply identically when $(x_1 - ax_2) < 0$ (I is now analytic in the lower half plane), and the symmetries of Eq. (16) keep their validity for all (z_1, z_2) values. Physically speaking Eqs. (16a) and (16b), together with the first term of Eq. (9), say that the transitions $i \to l' \to l$ give the same contribution as the transitions $l \to l' \to i$ to the line broadening. This fact can be understood on the basis of scattering theory,[12] which indicates that in the presence of weak coupling the electron scattering amplitudes are symmetric under a permutation of initial and final states. It is quite interesting to point out that the analog of Eq. (9) for fourth order will not show this property, even if the A and B functions remain symmetric. This clearly indicates that the corresponding electron scattering amplitude describes the so-called strong collisions in line-broadening theory.

Equations (15) and (16) will appear to be useful due to the simplifications they allow in complete profile calculation. Particularly, we can restrict ourselves to examining A and B as functions of z_1, while z_2 is considered as a parameter. The variations of A [Eq. (12)] and B [Eq. (13)] are shown in Figs. 2(a) and 2(b), respectively, as functions of

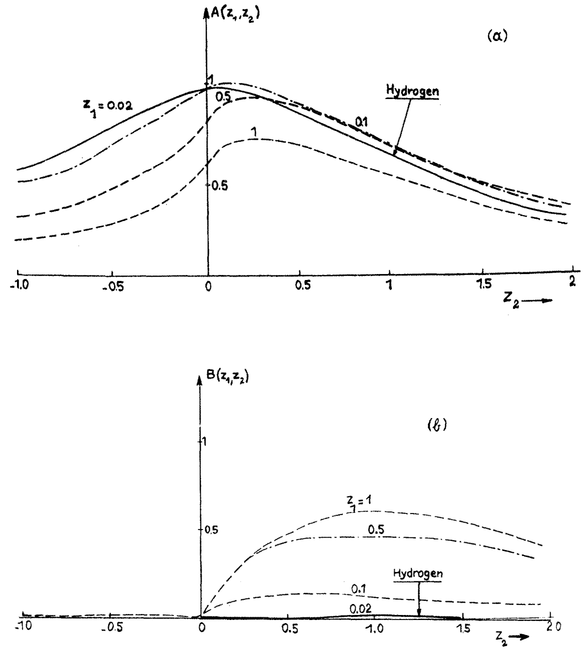

FIG. 2. Graphs of $A(z_1, z_2)$ and $B(z_1, z_2)$ as a function of z_2 corresponding to different z_1 values. The latter can be seen on the curves.

z_1 for several values of z_2. The numerical integrations are described in detail elsewhere.[13] The values of $A(z)$ [Eq. (12′)] and $B(z)$ [Eq. (13′)] were used to recalculate the values tabulated by Griem.[14] Our B values are 0.5% larger than Griem's numerical results.

B. Shifts and Widths

(1) *Completely Degenerate Levels*

Completely degenerate levels are shown in Figs. 2(a) and 2(b) by the curves with $z = 0.02$. It is important to note that the central parts with $-0.25 \leqslant z_1 \leqslant 0.25$ can be related to hydrogen broadening under conditions found very often in plasma spectroscopy, namely $N_e \cong 2\times10^{17}$ cm^{-3} and $T \cong 2\times10^4$ °K. The numerical values $A(z_1, 0.02) \simeq 1$ and $B(z_1, 0.02) \simeq 0.001$ confirm the full validity of the $\exp(\pm izx) \simeq 1$ approximation applied when treating that latter problem. Nevertheless, for the sake of theoretical completeness, it is important to realize that B is not identically zero. This fact indicates clearly that the generalized impact theory is able to give a small contribution to the shift in the limit of nearly completely degenerate levels.

This imaginary part is produced by the quasi-static Stark effect and vanishes with it, as Van Regemorter[15] has suggested previously. It is completely different from the imaginary part found recently by Smith[16] for the collision operator, in the "relaxation" theory of line broadening.

