Analysis of the 95-MeV Proton-Proton Scattering Data*

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The new Harwell data are found to reduce most of the 95-MeV phase-shift uncertainties by moderate amounts, but the important and poorly known $^3P_0$ phase shift is not affected significantly. A precise measurement of spin depolarization is found to be the only single experiment which would properly delineate this phase shift.

I. INTRODUCTION

NEW measurements of the proton-proton differential cross sections and polarizations near 98 MeV have recently been made at Harwell,[1] with increased accuracy. Apart from the desirability of improving knowledge of the nucleon-nucleon interaction in general, this energy is in a unique region for the $^3P_0$ phase shift. It reaches a maximum somewhere near this energy, as shown in Fig. 1, so one has been prevented from simply interpolating between the much more accurately known single-energy-analysis values at 50 and 142 MeV. Although a recent multienergy analysis[2] has appeared to yield an extremely precise value of the $^3P_0$ phase shift at 95 MeV, a cursory examination of the *extra* data used in that analysis makes it appear plausible that virtually all of the apparent increase in accuracy came from the use of an insufficient number of free parameters. In view of these uncertainties and of the importance of the $^3P_0$ state for nuclear physics, a detailed analysis with the new data seemed desirable.

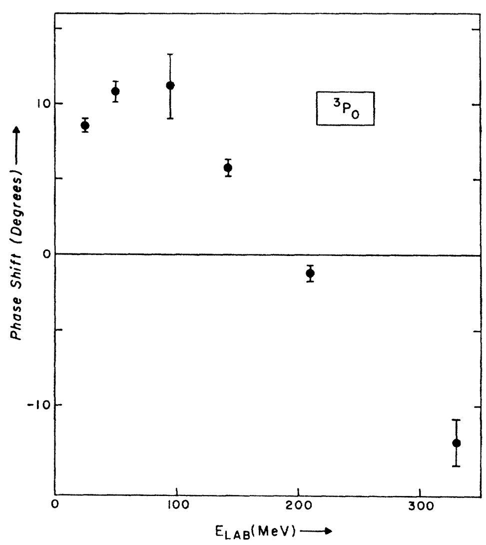

FIG. 1. The $^3P_0$ phase shifts from single-energy analysis of Ref. 2.

II. PHASE-SHIFT ANALYSES

The data used in our preliminary analyses are shown in Table I. In order to represent the phase shifts over the energy range of the data, 91–98.8 MeV, each phase shift was assumed as usual[2] to vary linearly with energy over this interval, with slopes as given by a previous multienergy analysis[3] of the 0–330 MeV data. The method of analysis was similar to that used previously[4]: The higher-angular-momentum phase shifts were set equal to their one-pion- (1π)-exchange values[5] and the lower-angular-momentum phases at 95 MeV were freely varied in order to obtain a least-squares fit to the data. Even when all of the phases with $L\leqslant 3$ were free to vary, however, the 91-MeV integrated cross section datum gave a χ^2 contribution of 16.

The complete set of Harvard-cyclotron integrated cross sections at various energies is shown in Fig. 2, along with a predicted curve from a previous multienergy analysis[3] (CR21) designed to simulate predictions of potentials. The analysis prediction is seen to be somewhat low, but is a smooth function of energy. The value predicted by the total data set in our preliminary analysis, which *included* the 91-MeV datum, is shown by the open circle. It is seen to form a smooth curve with

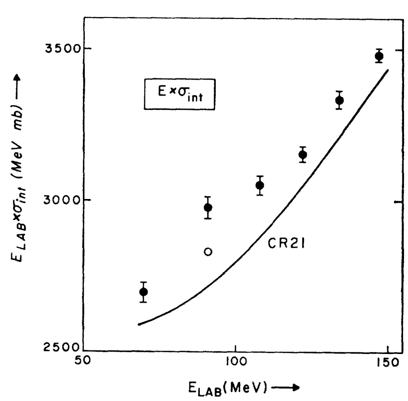

FIG. 2. The several Harvard integrated cross-section data multiplied by the corresponding energies of measurement. The "CR21" curve is from the multienergy analysis of Ref. 5.

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[1] See Refs. g and l of Table I.
[2] M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. **169**, 1128 (1968).
[3] P. Signell and N. R. Yoder, Phys. Rev. **134**, B100 (1964).
[4] P. Signell, N. R. Yoder, and J. E. Matos, Phys. Rev. **135**, B1128 (1964).
[5] P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959).

