experimental values are also listed for comparison. The disagreement between the predicted and experimental values is so large that we cannot possibly attribute the discrepancy to inaccuracies of the Glauber corrections. We feel that the disagreement stems from incompatibility of Kim's results with FESR.

ACKNOWLEDGMENTS

We wish to thank Dr. J. K. Kim for making available to us tabulated forms of the K^-N scattering amplitude. We also wish to thank Dr. T. Kycia for making available to us his results on KN total cross sections in the energy range 0.5 to 3.5 BeV prior to publication.

PHYSICAL REVIEW VOLUME 178, NUMBER 5 25 FEBRUARY 1969

Broad-Area Subtraction Dispersion Relation for Forward pp Scattering*

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(Received 26 June 1968)

Dispersion relations in which the real part of an amplitude appears in a finite range of the dispersion integral are studied, particularly for forward pp scattering. Lindenbaum's measurement of $\alpha \equiv \mathrm{Re}C_{pp}(\nu)/\mathrm{Im}C_{pp}(\nu)$ is tested.

KNOWING the analytic property and the high-energy behavior of a scattering amplitude, several extensions of ordinary dispersion relations have been possible. One is the finite-energy sum rule,[1] in which the high-energy behavior is assumed to be exactly given by the Regge-pole model. The other is a new superconvergent dispersion relation,[2] in which both the real and the imaginary parts appear in the same dispersion integral. The distinction between a finite-energy sum rule and a superconvergent dispersion sum rule is that, whereas the former will break down if the Regge behavior is invalid at the cutoff energy, the latter uses the asymptotic Regge behavior only as a guide (other bounds are also possible), but, will not, however, be able to determine the Regge parameters in the ordinary sense, because the high-energy tail of the dispersion integral is too small to render such information.

In this paper a third kind of extension is studied.[3] The real part of the amplitude appears only in a finite range of the dispersion integral, which is more plausible to deal with experimentally. This bears the name broad-area subtraction method.[4] The sum rule is not superconvergent, but relies on the Pomeranchuk theorem.[5] The unknown parameters, e.g., subtraction constants, are also minimal.

We consider forward proton-proton elastic scattering as a specific example. Other cases like K^+p scattering can be done in a similar way.

Following the notation of Goldberger, Nambu, and Oehme,[6] the forward, unpolarized, pp scattering amplitude is given by

$$C_{pp}(\nu)=G_1(\nu)-2\nu G_2(\nu)+\nu^2G_3(\nu)\,,$$

with $\mathrm{Im}C_{pp}(\nu)=(1/2m)(\nu^2-m^2)^{1/2}\sigma_{pp}(\nu)$. In this expression ν is the laboratory energy of the incident proton, and m is the proton rest mass. The amplitude $C_{pp}(\nu)$ has a pion pole at $\nu_\pi=-\nu_\mu=m-\mu^2/2m$ ($\mu=$pion mass), a right-hand cut starting from m to ∞, a crossing cut from $-m$ to $-\infty$, and an unphysical cut from $\nu_{2\pi}=-\nu(2\mu)=m-2\mu^2/m$ to $-\infty$. The crossing relation is $C_{pp}(-\nu)=C_{\bar{p}p}{}^*(\nu)$. The ordinary (unsubtracted) dispersion relation[7] is of the form

$$\mathrm{Re}C_{pp}(\nu)=\frac{g_\pi{}^2}{2m}\frac{\mu^2}{8m^2}\frac{1}{\nu-\nu_\pi}+\frac{1}{\pi}\int_{-\nu_{2\pi}}^{m}\frac{\mathrm{Im}C_{\bar{p}p}(\nu')}{\nu'+\nu}d\nu'+\frac{1}{\pi}\int_m^\infty\left[\frac{\mathrm{Im}C_{pp}(\nu')}{\nu'-\nu}+\frac{\mathrm{Im}C_{\bar{p}p}(\nu')}{\nu'+\nu}\right]d\nu',\quad(1)$$

where $g_\pi{}^2/4\pi=14.6$.

* Work supported in part by the U. S. Atomic Energy Commission.

[1] A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967); K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967); R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).

[2] Yu-Chien Liu and S. Okubo, Phys. Rev. Letters **19**, 190 (1967).

