High-Energy Contributions to Current-Algebra Sum Rules. I

KASHYAP V. VASAVADA
Department of Physics, University of Connecticut, Storrs, Connecticut 06268
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A method to evaluate high-energy contributions to current-algebra sum rules by combining them with finite-energy sum rules is suggested. Applications are made to the Adler-Weisberger-type sum rules for the π-π and the K-π systems, and to various sum rules for the pion photoproduction process. It is shown that the former sum rules can be approximately satisfied without requiring very large scalar contributions. Some interesting results are found for the latter sum rules.

I. INTRODUCTION

IN the last few years a large number of sum rules based on various assumptions of current algebra, unsubtracted dispersion relations, Regge-pole theory, etc. have been considered.[1] Some of these, like the original Adler-Weisberger sum rule, have been remarkably successful. Some others have met with rather marginal successes or failures. The current commutation relations and the hypothesis of the partially conserved axial-vector current (PCAC) provide low-energy theorems which, coupled with assumptions of unsubtracted dispersion relations, give the usual current-algebra sum rules. These sum rules equate integrals over certain functions of scattering amplitudes to certain known quantities. Since in most cases the relevant amplitudes are known only in the low-energy regions (less than 1 or 2 BeV), the high-energy contributions to the integrals are either thrown away or very crudely estimated.[2] In the present work we suggest that this high-energy contribution can be evaluated from the same low-energy data by using the so-called finite-energy sum rules (FESR).[3] FESR and their generalizations (continuous moment sum rules[4]) are just consistency conditions imposed by analyticity on functions of scattering amplitudes which have Regge-like behavior asymptotically. Combining FESR with the current-algebra sum rules (CASR), one can get an estimate of the high-energy contribution to CASR-independent of any high-energy fits to the amplitudes. In many cases such as the ones considered in the present work, high-energy fits for the relevant amplitudes are not available. Moreover, even when high-energy fits are available, they are for certain combinations of the amplitudes and at different energies up to which low-energy data are not generally available. This fact makes our procedure necessary.

In Sec. II we discuss the procedure. In Sec. III application is made to the Adler-Weisberger-type sum rules for the $\pi-\pi$ and the $K-\pi$ systems. The original Adler-Weisberger (A-W) sum rule is also briefly discussed. Some other sum rules are considered in Sec. IV. In particular, the pion photoproduction sum rules first suggested by Fubini, Furlan, and Rossetti[5] are discussed.

II. GENERAL FORMALISM

Consider a reaction $a+b\rightarrow c+d$. Let $s=(p_a+p_b)^2$, $t=(p_a-p_c)^2$, $u=(p_a-p_d)^2$ be the usual Mandelstam variables and define $\nu=\frac{1}{4}(s-u)$. Then low-energy theorems following from current algebra, PCAC, and unsubtracted dispersion relations lead to sum rules of the type

$$\int_0^\infty \frac{F(\nu,t)}{\nu^m}d\nu=G(t)\,, \tag{1}$$

where m is an integer, $F(\nu,t)$ being some function of the scattering amplitude. As shown in Ref. 3, if for $\nu>N$, $F(\nu,t)$ is dominated by the exchange of a single Regge trajectory in the t channel [i.e., $F(\nu,t)\rightarrow\beta(t)\nu^{\alpha(t)}$], the following FESR can be shown to hold good to the same approximation;

$$\int_0^N \nu^n F(\nu,t)d\nu=\frac{\beta(t)N^{\alpha(t)+n+1}}{\alpha(t)+n+1}\,, \tag{2}$$

where n is an even (odd) integer according to whether $F(\nu)$ is even (odd) under crossing ($s\leftrightarrow u$, $\nu\leftrightarrow -\nu$). $\alpha(t)$ and $\beta(t)$ are the trajectory parameter and the residue function of the t-channel trajectory. The right-hand side of Eq. (2) contains many factors depending on the normalization in ν, ghost-eliminating mechanisms,[4] etc., however, for our purpose here we have conveniently absorbed these in $\beta(t)$. Then Eq. (1 can)

[1] See, for example, S. L. Adler and R. F. Dashen, *Current Algebra and Applications to Particle Physics* (W. A. Benjamin, Inc., New York, 1968).