TABLE I. Data considered for this analysis. The normalization N is for data on following line. Cross sections and polarizations not preceded by a normalization are relative only. Note that, in the case of relative measurements, the effective number of data in a set is one less than the number given.

E_{LAB} (MeV)	Type	Number	Reference
91	σ_{int}	1	a
93.2	N	1	b
93.2	P	10	b
95	σ_{abs}	1	c
95	σ	13	d
95	N	1	e
95	P	14	d
95	σ	6	c
95	σ	5	c
97	P_{abs}	1	f
97.7	N	1	g
97.7	P	13	g
98	σ	6	h, i
98	N	1	e
98	P	14	h
98	D	5	j
98	R	5	k
98	$R'R$	4	k
98	C_{nn}	1	l
98.8	N	1	g
98.8	σ	19	g

[a] R. Goloskie and J. N. Palmieri, Nucl. Phys. **55**, 463 (1964). Not used in final analyses (see text).
[b] M. R. Wigan and P. J. Martin (unpublished). Omitted in favor of 97.7-MeV p on recommendation of O. N. Jarvis (private communication).
[c] U. E. Kruse, J. M. Teem, and N. F. Ramsey, Phys. Rev. **101**, 1079 (1956).
[d] J. N. Palmieri, A. M. Cormack, N. F. Ramsey, and R. Wilson, Ann. Phys. (N. Y.) **5**, 299 (1958).
[e] O. N. Jarvis, B. Rose, and J. P. Scanlon, Nucl. Phys. **77**, 161 (1966).
[f] P. Christmas and A. E. Taylor, Nucl. Phys. **41**, 388 (1963).
[g] M. R. Wigan, R. A. Bell, P. J. Martin, O. N. Jarvis, J. P. Scanlon, AERE-R 5685 1968 (to be published).
[h] A. E. Taylor, E. Wood, and L. Bird, Nucl. Phys. **16**, 320 (1960).
[i] 98-MeV cross section was omitted in favor of the 98.8-MeV one on the recommendation of O. N. Jarvis (private communication).
[j] E. H. Thorndike and T. R. Opher, Phys. Rev. **119**, 362 (1960).
[k] O. N. Jarvis, B. Rose, C. F. Cox, and G. H. Eaton, Nucl. Phys. **61**, 194 (1965).
[l] O. N. Jarvis, T. W. P. Brogden, B. Rose, J. P. Scanlon, J. Orchard-Webb, and M. R. Wigan, Nucl. Phys. **A108**, 63 (1968).

the other data, paralleling the multienergy-analysis curve, and in disagreement with the measured value. Finally, Chauvenet's criterion[6] allows a maximum of about 2.8 standard deviations from the mean for a data set of this size, in order to maintain the normal distribution of deviations: The 91-MeV datum is an unacceptable four standard deviations away. In view of all of the above evidence against the 91-MeV integrated cross section, we decided to remove it from the data set for all subsequent analyses.

In the absence of reliable knowledge of the two-pion-exchange contributions, the appropriate dividing line in orbital angular momentum L between the free phases and the fixed 1π-exchange phases is somewhat ambiguous. On the basis of current knowledge, however, one would certainly not expect 2π-exchange contributions to be substantial at 95 MeV for $L>3$. Whether the $L=3$ phases are adequately given by 1π exchange is examined in the first two lines of Table II. The release of the ${}^3F_3$ and ${}^3F_4$ phases from their 1π-exchange values is seen to result in a substantial drop in the ratio of χ^2 to its expected value, so one concludes that these phases must not be held fixed. It has been noted previously[4] that the 1π-exchange mechanism is anomalously weak in states with total angular momentum $J=L+1$, and this is certainly borne out by the ${}^3F_4$ phase in this analysis. The value found in the final analysis is consistent with the best current meson-theoretical estimates of 0.49° at 95 MeV by Furuichi[7] and 0.50° by Signell and Durso.[8] These numbers include the 1π-exchange value of 0.23°.

Comparison of the second and third lines of Table II shows that the new data have resulted in modest reductions in some of the other phase-shift uncertainties, but have barely touched the ${}^3P_0$ error bar.