[3] In the course of a more general survey (to appear elsewhere), the author came across the following papers in which the real part also appears only in a finite region of the dispersion integral: S. L. Adler, Phys. Rev. **137**, B1022 (1965); A. D. Martin and F. Poole, Phys. Letters **25B**, 343 (1967); C. H. Chan and F. T. Meiere, Phys. Rev. Letters **20**, 568 (1968). The derivation of the sum rule itself is very simple. It is how to extract physics from the sum rule that is important, e.g., the particular combinations of Eq. (2).

[4] S. L. Adler, Ref. 3. The author wishes to thank N. Zovko for bringing his attention to this paper and to the name "broad-area subtraction dispersion relation."

[5] I. Ya. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. **34**, 725 (1958) [English transl.: Soviet Phys.—JETP **7**, 499 (1958)].

[6] M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. (N. Y.) **2**, 226 (1957).

[7] The presence of the unphysical region has obscured the detection of either a wrong expression or a wrong factor in the dispersion relation for NN (KN) scattering in much of the previous work. The unsubtracted dispersion relation Eq. (1) helps to clarify these ambiguities. (Once one knows the unsubtracted dispersion relation, one can "derive" everything.)

Now let us consider

$$F(\nu)=\frac{C_{pp}(\nu)}{(\nu-m)(\nu-\nu_a)^\beta(\nu-\nu_b)^{1-\beta}},\quad (0<\beta<1) \quad (2)$$

where β is real, ν_a and ν_b are also real, and are chosen such that $m<\nu_a\leqslant\nu_b\ll\infty$. If we normalize $(\nu-\nu_a)^\beta$ $[(\nu-\nu_b)^{1-\beta}]$ to be real and positive as $\nu>\nu_a+i0$ $(\nu>\nu_b+i0)$ and introduce a cut for it to run from ν_a to ∞ (ν_b to ∞), then $F(\nu)$ is an analytic function in the complex ν plane, except for poles at $\nu=m$ and at $\nu=\nu_\pi$, and for cuts specified above. Using the Pomeranchuk theorem, i.e., $\sigma_{pp}(\infty)=\sigma_{\bar{p}p}(\infty)$, one arrives at the following broad-area subtraction dispersion sum rule fo $C_{pp}(\nu)$:

$$\pi\,\mathrm{Re}C_{pp}(m)\frac{1}{(\nu_a-m)^\beta(\nu_b-m)^{1-\beta}}-\left[\pi\frac{g_\pi^2}{2m}\frac{\mu^2}{8m^2}\frac{1}{(m-\nu_\pi)(\nu_a-\nu_\pi)^\beta(\nu_b-\nu_\pi)^{1-\beta}}+\int_{-\nu_{2\pi}}^{m}d\nu\frac{\mathrm{Im}C_{\bar{p}p}(\nu)}{(\nu+m)(\nu+\nu_a)^\beta(\nu+\nu_b)^{1-\beta}}\right]$$
$$-\int_m^{\nu_a}d\nu\frac{\mathrm{Im}C_{pp}(\nu)}{(\nu-m)(\nu_a-\nu)^\beta(\nu_b-\nu)^{1-\beta}}-\int_{\nu_a}^{\nu_b}d\nu\frac{\cos\pi\beta\,\mathrm{Im}C_{pp}(\nu)}{(\nu-m)(\nu-\nu_a)^\beta(\nu_b-\nu)^{1-\beta}}-\int_{\nu_a}^{\nu_b}d\nu\frac{\sin\pi\beta\,\mathrm{Re}C_{pp}(\nu)}{(\nu-m)(\nu-\nu_a)^\beta(\nu_b-\nu)^{1-\beta}}$$
$$+\int_{\nu_b}^{\infty}d\nu\frac{\mathrm{Im}C_{pp}(\nu)}{(\nu-m)(\nu-\nu_a)^\beta(\nu-\nu_b)^{1-\beta}}-\int_m^{\infty}d\nu\frac{\mathrm{Im}C_{\bar{p}p}(\nu)}{(\nu+m)(\nu+\nu_a)^\beta(\nu+\nu_b)^{1-\beta}}=0,\quad (3)$$

using the same method as in the derivation of ordinary dispersion relation.