[2] An exception is the original Adler-Weisberger sum rule which requires only the πN total cross-section data. These are available up to an energy of about 20 BeV. The high-energy contributions to $K-N$ and $\bar{K}-N$ sum rules have been also estimated. In this paper, by "high-energy region," we shall mean the energy region beyond the low-energy-resonant or nonresonant region for which there is detailed experimental information and not just the asymptotic region.

[3] K. Igi, Phys. Rev. Letters **9**, 76 (1962); K. Igi and S. Matsuda, *ibid.* **18**, 625 (1967); A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967); R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).

[4] K. V. Vasavada and K. Raman. Phys. Rev. Letters **21**, 577 (1968); K. Raman and K. V. Vasavada, Phys. Rev. **175**, 2191 (1968); M. G. Olsson, Phys. Rev. **171**, 1681 (1968) and University of Wisconsin reports of work prior to publication.

[5] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, 1171 (1965); **43**, 161 (1966).

be written as

$$\int_0^N \frac{F(\nu,t)d\nu}{\nu^m} - \frac{\beta(t)N^{\alpha(t)-m+1}}{\alpha(t)-m+1} = G(t) \quad (3)$$

with the same assumptions as above. Eliminating $\beta(t)$ from (2) and (3), we get

$$\int_0^N \frac{F(\nu,t)d\nu}{\nu^m} - \frac{\alpha(t)+n+1}{\alpha(t)-m+1}\frac{1}{N^{m+n}} \times \int_0^N \nu^n F(\nu,t)d\nu = G(t). \quad (4)$$

Thus, if α is known, the left-hand side of the sum rule (4) can be evaluated without throwing away the high-energy contribution. Again, as is known, α itself can be determined by considering Eq. (2) for two different values of n. This has been done by various authors, leading to reasonable results approximately agreeing with the results of Regge analysis of high-energy scattering.[3,4] However, for application of Eq. (4) one does not need any detailed results from high-energy analysis, apart from the general idea of dominance of certain trajectories. In many cases the amplitudes under consideration permit particles with only restricted sets of quantum numbers to be exchanged in the t channel. Thus, the procedure is simplified.

In the above we have considered dominance of a single trajectory only for the sake of simplicity. The procedure can be readily generalized for an arbitrary number of trajectories. In the general case, Eqs. (2) and (3) become

$$\int_0^N \nu^n F(\nu,t)d\nu = \sum_i \frac{\beta_i(t)N^{\alpha_i(t)+n+1}}{\alpha_i(t)+n+1} \quad (2')$$

and

$$\int_0^N \frac{F(\nu,t)d\nu}{\nu^m} - \sum_i \frac{\beta_i(t)N^{\alpha_i(t)-m+1}}{\alpha_i(t)-m+1} = G(t). \quad (3')$$

To determine α_i and β_i, one can write number of equations of the type (2′) with different moments n and solve them simultaneously. Equation (3′) then gives the high-energy contribution to the sum rule. In practice, of course, owing to the limited amount of accuracy in the low-energy data, sum rules with very high moments may not lead to meaningful results. Also in some cases fixed poles may be present, or indeed required to be present, on some theoretical grounds. If these do affect the asymptotic behaviors of the amplitudes for which the current algebra sum rules are written, they should be considered in Eq. (2′) and (3′) to be like any other Regge trajectory. For the sum rules under consideration in this paper, we assume that fixed poles are not present.[6] It should be noted also that the usual "double-counting" problem while adding Regge contributions does not arise here because of the use of FESR.

[6] We are investigating at present the effect of the existence of fixed poles on the current-algebra sum rules and the present method of evaluating the high-energy contribution.

III. ADLER-WEISBERGER TYPE SUM RULES

Now we consider application of Eq. (4) to the Adler-Weisberger-type sum rules for $\pi-\pi$ and $K-\pi$ systems. According to the hypothesis of the chiral $SU(2)\otimes SU(2)$ algebra[1] of Gell-Mann, the axial-vector charges

$$Q_5^{\pm} = \int A_0^{\pm}(x,t)d^3x \quad (5)$$

satisfy the commutation relation

$$[Q_5^+, Q_5^-] = 2Q^{(3)}, \quad (6)$$

where $A_0(x,t)$ is the time component of the axial-vector current, $Q^{(3)}$ denotes the vector charge, and $\pm$ refer to the $1\pm i2$ components of isospin. Sandwiching the above commutator between various single-particle states and using PCAC and unsubtracted dispersion relations, one gets different Adler-Weisberger-type sum rules.[7] These sum rules for $\pi-N$ and $K-N$ systems have been very well discussed. Here we consider the cases of $\pi-\pi$ and $K-\pi$ systems.