(2) *Partially Degenerate Levels*

Other curves in Figs. 2(a) and 2(b) show the A

and B values pertaining to the excited levels of He I. More precisely, the ratio $-0.3 \leq z_2/z_1 \leq 1.8$ gives the electron broadening of $4Q(Q=P,D,F)$ sublevels subjected to an electric field of 100 kV/cm. The value $z_2 = z_D \simeq 1$ under these conditions, is obtained with $\rho = \lambda_D$ and $v=(2kT/m_e)^{1/2}$. The more striking fact shown by Fig. 2(b) is the relation

$$B(z_1, z_2) \simeq 0, \quad (z_1 \leq 0), \tag{20}$$

satisfied for all z_2 values. The value of $I(0, z_2)$ is readily obtained from Eq. (11)

$$I(0, z_2) = I(z_2, 0) = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dx_1 e^{iz_2x_1}}{(1+x_1^2)^{3/2}} \times\left(\frac{-x_1}{(1+x_1^2)^{1/2}} + \frac{x_2}{(1+x_2^2)^{1/2}}\right) = |z_2| K_1(|z_2|), \tag{21}$$

with $B(0, z_2) = 0$.

Apparent in Fig. 1(b) is the main content of Eqs. (20) and (21) for $z_1 \cdot z_2 \leq 0$. In that case, we find the most inelastic transitions which broaden the spectral line without shifting it: This is the situation described by the Lorentz theory.[17] Numerically speaking, this result is very important for complete calculations of entire profiles. For the B functions, which are the most cumbersome to evaluate, this simplification will reduce the computation time considerably. It is also very instructive to contemplate the complementary relations

$$A(z, az) \geq A(z, z) \tag{22a}$$

and
$$B(z, az) \leq B(z, z), \tag{22b}$$

with $0 \leq a \leq 1$. When $|a| > 1$, the above inequalities remain unchanged if one considers $z' = az$ and $a' = a^{-1}$. Physically speaking, Eq. (22a) says that the width contribution of a second-order transition $i \to l' \to l$ is minimum for a diagonal matrix element (isolated line), the shift contribution being then maximum as shown by (22b).

5. QUADRUPOLE CORRECTION

It is possible to improve the second-order evaluation of Eq. (5) by taking into account the effect of the monopole-quadrupole interaction

$$V_Q = \tfrac{3}{2} a_0 |\vec{r}(u)|^{-3} \{[\vec{r}(u) \cdot \vec{r}]^2 / |\vec{r}(u)|^2 - \tfrac{1}{3} \vec{r} \cdot \vec{r}\}, \tag{23}$$

where a_0 denotes the Bohr radius.

Performing the angular average with the aid of the relations

$$\{\rho_\sigma \rho_\lambda\} = \frac{\rho^2}{3}\delta_{\sigma\lambda}, \quad \{v_\sigma v_\lambda\} = \frac{v^2}{3}\delta_{\sigma\lambda}, \quad \{\rho_\mu{}^4\} = \frac{\rho^4}{5}, \quad \{v_\mu{}^4\} = \frac{v^4}{5}, \quad \{\rho_\mu^2 v_\mu{}^2\} = \frac{2\rho v^2}{9},$$

$$\{\rho_\mu{}^2 \rho_\sigma{}^2\} = \frac{\rho^4}{15}, \quad \{v_\mu{}^2 v_\sigma{}^2\} = \frac{v^4}{15}, \quad \{\rho_\mu{}^2 v_\sigma{}^2\} = \frac{\rho^2 v^2}{9}, \quad \{\rho_\mu v_\mu \rho_\sigma v\ \} = -\frac{2\rho^2 v^2}{9}, \tag{24}$$

where $\mu \neq \sigma$, we are led to add the following to Eq. (9):

$$\left\{\langle ni | S_n{}^Q(0) | nl\rangle\right\}_{\substack{\text{angular}\\ \text{average}}} = -\frac{9}{4}\left(\frac{e^2}{\hbar\rho v}\right)^2 \left(\frac{a_0}{\rho}\right)^2 \left[\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{x_1} dx_2 e^{i(z_1x_1 - z_2x_2)}\right]$$

$$\times\Bigg\{\sum_{\mu, l'} \langle i | r_\mu{}^2 | l'\rangle\langle l' | r_\mu{}^2 | l\rangle \{[\tfrac{1}{5}(1+x_1{}^2x_2{}^2) + \tfrac{2}{9}(x_1{}^2 + x_2{}^2 + 4x_1x_2)](1+x_1{}^2)^{-\frac{5}{2}}(1+x_2{}^2)^{-\frac{5}{2}}$$