The numerical result for the sum rule (3) is shown in Table I, for $T_a=\nu_a-m=9$ BeV and $T_b=\nu_b-m=22$ BeV. Within this region the α in $\mathrm{Re}C_{pp}(\nu)=\alpha\,\mathrm{Im}C_{pp}(\nu)$ is measured by Lindenbaum.[8] Other quantities in the sum rule are obtained as follows.

For pp: Because of our normalization

$$\mathrm{Im}C_{pp}(\nu)=(1/2m)(\nu^2-m^2)^{1/2}\sigma_{pp}(\nu),$$

we have $\mathrm{Re}C_{pp}(\nu)=(2\pi/m)a_{pp}$,[9] $a_{pp}=$(nuclear) pp scattering length$=a_{nn}=16.1$ F.[10] Between 0 and 20 MeV, the effective-range theory $k_{\text{c.m.}}\cot\delta=1/a_{pp}+\frac{1}{2}r_0k_{\text{c.m.}}^2$, $r_0=2.7$ F, is used. From 30 to 160 MeV the relation[10]

$$\sigma_{pp}(T)=3360.4/T+0.1527T-19.18,$$

where $T=\nu-m$, is a good parametrization. (To compare with the actual experimental values of these low-energy pp total cross sections, see Ref. 11.) In the intermediate energy range, data are taken from Ref. 12. Above 20 BeV, Lindenbaum's fit[13]

$$\sigma_{pp}(\nu)=38.151+14.16(\nu^2-m^2)^{-0.46}$$

is assumed.

For $\bar{p}p$, experimental data are lacking for $T<50$ MeV. The intermediate energy data are from Ref. 14, while above 20 BeV again Lindenbaum's fit[13]

$$\sigma_{\bar{p}p}(\nu)=38.151+91.5(\nu^2-m^2)^{-0.37}$$

is used. The unphysical integral, unlike the K^-p case, cannot be obtained from analytic continuation. A phenomenological approach is to assume the saturation of 0^-, 0^+, 1^-, $\cdots$, mesons. Then

$$-\int_{-\nu_{2\pi}}^{m+50}d\nu\frac{\mathrm{Im}C_{\bar{p}p}(\nu)}{(\nu+m)(\nu+\nu_a)^\beta(\nu+\nu_b)^{1-\beta}}\simeq-\pi\sum_j\frac{R_j}{2m}\frac{1}{(m-\nu_j)(\nu_a-\nu_j)^\beta(\nu_b-\nu_j)^{1-\beta}},\quad (4)$$

where $\nu_j=m-m_j^2/2m$ ($m_j=$mass of the jth meson), and R_j, the residue, is given by[15]

TABLE I. Numerical result for the broad-area subtraction sum rule (3) [and (4)], in units of BeV^{-3}. Low-energy parameters used are $a_{pp}=16.1$ F, $r_0=2.7$ F, $g_\pi^2/4\pi\simeq14.6$, $g_\eta^2/4\pi\simeq10$, $g_\rho^2/4\pi\simeq0.6$, $g_{2\rho}^2/4\pi\simeq2.82$, $g_{1\omega}^2/4\pi\simeq1.5$, and $g_{2\omega}^2/4\pi\simeq0.0$. When $\beta=0.0$ and $\beta=1.0$, principal values have been evaluated.

β	1st term	2nd term	3rd term	4th term	5th term	6th term	7th term	Sum
0.0	78	−14	−101	−188	29	326	−131	−2
0.1	85	−15	−114	−542	32	692	−135	2
0.2	93	−17	−130	−224	35	407	−140	5
0.3	102	−18	−148	−129	38	308	−144	8
0.4	112	−20	−171	−58	41	256	−149	10
0.5	122	−22	−201	0	44	223	−154	12
0.6	133	−24	−242	59	47	200	−159	14
0.7	146	−26	−305	134	50	182	−164	16
0.8	160	−28	−419	255	53	168	−170	20
0.9	174	−31	−735	576	56	156	−176	22
1.0	191	−33	−350	194	59	147	−182	25

[8] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 857 (1967).

[9] Compare with L. I. Lapidus, Zh. Eksperim. i Teor. Fiz. **36**, (1959) [English transl.: Soviet Phys.—JETP **9**, 193 (1959)].