A. $\pi-\pi$ Sum Rule

Sandwiching the commutator (6) between two π states, we get the sum rule

$$\frac{1}{\pi}\int_0^\infty \frac{\text{Im}[T_-(\nu,0)-T_+(\nu,0)]d\nu}{\nu^2} = \frac{4}{f_\pi^2}, \quad (7)$$

where

$$f_\pi^2 = 2g_A^2 m^2/g^2K^2(0), \quad f_\pi \approx 135 \text{ MeV (expt.)}.$$

$T_\pm(\nu,0)$ is the forward invariant amplitude for scattering of a zero-mass $\pi_\pm$ by a physical π^+ meson. The amplitude $T_- - T_+$ can be readily shown to be a pure $I=1$ t-channel amplitude and odd under crossing. Thus, among the known Regge trajectories only the ρ will contribute to this amplitude at high energies.

This sum rule has been discussed by various authors in the resonance approximation.[8,9] Saturating by ρ and f^0 resonances, Adler finds that a large S-wave scattering length ($a_0 > 1.3/m_\pi$) is necessary to satisfy the sum rule. In contrast, Weinberg's calculation of the $\pi-\pi$ scattering length from current algebra suggests a considerably smaller scattering length ($\approx 0.2/m_\pi$). Several authors have introduced a scalar-meson resonance σ of

[7] S. L. Adler, Phys. Rev. **140**, B736 (1965); **149**, 1294(E) (1966); **175**, 2224(E) (1968); W. I. Weisberger, *ibid.* **143**, 1302 (1966). Also see Ref. 5.

[8] S. L. Adler, Ref. 6; G. Furlan and C. Rossetti, Phys. Letters **23**, 499 (1966); I. J. Muzinich and S. Nussinov, *ibid.* **19**, 248 (1966); K. Kawarabayashi, W. D. McGlinn, and W. W. Wada, Phys. Rev. Letters **15**, 897 (1965).

[9] F. J. Gilman and H. Harari, Phys. Rev. **165**, 1803 (1968).

large width in order to account for the discrepancy. In Ref. 9, for example, the existence of a σ meson with $m_\sigma = 765$ MeV and $\Gamma_\sigma = 650$ MeV is shown to be necessary to satisfy the sum rule. Experimentally, the existence of scalar mesons is not well established.[10] We will show in the following that if the high-energy contributions are taken into account, the sum rule can be approximately satisfied without requiring very large scalar contribution.

For this purpose we divide the integral into high- and low-energy parts. Since a reliable phase-shift analysis of $\pi-\pi$ scattering is as yet not available, we will adopt in this paper the resonance approximation as done by previous authors. For the low-energy region, we retain ρ and f^0 contributions.

In the narrow-resonance approximation we obtain

$$4\pi\left(\frac{3m_\rho^2\Gamma_\rho}{\nu_\rho^2 q_\rho}+\frac{10}{3}\frac{m_f^2\Gamma_f}{\nu_f^2 q_f}\right)+I_H=\frac{4}{f_\pi^2}, \tag{8}$$

where m_ρ, m_f, Γ_ρ, and Γ_f are the masses and widths of ρ and f^0 mesons, and $\nu=\frac{1}{4}(s-u)=\frac{1}{2}(s-m_\pi^2)$. Here q_ρ, q_f, ν_ρ, and ν_f are the values of the c.m. momenta and ν of the $\pi-\pi$ system at the position of the resonances. I_H denotes the high-energy contribution. Relating the widths to the coupling constants by

$$\Gamma_\rho=\tfrac{2}{3}(g_{\rho\pi\pi}^2/4\pi)q_\rho^3/m_\rho^2, \tag{9}$$

$$\Gamma_f=\tfrac{1}{10}(g_{f\pi\pi}^2/4\pi)q_f^5/m_f^2, \tag{10}$$

this becomes

$$\frac{2g_{\rho\pi\pi}^2}{m_\rho^2}+\frac{g_{f\pi\pi}^2(m_f^2-m_\pi^2)^2}{12m_f^4}+I_H=\frac{4}{f_\pi^2}. \tag{11}$$