III. UNCERTAINTY IN THE ${}^3P_0$ PHASE SHIFT

The uncertainty in the binding energy per nucleon in nuclear matter, due solely to the present uncertainty in the single-energy-analysis ${}^3P_0$ phase shift at 95 MeV, has been conservatively estimated[9] to be

TABLE II. Results of the phase-shift analyses of the 95-MeV data. The preferred set is that in the second line. The values of the "nuclear bar" phase parameters are in degrees. Those in parentheses were kept fixed at the values shown. The third line (Liv.-VII) is from Ref. 2, the latest previous analysis at this energy. The ${}^3F_2$ phase was not fixed along with the other $L=3$ phase in the first because it is coupled to the ${}^3P_2$ by ϵ_2.

Analysis	χ^2	$\chi^2/(\chi^2)_{\mathrm{exp}}$	${}^1S_0$	${}^1D_2$	${}^3P_0$	${}^3P_1$
${}^3F_3$, ${}^3F_4$ fixed	74	0.91	24.83±0.63	4.26±0.14	11.71±1.69	−12.62±0.30
Final analysis	61	0.76	26.33±1.09	3.65±0.19	12.56±1.79	−12.90±0.44
Liv.-VII	79		26.87±1.44	3.77±0.26	11.17±2.15	−13.12±0.66

Analysis	${}^3P_2$	ϵ_2	${}^3F_2$	${}^3F_3$	${}^3F_4$
${}^3F_3$, ${}^3F_4$ fixed	10.60±0.25	−3.18±0.15	0.46±0.15	(−1.73)	(0.23)
Final analysis	10.11±0.32	−2.76±0.19	0.61±0.57	−1.08±0.46	0.63±0.16
Liv.-VII	9.70±0.50	−2.73±0.32	−0.19±0.92	−0.62±0.92	0.40±0.28
1π exchange		−3.19	0.87	−1.73	0.23

[6] L. G. Parratt, *Probability and Experimental Errors in Science* (Wiley-Interscience, Inc., New York, 1961), p. 176.
[7] S. Furuichi, Progr. Theoret. Phys. (Kyoto) Suppl. **39**, 190 (1967).
[8] P. Signell and J. Durso, Rev. Mod. Phys. **39**, 635 (1967).
[9] By use of the phase-shift aproximation for the K matrix in the equations of K. A. Brueckner and K. S. Masterson, Phys. Rev. **128**, 2267 (1962). Similar calculations were reported by P. Signell [Bull. Am. Phys. Soc. **10**, 1212 (1965)].

0.40 MeV/nucleon. Thus, it is of some importance to pin down this phase shift more accurately.

Recently, MacGregor *et al.*[2] indeed found the $^3P_0$ phase shift to have the very precise value of 10.34±0.23° at 95 MeV from their multienergy analysis. However, their single-energy analysis gave 11.17±2.15°: The uncertainties from the two analyses differ by an order of magnitude. One must then determine whether the huge apparent increase in accuracy of the multienergy analysis is due to the inclusion of data which is near in energy but is omitted from the single-energy analysis. Examination of the data at energies near 95 MeV, used in the multienergy analysis but omitted from the single-energy analysis, reveals only a few 4–7% polarization and 2% cross-section measurements. From Table III, it is obvious that these data would have little effect on the $^3P_0$ uncertainty, even in conjunction with all of the rest of the "single-energy" data near 95 MeV. The alleged accuracy of the multienergy analysis thus seems to be due almost solely to the use of a too restrictive function of energy to represent the phase shift, or to the use of too few free parameters in that phenomenological function.[10] One concludes that only the single-energy-analysis value, which is virtually model-independent, is a physically meaningful quantity. If one desires the phase shift at energies between those of the single-energy analyses, a line drawn on Fig. 1 by hand, smoothly connecting the points, would appear to be more accurate and meaningful than the use of a too-restrictive multienergy-analysis curve.[11]

IV. POSSIBLE EXPERIMENTS

Since the $^3P_0$ phase has some importance for nuclear physics, we investigated what experiments would bring our knowledge of it into line with that of the other phases. Again, the method was similar to that previously employed.[4] To the data set, we added hypothetical data with accompanying hypothetical errors.

TABLE III. The effect, on the $^3P_0$ uncertainty, of various types of simulated data and errors. The "min per datum" column shows the minimum value, for the set, of the common standard deviation divided by each individual datum.

