PHYSICAL REVIEW VOLUME 178, NUMBER 5 25 FEBRUARY 1969

Generalized $U(6)\times U(6)$ Kinematical Transformations and Properties of 2^+ Mesons*

SHAU-JIN CHANG
Physics Department, Harvard University, Cambridge, Massachusetts 02138
and
Institute for Advanced Study, Princeton, New Jersey† *08540*

(Received 5 September 1968)

Schwinger's comparative kinematical transformations are generalized to include semilocal transformations. By identifying the 2^+ mesons as bound states of two fundamental Dirac fields ψ through $h_{\mu\nu}\sim\bar{\psi}\gamma_\mu\partial_\nu\psi$, we obtain various trilinear meson-meson interactions among 2^+ mesons and 0^- and 1^- mesons. The agreement between the theoretical predictions and the experimental measurements for various decay widths is satisfactory.

1. INTRODUCTION

SINCE the proposal of the nonrelativistic $SU(6)$ theory by Sakita, and by Gürsey and Radicati,[1] there have been various attempts to construct a generalized relativistic theory. It has now been recognized that these theories have grave difficulties.[2] The explanation of the successful results of $SU(6)$, therefore, has to be sought on grounds other than $SU(6)$. Some time ago, Schwinger proposed a model in which the physical 0^- and 1^- mesons are repersented as the low-lying bound states of pairs of a more fundamental triplet of fermion fields.[3] In his model, 0^- and 1^- mesons are local objects. The relativistic dynamics of these mesons in the idealization of $U(3)$ symmetry is derived from the hypothesis that a compact group of transformations on fundamental fields induces a predominantly local and linear transformation on the phenomenological fields that are associated with particles. It is verified that the meson-interaction term derived is invariant under the parity-conserving group $U(6)\times U(6)$. In this paper, these local kinematical transformations are generalized to arbitrary semilocal transformations. Making use of the technique of comparative kinematical transformations, and assuming that the 2^+ (and the 1^+) mesons can be represented by some semilocal ψ-combinations, we can determine the dynamics of 2^+ (and 1^+) mesons easily.

* This work is based on part of a dissertation submitted to Harvard University in partial fulfillment of the requirements for the degree of Doctor of Philosophy. Publication of this paper is supported in part by the National Science Foundation.

† Present address.

[1] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, *ibid.* **13**, 175 (1964); F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1964); B. Sakita, *ibid.* **13**, 643 (1964); Phys. Rev. **136**, B1759 (1964); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965).

[2] S. Coleman, talk given at the East Coast Theoretical Physics Conference, November, 1965 (unpublished). More comprehensive discussion can be found in *Proceedings of the International Seminar in High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965).

[3] J. Schwinger, Phys. Rev. **140**, B158 (1965). The basic concept of the comparative-kinematical-transformation technique is contained in this paper and in *Proceedings of the Second Coral Gables Conference on Symmetry Properties at High Energy*, edited by B. Kursunoglu, A. Perlmutter, and I. Sakmar (W. H. Freeman and Co., San Francisco, Calif., 1965). The notations used in this paper follow this reference closely.

2. GENERALIZED KINEMATICAL TRANSFORMATIONS

Infinitesimal semilocal kinematical transformations on $\psi_{a\alpha}(x)$ and $\psi_{a\alpha}^\dagger(x)$ at a fixed time x^0 are given by

$$\delta\psi(\mathbf{x})=\int i\delta\lambda(\mathbf{x},\mathbf{y})\psi(\mathbf{y})d^3y\,,$$
$$\delta\psi^\dagger(\mathbf{x})=\int -i\psi^\dagger(\mathbf{y})\delta\lambda(\mathbf{y},\mathbf{x})d^3y\,, \tag{1}$$

where a and α are the unitary and the Dirac indices, respectively, and $\delta\lambda_{a\alpha,a'\alpha'}(\mathbf{x},\mathbf{y})$ is a 12×12 semilocal matrix which is Hermitian in the general sense

$$\delta\lambda_{a'\alpha',a\alpha}(\mathbf{y},\mathbf{x})^*=\delta\lambda_{a\alpha,a'\alpha'}(\mathbf{x},\mathbf{y})\,.$$

The meaning of semilocal is that $\delta\lambda(\mathbf{x},\mathbf{y})$ is nonvanishing only in a small neighborhood around $x=y$. These transformations are generated by the generators

$$\int\delta\lambda(\mathbf{x},\mathbf{y})m(\mathbf{x},\mathbf{y})d^3xd^3y\,.$$

The parity-preserving generators are

$$m_{AB}{}^{(\pm)}(x,y)=\psi_a{}^\dagger(x)\tfrac{1}{2}(1\pm\gamma^0)\tfrac{1}{2}\begin{pmatrix}1+\sigma_3 & \sigma_1+i\sigma_2\\ \sigma_1-i\sigma_2 & 1-\sigma_3\end{pmatrix}\psi_b(y)$$
$$=[m_{BA}{}^{(\pm)}(y,x)]^\dagger\,, \tag{2}$$

where A, for example, is a sextuple-valued index that combines a and the double-valued spin label; and the parity-changing generators are

$$m_{AB}(x,y)=-\psi_a{}^\dagger(x)\gamma_5\tfrac{1}{2}(1+\gamma^0)\times\tfrac{1}{2}\begin{pmatrix}1+\sigma_3 & \sigma_1+i\sigma_2\\ \sigma_1-i\sigma_2 & 1-\sigma_3\end{pmatrix}\psi_b(y)\,, \tag{3}$$

$$m_{AB}{}^\dagger(x,y)=\psi_a{}^\dagger(x)\tfrac{1}{2}(1+\gamma^0)\gamma_5\tfrac{1}{2}\begin{pmatrix}1+\sigma_3 & \sigma_1+i\sigma_2\\ \sigma_1-i\sigma_2 & 1-\sigma_3\end{pmatrix}\psi_b(y)$$
$$=[m_{BA}(y,x)]^\dagger\,. \tag{4}$$

