Calculation of the Coupling Constants $g_{\Lambda KN}^2$ and $g_{\Sigma KN}^2$ Using Kaon-Nucleon Dispersion Relations*

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We apply forward kaon-nucleon dispersion relations to determine whether the known energy dependence of the low-energy $K p$ elastic scattering amplitude uniquely and consistently prescribes the coupling constants $g_{\Lambda K^- p}^2$ and $g_{\Sigma^0 K^- p}^2$. In one application, we check the consistency of the multichannel effective-range continuation below the physical threshold by calculating the coupling constants over a range of energies. In the second application, we use a theorem which places bounds on the energy dependence of certain types of unknown dispersion integrals, to find bounds on the coupling constants.

I. INTRODUCTION

IN 1957, Mathews and Salam[1] wrote the following dispersion relations for the forward elastic scattering amplitudes of kaons on protons:

$$\text{Re}[M^{\pm}(\omega)] = \sum_{Y=\Lambda,\Sigma} B_Y^{\pm}(\omega) + \frac{1}{\pi} \int_{\bar{\omega}}^{K} \frac{\text{Im}[M_{\text{abs}}^{-}(\omega')]}{\omega' \pm \omega} d\omega'$$

$$+ \frac{1}{\pi} \int_{K}^{\infty} \frac{\text{Im}[M^{+}(\omega')]}{\omega' \mp \omega} + \frac{\text{Im}[M^{-}(\omega')]}{\omega' \pm \omega} d\omega' + C, \quad (1.1)$$

where the $\pm$ signs refer to $K^{\pm}p$ scattering, ω is the lab energy of the kaon, C is an undetermined constant, and $B_Y^{\pm}$ represents the hyperon pole terms with the structure

$$B_Y^{\pm} = \frac{g_{YNK}^2}{2N} \frac{X_Y}{\omega_Y \pm \omega}, \quad (1.2a)$$

$$X_Y = \frac{(Y-N)^2 - K^2}{2N}, \quad (1.2b)$$

$$\omega_Y = \frac{Y^2 - N^2 - K^2}{2N}. \quad (1.2c)$$

We use capital letters to stand for the mass of the corresponding particle. An exactly analogous representation exists for the forward elastic kaon-neutron scattering amplitude.

Although these relations are divergent as they stand, all other useful KN dispersion relations may be derived from them by taking suitable combinations.

Every attempt to use these equations is plagued by the existence of the integral over the unphysical region, which extends from the beginning of the $\pi\Lambda$ cut, at $\omega = -\bar{\omega}$, to the physical threshold at $\omega = -K$. We know that there are resonances which should contribute to $\text{Im}M_{\text{abs}}^{-}$ in this region, and the crudest models suggest that this part of the integral will give a very important contribution. This is especially true for the problem which occupies the remainder of this paper, namely, the calculation of the ΛKN and ΣKN coupling constants. In this problem the closeness of the poles to the unphysical region forces one to take into account the unphysical integral.

There are two ways in which one might do this: (i) write down explicit forms for $\text{Im}M_{\text{abs}}^{-}$ as derived from some model, and (ii) utilize only very general properties of $\text{Im}M_{\text{abs}}^{-}$.

The latter may not seem very useful, since we know that the calculated value of the coupling constants, using a dispersion relation at one energy, depends very sensitively on the exact form that we use in the unphysical region. However, it is also possible to correlate via the dispersion relation the energy dependence of the pole terms with that of the unphysical integral, and this latter dependence may be fairly insensitive to the exact form of $\text{Im}M_{\text{abs}}^{-}$.

A technique for doing this is given in Sec. III, and there we do impose useful limits on the coupling constants.