Simulated data type	c.m. angles of the hypothetical data (in deg.)	Assumed errors of the hypothetical data: Std. dev. (mb/sr)	Min per datum	Resulting $^3P_0$ std. dev.
None				1.8°
σ	20, 30, 40, 50, 60, 70, 80, 90	0.5	1%	1.6°
	with normalization	0.01	1%	
C_{nn}	90	0.02	3%	1.8°
C_{nn}	45	0.006	3%	1.8°
P	20, 30, 40, 50, 60, 70, 80	0.002	2%	1.7°
	with normalization	0.02	2%	
A	20, 30, 40, 50, 60	0.01	4%	1.3°
R	20, 30, 40, 50, 60	0.01	3%	1.2°
D	20, 30, 40, 50, 60	0.02	13%	0.8°
D	90	0.02	6%	0.7°
A'	20, 30, 40, 50, 60	0.01	2.5%	1.0°
R'	20, 30, 40, 50, 60	0.01	2%	1.3°

The phase-shift analysis was then remade. Although the resulting phase-shift *values* are rather meaningless, their *uncertainties* should be a rather accurate estimate of the improved knowledge of the values which would result from the proposed accuracy of the proposed experiment.

In the final column of Table III we show the standard deviation of the $^3P_0$ phase shift which can be expected to result from each of a number of proposed experiments. The angular ranges and assumed accuracies were chosen to roughly correspond with measurements which have been made at other energies. It is apparent from the table that an accurate measurement of spin depolarization would seem to be the only feasible single experiment which would provide the required restriction on the $^3P_0$ phase shift. From the precisely defined value at 50 MeV, however, it is obvious that combinations of other types of data might also be sufficient for this purpose.

Finally, we note that the use of trustworthy *theoretical* values for the $^3F_3$ and $^3F_4$ phase shifts at 95 MeV would drastically affect the last three lines of Table III. For example, a measurement of D to ±0.01 would then reduce the $^3P_0$ uncertainty to a magnificent ±0.4°.

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[10] The analysis of Ref. 2 used two phenomenological parameters to represent the $^3P_0$ phase shift from 5 to 400 MeV, but the values of those parameters were not reported. We have examined the effect of parameter restriction using the published parameters of a previous multienergy representation labeled CR(21) [P. Signell and N. R. Yoder, Phys. Rev. **134**, B100 (1964)]. We analyzed the data in a manner similar to that of Ref. 2, using 2, 3, and then 4 parameters to represent the $^3P_0$ phase shift. The resulting errors on the value at 95 MeV were 0.37°, 0.43°, and 0.56°, respectively, illustrating the lack of sufficient freedom with two parameters. Releasing more parameters would undoubtedly bring the error up to the single-energy value, but the computer time needed looked prohibitive. Finally, we examined the influence of the 60–127-MeV data included in the multienergy analysis but not in the single-energy analysis, by simply removing them from that multienergy analysis mentioned above in which three parameters represented the $^3P_0$ phase shift. The $^3P_0$ error changed from 0.43° to 0.46°, demonstrating that the extra data were not a factor in the over-restriction of the $^3P_0$ error in the multienergy analyses.

[11] A direct test was made by drawing a single smooth curve by hand through the six single-energy-analysis $^3P_0$ phases given in Ref. 2. Used as a replacement for the 60–127 MeV multienergy-analysis $^3P_0$ phases of Ref. 2, it yielded a slightly smaller value of χ^2 for those data than did the original (two-parameter representation) of Ref. 2.

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Dip Mechanisms at $J=-1$*

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Possible dip mechanisms at $J=-1$ are discussed and then applied to the case of πN charge-exchange scattering.

ONE success[1] of the Regge-pole hypothesis has been the explanation of the dip in the πN charge-exchange differential cross section at $t=-0.6$ GeV.[2] At that value of t, the spin-flip amplitude vanishes because the ρ trajectory passes through $J=0$. At more negative values of t, the ρ trajectory may pass through $J=-1$, possibly causing a dip at that value of t in this reaction. In this paper, possible dip mechanisms at $J=-1$ are discussed for the case of nucleon-antinucleon scattering, and the results will be applied to πN charge-exchange scattering via factorization. However, the results obtained may be generalized to other processes involving particles of higher spin.