In Eqs. (2)–(4), all x and y are assumed to be at equal time. For simplicity, we denote the generator $m(x,y)|_{x^0=y^0}$ by $m(x,y)$. These generators have the following equal-time commutator structure:

$$[m_{AB}^{(+)}(x,y),m_{CD}^{(-)}(x',y')]=0\,, \tag{5}$$

$$[m_{AB}^{(\pm)}(x,y),m_{CD}^{(\pm)}(x',y')]=\delta(\mathbf{x}'-\mathbf{y})\delta_{BC}m_{AD}^{(\pm)}(x,y') -\delta(\mathbf{x}-\mathbf{y}')\delta_{AD}m_{CB}^{(\pm)}(x',y)\,, \tag{6}$$

$$[m_{AB}(x,y),m_{CD}^{(+)}(x',y')]=\delta(\mathbf{x}'-\mathbf{y})\delta_{BC}m_{AD}(x,y')\,, \tag{7}$$

$$[m_{AB}(x,y),m_{CD}^{(-)}(x',y')] =-\delta(\mathbf{x}-\mathbf{y}')\delta_{AD}m_{CB}(x',y)\,, \tag{8}$$

$$[m_{AB}^{\dagger}(x,y),m_{CD}^{(+)}(x',y')] =-\delta(\mathbf{x}-\mathbf{y}')\delta_{AD}m_{CB}^{\dagger}(x',y)\,, \tag{9}$$

$$[m_{AB}^{\dagger}(x,y),m_{CD}^{(-)}(x',y')] =\delta(\mathbf{x}'-\mathbf{y})\delta_{BC}m_{AD}^{\dagger}(x,y')\,, \tag{10}$$

and

$$[m_{AB}(x,y),m_{CD}^{\dagger}(x',y')]=\delta(\mathbf{x}'-\mathbf{y})\delta_{BC}m_{AD}^{(-)}(x,y') -\delta(\mathbf{x}-\mathbf{y}')\delta_{AD}m_{CB}^{(+)}(x',y)\,. \tag{11}$$

In this paper, all the generators of the generalized kinematical group are undertsood to be semilocal, i.e., $x\approx y$ and $x'\approx y'$, etc. After the identification

$$m_{AB}(x)=m_{AB}(x,x)\,,$$

we reproduce the local $U(6)\times U(6)$ kinematical group structure considered by Schwinger.[3]

Since the physical mesons and baryons are local bound states of the fundamental ψ fields, a semilocal and linear transformation on ψ will induce predominantly a corresponding semilocal transformation on the phenomenological meson and baryon fields $M(x)$ and $B(x)$, through

$$\delta M(x)=i\int T\delta\lambda(\mathbf{x},\mathbf{y})M(y)d^3y\,,\qquad \delta B(x)=i\int T\delta\lambda(\mathbf{x},\mathbf{y})B(y)d^3y\,, \tag{12}$$

where

$$T\delta\lambda=\sum T_{AB}\delta\lambda_{AB}\,.$$

The associated transformation matrices $T\alpha$ obey the corresponding group commutation relations. One should notice that the parity-preserving subgroup, which has the group structure of $U(6)\times U(6)$, is of special physical interest.[3] This subgroup leads to linear kinematical transformations among the independent components of the 0^- and 1^- meson fields. This property ensures that the corresponding transformations on the meson fields can be generated by alternative generators in terms of the phenomenological 0^- and 1^- fields alone. In other words, we are led to the conjecture that the independent components of the meson variables form a separate basis for this generalized kinematical group. Following Schwinger, we can compile these 72 independent components of the 0^- and 1^- meson variables into the following matrix forms:

$$M_{AB}(x)=(\tfrac{1}{2}m)^{3/2}/(-g)(\phi_{ab}^{+}+\sigma_k{}^{T}U_{kab}^{+})\,, \tag{13}$$

$$M_{AB}^{\dagger}(x)=(M_{BA})^{\dagger}=(\tfrac{1}{2}m)^{3/2}/(-g) \times(\phi_{ab}^{-}+\sigma_k{}^{T}U_{kab}^{-})\,, \tag{14}$$

where

$$\phi^{\pm}=(\tfrac{1}{2}m)^{1/2}\phi\mp i(2m)^{-1/2}\phi^0$$

and

$$U_k{}^{\pm}=(2m)^{-1/2}U_k{}^0\pm i(\tfrac{1}{2}m)^{1/2}U_k$$

are non-Hermitian combinations of the independent components of 0^- and 1^- mesons, m is the mass of the 1^- mesons, and g is a universal coupling constant. These M's satisfy the equal-time commutator relations

$$[M(x),M(x')]=[M^{\dagger}(x),M^{\dagger}(x')]=0\,,\qquad [M_{AB}(x),M_{CD}^{\dagger}(x')]=(m^3/4g^2)\delta_{BC}\delta_{AD}\delta(\mathbf{x}-\mathbf{x}')\,. \tag{15}$$