Now if only the ρ trajectory dominates,

$$T_-(\nu,0)-T_+(\nu,0)\underset{\nu>N}{\longrightarrow}\beta(t)\nu^{\alpha_\rho(0)}.$$

Writing a zero-moment FESR, we can obtain I_H as explained in Sec. II and find

$$\frac{2g_{\rho\pi\pi}^2}{m_\rho^2}\left(1-\frac{\nu_\rho^2}{N^2}\frac{\alpha_\rho+1}{\alpha_\rho-1}\right)+\frac{g_{f\pi\pi}^2(m_f^2-m_\pi^2)^2}{12m_f^4}\times\left(1-\frac{\nu_f^2}{N^2}\frac{\alpha_\rho+1}{\alpha_\rho-1}\right)=\frac{4}{f_\pi^2}. \tag{12}$$

First of all, note that, since $\alpha_\rho(0)<1$, the high-energy contribution is in the right direction to correct the discrepancy. The question of choice of N is a rather delicate one, especially since we are using a crude narrow-resonance approximation. More careful integration over resonant cross sections is certainly possible. But in view of the large uncertainty in the experimentally determined values of the width of ρ, we have not attempted such a procedure. In conformity with the recent discussions on the FESR bootstrap,[11] we choose N to be about halfway between the last resonance included and the next higher one. Thus when only the ρ is retained, we choose $N=27.5$.[12] After f^0, the next higher resonance is the $g(1650)$ meson. So when we retain both ρ and f^0, $N=54.9$. The results are shown in Table I. It can be seen that without the high-energy contribution (I_H) the sum rule is off by about a factor of 2 or more. Inclusion of I_H leads to a reasonable agreement, which is particularly good for larger values of ρ width. It is interesting to note that variation of N from the value between ν_ρ and ν_f to that between ν_f and ν_g leads to a quite small variation in the results. This seems to confirm the reasonableness of the scheme.

There are large number of resonances coupled to the $\pi-\pi$ system at higher energies. Thus it is not surprising that I_H, which takes into account the contributions of these resonances, makes an important contribution to the sum rule. This idea is also supported by recent results of Meiere and Sugawara.[13] They first write dispersion integrals for S-wave scattering lengths and, using these, rewrite the Adler sum rule so as to suppress the high-energy contribution still further. As a result, they find that an S-wave scattering length of magnitude only about a fourth of that suggested by Adler is sufficient to saturate the sum rule. Thus it appears that if proper account of the high-energy contribution is taken, the sum rule can be saturated by only a small S-wave scattering length consistent with current algebra and there is no particular need for very large scalar contribution as far as this sum rule is concerned.[14] Of course, we have made several approximations. Neglect of the continuum may be the most serious one. However, qualitatively, the results appear to be reliable. Next, we discuss the $K-\pi$ sum rule in an exactly similar fashion.

B. $K-\pi$ Sum Rule

Taking the matrix element of the charge algebra commutator (6) between two K^+ states yields the sum rule

$$\frac{1}{\pi}\int_0^\infty\frac{\mathrm{Im}[T_-(\nu,0)-T_+(\nu,0)]d\nu}{\nu^2}=\frac{2}{f_\pi^2}, \tag{13}$$

[10] Several phase-shift analyses for the π-π system based on the study of pion-production reactions have appeared recently. There is considerable disagreement between them at present but some of them do show existence of σ at around the ρ mass region. Evaluation of the π-π sum rule using these analyses is under consideration and will be reported later.

[11] C. Schmid, Phys. Rev. Letters **20**, 628 (1968).

[12] All quantities are in units of $\hbar=c=m_\pi=1$ unless otherwise mentioned.

[13] F. T. Meiere and M. Sugawara, Phys. Rev. **153**, 1702 (1967).