The model calculations of $\text{Im}M_{\text{abs}}^{-}$ have been gradually refined until at present there are two which seriously compete for attention. They both involve parametrizations of the low-energy $K^- p$ data in a form suitable for continuation below the physical threshold, and they may be referred to as (i) the constant scattering-length approximation[2] (CSL) and (ii) the multi-channel effective-range approximation[3] (MER). These have been extensively described elsewhere and we shall not reproduce the details here. For our purposes it is sufficient to note that a "world average" of the results of these two models might be given as follows. Let

$$g_{\Lambda}^2 = g_{\Lambda K^- p}^2, \quad g_{\Sigma}^2 = g_{\Sigma^0 K^- p}^2 = \tfrac{1}{2}g_{\Sigma^- K^- n}^2. \quad (1.3)$$

[1] P. T. Mathews and Abdus Salam, Phys. Rev. **110**, 569 (1958); see also P. T. Mathews and Abdus Salam, *ibid.* **110**, 565 (1958).

[2] An incomplete list of the applications of this model includes M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters **21**, 229 (1966); N. Zovko, *ibid.* **23**, 143 (1966); A. D. Martin and F. Poole, *ibid.* **25B**, 343 (1967); G. H. Davies, N. M. Queen, M. Lusignoli, M. Restignoli, and G. Violini, Nucl. Phys. **3B**, 616 (1967).

[3] J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967); **19**, 1079 (1967).

Then

$$\text{MER: } g_\Lambda{}^2 \approx 13.5 \pm 2.5, \quad g_\Sigma{}^2 \leqslant 1;$$
$$\text{CSL: } g_\Lambda{}^2 \approx 6.5 \pm 2.5, \quad g_\Sigma{}^2 \lesssim 3.$$

The MER values are consistent with an $SU(3)$ mixing parameter α [$=F/(F+D)$] of approximately 0.35, whereas there is no value of α consistent with the pair of values given by CSL.

However, the quoted errors are derived purely from experimental errors, and give no indication of the uncertainty due to the approximations inherent in the models themselves. These could be extremely large, because a small variation in the shape of $\mathrm{Im}M_{\mathrm{abs}}{}^-$ changes the couplings considerably.[3] Therefore, before accepting either model as giving a useful result, we should at least check to see whether it is self-consistent.

Since both the CSL and the MER models give expressions for the real and the imaginary parts for $\bar\omega < \omega < 0.7$ GeV, one may check that the values of $g_\Lambda{}^2$ and $g_\Sigma{}^2$ calculated from the dispersion relation are independent of the energy at which the equation is evaluated.

Tests of this type have been hindered by the lack of

reasonably precise experimental values for the real parts. Typical errors are at least 25%. However, a recent phase-shift analysis[4] of K^+p scattering for the K^+ lab momentum less than 1500 MeV/c has supplied us with some real parts which we might expect to be reasonably accurate.

Together with these real parts we may use those derived from low-energy K^-p data ($\omega < 0.7$ GeV) and expressible in terms of the parameters of the MER model. Finally, we may compute the MER extrapolated real parts and use these also. In this way we may make many different estimates of the coupling constants and check their consistency. This is done in Sec. II.

Before doing this, we should like to note that a more restricted consistency test has already been carried out in Ref. 5. Briefly, the argument is as follows.

If $M(\omega)$ satisfies (1.1), then the function $M(\omega)/\alpha(\omega)$, where

$$\alpha(\omega) = (\omega + \bar\omega)^\beta (\omega + K)^{1-\beta} \tag{1.4}$$

and

$$0 < \beta < 1,$$

satisfies the new equation

$$\mathrm{Re}\frac{M^+(\omega)}{\alpha(\omega)} = \sum_{Y=\Lambda,\Sigma}\frac{X_Y}{\alpha(-\omega_Y)}\frac{g_Y{}^2}{2N}\frac{1}{\omega + \omega_Y} + \frac{1}{\pi}\int_N^\infty\frac{\mathrm{Im}[M^+(\omega')]}{\alpha(\omega')(\omega'-\omega)} + \frac{\mathrm{Im}[M^-(\omega')]}{\alpha(-\omega')(\omega'+\omega)}d\omega'$$
$$+ \frac{1}{\pi}\int_{\bar\omega}^K\frac{\mathrm{Re}[M^-(\omega')]\sin(\pi\beta) + \mathrm{Im}[M^-(\omega')]\cos(\pi\beta)}{(\omega'-\bar\omega)^\beta(K-\omega')^{1-\beta}(\omega'+\omega)}d\omega'. \tag{1.5}$$

This is valid for all ω, and β in the range $0 < \beta < 1$. (There is a concealed subtraction in this equation, due to a divergence in the unphysical integral.)