The required generators can be expressed in terms of these special combinations as

$$m_{AB}^{(+)}(x,y)\simeq(4g^2/m^3)[M^{\dagger}(x)M(y)]_{AB}\,,\qquad m_{AB}^{(-)}(x,y)\simeq-(4g^2/m^3)[M(x)M^{\dagger}(y)]_{AB}\,, \tag{16}$$

which possess all the commutator properties required by the generalized $U(6)\times U(6)$ generators. In these expressions and hereafter, we denote the generator on the fundamental level by m, and the corresponding generator linear in the phenomenological field variables by M.

3. 0^+, 1^+, AND 2^+ MESONS

In contrast to the 0^- and 1^- mesons, the physical 2^+ mesons can not be identified with the simple bilinear ψ combinations,

$$\int f(|\xi|)\bar{\psi}(x+\tfrac{1}{2}\xi)\Gamma\psi(x-\tfrac{1}{2}\xi)d^3\xi\,,$$

with Γ being some Dirac matrices, because the latter can not carry an extra unit of angular momentum. It is very natural to identify them with the next simplest expression where only one gradient is involved[4]:

$$m^{5/2}m_s{}^{1/2}[\phi_{\mu\nu}(x)]_{ab}\leftrightarrow -\int i\partial_\nu f(|\xi|)\bar{\psi}_a(\mathbf{x}+\tfrac{1}{2}\xi)\gamma_\mu\psi_b(\mathbf{x}-\tfrac{1}{2}\xi)d^3\xi\,,$$
$$m^{5/2}m_s{}^{-1/2}[{}_\lambda\phi_{\mu\nu}(x)]_{ab}\leftrightarrow \int i\partial_\lambda f(|\xi|)\bar{\psi}_a(\mathbf{x}+\tfrac{1}{2}\xi)\sigma_{\mu\nu}\psi_b(\mathbf{x}-\tfrac{1}{2}\xi)d^3\xi\,. \tag{17}$$

---

[4] Under the local approximation $f(\xi)=f\delta(\xi)$, these meson fields can be identified as $\phi_{\mu\nu}\sim\bar{\psi}\gamma_\mu i\partial_\nu\psi$, ${}_\lambda\phi_{\mu\nu}\sim\bar{\psi}\sigma_{\mu\nu}i\partial_\lambda\psi$. The original concept for representing spin-2 mesons by these expressions was invented by J. Schwinger. See *Note added in proof* of Ref. 3.

In Eq. (17), m_s is a mass scaling factor, and $f(|\xi|)$ is a spherically symmetric scalar function related to the internal wave function of the 2^+ mesons. It should be noted that the field variables $\phi_{\mu\nu}$, ${}_\lambda\phi_{\mu\nu}$ describe more than a single 2^+ meson. They actually describe the superpositions of a 0^+, a 1^+, and a 2^+ meson field[5] whose neutral members all have the same spatial parity +1 and C parity +1. The mass factors, m_s's, and the internal wave functions are in general different for mesons with different spins. We express them collectively only for the purpose of simplicity, and we should always keep in mind that they actually represent different factors. $\phi_{\mu\nu}$ and ${}_\lambda\phi_{\mu\nu}$ are related to the phenomenological 0^+, 1^+, and 2^+ meson fields, $\Phi^{(0)}$, $\Phi^{(1)}$, and $\Phi^{(2)}$, through

$$\phi_{\mu\nu}=\frac{1}{\sqrt{3}}\left(g_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{m_0^2}\right)\Phi^{(0)}+\frac{1}{2\sqrt{2}m_1}\epsilon_{\mu\nu\lambda\sigma}\Phi^{\lambda\sigma(1)}+\Phi_{\mu\nu}{}^{(2)},$$

$${}_\lambda\phi_{\mu\nu}=\frac{1}{\sqrt{3}}\left(g_{\lambda\nu}-\frac{\partial_\lambda\partial_\nu}{m_0^2}\right)\Phi_\mu{}^{(0)}-\frac{1}{\sqrt{3}}\left(g_{\lambda\mu}-\frac{\partial_\lambda\partial_\mu}{m_0^2}\right)\Phi_\nu{}^{(0)}+\frac{m_1}{\sqrt{2}}\left(g_{\lambda\rho}-\frac{\partial_\lambda\partial_\rho}{m_1^2}\right)\epsilon^\rho{}_{\mu\nu\sigma}\Phi^{\sigma(1)}+{}_\lambda\Phi_{\mu\nu}{}^{(2)},\qquad(18)$$