The partial-wave crossed-channel helicity amplitudes for nucleon-antinucleon scattering will be denoted by

$$F_{\lambda\mu}(J,t,\eta,\xi),$$

where μ and λ are the total initial and final helicities, η is the eigenvalue of $(-1)^J P$, $\xi=\pm 1$ is the signature of the amplitude, and t is the square of the c.m. energy in the crossed channel. From the properties of the rotation functions, it can be shown that the nonsense-nonsense amplitude satisfies[2]

$$F_{11}(0,t,\eta,\xi)=F_{11}(-1,\,t,\,-\eta,\,-\xi) \tag{1}$$

at the nonsense value $J=0$. The Regge-pole representation of the partial-wave amplitude,

$$F_{11}(J,t,\eta,\xi)=\sum_{\text{trajectories}}\frac{\beta(t,\eta,\xi)}{J-\alpha(t,\eta,\xi)}, \tag{2}$$

gives rise to a relation between residues and trajectory functions of poles of opposite parity and signature:

$$\sum_{\text{trajectories}}\frac{\beta_{11}(t,\eta,\xi)}{\alpha(t,\eta,\xi)}=\sum_{\text{trajectories}}\frac{\beta_{11}(t,\,-\eta,\,-\xi)}{\alpha(t,\,-\eta,\,-\xi)+1}. \tag{3}$$

Now, suppose that at $t=t_0$, a trajectory is passing through $J=-1$. If there is no other trajectory of the same signature and parity passing through $J=-1$ at $t=t_0$, then the right-hand side of Eq. (3) has a pole. If there is no trajectory of opposite signature and parity passing through $J=0$ at $t=t_0$, then the left-hand side of Eq. (3) has no pole. Therefore, the residue function of the trajectory passing through $J=-1$ *must vanish at* $t=t_0$, *at least linearly in* $[\alpha(t)+1]$.

$$\beta_{11}(t,\eta,\xi)=[\alpha(t,\eta,\xi)+1]b_{11}(t,\eta,\xi). \tag{4}$$

This same result may also be derived from finite-energy sum rules and a consideration of the analytic properties of the scattering amplitude. Consider the asymptotic Regge-pole representation of the crossed-channel nucleon-antinucleon helicity amplitude[3]:

$$\bar{F}_{\lambda\mu}(s,t)=\sum_{\text{trajectories}}\frac{\exp(-i\pi\alpha)+\xi}{\sin\pi\alpha}\left(\frac{s}{4q^2}\right)^{\alpha-M}$$
$$\times[\Gamma(\alpha+\lambda+1)\Gamma(\alpha-\lambda+1)\Gamma(\alpha+\mu+1)\Gamma(\alpha-\mu+1)]^{-1/2}$$
$$\times\Gamma(\alpha+1)\beta_{\lambda\mu}(t,\eta,\xi), \tag{5}$$

where α is a function of t, η, ξ as before,

$$M=\max(|\lambda|,|\mu|),\quad 4q^2=(t-4m^2)^{1/2},$$

m is the nucleon mass, and s is the square of the c.m. energy in the direct channel. The nonsense-nonsense amplitude has the following form:

$$\bar{F}_{11}(s,t)=\sum_{\text{trajectories}}\frac{[\exp(-i\pi\alpha)+\xi]\Gamma(1-\alpha)}{\pi(\alpha+1)}\times\left(\frac{s}{4q^2}\right)^{\alpha-1}\beta_{11}(t). \tag{6}$$

Now, suppose that one of the odd-signature trajectories is passing through $J=-1$ at $t=t_0$. If there is no other odd-signature trajectory passing through $J=-1$ at $t=t_0$, then the nonsense-nonsense amplitude will have a pole at $t=t_0$. The amplitude cannot have the pole at zero or a negative value of t (it would be in the physical region of the direct channel), and the pole cannot occur at a positive value of t (it would represent a particle of spin -1). Therefore, the residue function of the negative-signature trajectory passing through $J=-1$ must vanish at least linearly in $[\alpha(t)+1]$.

* Supported by the National Science Foundation under Grant No. NSF GP 9273.
† National Science Foundation Graduate Fellow.
[1] F. Arbab and C. B. Chiu, Phys. Rev. 147, 1045 (1966).
[2] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).
[3] Kinematic singularities in the variable s have been removed.