In Ref. 5, Eq. (1.5) was tested for one value of ω and β in the range (0,1), and it was found that the MER model gave values of the coupling constants which were relatively stable against β, while those of the CSL model were not. We shall therefore restrict ourselves to checking further the self-consistency of the MER model.

II. CALCULATION OF $\bar g_\Lambda{}^2$

In this section, we carry out the detailed calculation of the ΛKN coupling constant, using the MER parametrization of the low-energy K^-p data and the unphysical region. The basic equation is derived from (1.1) by performing a subtraction at $\omega_S = -y$, with $y > 0$, and setting $\omega = x$ ($x > 0$):

$$\mathrm{Re}M^+(x) - \mathrm{Re}M^-(y) = -(x+y)\sum_{Y=\Lambda,\Sigma}\frac{a_Y}{(\omega_Y+x)(\omega_Y-y)}$$
$$+\frac{x+y}{\pi}\int_{\bar\omega}^\infty\frac{\mathrm{Im}M^+(\omega')}{(\omega'-x)(\omega'+y)} - \frac{\mathrm{Im}M^-(\omega')}{(\omega'+x)(\omega'-y)}d\omega', \tag{2.1}$$

where

$$a_Y = (X_Y/2N)g_Y{}^2. \tag{2.2}$$

In the physical region $\mathrm{Im}M^\pm$ will be calculated from the total cross sections for $K^\pm p$ scattering, according to the formulas

$$\mathrm{Im}M^\pm(\omega) = \frac{k}{4\pi}\sigma^\pm(\omega), \tag{2.3}$$

where k is the $K^\pm$ lab momentum.

To isolate the Λ pole, we should need to consider the isospin-zero combination of the Kp and Kn amplitudes, but we do not do this for the following reasons.

Using the MER model, one finds that $g_\Sigma{}^2$ is less than unity; also the ratio of the coefficients of $g_\Lambda{}^2$ and $g_\Sigma{}^2$ is constant to within a few percent, for all the energies which we use. Since we are primarily interested in the constancy of the couplings, we may consider a composite coupling $\bar g_\Lambda{}^2$ given by

$$\bar g_\Lambda{}^2 = g_\Lambda{}^2 + \frac{X_\Sigma(\omega_\Lambda + x)(\omega_\Lambda - y)}{X_\Lambda(\omega_\Sigma + x)(\omega_\Sigma - y)}g_\Sigma{}^2. \tag{2.4}$$

[4] A. T. Lea, B. R. Martin, and G. C. Oades, Phys. Rev. **165**, 1770 (1968). I should like to thank Dr. B. R. Martin for making available to me the unpublished parameters of this phase-shift analysis.

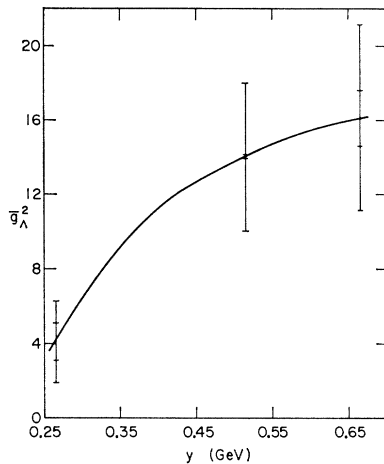[5] C. H. Chan and F. T. Meiere, Phys. Rev. Letters **20**, 568 (1968).

FIG. 1. Smooth curve through the values of $\bar{g}_A{}^2$, which are calculated from the dispersion relation (2.1) at different values of the energy y. The double error bars distinguish the source of the errors as discussed under (a) and (b) of Sec. II. The larger error bar shows the combined error from (a) and (b), the smaller error bar that from (b) alone.