where m_0 and m_1 are the physical masses of 0^+ and 1^+ mesons, and will be denoted collectively as $\mathfrak{M}$ hereafter. The independent components of the 0^+, 1^+, and 2^+ mesons are associated with ϕ_{kl} and ${}_k\phi_{0l}$, and their corresponding equal-time commutator relations can be derived from Eq. (17) through the anticommutator relations among the fundamental fields. These commutators should be identical to those derived from the phenomenological fields. This is a consistency test for our dynamical assumption of representing composite systems by phenomenological fields. Under the local approximation, we can replace $f(|\xi|)$ by $f\delta^3(\xi)$. We will keep $f(|\xi|)$ explicitly for computational convenience, and equate it to $f\delta^3(\xi)$ only in the final expressions. The equal-time commutator relations derived from the fundamental level are

$$\langle[(\phi_{ab})_{kl}(\mathbf{x}),(\phi_{cd})_{pq}(\mathbf{x}')]\rangle=\langle[{}_k(\phi_{ab})_{0l}(\mathbf{x}),{}_p(\phi_{cd})_{0q}(\mathbf{x}')]\rangle=0,$$

$$\langle i[(\phi_{ab})_{kl}(\mathbf{x}),{}^p(\phi_{cd})^{0q}(\mathbf{x}')]\rangle=\frac{i}{m^5}\int\int d^3\xi d^3\xi'\partial_l f(\xi)\partial_p f(\xi')\langle[\bar\psi_a(\mathbf{x}+\tfrac12\xi)\gamma_k\psi_b(\mathbf{x}-\tfrac12\xi),\bar\psi_c(\mathbf{x}'+\tfrac12\xi')\sigma^0{}_q\psi_d(\mathbf{x}'-\tfrac12\xi')]\rangle$$

$$=\frac{\delta_{kq}}{m^5}\int\int d^3\xi d^3\xi'\partial_l f(\xi)\partial_p f(\xi')[\delta_{bc}\delta(\mathbf{x}-\mathbf{x}'-\tfrac12(\xi+\xi'))\langle\bar\psi_a(\mathbf{x}+\tfrac12\xi)\psi_d(\mathbf{x}'-\tfrac12\xi')\rangle$$
$$+\delta_{ad}\langle\bar\psi_c(\mathbf{x}'+\tfrac12\xi')\psi_b(\mathbf{x}-\tfrac12\xi)\rangle\delta(\mathbf{x}-\mathbf{x}'+\tfrac12(\xi+\xi'))],\qquad(19)$$

where use is made of

$$\langle\bar\psi(\mathbf{x})\gamma_k\gamma_l\psi(\mathbf{x}')\rangle=-\delta_{kl}\langle\bar\psi(\mathbf{x})\psi(\mathbf{x}')\rangle.$$

Since $\langle\bar\psi(\mathbf{x})\psi(\mathbf{x}')\rangle$ is an even function of $\mathbf{x}-\mathbf{x}'$, our expression can be reduced to

$$\delta_{ad}\delta_{bc}\frac{\delta_{kq}}{m^5}\langle\bar\psi(2\mathbf{x})\psi(2\mathbf{x}')\rangle\int\int d\xi d\xi'\partial_l f(\xi)\partial_p f(\xi')[\delta(\mathbf{x}-\mathbf{x}'+\tfrac12(\xi+\xi'))+\delta(\mathbf{x}-\mathbf{x}'-\tfrac12(\xi+\xi'))]$$
$$=\frac{\delta_{kq}}{2m^5}\langle\bar\psi(2\mathbf{x})\psi(2\mathbf{x}')\rangle\frac{\partial}{\partial x^l}\frac{\partial}{\partial x^p}f^{(2)}(\mathbf{x}-\mathbf{x}')\delta_{ad}\delta_{bc},$$

where $f^{(2)}(\mathbf{x})$ is the convolution of $f(\mathbf{x})$ with itself. Under the approximation $f(\mathbf{x})=f\delta(\mathbf{x})$, we have

$$f^{(2)}(\mathbf{x})=f^2\delta(\mathbf{x});$$

consequently, our commutator can be reduced to

$$x^0=x^{0\prime}:\quad\langle i[(\phi_{ab})_{kl}(x),{}^p(\phi_{cd})^{0q}(x')]\rangle$$
$$=\frac{\delta_{kq}}{2m^5}[\tfrac43 f^2\langle\bar\psi\nabla^2\psi\rangle\delta_{pl}\delta(\mathbf{x}-\mathbf{x}')+f^2\langle\bar\psi\psi\rangle\partial_p\partial_l\delta(\mathbf{x}-\mathbf{x}')]\delta_{ad}\delta_{bc}.$$

[5] The introduction of 0^+ and 1^+ mesons here is purely formal. Our final results do *not* depend on the existence of these mesons. The essential point is that the 2^+ mesons can be described by the traceless and symmetric part of $\phi_{\mu\nu}\sim\bar\psi\gamma_\mu i\partial_\nu\psi$.