Use of the finite-energy sum rule on Eq. (5) gives

$$\int_{-N}^{+N} s^a ds\, \mathrm{Im}\bar{F}_{\lambda\mu}(s,t) = \sum_{\text{trajectories}} \left(\frac{1}{4q^2}\right)^{\alpha-M} [1+\xi(-1)^{M+a}] \times [\Gamma(\alpha+\lambda+1)\Gamma(\alpha-\lambda+1)\Gamma(\alpha+\mu+1)\Gamma(\alpha-\mu+1)]^{-1/2} \times \Gamma(\alpha+1)\beta_{\lambda\mu}\frac{N^{\alpha-M+a+1}}{\alpha-M+a+1}, \quad (7)$$

where a is zero or a positive integer. The sum rule for the nonsense-nonsense amplitude has the following form:

$$\int_{-N}^{+N} s^a ds\, \mathrm{Im}\bar{F}_{11}(s,t) = \sum_{\text{trajectories}} \left(\frac{1}{4q^2}\right)^{\alpha-1} [1-\xi(-1)^{a}] \times \frac{\alpha\beta_{11}}{(\alpha+a)\Gamma(\alpha+2)} N^{\alpha+a}. \quad (8)$$

Consider the $a=1$ sum rule. If one even-signature trajectory is passing through $J=-1$ at $t=t_0$, then the right-hand side of Eq. (8) has a pole at $t=t_0$. By the very same argument presented above, this cannot happen, and so the residue function of the positive-signature trajectory passing through $J=-1$ must vanish at least linearly in $[\alpha(t)+1]$.

Factorization of the residues,

$$\beta_{00}\beta_{11}=\beta_{01}^2, \quad (9)$$

as well as the information[4] that the sense-nonsense residue must vanish at least as $[\alpha(t)+1]^{1/2}$, imply one of the following mechanisms for the behavior of the residue functions near $\alpha(t)=-1$:

$$\begin{array}{llll} \text{(i)} & \beta_{11}\propto(\alpha+1), & \beta_{00}\propto 1, & \beta_{01}\propto(\alpha+1)^{1/2}; \\ \text{(ii)} & \beta_{11}\propto(\alpha+1), & \beta_{00}\propto(\alpha+1)^2, & \beta_{01}\propto(\alpha+1)^{3/2}; \\ \text{(iii)} & \beta_{11}\propto(\alpha+1)^2, & \beta_{00}\propto(\alpha+1), & \beta_{01}\propto(\alpha+1)^{3/2}. \end{array} \quad (10)$$

Note that the following behavior[5] is not allowed:

$$\beta_{11}\propto 1, \quad \beta_{00}\propto(\alpha+1), \quad \beta_{01}\propto(\alpha+1)^{1/2}. \quad (11)$$

In the case of πN charge-exchange scattering, the crossed-channel helicity amplitudes are assumed to be dominated by the ρ trajectory and may be written as

$$\bar{F}_{00} = -\frac{\exp(-i\pi\alpha)-1}{\pi}\left(\frac{s}{4q^2}\right)^{\alpha}\Gamma(-\alpha)\beta_{00}(t)$$

and (12)

$$\bar{F}_{01} = -\frac{\exp(-i\pi\alpha)-1}{\pi}\left(\frac{s}{4q^2}\right)^{\alpha-1}\left(\frac{\alpha}{\alpha+1}\right)^{1/2}\Gamma(-\alpha)\beta_{01}(t).$$

Since the ρ is a leading trajectory, it must choose one of the mechanisms (10). If (i) is chosen, then neither amplitude vanishes at $t=t_0$. In that case, the differential cross section would not have a dip at $t=t_0$. If either (ii) or (iii) is chosen, then the differential cross section would have a deep dip at $t=t_0$, since the ρ contribution to both amplitudes vanishes there. Reeder and Sarma[6] choose mechanism (iii) and assume that only the ρ trajectory contributes. They predict a zero in the differential cross section at $t\simeq-2.5$ GeV^2.

The author gratefully acknowledges the help of Professor John D. Stack and thanks him for many helpful discussions.

[4] The sense-nonsense amplitude has a factor $[\alpha(\alpha+1)]^{1/2}$ from the rotation function d_{01}^{α}. In order that the amplitude does not have a cut due to this factor, the sense-nonsense residue function must have a factor $[\alpha(\alpha+1)]^{1/2}$.

[5] C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. **161**, 1563 (1967).

[6] D. D. Reeder and K. V. L. Sarma, Phys. Rev. **172**, 1566 (1968).