[14] Recently G. Patsakos [Phys. Rev. **165**, 1667 (1968)] has considered the evaluation of a high-energy Regge contribution to this sum rule and concludes that appreciable contribution is obtained. However, he takes the residue function $\bar{\beta}(0)=\bar{\beta}(m^2)$. The value of $\bar{\beta}$ depends on the normalization energy and this can be a rather unreliable approximation for certain values of this energy. See, for example, Ref. 11.

where now $T_{\pm}(\nu,0)$ denotes the forward invariant amplitude for scattering of a zero-mass $\pi^{\pm}$ by a physical K^+ meson. Also we have

$$\nu=\tfrac{1}{2}(s-m_K^2).$$

Saturation of the low-energy part by $K^*(890)$ and $K^{**}(1420)$ mesons gives

$$\frac{2}{3}\left[\frac{2g_{K^*K\pi}^2}{m_{K^*}^2}+\frac{1}{12}\frac{g_{K^{**}K\pi}^2}{m_{K^{**}}^4}(m_{K^{**}}^2-m_K^2)^2\right]+I_H=2/f_\pi^2. \quad (14)$$

The coupling constants are related to the widths in the same way as above.

Following the same procedure as in the $\pi-\pi$ case and assuming that only the ρ trajectory dominates for $\nu>N$, we get

$$\frac{8g_{K^*K\pi}^2}{m_K^2}\left(1-\frac{\nu_{K^*}^2}{N^2}\frac{\alpha_\rho+1}{\alpha_\rho-1}\right)+\frac{g_{K^{**}K\pi}^2(m_{K^{**}}^2-m_K^2)^2}{2m_{K^{**}}^4}\times\left(1-\frac{\nu_{K^{**}}^2}{N^2}\frac{\alpha_\rho+1}{\alpha_\rho-1}\right)=\frac{12}{f_\pi^2}. \quad (15)$$

When only $K^*(890)$ is retained, we take $N=29.8$. The next higher resonance coupled to the $K-\pi$ system appears to be the L meson (1800). So when both $K^*(890)$ and $K^{**}(1420)$ are retained, N is taken to be 61.1. The right-hand side of (15) is numerically equal to 12.84. The left-hand side has the following values: (a) When I_H is neglected, the left-hand side$=7.12$; (b) when only K^* is retained and I_H is included, the left-hand side$=10.03$ with $\alpha_\rho=0.5$ and 11.36 with $\alpha_\rho=0.6$; (c) when K^{**} and I_H are also included, the left-hand side$=9.95$ with $\alpha_\rho=0.5$ and 10.87 with $\alpha_\rho=0.6$. Thus inclusion of I_H certainly leads to a better agreement. Also the results are not sensitive to the values of N chosen as above. The remaining discrepancy can be perhaps cured by including continuum contributions. As in the π-π case, several authors have concluded that a large scalar contribution in the form of a resonance is required to saturate the sum rule.[8] We find that this need not be the case.

It should be noted that our results are not inconsistent with the existence of scalar resonances. If they do exist, they should be included on the left-hand side of the sum rules. Our results only imply that there is a very substantial high-energy contribution to these sum rules and that there is no great need for large scalar contributions to the sum rules.

A natural question arises at this point: What happens if we apply our procedure to the original Adler-Weisberger relation for the π-N system, for which the high-energy contributions have been accurately evaluated due to availability of the total cross-section data? The sum rule is given by

TABLE I. Left-hand side of Eq. (12) for various values of N, α_ρ, and Γ_ρ. Right-hand side$=4/f_\pi^2=4.28$.

α_ρ	N	$\Gamma_\rho=90$ MeV	$\Gamma_\rho=125$ MeV	$\Gamma_\rho=150$ MeV
0.5	∞	1.95	2.49	2.73
0.5	27.5	2.67	3.68	4.12
0.5	54.9	3.10	3.76	4.04
0.6	27.5	3.08	4.24	4.75
0.6	54.9	3.47	4.16	4.47

$$\frac{2m_n^2}{\pi g^2}\int_{m_\pi}^{\infty}\frac{kd\nu[\sigma_{\pi^+p}(\nu)-\sigma_{\pi^-p}(\nu)]}{\nu^2}=1-\frac{1}{g_A^2},$$