These are indeed identical to the results obtained from the canonical commutators taken between the phenomenological fields,

$$x^0=x^{0\prime}:\quad [(\phi_{ab})_{kl}(x),(\phi_{cd})_{pq}(x')] =[{}_k(\phi_{ab})_{0l}(x),{}_p(\phi_{cd})_{0q}(x')]=0\,,$$

$$i[(\phi_{ab})_{kl}(x),{}^p(\phi_{cd})^{0q}(x')] =\delta_{ad}\delta_{bc}\delta_{kq}\left(\delta_{pl}-\frac{\partial_k\partial_l}{\mathfrak{M}^2}\right)\delta(\mathbf{x}-\mathbf{x}')\,, \tag{20}$$

provided that

$$\frac{f^2\langle\bar{\psi}\psi\rangle}{2m^5}=-\frac{1}{\mathfrak{M}^2},\quad \frac{2f^2\langle\bar{\psi}\nabla^2\psi\rangle}{3m^5}=1\,. \tag{21}$$

The agreement between the canonical commutators on both levels confirms our identification of $\phi_{\mu\nu}$ and ${}_\lambda\phi_{\mu\nu}$ as the superposition of 0^+, 1^+, and 2^+ mesons. Under the assumption that the vacuum expectation value of $\bar{\psi}\psi$, i.e., $\langle\bar{\psi}(x)\psi(x')\rangle_{x=x'}$, takes the same numerical value for the system describing the 0^- and 1^- mesons, we obtain a relation between the coupling constant f defined here and the coupling constant g defined in Ref. 3:

$$\frac{f^2}{4\pi}=\left(\frac{g^2}{4\pi}\right)\left(\frac{2m}{\mathfrak{M}}\right)^2. \tag{22}$$

In deriving Eq. (22), we have made use of[6] $2g\langle\bar{\psi}\psi\rangle=-m^3$. As we shall see, relation (22) is satisfied experimentally. Another important consequence is that the nonuplet of 2^+ mesons are degenerate under the limit of unitary symmetry—a result which is also realized experimentally.

4. DYNAMICS OF 2^+ (and 1^+) MESONS

To compute the interaction between the 2^+ (and 1^+) mesons and the 0^-, 1^- mesons, we express the dependent fields of the 2^+ (and 1^+) meson variables in terms of the independent fields of the 2^+ (and 1^+) meson variables through the field equations

$$\phi^0{}_p=-\frac{1}{\mathfrak{M}^2}\partial_k{}^p\phi^{0k}+\text{further structure associated with other systems}. \tag{23}$$

$${}_k\phi_{lm}=\partial_l\phi_{mk}-\partial_m\phi_{lk}+\text{further structure associated with other systems}. \tag{24}$$

The further structures in Eqs. (23) and (24) can be determined through the correspondence of the generators expressed in Eq. (16). With the help of Eqs. (2), (13), and (14), Eq. (16) can be expressed explicitly as

$$\psi_a{}^\dagger(x)\tfrac{1}{2}(1+\gamma^0)\tfrac{1}{2}\begin{pmatrix}1+\sigma_3 & \sigma_1+i\sigma_2\\ \sigma_1-i\sigma_2 & 1-\sigma_3\end{pmatrix}\psi_b(y)\leftrightarrow \tfrac{1}{2}[(\phi^-(x)+\sigma_k{}^TU_k{}^-(x))(\phi^+(y)+\sigma_l{}^TU_l{}^+(y))]_{AB}\,,$$

$$\psi_a{}^\dagger(x)\tfrac{1}{2}(1-\gamma^0)\tfrac{1}{2}\begin{pmatrix}1+\sigma_3 & \sigma_1+i\sigma_2\\ \sigma_1-i\sigma_2 & 1-\sigma_3\end{pmatrix}\psi_b(y)\leftrightarrow -\tfrac{1}{2}[(\phi^+(x)+\sigma_k{}^TU_k{}^+(x))(\phi^-(y)+\sigma_l{}^TU_l{}^-(y))]_{AB}\,,$$

which leads to, in particular,

$$\bar{\psi}_a(x)\gamma^0\psi_b(y)\leftrightarrow[\phi^-(x)\phi^+(y)-\phi^+(x)\phi^-(y) +U_k{}^-(x)U_k{}^+(y)-U_k{}^+(x)U_k{}^-(y)]_{ab}\,,$$

$$\bar{\psi}_a(x)\sigma_{[k]}\psi_b(y)\leftrightarrow\{\phi^-(x)U_k{}^+(y)+U_k{}^+(x)\phi^-(y) +U_k{}^-(x)\phi^+(y)+\phi^+(x)U_k{}^-(y) -i\epsilon_{klm}[U_l{}^-(x)U_m{}^+(y)-U_m{}^+(x)U_l{}^-(y)]\}_{ab}\,.$$

Then, making use of

$$(\phi_{ab})^0{}_p(x)=-ifm^{-5/2}m_s{}^{-1/2}\times\int\partial_p\delta(\xi)\bar{\psi}_a(x+\tfrac{1}{2}\xi)\gamma^0\psi_b(x-\tfrac{1}{2}\xi)d^3\xi\,,$$

$${}_k(\phi_{ab})_{lm}(x)=ifm^{-5/2}m_s{}^{1/2}\times\int\partial_k\delta(\xi)\bar{\psi}(x+\tfrac{1}{2}\xi)\sigma_{lm}(x-\tfrac{1}{2}\xi)d^3\xi\,,$$

and after including the derivative terms, we have

$$(\phi_{ab})^{0p}=-\mathfrak{M}^{-2}\partial_k{}^p(\phi_{ab})^{0k}+fm^{-5/2}m_s{}^{-1/2} \times[\phi^0\partial^p\phi-\phi\partial^p\phi^0+U^0{}_m\partial^pU_m-U_m\partial^pU^0{}_m]_{ab}\,, \tag{25}$$

$${}_k(\phi_{ab})_{[m]}=\epsilon_{mnl}\partial_n(\phi_{ab})_{lk}+ifm_s{}^{1/2}m^{-5/2} \times[U^0{}_m\partial_k\phi+\phi\partial_kU^0{}_m-U_m\partial_k\phi^0-\phi^0\partial_kU_m -i\epsilon_{mpl}(m^{-1}U^0{}_p\partial_kU^0{}_l+mU_p\partial_kU_l)]_{ab}\,. \tag{26}$$