If $g_A{}^2$ and $g_\Sigma{}^2$ are independent of energy, then $\bar{g}_A{}^2$ will be also, to within about 1%.

This might be unnecessary if the Kn data were not far less accurately known than the Kp data, but this loss of accuracy compensates any gain in knowledge of $g_A{}^2$ which one might achieve by eliminating $g_\Sigma{}^2$.

To check the consistency of the MER parametrization, we set $x = 0.494$ GeV, and calculate $\bar{g}_A{}^2$ for various values of y in the range $0.264 \leqslant y \leqslant 0.714$ GeV. This includes both the unphysical and the low-energy $K^- p$ regions, since the physical threshold occurs at 0.494 GeV.

Secondly, we check that the MER model is consistent with the latest phase-shift analysis of the low-energy $K^+ p$ data.[4] Thus, we set $y = 0.464$ GeV and calculate $\bar{g}_A{}^2$ for $0.494 \leqslant x \leqslant 1.64$ GeV.

We call these two calculations solutions I and II, respectively, and we plot the results in Figs. 1 and 2.

Before any meaningful analysis of the results may be made, we must look carefully at the errors, which may be classified into three types.

(a) *Error in* $\sigma^\pm$. For energies below 20 GeV, σ^+ is known to within 3% accuracy.[6] Although some experiments claim much better accuracy, e.g., Ref. 7, we believe that the apparent discontinuities where one experimental set adjoins another oblige us to be more cautious.

The total cross section σ^- is less accurately known in the low-energy regions. In the range $0.5 \leqslant \omega \leqslant 0.85$ GeV, there are no precise measurements and we feel obliged

[6] We have used the latest and most accurate total cross sections available and most of these are listed in one of the following: C. H. Chan and W. L. Yen, Phys. Rev. **165**, 1565 (1968); Y. Sumi, Progr. Theoret. Phys. (Kyoto) Suppl., **43** (1967). A notable exception is Ref. 7.

[7] D. V. Bugg *et al.*, Phys. Rev. **168**, 1466 (1968).

to attribute to σ^- the large error of 15%. From 0.85 to 1.0 GeV we give an error of 5%, and above 1.0 GeV we give an error of 3%.

The extrapolation of $\sigma^\pm$ beyond 20 GeV was carried out by use of the Regge poles and parameters given in Ref. 8. The error here is fairly arbitrary and we chose 3%. Because of the cancellation between the terms involving σ^+ and σ^-, the net contribution is small but not insignificant.

(b) *Error in the real parts*. For the real parts calculated from the phase-shift analysis an estimate of the error was obtained as follows. There are three sets of phase shifts which account for the data, and at any energy these yield three different estimates of the real part. The maximum discrepancy was taken as the error, and the mean value used as the real part. The error is typically of the order of 5%, and its contribution to the final error in $\bar{g}_A{}^2$ is usually small.

In calculating the $K^- p$ real parts from the MER model, we have made use of the parameter uncertainties quoted in Kim's analysis[3] to obtain an estimated error. As one would expect, this gives a small error near the physical threshold, but allows considerable uncertainty at the $\pi\Lambda$ threshold.

(c) *Error in the integral over the unphysical region*. We have no way of calculating this, but it is useful to see how a given error reflects in the error on $\bar{g}_A{}^2$. We have calculated the uncertainty in $\bar{g}_A{}^2$ due to an uncertainty of 20% in the integral.

In Figs. 1 and 2, we show a smooth curve representing $\bar{g}_A{}^2$ as a function of the energies x or y. Also shown are some typical errors, derived only from sources (a) and (b) above, and labeled in the figures accordingly.