$$-\tfrac{1}{9}(1+x_1{}^2)^{-\frac{3}{2}}(1+x_2{}^2)^{-\frac{3}{2}}\} + \sum_{\mu, l', \sigma} \langle i | r_\mu{}^2 | l'\rangle\langle l' | r_\sigma{}^2 | l\rangle$$

$$\times\{[\tfrac{1}{15}[1+x_1{}^2x_2{}^2 + \tfrac{2}{9}(x_1{}^2 + x_2{}^2 - 4x_1x_2)](1+x_1{}^2)^{-\frac{5}{2}}(1+x_2{}^2)^{-\frac{5}{2}} - \tfrac{1}{9}(1+x_1{}^2)^{-\frac{3}{2}}(1+x_2{}^2)^{-\frac{3}{2}}\}$$

$$+\sum_{\mu, \sigma, l'} \langle i | r_\mu r_\sigma | l'\rangle\langle l' | r_\mu r_\sigma + r_\sigma r_\mu | l\rangle [\tfrac{1}{15}(1+x_1{}^2x_2{}^2) - \tfrac{1}{9}(x_1{}^2 + x_2{}^2)](1+x_1{}^2)^{-\frac{5}{2}}(1+x_2{}^2)^{-\frac{5}{2}}\Bigg\}. \tag{25}$$

As before, it is possible to evaluate the real parts of Eq. (25) in a straightforward way in terms of K_0 and K_1 functions. But the imaginary parts are no longer treatable with the methods developed in the Appendix. This is not a serious drawback: in most spectroscopic investigations of neutral lines in plasmas we have $kT_e \leq 10$ eV, so that

$$\rho \geqslant \lambda_{\text{de Broglie}} \simeq 10a_0,$$

and the $(a_0/\rho)^2$ factor limits the z and z' values of practical interest to be close to zero. Then the dispersion relation (19b) immediately shows that the quadrupole contribution to B is negligible. Physically this means that interactions stronger than dipole ones will give only a weak contribution to the width. Thus, neglecting the imaginary part and also the vanishing real parts, [Eq. (25)] will be well approximated by

$$\left\{\langle ni|S_n^{\,Q}(0)|nl\rangle\right\}_{\substack{\text{angular}\\ \text{average}}} \simeq -\frac{1}{5}\left(\frac{e^2}{\hbar\rho v}\right)^2\left(\frac{a_0}{\rho}\right)^2 \left\{ 2\sum_{\mu,\,l'} \langle i|r_\mu^{\,2}|l'\rangle\langle l'|r_\mu^{\,2}|l\rangle \right.$$

$$\times\left[A(z_1,z_2)+\tfrac{2}{27}|z_1||z_2|K_1(|z_2|)K_1(|z_1|)\right] - \sum_{r,\,l',\,\sigma} \langle i|r_r^{\,2}|l'\rangle\langle l'|r_\sigma^{\,2}|l\rangle\left[A(z_1,z_2)+\tfrac{8}{27}|z_1||z_2|K_1(|z_1|)K_1(|z_2|)\right]$$

$$\left. +2\sum_{\mu,\,\sigma,\,l'}\langle i|r_\mu r_\sigma|l'\rangle\langle l'|r_\mu r_\sigma + r_\sigma r_\mu|l\rangle\left[\tfrac{2}{27}|z_1||z_2|K_1(|z_1|)K_1(|z_2|)\right]\right\} . \tag{26}$$

6. AN ILLUSTRATIVE EXAMPLE

It is quite worthwhile to emphasize that the effect of an external (constant) magnetic field $\vec{H}$ may be described, in a natural way, within the frame of the above generalization when the electron Debye radius λ_D remains smaller than its Larmor radius r_g, i.e.,

$$\frac{r_g}{\lambda_D} = \frac{4.544\times10^{-3} N_e^{\,1/2}\ (\text{cm}^{-3})}{H\ (\text{Gauss})} \geqslant 1, \tag{27}$$