Through the machinery of relativistic field theory, Eqs. (25) and (26) are sufficient to determine the interaction of 2^+ mesons with 0^- and 1^- mesons. Comparing Eqs. (25) and (26) with[7]

$$\mathfrak{M}^2(\phi_{ab})^{0k}=-\partial_l{}^k(\phi_{ab})^{0l}-\partial L_{\text{int}}/\partial(\phi_{ba})^{0k}\,,$$

$${}_k(\phi_{ab})_{lm}=\partial_l(\phi_{ab})_{mk}-\partial_m(\phi_{ab})_{lk}-2\partial L_{\text{int}}/\partial_k(\phi_{ba})_{lm}\,,$$

---

[6] This relation can be obtained by commuting ϕ with ϕ^0 or U_k with $U_k{}^0$ in the fundamental level.

[7] These two expressions should be understood as

$$m_2{}^2\phi^{(2)0k}=-\partial_l{}^k\phi^{(2)0l}-\partial L_{\text{int}}/\delta\phi^{(2)0k},$$

$$\phi^{(1)}{}_{lm}=\partial_l\phi^{(1)}{}_m-\partial_m\phi^{(1)}{}_l-2\partial L_{\text{int}}/\partial\phi^{(1)}{}_{lm},\text{ etc.}$$

and after relativistic completion, we have[8]

$$\begin{aligned}L_{\rm int}=&f\mathfrak{M}^2m^{-5/2}m_s^{-1/2}\,\mathrm{Tr}\phi_{\mu\nu}[\phi^\mu\partial^\nu\phi-\phi\partial^\nu\phi^\mu+U^\mu{}_\lambda\partial^\nu U^\lambda\\&-U_\lambda\partial^\nu U^{\mu\lambda}+(\text{possible } g_{\mu\nu}\text{ term})]\\&-\tfrac{1}{2}ifm_s^{1/2}m^{-5/2}\,\mathrm{Tr}^\lambda\phi^{\mu\nu}\{\epsilon_{\alpha\beta\mu\nu}[\tfrac{1}{2}U^{\alpha\beta}\partial_\lambda\phi\\&+\tfrac{1}{2}\phi\partial_\lambda U^{\alpha\beta}-U^\beta\partial_\lambda\phi^\alpha-\phi^\alpha\partial_\lambda U^\beta]\\&+i(mU_\mu\partial_\lambda U_\nu-mU_\nu\partial_\lambda U_\mu-m^{-1}U_{\sigma\mu}\partial_\lambda U^\sigma{}_\nu\\&+m^{-1}U_{\sigma\nu}\partial_\lambda U^\sigma{}_\mu)\}\,.\qquad(27)\end{aligned}$$

The possible existence of the $g_{\mu\nu}$ terms in the interaction Lagrangian indicates that the interactions of the 0^+ meson with 0^- and 1^- mesons can not be fully determined. We would also like to point out that, in Eq. (27), the $\Phi_{\mu\nu}^{(2)}$ is coupled to the stress tensor of the system. This is in striking analogy with the well-known 1^- mesons. The dynamical couplings of 2^+ mesons should, therefore, have a universal character similar to those of 1^- mesons, since the former provided a basis for the representation of gravitational interactions analogous to the electromagnetic use of vector meson.[3]

Under the assumption that the independent components of the 0^+, 1^+, and 2^+ mesons form a separate basis for the generalized $U(6)\times U(6)$ kinematical group, we are able to compute the interaction between one 0^- or 1^- meson and two 2^+ (and/or 1^+) mesons through the technique of comparative kinematical transformations. We shall omit the detailed calculations here, and only copy down their final results:

$$\begin{aligned}L_{\rm int}=&ig\,\mathrm{Tr}\,{}^\lambda\phi^{\mu\nu}[U_\mu,\phi_{\lambda\nu}]-(g/m)\,\mathrm{Tr}\phi^\lambda\{\phi^{\mu\nu},{}_\mu\phi_{\tilde{\nu\lambda}}\}\\&+(igm_s/m)\,\mathrm{Tr}U_{\mu\nu}\phi^{\lambda\mu}\phi_\lambda{}^\nu-(ig/mm_s)\,\mathrm{Tr}U_{\mu\nu}{}^\lambda\phi^{\sigma\mu}{}_\lambda\phi_\sigma{}^\nu\end{aligned}$$

with

$${}_\mu\phi_{\tilde{\gamma\lambda}}=\tfrac{1}{2}\epsilon_{\nu\lambda\sigma\rho}\,{}_\mu\phi^{\sigma\rho}\,.$$