From Fig. 1 it would appear that $\bar{g}_A{}^2$ is not constant, even within the errors allowed to us by the model. The
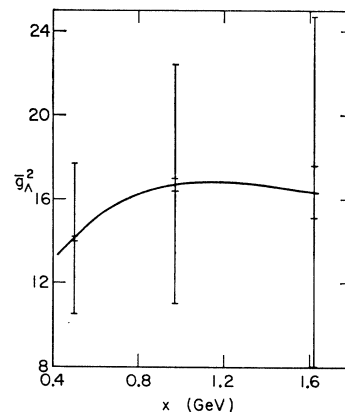

FIG. 2. Smooth curve through the values of $\bar{g}_A{}^2$, which are calculated from the dispersion relation (2.1) at different values of the energy x. The double error bars distinguish the source of the errors as discussed under (a) and (b) of Sec. II. The larger error bar shows the combined error from (a) and (b), the smaller error bar that from (b) alone.

[8] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).

parameters of the model therefore require some modification, and this could be achieved in one of two ways.

(i) It is conceivable that the energy dependence of the amplitude can be altered in such a way that the value near the physical threshold is approximately unchanged, while the value near the $\pi\Lambda$ threshold changes considerably. Thus, the predictions of the mass and width of the $Y_0{}^*(1405)$ would remain unchanged, and, since this resonance dominates the unphysical integral, the values of $\bar{g}_\Lambda{}^2$ near the physical threshold would not change either. In this way it might be possible to have $\bar{g}_\Lambda{}^2$ constant at the value of approximately 14.

(ii) It may be that the change in the unphysical absorptive part is sufficient to produce a significant change in the unphysical integral, and it is therefore interesting to have some idea as to how important this might be.

With this in mind we have computed the change in $\bar{g}_\Lambda{}^2$ (Δg^2) for a 20% change in the unphysical integral. The results are shown in Table I. They suggest that the value of $\bar{g}_\Lambda{}^2$ is very sensitive to changes in the unphysical integral, especially when evaluated near the physical threshold.

Thus, it is very much a complicated computational problem as to whether one may remove the apparent energy dependence of $\bar{g}_\Lambda{}^2$ without significantly altering its value at the physical threshold.

The results of Fig. 2 are more reassuring. They indicate that the predicted values of the unphysical absorptive part are consistent with the K^+p real parts. One might worry that there is a certain degree of circularity, which arises because of the developing habit of using dispersion relations to eliminate the well-known Yang ambiguity inherent in phase-shift determinations.

However, if some of the terms in the dispersion relation have very large errors, the constraint will be very weak, and will only decide between two types of phase shift, and not strongly limit their energy dependence. This was true of the analysis whose results we use. The assumed coupling constants, unphysical absorptive part, and s-wave K^+p scattering length were all given a large error, and the real part calculated from the dispersion integral entered the analysis with a very low statistical weight. The phase shifts are more or less completely determined by the other experimental information, and it is by no means certain that they will be compatible with the MER absorptive part and an energy-independent coupling constant.

To conclude this section, we may say that there is still some doubt about the reliability of the MER model for the purpose of calculating $\bar{g}_\Lambda{}^2$. Within present errors it is consistent with the known experimental real parts, but is not completely self-consistent, to the extent that the extrapolated real parts appear to diverge from those calculated via the dispersion relation.

TABLE I. Change Δg^2 in $\bar{g}_\Lambda{}^2$ caused by changing the value of the unphysical region contribution by 20%.

y (GeV)	Δg^2
0.264	2
0.514	8
0.714	3.5

III. DETERMINATION OF UPPER AND LOWER BOUNDS ON $\bar{g}_\Lambda{}^2$

In this section we attempt to put bounds on $\bar{g}_\Lambda{}^2$ which are independent of any particular model for the unphysical region. The only information used is the positivity of the unphysical absorptive part of the K^-p elastic amplitude, and the experimentally determined real and imaginary parts.

The positivity condition is by no means guaranteed to be satisfied, and in Ref. 1 it was shown that an approximate calculation suggests that $\mathrm{Im}M_{\mathrm{abs}}$ changes sign in the unphysical region if the relative $K\Lambda$ parity is even, and does not do so if that parity is odd. With the accepted odd $K\Lambda$ parity, no model or physical condition requires that $\mathrm{Im}M_{\mathrm{abs}}$ should change sign, and we believe that our assumption is very probably justified.