where $\nu=(s-m_n^2)/2m_n$ is the lab energy of the pion, and k is the lab momentum. According to the numerical estimates of Weisberger,[7] the (3,3) resonance in above gives the right-hand side$=0.451$, resulting in $|g_A|=1.35$. The higher resonances, continuum, and high-energy tail contributions bring this value down to 0.246 which gives $|g_A|=1.15$. Thus, because of the peculiar way in which g_A occurs, namely as $(1-1/g_A^2)$ in the right-hand side (unlike $1/g_A^2$ as for the π-π and K-π cases), a change in the right-hand side by a factor of about 2 brings only a few percent change in the value of g_A. Thus, states other than the (3,3) resonance contribute quite significantly to the left-hand side, even though this is not reflected significantly by the change in value of g_A.

Now according to our procedure the correction term will be given by

$$-\frac{\alpha_\rho+1}{\alpha_\rho-1}\frac{2m_n^2}{\pi g^2N^2}\int_{m_\pi}^{N}k[\sigma_{\pi^+p}(\nu)-\sigma_{\pi^-p}(\nu)]d\nu.$$

Here N should be large enough so that the Regge term can be expected to give a reasonable approximation to the amplitudes. If N is given a value just after the (3,3) resonance, as was done in the case of the π-π system, a large contribution in opposite direction to that required to satisfy the sum rule will be obtained. It is clear, however, that such a low value of $N\approx400$–500 MeV is completely unjustified. For $t=0$ we have $\nu=-m_\pi\cos\theta_t$, where $\cos\theta_t$ is the cosine of the scattering angle in the t channel. In the π-π case, near the ρ resonance, $\cos\theta_t$ is already very large (our N in the $\pi\pi$ case corresponds to $|\cos\theta_t|\approx27$) whereas in the π-N case, near N^*, it is about 3 or 4 and very low values of N are not justified. In other words, this implies that energy at which Regge behavior takes over should be high enough relative to the threshold. Thus, in the π-N case N should be taken to be at least at 1.5 to 2 BeV. In the usual treatments on FESR for π-N scattering, N is taken to have some value in the range 1.5 to 3 or 4 BeV.[3] Although we have not done any numerical calculation of the A-W relation for the π-N case, we can show that with any reasonable choice of N, the correction term will be small. It is known that the major contribution comes from

$N^*(1238)$, $N^{**}(1520)$, and some other low-energy resonances, and that there is a cancellation between contributions of $N^*(1238)$ and other resonances in the A-W relation. This cancellation will be more effective in the correction term given above because of the absence of the factor $1/\nu^2$ in the integrand. This, along with the factor $1/N^2$ in the denominator, will make the correction term small. Furthermore, the large contribution to the correction integral from the $I=\frac{1}{2}$ resonances can be seen to give the right sign to the correction term. Thus, the fact that ρ and f^0 do not saturate the π-π sum rule, but a few low-energy π-N resonances do saturate the π-N sum rule, may be due to the essential differences between the π-π and the π-N systems.

In Sec. IV we consider some sum rules for the pion photoproduction process.

IV. PION PHOTOPRODUCTION SUM RULES

As another set of sum rules we now consider the pion-photoproduction sum rules first suggested by Fubini, Furlan, and Rossetti.[5] The commutators of the $SU(2)\otimes SU(2)$ algebra used to derive these sum rules are given by

$$[Q_5^{(3)},j_\mu^{(v)}]=0\,,\quad [Q_5^{(3)},j_\mu^{(s)}]=0\,, \tag{16}$$

where $Q_5^{(3)}$ is the axial-vector charge and $j_\mu^{(v,s)}$ are the isovector and isoscalar electromagnetic currents. These commutators taken between two proton states, together with the use of PCAC and unsubtracted dispersion relations for the pion-photoproduction invariant amplitudes, give rise to the following sum rules:

$$\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}A_1^+(\nu)d\nu}{\nu}=\frac{(\mu_p-\mu_n)g}{4m} \tag{17}$$

and

$$\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}A_1^0(\nu)d\nu}{\nu}=\frac{(\mu_p+\mu_n)g}{4m}\,. \tag{18}$$