5. COMPARISON WITH EXPERIMENTS

It is now well established that the 2^+ mesons form an octet, plus a singlet with approximately equal masses of about 1300 MeV. They are identified experimentally as A_2(1300 MeV, $T=1$), K^{**}(1420 MeV, $T=\frac{1}{2}$), f(1260 MeV, $T=0$), and f'(1514 MeV, $T=0$), which have the C parity $+1$ and $J^P=2^+$.[3,7] The f and f' are linear combinations of the unitary singlet and the octet singlet. Under the assumption that the symmetry-breaking interaction transforms like a component of an octet, it is sufficient to determine the mixing angle between f (and f') and the unitary singlet from the masses of these $8+1$ mesons. The f and f' are then found predominantly as

$$f\sim(|11\rangle+|22\rangle)/\sqrt{2},\quad f'\sim|33\rangle,$$

which is analogous to ω and ϕ in the 1^- mesons. It has been shown by Nieh[10] that this special mixing pattern for vector mesons is a consequence of the vector-current conservation, if the symmetry-breaking effect is dominated by S_{33}, the vacuon. This argument can be applied to the 2^+ mesons, where the tensor currents coupled to the 2^+ mesons are also approximately conserved. The 2^+ mesons can then be displayed in the square array as

$$(H)=\begin{pmatrix}(-A_2^0+f)/\sqrt{2} & A_2^+ & K^{**+}\\ -A_2^- & (A_2^0+f)/\sqrt{2} & K^{**0}\\ \bar{K}^{**-} & \bar{K}^{**0} & f'\end{pmatrix}.$$

Since the present situation about the 1^+ mesons are not yet clear, and the available data are not sufficient to give a crucial test on our proposed interactions; we will not discuss the interactions relating the 1^+ mesons hereafter. Tests of the prediction that $H\phi\phi$, $H\phi U$, and HUU are all governed by a single coupling constant are avilable only for the first two, which are involved in various 2^+ meson decays. After the spin indices are suppressed, the interaction Lagrangians have the following protoforms:

$$\mathrm{Tr}H\phi\phi\quad\text{and}\quad\mathrm{Tr}H(U\phi-\phi U)\,,$$

where ϕ and U are nonuplets of 0^- and 1^- mesons displayed in the matrix forms[11]

$$\phi=\begin{pmatrix}-\pi^0/\sqrt{2}+\frac{1}{2}(\eta+\delta) & \pi^+ & K^+\\ -\pi^- & \pi^0/\sqrt{2}+\frac{1}{2}(\eta+\delta) & K^0\\ \bar{K}^- & \bar{K}^0 & (-\eta+\delta)/\sqrt{2}\end{pmatrix},$$

$$U=\begin{pmatrix}(-\rho^0+\omega)/\sqrt{2} & \rho^+ & K^{*+}\\ -\rho^- & (\rho^0+\omega)/\sqrt{2} & K^{*0}\\ \bar{K}^{*-} & \bar{K}^{*0} & \phi\end{pmatrix}.$$

The unitary structures can be written out explicitly as

$$\begin{aligned}\mathrm{Tr}H\phi\phi=&\bar{A}\pi(\eta+\delta)-\sqrt{2}A\bar{K}tK+(1/\sqrt{2})f\bar{\pi}\pi\\&+\tfrac{1}{2}(f'+f/\sqrt{2})(\eta\eta+\delta\delta+2\bar{K}K)\\&-(f'-f/\sqrt{2})\eta\delta-\sqrt{2}\pi(\bar{K}^{**}tK+\bar{K}tK^{**})\\&+(\bar{K}K^{**}+\bar{K}^{**}K)[\tfrac{1}{2}(\delta+\eta)+(1/\sqrt{2})(\delta-\eta)]\end{aligned}$$

and

$$\begin{aligned}&\mathrm{Tr}H(U\phi-\phi U)\\&=\sqrt{2}A\bar{\rho}t\pi+\sqrt{2}A(\bar{K}^*tK-\bar{K}tK^*)\\&+(f/\sqrt{2}-f')(\bar{K}K^*-\bar{K}^*K)\\&+\sqrt{2}\pi(\bar{K}^{**}tK^*-\bar{K}^*tK^{**})+[\tfrac{1}{2}(\eta+\delta)+(\eta-\delta)/\sqrt{2}]\\&\times(\bar{K}^*K^{**}-\bar{K}^{**}K^*)+\sqrt{2}\rho(\bar{K}tK^{**}-\bar{K}^{**}tK)\\&+(\phi-\omega/\sqrt{2})(\bar{K}K^{**}-\bar{K}^{**}K)\,,\end{aligned}$$

where t represents the appropriate isotopic spin matrices and

$$\bar{\pi}=-e^{\pi it_2}\pi=(-\pi^-,-\pi^+,\pi^0)\,,$$

[8] The possible existence of terms like

$$\mathrm{Tr}^\lambda\phi^{\mu\nu}\phi_\mu\partial_\lambda\phi_\nu\ \text{ and }\ \mathrm{Tr}\phi_{\mu\lambda}\phi_\nu\partial^\lambda U^{\mu\nu},$$

which can not be determined by means of these comparative kinematical transformations, is suppressed in this simplified model.