a condition offering a wide range, $N_e \geqslant 5.10^{14}$ cm^{-3} and $H \leqslant 10^5$ G, for practical applications. The magnetic field introduces deep modifications in the Stark patterns[3,4] of overlapping He I lines. A typical example of this new situation can be seen in Fig. 3. Figure 3(a) shows the Stark pattern of the transitions $2^1P-4^1Q(Q=P,D,F)$ produced by a static electric field of 100 kV/cm, which corresponds to $N_e = 1.4\times10^{17}$ cm^{-3}. Here the components are few in number, and it is possible, with a limited loss of accuracy, to treat their electron-impact broadening in the complete degeneracy approximation. The modifications produced by a 200-G magnetic field orientated at 45° with respect to the electric field is shown on Fig. 3(b). Components with magnetic quantum number $m \neq 0$ are replaced by doublets. With increasing field strength their number increases, and they spread out on the abscissa. In the presence of a strong magnetic field [Fig. 3(c), $H = 40$ kG] we see many components of comparable magnitude standing over an appreciable "distance." Figures 3(a) and 3(b) show clearly that it is necessary to take into account off-diagonal matrix elements of the electron-collision operator.

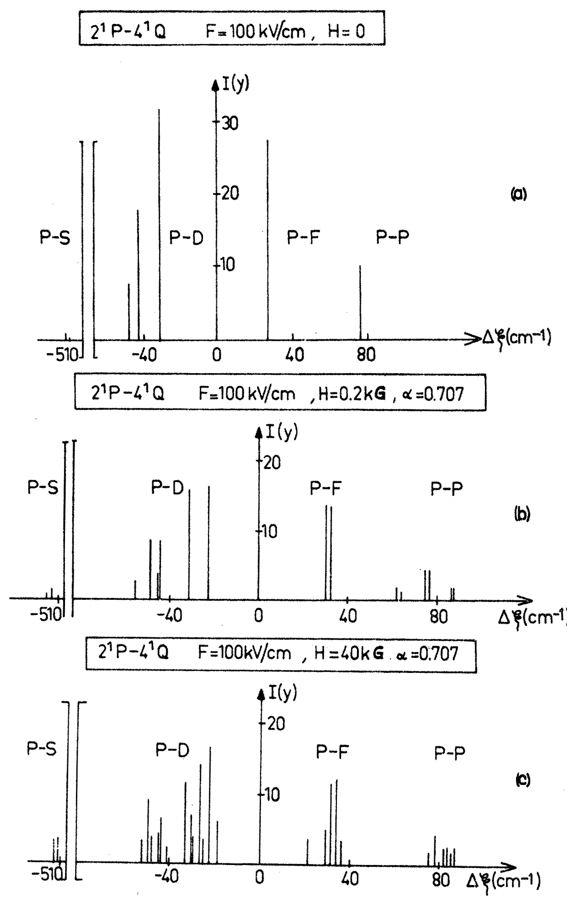

FIG. 3. Stark patterns, with emitted intensities polarized perpendicularly to $\vec{F}$ and $\vec{H}$ (see the text for the orientations of the fields) of the transitions 2^1P-4^1Q $(Q=S,P,D,F)$ for an electric field strength $\vec{F}$ of 100 kV/cm. The normalization of the intensities is given by the condition.

$$\sum_{Q=S,P,D,F} (2^1P-4^1Q) = 100\%.$$

APPENDIX

The integrals appearing in the impact-broadening operator are evaluated in this Appendix. According to Eq. (11), one has

$$I(z_1, z_2)=A(z_1, z_2)+iB(z_1, z_2)=2^{-1}\int_{-\infty}^{\infty} dx_1\int_{-\infty}^{x_1} dx_2 \frac{e^{i(z_1x_1-z_2x_2)}(1+x_1x_2)}{(1+x_1^2)^{3/2}(1+x_2^2)^{3/2}}\,. \tag{A.1}$$

First we look for a transformation of the integrand, which will replace the variable limit x_2 by a fixed one. It is in fact possible[18] to represent the algebraic part of (A.1) as a sum of Laplace transforms of Bessel functions. In consequence, our first aim will be to put Eq. (A.1) in the form

$$\int_0^{\infty} dx_1 \int_0^{\infty} dx_2 \pm \int_0^{\infty} dx_1 \int_{x_1}^{\infty} dx_2\,.$$

It proves convenient to start from

$$I(z_1, z_2)=\int_{-\infty}^{\infty} dx_1\int_{-\infty}^{x_1} \frac{dx_2\, e^{i(z_1x_1-z_2x_2)}}{(1+x_1^2)^{3/2}(1+x_2^2)^{3/2}}+\int_{-\infty}^{\infty} dx_1\int_{-\infty}^{x_1} dx_2 \frac{e^{i(z_1x_1-z_2x_2)}x_1x_2}{(1+x_1^2)^{3/2}(1+x_2^2)^{3/2}}\equiv I_1+I_2\,. \tag{A.2}$$