We have adopted the notation of Ball.[15] A_1 is one of the four invariant amplitudes for the pion-photoproduction reaction. μ_p and μ_n are the anomalous magnetic moments of the proton and neutron and g is the usual π-N-N coupling constant. $+$ and 0 refer to the isospin-symmetric part of the amplitudes for scattering of isovector and isoscalar photons. The first sum rule has been numerically evaluated by Adler and Gilman[16] using the photoproduction analyses of Walker and of Schmidt and Hohler.[17] Apart from a special model used for the S-wave contribution, they take into account only the resonant terms and find the left-hand side of (17) to be about 86% of the right-hand side. We have evaluated these sum rules using the more recent fits of Walker in which the amplitudes are given as sums of Born terms, resonant contributions, and background terms.

For zero-mass pions, we express the amplitude A_1 in terms of the multipole amplitudes

$$A_1(\nu,0)=(8\pi W/W^2-m^2)(E_{0+}+M_{1+}+3E_{1+}+3M_{2+} +6E_{2+}-3M_{2-}+E_{2-}-6M_{3-}+3E_{3-})\,. \tag{19}$$

W is the c.m. energy and the multipoles are taken with the appropriate isospin factors for the $+$ and 0 cases separately. Furthermore, since they refer to the final pions of zero mass, we multiply them by factors $(|\mathbf{q}|_{m_\pi=0}/|\mathbf{q}|)^l$ following Ref. 16. A factor $[g_{\mathrm{physical}}/g(0)]$ has been also eliminated from the left-hand sides of the sum rules.

The integrals are evaluated up to a photon lab energy of 1.2 BeV. It can be readily shown that among the known leading trajectories, only ω contributes to A_1^+ whereas both ρ and B contribute to A_1^0. High-energy contributions can be evaluated as in the previous cases by writing first-moment FESR. This gives

$$\frac{1}{\pi}\int_0^N \frac{\mathrm{Im}A_1^+(\nu)}{\nu}d\nu-\frac{1}{N^2}\frac{\alpha_\omega+1}{\alpha_\omega-1}\frac{1}{\pi}\int_0^N \nu\,\mathrm{Im}A_1^+(\nu)d\nu =\frac{(\mu_p-\mu_n)g}{4m}, \tag{20}$$

$$\frac{1}{\pi}\int_0^N \frac{\mathrm{Im}A_1^0(\nu)}{\nu}d\nu-\frac{1}{N^2}\frac{\alpha_\rho+1}{\alpha_\rho-1}\frac{1}{\pi}\int_0^N \nu\,\mathrm{Im}A_1^0(\nu)d\nu =\frac{(\mu_p+\mu_n)g}{4m}\,. \tag{21}$$

Since the B trajectory is expected to lie considerably below the ρ trajectory, we have neglected it in writing (21). N corresponds to the photon lab energy of 1.2 BeV, up to which Walker's multipole fits are available.

Numerically we find that the first term on the left-hand side of (20) is 0.0389 (with $m_\pi=1$), whereas the right-hand side is 0.0419. Thus there is about 7% discrepancy. The second term gives the high-energy correction. Taking $\alpha_\omega=0.5$, we find it to be $+0.0127$. Thus the correction term is of the right sign but rather large in magnitude. However, this discrepancy could be cured by a more precise multipole analysis.

In the case of the sum rule (21) we find that the first term on the left-hand side is -0.00069 while the right-hand side is -0.00136. Again we take $\alpha_\rho=0.5$ and find that the correction term is -0.00062. Thus the left-hand side becomes -0.00131 in remarkable agreement with the right-hand side. This can be accidental to some extent. However, in both cases the correction terms

[15] J. S. Ball, Phys. Rev. 124, 2014 (1961). See also G. F. Chew, F. E. Low, M. L. Goldberger, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

[16] S. L. Adler and F. J. Gilman, Phys. Rev. 152, 1460 (1966).

[17] R. L. Walker (private communication) and Phys. Rev. (to be published); W. Schmidt and G. Hohler, Ann. Phys. (N. Y.) 28, 34 (1964). The author is grateful to Professor Walker for communications regarding the pion-photoproduction multipole fits.

have the right sign and reasonable magnitudes. This would indicate the general validity of the assumptions.