[9] S. U. Chung *et al.*, Phys. Rev. Letters **15**, 325 (1965); S. L. Glashow and R. H. Socolow, *ibid.* **15**, 329 (1965).

[10] H. T. Nieh, Phys. Rev. Letters **15**, 902 (1965); Phys. Rev. **146**, 1012 (1966).

[11] J. Schwinger, Phys. Rev. **135**, B816 (1964).

etc. The coefficients in these expansions describe the intrinsic strength of various couplings, and are denoted by g_i.

In order to compare our results with experiment, we have to know the $SU(3)$ symmetry-breaking effect. We make the simple physical assumption that the actual interaction takes the same foım even in the presence of symmetry-breaking interaction. All the symmetry-breaking effects can be taken care of by the proper mass terms. Then, the theoretical prediction for various decay widths are

$$\Gamma_{(2^+\to 0^-+0^-)}=\frac{f^2}{4\pi}\left(\frac{4}{15}g_i^2\right)\left(\frac{p}{m}\right)^5\frac{M_2^2}{m_2},$$

$$\Gamma_{(2^+\to 0^-+1^-)}=\frac{f^2}{4\pi}(\tfrac{2}{5}g_i^2)\left(\frac{p}{m}\right)^5 m_2,$$

where $\Gamma_{(2^+\to 0^-+0^-)}$ denotes the partial width of a 2^+ meson decaying into two 0^- mesons, etc., m_2 and M_2 are the scaling masses and the physical masses of 2^+ mesons, respectively, and p is the magnitude of the spatial momentum of the decay products in the center-of-mass system. In these calculations, only the lowest-order perturbation theory has been used.[12] Now we have a two-parameter fit for all the 2^+ decay modes. We take the experimental widths of $A_2\to\rho\pi$ and $K^{**}\to K\pi$ as our input, then the coupling constant f and the mass factor m_2 are determined as

$$f^2/4\pi\approx\tfrac{1}{2}\quad\text{and}\quad m_2=2542\text{ MeV},$$

while the coupling constant computed from Eq. (22) is

$$f^2/4\pi\approx\tfrac{2}{3}.$$

The agreement between these two values of f is fair. Making use of the $f^2/4\pi$ and m_2 determined above, we can compute all decay rates easily. The comparisons between the calculated and the observed partial widths are listed in Table I. The agreement is, in general, very good.

TABLE I. Various decay modes of 2^+ mesons.

Decaying particles	Products	p (MeV)	$\Gamma_{\rm cal}$ (MeV)	$\Gamma_{\rm expt}$[a] (MeV)	Remarks
$A_2(1300)$	$\rho\pi$	410	85	85 ±2.2	input
	$\bar{K}K$	424	4.4	4.0±1.1	
	$\eta\pi$	526	13	10.9±2.1	
	$\delta\pi$	275	0.5	<10	$\delta=\eta'$
	total			90 ±10	
$K^{**}(1420)$	$K\pi$	615	50	51 ±3	input
	$K^*\pi$	413	33	33 ±3	
	ρK	320	9.2	12 ±3	$S=1.5$
	$K\omega$	304	2.4	2.4±1	
	$K\eta$	482	0.4	2.1±1	
	total			89.1±5.1	
$f(1260)$	$\pi\pi$	616	80	large	
	$\bar{K}K$	388	2.7	<2.5	seen
	4π			<4	
	total			141 ±13	$S=2.3$
$f'(1514)$	$\bar{K}K$	570	52	72 ±12	
	$\bar{K}^*K,K^*\bar{K}$	292	16	10 ±10	
	$\pi\pi$	744	0[b]	<14	
	$\eta\eta$	522	8.4	<40	
	total			73 ±23	$S=1.8$

[a] See Ref. 13.
[b] Based on the assumption $f'\sim<33|$.

ACKNOWLEDGMENTS

I wish to express my sincere gratitude to Professor Julian Schwinger for his guidance, suggestions, and encouragement.

Note added in manuscript. After submission of this paper, we became aware of the recent dramatic change of the A_2 data,[14] which indicates that A_2 is composed of two peaks. There are several possible explanations: (1) One obvious explanation is that the observed A_2 resonance is indeed composed of two kinds of meson and both kinds of meson have the spin-parity assignment of 2^+. This will make the good agreement of this paper, which is based on a theory of an octet plus a singlet of 2^+ mesons, quite accidental unless all $8+1$ 2^+ mesons are doublets. It is certainly of great physical interest to verify experimentally whether all these nine 2^+ mesons are doublets. The same conclusion can be reached if the A_2 resonance belongs to a double pole. (2) Since the splitting of A_2 resonance so far observed is based on the data involving processes of large momentum transfer, it may happen that the splitting of A_2 actually depends on its creation process and that the A_2 resonance remains a single peak for creation processes involving only small momentum transfer. If this is indeed the case, the splitting of A_2 at large momentum transfer should be merely an interference effect (interference between A_2 and its background or some other nearby resonances). Then, the conclusion obtained in this paper is not affected.

The author wishes to thank Professor R. Dashen for a very helpful discussion on this point.

[12] We follow the spirit that the renormalization effects have already been taken care of in the phenomenological Lagrange function. One only needs to compute the tree diagrams for any given physical process.

[13] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

[14] For a summary of recent experiments on the A_2 resonance, see Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).