A. Evaluation of I_1

First, we write

$$I_1=\int_{-\infty}^{0}\frac{dx_1\, e^{iz_1x_1}}{(1+x_1^2)^{3/2}}\int_{-\infty}^{x_1}\frac{dx_2\, e^{-iz_2x_2}}{(1+x_2^2)^{3/2}}+\int_0^{\infty}\frac{dx_1\, e^{iz_1x_1}}{(1+x_1^2)^{3/2}}\int_{-\infty}^{x_1}\frac{dx_2\, e^{-iz_2x_2}}{(1+x_2^2)^{3/2}}\,. \tag{A.3}$$

Decomposing the exponential[18] (A.3) becomes

$$\begin{aligned} I_1 &= 2\int_0^{\infty}\frac{dx_1 \cos z_1x_1}{(1+x_1^2)^{3/2}}\int_{x_1}^{\infty}\frac{dx_2 \cos z_2x_2}{(1+x_2^2)^{3/2}}+2i\int_0^{\infty}\frac{dx_1 \sin z_1x_1}{(1+x_1^2)^{3/2}}\int_0^{\infty}\frac{dx_2\cos z_2x_2}{(1+x_2^2)^{3/2}}\\ &\quad +2i\int_0^{\infty}\frac{dx_1\cos z_1x_1}{(1+x_1^2)^{3/2}}\int_{x_1}^{\infty}\frac{dx_2 \sin z_2x_2}{(1+x_2^2)^{3/2}}-2i\int_0^{\infty}\frac{dx_1\sin z_1x_1}{(1+x_1^2)^{3/2}}\int_{x_1}^{\infty}\frac{dx_2\cos z_2x_2}{(1+x_2^2)^{3/2}}\\ &= 2|z_1||z_2|K_1(|z_1|)K_1(|z_2|)+2iz_1[{}_1F_2(1;\tfrac12,\tfrac32;\tfrac14 z_1^2)-\tfrac14\pi|z_1|\,{}_1F_2(\tfrac32;2,\tfrac32;\tfrac14 z_1^2)]|z_2|K_1(|z_2|)\\ &\quad +2i\int_0^{\infty}\frac{dx_1\cos z_1x_1}{(1+x_1^2)^{3/2}}\int_{x_1}^{\infty}\frac{dx_2\sin z_2x_2}{(1+x_2^2)^{3/2}}-2i\int_0^{\infty}\frac{dx_1\sin z_1x_1}{(1+x_1^2)^{3/2}}\int_{x_1}^{\infty}\frac{dx_2\cos z_2x_2}{(1+x_2^2)^{3/2}}\,, \end{aligned} \tag{A.4}$$

where we used (cf. Ref. 9, p. 429)

$$\int_0^{\infty}\frac{dx_2 \sin zx_2}{(1+x_2^2)^{3/2}}=z[{}_1F_2(1;\tfrac12,\tfrac32;\tfrac14 z^2)-\tfrac14\pi|z|\,{}_1F_2(\tfrac32;2,\tfrac32;\tfrac14 z^2)] \tag{A.5}$$

with ${}_1F_2(\alpha;\beta;\gamma;u)=\dfrac{\Gamma(\beta)\Gamma(\gamma)}{\Gamma(\alpha)}\displaystyle\sum_{n=0}^{\infty}\frac{\Gamma(\alpha+n)}{\Gamma(\beta+n)\Gamma(\gamma+n)}\frac{u^n}{n!}\,.$

1. *Evaluation of* $I_{11}=\int_0^{\infty}dx_1[\cos z_1x_1/(1+x_1^2)^{3/2}]\int_{x_1}^{\infty}dx_2\sin z_2x_2/(1+x_2^2)^{3/2}$. Now, it is convenient to utilize[18]

$$(1+x_2^2)^{-3/2}=\int_0^{\infty}dt\, tJ_1(t)e^{-x_2t} \tag{A.6}$$

with $x_2>0$, and to permute $\int_0^{\infty}dt$ and $\int_{x_1}^{\infty}dx_2$. So we obtain

$$\int_0^{\infty}dx_2\sin z_2x_2/(1+x_2^2)^{3/2}=\int_0^{\infty}dt\, tJ_1(t)\int_{x_1}^{\infty}dx_2\, e^{-x_2t}\sin z_2x_2. \tag{A.7}$$