Finally we make some remarks about the superconvergence sum rules (SCSR). In many cases when attempts are made to saturate these sum rules by low-energy data alone, some inconsistencies appear. For example, Halpern[18] has considered the SCSR

$$\frac{1}{\pi}\int_0^\infty \mathrm{Im}(A_1^- + tA_2^-)d\nu = 0, \qquad (22)$$

where A_1^-, A_2^- are two of the invariant amplitudes for pion photoproduction. This amplitude gets a contribution from only the π trajectory. The SCSR follows from the assumption that $\alpha_\pi(0)<0$ for the pion trajectory. Using Walker's fits on multipoles, Halpern finds a rather large discrepancy. This sum rule has been studied by using a FESR and its generalization, a continuous-moment sum rule.[19] These take into account the high-energy contribution, and reasonable results are obtained without any discrepancy.

Similarly, Choudhury and Nussinov[20] have studied the SCSR

$$I = \frac{1}{\pi}\int_0^\infty \mathrm{Im}[A_3^+(\nu,0) + \tfrac{1}{3}A_3^0(\nu,0)]d\nu = 0. \qquad (23)$$

Here A_3 is again an invariant amplitude for the pion-photoproduction process. These authors find some discrepancy in the sum rule. Using the more recent fits by Walker, we have reevaluated the sum rule and find that $I=-0.0203$ with the cutoff N corresponding to the lab photon energy of 1.2 BeV and I/(nucleon pole term)$\approx -\frac{1}{2}$. Thus there is about 50% discrepancy. It is quite possible that this may be due to inaccuracies in the multipole fits. However, it can also be explained by writing a FESR if a Regge cut is present.[21]

In the presence of a cut, the last relation can be approximately written as

$$\frac{1}{\pi}\int_0^N \mathrm{Im}[A_3^+(\nu) + \tfrac{1}{3}A_3^0(\nu)]d\nu - \frac{1}{N^2}\frac{\alpha+2}{\alpha}\frac{1}{\pi}\int_0^N \nu^2\,\mathrm{Im}[A_3^+(\nu) + \tfrac{1}{3}A_3^0(\nu)]d\nu = 0. \qquad (24)$$

Numerically we estimate the second term to be $+0.0120\ (\alpha+2)/\alpha$. Thus for $\alpha_{\mathrm{cut}}>0$, the second term goes in the right direction to remove the discrepancy.
goes in the right direction to remove the discrepancy.
When the cut is present, the above treatment may be at best qualitatively correct. However, we can regard these examples as indicative of the importance of the high-energy contributions.

In the present work, we have examined the possibility of evaluating the high-energy contributions to the current algebra sum rules by combining them with the finite-energy sum rules. In many cases we arrived at interesting conclusions. Also it was shown that the discrepancies in some of the superconvergence relations could be removed. With the assumption of FESR, one needs only the low-energy data and not any detailed high-energy fits. Thus the present method seems to be quite useful in studying the sum rules. Further applications of this method to some other sum rules are in progress and will be discussed later.

Note added in proof. We have now completed the evaluation of the $\pi-\pi$ A-W sum rule using the available $\pi-\pi$ phase shifts, which strongly indicates existence of a scalar meson σ resonance near the ρ mass. The scalar contribution is substantial but the high-energy contribution is equally important. These results, along with consideration of some other sum rules, will be discussed in a forthcoming publication. Here we just discuss the scalar contribution in a narrow resonance approximation. σ contribution to the left-hand side of Eq. (12) is given by

$$\tfrac{1}{2}g_{\sigma\pi\pi}^2\{1+(\nu_\sigma^2/N^2)[(1+\alpha_\rho)/(1-\alpha_\rho)]\},$$

using the notation of Ref. 9. With $N=54.9$, $m_\sigma=730$ MeV, and $\alpha_\rho=0.5$ this gives 0.66, 1.1, and 2.86 for $\Gamma_\sigma=150$, 250, and 650 MeV, respectively. Thus the sum rule is consistent with only small values of Γ_σ ($\approx 150-250$ MeV) and not with the large value (≈ 650 MeV), that was obtained in Ref. 9. Our treatment of the sum rule seems to be more satisfactory in view of this fact.

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