Since the method requires fairly accurate real parts at several different energies, we have had to use the results of the phase-shift analysis already quoted. However, the results of this section strongly reinforce our remarks in Sec. II, namely, that the resulting phase-shifts limit the possible values of the coupling constants to a much smaller range than was assumed at the start of the phase-shift analysis.

The technique that we use is based upon some relations given in a paper by Tiktopoulos and Treiman.[9] We extract from that paper the following results.

Consider the integral

$$\psi(\lambda) = \int_{-1}^{1} \frac{\sigma(\alpha)}{1-\alpha\lambda}\,d\alpha, \qquad (3.1)$$

where $\sigma(\alpha) \geqslant 0$ for $-1 \leqslant \alpha \leqslant 1$, and we restrict λ to the range $-1 < \lambda < 1$. Suppose that we evaluate the integral at a set of M points, so that we have

$$\psi_i = \psi(\lambda_i), \quad i=1, 2, \cdots, M. \qquad (3.2)$$

We may then show that the ψ_i must satisfy certain relations among themselves, which are summarized as follows. If out of the set $\{\psi_i\}$ we choose a subset containing m members $(m \leqslant M)$, then the elements of that subset satisfy either condition (a) or (b) below. We order the parameters λ_i so that $-1 < \lambda_1 < \lambda_2 < \cdots < \lambda_m < 1$.

[9] G. Tiktopoulos and S. Treiman, Phys. Rev. **167**, 1437 (1968). I should like to thank Professor Tiktopoulos for bringing this paper to my notice and suggesting its relevance to this problem.

(a) $m=2N$, N integral. Define

$$D_{2N}[\psi(\lambda)]$$
$$\equiv (-1)^N \begin{vmatrix} \psi_1, & \lambda_1\psi_1, & \cdots, & \lambda_1^{N-1}\psi_1, & 1, & \lambda_1, & \cdots, & \lambda_1^{N-1} \\ \psi_2, & \lambda_2\psi_2, & \cdots, & & 1, & \lambda_2, & \cdots, & \lambda_2^{N-1} \\ \cdots & & & & & & & \\ \psi_{2N} & & & & & & & \end{vmatrix}.$$

(3.3)

Then the conditions are given by

$$D_{2N}[(1-\lambda)\psi(\lambda)] \leqslant 0 \leqslant D_{2N}[(1+\lambda)\psi(\lambda)]. \quad (3.4)$$

(b) $m=2N+1$, N integral. Define

$$D_{2N+1}[\psi(\lambda)]$$
$$\equiv (-1)^N \begin{vmatrix} \psi_1, & \lambda_1\psi_1, & \cdots, & \lambda_1^N\psi_1, & 1, & \lambda_1, & \cdots, & \lambda_1^{N-1} \\ \psi_2, & \lambda_2\psi_2, & \cdots & & & & & \\ \cdots & & & & & & & \\ \psi_{2N+1} & & & & & & & \end{vmatrix}.$$

(3.5)

The required conditions are given by

$$D_{2N+1}[(1+\lambda)\psi(\lambda)] \geqslant 0,$$
$$D_{2N+1}\{[(1-\lambda)\psi(\lambda)]^{-1}\} \geqslant 0. \quad (3.6)$$

Furthermore, setting $m=1, 2, \cdots, M$ and choosing all possible subsets for a given m, we deduce a hierarchy of relations, all of which must be satisfied. As an illustration, and for future reference, we work out the results for $m=1, 2, 3$.