This operation is permissible because

$$\int_{x_1}^{\infty} |tJ_1(t)|\, dt \int_{x_1}^{\infty} e^{-x_2 t}\, dx_2 = \int_0^{\infty} dt\, |J_1(t)|\, e^{-x_1 t}$$

has a meaning whenever $x_1 > 0$. The x_2 integration of (A. 7) allows us to write (cf. Ref. 9, p. 196)

$$\int_{x_1}^{\infty} dx_2 \sin z_2 x_2/(1+x_2^2)^{3/2} = \int_0^{\infty} dt\, tJ_1(t)(t \sin z_2 x_1 + z_2 \cos z_2 x_1) e^{-x_1 t}/(t^2+z_2^2) \tag{A.8}$$

with $x_1 > 0$. Its two members being continuous functions of x_1, it is easy to convince oneself that (A.8) keeps its validity in the $x_1 = 0$ limit. Thus whenever $x_1 \geq 0$,

$$I_{11} = \int_0^{\infty} dx_1 [\cos z_1 x_1/(1+x_1^2)^{3/2}] \int_0^{\infty} dt\, tJ_1(t) e^{-x_1 t} (t \sin z_2 x_1 + z_2 \cos z_2 x_1)/(t^2+z_2^2) \tag{A.9}$$

2. *Evaluation of* $I_{12} = \int_0^{\infty} dx_1 [\sin z_1 x_1/(1+x_1^2)^{3/2}] \int dx_2 \cos z_2 x_2/(1+x_2^2)^{3/2}$. Identical manipulations lead to

$$I_{12} = \int_0^{\infty} dx_1 [\sin z_1 x_1/(1+x_1^2)^{3/2}] \int_0^{\infty} dt\, tJ_1(t)(t \cos z_2 x_1 - z_2 \sin z_2 x_1) e^{-x_1 t}/(t^2+z_2^2). \tag{A.10}$$

When (A.9) and (A.10) are put together, the last line of the second expression of (A.4) reads

$$2i(I_{11} - I_{12}) = 2i \int_0^{\infty} dt [tJ_1(t)/(t^2+z_2^2)]$$

$$\times [z_2 \int_0^{\infty} dx_1\, e^{-x_1 t} \cos(z_2 - z_1)x_1/(1+x_1^2)^{3/2} + t \int_0^{\infty} dx_1\, e^{-x_1 t} \sin(z_2 - z_1)x_1/(1+x_1^2)^{3/2}], \tag{A.11}$$

where use has again been made of the permutation of $\int_0^{\infty} dt$ and $\int_0^{\infty} dx_1$. A more symmetrical expression for (A.11) is obtained with the help of Eq. (A.6) which gives

$$2i(I_{11} - I_{12}) = 2iz_2 \int_0^{\infty} dt [tJ_1(t)/(t^2+z_2^2)] \int_0^{\infty} dt'\, t'(t+t') J_1(t')/[(t+t')^2 + (z_1 - z_2)^2]$$

$$+ 2i(z_2 - z_1) \int_0^{\infty} dt [t^2 J_1(t)/(t^2+z_2^2)] \int_0^{\infty} dt'\, t' J_1(t')/[(t+t')^2 + (z_1 - z_2)^2]. \tag{A.12}$$

B. *Evaluation of* I_2

In this paragraph, the tricks shown above are re-utilized. By separating the real and imaginary parts, we obtain the following relations:

$$I_2 = 2\int_0^{\infty} \frac{dx_1\, x_1 \sin z_1 x_1}{(1+x_1^2)^{3/2}} \int_0^{\infty} \frac{dx_2\, x_2 \sin z_2 x_2}{(1+x_2^2)^{3/2}} - 2i \int_0^{\infty} \frac{dx_1\, x_1 \cos z_1 x_1}{(1+x_1^2)^{3/2}} \int_0^{\infty} \frac{dx_2\, x_2 \sin z_2 x_2}{(1+x_2^2)^{3/2}}$$

$$+ 2i \int_0^{\infty} \frac{dx_1\, x_1 \cos z_1 x_1}{(1+x_1^2)^{3/2}} \int_{x_1}^{\infty} \frac{dx_2\, x_2 \sin z_2 x_2}{(1+x_2^2)^{3/2}} - 2i \int_0^{\infty} \frac{dx_1\, x_1 \sin z_1 x_1}{(1+x_1^2)^{3/2}} \int_{x_1}^{\infty} \frac{dx_2\, x_2 \cos z_2 x_2}{(1+x_2^2)^{3/2}}$$

$$= 2z_1 z_2 K_0(|z_1|) K_0(|z_2|) - 2i[{}_1F_2(1; \tfrac{1}{2}, \tfrac{1}{2}; \tfrac{1}{4}z_1^2) - \tfrac{1}{2}\pi |z_1|\, {}_1F_2(\tfrac{3}{2}; 1, \tfrac{3}{2}; \tfrac{1}{4}z_1^2)] z_2 K_0(|z_2|)$$

$$+ 2i \int_0^{\infty} \frac{dx_1\, x_1 \cos z_1 x_1}{(1+x_1^2)^{3/2}} \int_{x_1}^{\infty} \frac{dx_2\, x_2 \sin z_2 x_2}{(1+x_2^2)^{3/2}} - 2i \int_0^{\infty} \frac{dx_1\, x_1 \sin z_1 x_1}{(1+x_1^2)^{3/2}} \int_{x_1}^{\infty} \frac{dx_2\, x_2 \cos z_2 x_2}{(1+x_2^2)^{3/2}}. \tag{A.13}$$

With the aid of (Ref. 9, p. 429)

$$\int_0^{\infty} dx_1\, x_1 \cos z_1 x_1/(1+x_1^2)^{3/2} = {}_1F_2(1; \tfrac{1}{2}, \tfrac{1}{2}; \tfrac{1}{4}z_1^2) - \tfrac{1}{2}\pi |z_1|\, {}_1F_2(\tfrac{3}{2}; 1, \tfrac{3}{2}; \tfrac{1}{4}z_1^2).$$

So we again obtain the difference of two integrals with the variable x_2 limit. Now, we use (see Ref. 18, p. 213) the relation

$$x_2/(1+x_2^2)^{3/2} = \int_0^{\infty} dt\, e^{-x_2 t} tJ_0(t) \quad (x_2 > 0) \tag{A.14}$$

to write

$$I_{21} \equiv \int_0^\infty dx_1[x_1\cos z_1x_1/(1+x_1^2)^{3/2}]\int_{x_1}^\infty dx_2\,x_2\sin z_2x_2/(1+x_2^2)^{3/2}$$

$$= \int_0^\infty dx_1[x_1\cos z_1x_1/(1+x_1^2)^{3/2}]\int_0^\infty dt\,tJ_0(t)(t\sin z_2x_1+z_2\cos z_2x_1)e^{-x_2t}/(t^2+z_2^2), \quad (A.15)$$

$$I_{22} \equiv \int_0^\infty dx_1[x_1\sin z_1x_1/(1+x_1^2)^{3/2}]\int_{x_1}^\infty dx_2\,x_2\cos z_2x_2/(1+x_2^2)^{3/2}$$

$$= \int_0^\infty dx_1[x_1\sin z_1x_1/(1+x_1^2)^{3/2}]\int_0^\infty dt\,tJ_0(t)(t\cos z_2x_1 - z_2\sin z_2x_1)e^{-x_1t}/(t^2+z_2^2). \quad (A.16)$$

Both expressions are valid for $x_1 \geq 0$. Using a relation analogous to (A.14), with x_2 replaced by x_1, gives

$$2i\,(I_{21}-I_{22}) = 2iz_2\int_0^\infty dt[tJ_0(t)/(t^2+z_2^2)]\int_0^\infty dt'\,t'(t+t')J_0(t')/[(t+t')^2+(z_2-z_1)^2]$$

$$+2i(z_2-z_1)\int_0^\infty dt[t^2J_0(t)/(t^2+z_2^2)]\int_0^\infty dt'\,t'J_0(t')/[(t+t')^2+(z_2-z_1)^2]. \quad (A.17)$$

Finally, with (A.4), (A.12), (A.13), and (A.17), one obtains Eqs. (12) and (13) of the main text.

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