One-Point Conditions

Every element of the set $\{\psi_i\}$ satisfies

$$(1+\lambda_i)\psi_i \geqslant 0$$

and

$$1/(1-\lambda_i)\psi_i \geqslant 0. \quad (3.7)$$

Two-Point Conditions

Every pair of elements ψ_j, ψ_k from $\{\psi_i\}$, such that $-1 < \lambda_j < \lambda_k < 1$, satisfies

$$(1+\lambda_j)\psi_j \leqslant (1+\lambda_k)\psi_k,$$
$$(1-\lambda_j)\psi_j \geqslant (1-\lambda_k)\psi_k. \quad (3.8)$$

Three-Point Conditions

Every possible triplet ψ_j, ψ_k, ψ_l from $\{\psi_i\}$, such that $-1 < \lambda_j < \lambda_k < \lambda_l < 1$, satisfies

$$\frac{\lambda_l-\lambda_k}{(1+\lambda_j)\psi_j} + \frac{\lambda_k-\lambda_j}{(1+\lambda_l)\psi_l} + \frac{\lambda_j-\lambda_l}{(1+\lambda_k)\psi_k} \leqslant 0$$

and

$$(\lambda_l-\lambda_k)(1-\lambda_j)\psi_j + (\lambda_k-\lambda_j)(1-\lambda_l)\psi_l + (\lambda_j-\lambda_l)(1-\lambda_k)\psi_k \leqslant 0. \quad (3.9)$$

The one-point conditions are trivially satisfied, and in the rest of this paper we shall make use of only the

two-point conditions. When used in conjunction with experimental quantities which have appreciable errors, the higher inequalities give meaningless results.

To use the two-point conditions we proceed as follows. Subtracting (1.1) at $\omega=\omega_s<0$, we obtain

$$\frac{\text{Re}M^+(\omega)-\text{Re}M^-(-\omega_s)}{\omega-\omega_S}$$

$$= -\sum_{Y=\Sigma,\Lambda} \frac{X_Y g_Y^2}{2N} \frac{1}{(\omega_Y+\omega)(\omega_Y+\omega_S)}$$

$$+ \frac{1}{\pi} \int_{\bar\omega}^{\infty} \frac{\text{Im}M^+(\omega')}{(\omega'-\omega)(\omega'-\omega_S)} - \frac{\text{Im}M^-(\omega')}{(\omega'+\omega)(\omega'+\omega_S)} d\omega'. \quad (3.10)$$

From the right-hand side of (3.10) we extract the unphysical integral and rewrite it as

$$-\int_{\bar\omega}^K \frac{\text{Im}M_{\text{abs}}^-(\omega')}{(\omega'+\omega)(\omega'+\omega_S)} d\omega' = \int_{\bar\omega}^K \frac{\chi(\omega')}{\omega'+\omega} d\omega' \equiv I(\omega). \quad (3.11)$$

If $\text{Im}M_{\text{abs}}^- \geqslant 0$ and $\omega_S < -K$, then

$$\chi(\omega) = \frac{\text{Im}M_{\text{abs}}^-(\omega)}{-(\omega+\omega_S)} \geqslant 0.$$

We reduce $I(\omega)$ to the form required, by the transformations

$$\alpha = \frac{2}{K-\bar\omega}\left(\omega' - \frac{K+\bar\omega}{2}\right),$$
$$\frac{1}{\lambda} = \frac{2}{K-\bar\omega}\left(-\omega - \frac{K+\bar\omega}{2}\right), \quad (3.12)$$

so that

$$I(\omega) = \lambda \int_{-1}^{1} \frac{\chi(\alpha)}{\lambda\alpha - 1} d\alpha. \quad (3.13)$$

Applying the two-point conditions to a pair of values of $I(\omega)$, we obtain

$$(\omega_1+\bar\omega)I_1 \leqslant (\omega_2+\bar\omega)I_2,$$
$$(\omega_1+K)I_1 \geqslant (\omega_2+K)I_2, \quad (3.14)$$

for $\omega_2 > \omega_1 > K$ and $\omega_S < -K$.

If (3.10) is written in the form

$$C(\omega)\bar g_\Lambda^2 = G(\omega) + I(\omega),$$

we have

$$\bar g_\Lambda^2[C(\omega_1)(\omega_1+\bar\omega) - C(\omega_2)(\omega_2+\bar\omega)]$$
$$\leqslant (\omega_1+\bar\omega)G(\omega_1) - (\omega_2+\bar\omega)G(\omega_2) \quad (3.15)$$

and

$$\bar g_\Lambda^2[C(\omega_1)(\omega_1+K) - C(\omega_2)(\omega_2+K)]$$
$$\geqslant (\omega_1+K)G(\omega_1) - (\omega_2+K)G(\omega_2). \quad (3.16)$$

It is easy to show that the coefficient of $\bar{g}_\Lambda{}^2$ in these inequalities is positive, so that (3.15) sets an upper bound on $\bar{g}_\Lambda{}^2$ and (3.16) sets a lower bound. These bounds are expressible in terms of experimental real parts and integrals over experimentally determined quantities.

For these relations to be useful, it is imperative that the pole terms give sizable contributions to the dispersion relation, and that $G(\omega)$ can be measured to significantly better accuracy than its change over some energy range.

Our bounds are derived by setting $\omega_S = -0.614$ GeV, $\omega_2 = 1.62$ GeV, and varying ω_1 in the range $0.55 \leqslant \omega_1 \leqslant 1.5$ GeV. The results for the lower bound ($g_{min}{}^2$) are shown in Fig. 3, where we have interpolated the calculated points with a smooth curve. Representative errors are shown at three energies, and we have distinguished the error arising from the real parts from that due to the integrals. The results for the upper limit ($g_{max}{}^2$) follow a similar curve, but vary from a value of 35 to 150. These latter numbers are not useful and we do not comment on them further.

The conclusions concerning $g_{min}{}^2$ are, on the contrary, very interesting. The most reliable estimate that we can form of $g_{min}{}^2$ is 14 ± 4, this being the lower limit of $g_\Lambda{}^2 + 1.05 g_\Sigma{}^2$. The errors derive almost equally from those on the total cross sections and on the real parts. We have tried to be cautious and to allow larger errors than might seem necessary, and the major source of uncertainty lies in the validity of the particular phase-shift parameters that we use. Perhaps further effort in phase-shift analyses will result in more reliable real-part estimates.

The important point, however, is that the problem is an experimental one and can, one hopes, be tackled in the near future.

In principle, the method is equally applicable to the amplitudes of definite isospin, and limits could be found on $g_\Lambda{}^2$ and $g_\Sigma{}^2$ separately. However, this will require knowing Kp and Kn real parts over a common range of energies to a high degree of accuracy. We suspect that this will not be achieved for some time.

IV. CONCLUSIONS

We have tried to decide whether the MER parametrization of the low-energy K^-p scattering amplitude
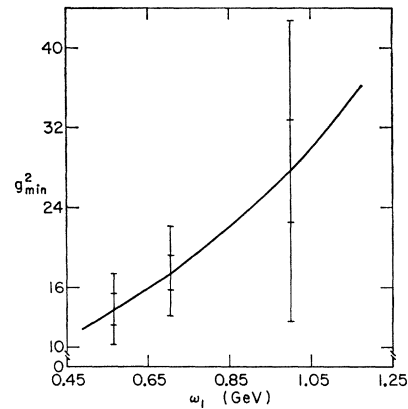

FIG. 3. Smooth curve through the values of the lower limit of $\bar{g}_\Lambda{}^2$, calculated from (3.16) for various values of ω_1. The double error bars distinguish the source of the errors as discussed under (a) and (b) of Sec. II. The larger error bar shows the combined error from (a) and (b), the smaller error bar that from (b) alone.

gives a completely self-consistent calculation of the Λ and Σ coupling constants $g_{\Lambda K - p}{}^2$ and $g_{\Sigma^0 K - p}{}^2$. In Sec. II we show that as yet it does not, but that it may well be possible to make minor modifications to achieve this, without significantly altering the presently calculated values of those constants.

Secondly, we have shown that it is possible to put bounds on the coupling constants, and that these may be calculated from a knowledge of experimental quantities only. At the present stage of experimental knowledge, this gives a useful result only for the lower bound of a combination of the Λ and Σ coupling constants. The result is consistent with the MER model, and indicates that in the future it should be possible to use the method to distinguish the $SU(3)$-satisfying values and the $SU(3)$-violating values.

ACKNOWLEDGMENTS