[10] A. A. Carter, Nuovo Cimento **48**, 15 (1967).

[11] F. F. Chen *et al.*, Phys. Rev. **103**, 211 (1956); B. Cork *et al.*, *ibid.* **80**, 321 (1950).

[12] F. F. Chen *et al.*, Ref. 11; D. V. Bugg *et al.*, Phys. Rev. **146**, 980 (1966), and references therein; K. J. Foley *et al.*, Ref. 8.

[13] S. J. Lindenbaum, Brookhaven National Laboratory Report, invited paper, Conference on πN Scattering, December 1–2, 1967, at University of California, Irvine, California (unpublished).

[14] U. Amaldi *et al.*, Nuovo Cimento **46**, 171 (1966); **34**, 825 (1964), and references therein; R. J. N. Phillips, Rev. Mod. Phys. **39**, 681 (1967), and references therein; W. Galbraith *et al.*, Phys. Rev. **138**, 913 (1965).

[15] The method for these calculations can be found in Ref. 6. The phenomenological Lagrangian is (suppressing isospin)

$$\mathcal{L}(0^-)=ig\bar{\psi}\gamma_5\psi\varphi,$$
$$\mathcal{L}(0^+)=g\bar{\psi}\psi\varphi,$$

and

$$\mathcal{L}(1^-)=ig_1\bar{\psi}\gamma_\mu\psi V_\mu+g_2\bar{\psi}(\sigma_{\mu\nu}/2m)\psi(\tfrac{1}{2})(\partial_\mu V_\nu-\partial_\nu V_\mu).$$

The term $g_{2j}^2(1/32)(m_j^4/m^4)$ is usually neglected.

0^-: $R_j=g_j^2(m_j^2/8m^2)$,

0^+: $R_j=g_j^2(-\frac{1}{2}+m_j^2/8m^2)$,

1^-: $R_j=g_{1j}^2(\frac{1}{2}+m_j^2/4m^2)+g_{1j}g_{2j}(\frac{3}{4}m_j^2/m^2)$
$$+g_{2j}^2(\frac{1}{4}m_j^2/m^2+\frac{1}{32}m_j^4/m^4).$$

The numbers shown in Table I involve η, ρ, and ω only, with[16] $g_\eta^2/4\pi\simeq10$, $g_{1\rho}^2/4\pi\simeq0.6$, $g_{2\rho}^2/4\pi\simeq2.82$, $g_{1\omega}^2/4\pi\simeq1.5$, and $g_{2\omega}^2/4\pi\simeq0$. The inclusion of 0^+ mesons will make the sum rule (3) worse, because it has an opposing contribution relative to 0^- and 1^- poles [i.e., the second term of Eq. (3) becomes positive]. This conclusion persists even when the pp scattering length a_{pp} is varied from $a_{pp}=a_{np}$ (singlet) $=23.68$ F[17] (assumption of charge independence) down to $a_{pp}=7.7$ F (presence of the Coulomb term), as long as the effective-range parametrization between 0 and 30 MeV is valid, since the first and the third terms both become large or small and cancel with each other. If one seriously saturates with the above η, ρ, and ω, then the situation is that, the smaller the value of a_{pp} (smaller than 16.1 F), the better is the sum rule (3).

When β approaches 0^+, Eq. (3) is an ordinary dispersion relation with one subtraction at $\nu_b\simeq23$ BeV. The sum rule in all values of a_{pp} is very good around here, as the low-energy contribution has been damped. When β approaches 1^-, the subtraction point moves to $\nu_a\simeq10$ BeV. It seems that this energy is not high enough to ignore the low-energy uncertainty [first through third terms of Eq. (3)]. However, the data in Table I are satisfactory, showing the validity of our broad-area subtracted dispersion sum rule (it is very good for the πN case), or, turning the thing around, Lindenbaum's measurement is consistent with the present method for the treatment of pp dispersion relations.

The author is indebted to Professor S. Okubo for guidance. He also wishes to thank I. J. Kim for partial participation in the derivation of pole residues and in computer programming, and to thank M. Samuel for improving the manuscript.

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[17] I. I. Levintov and G. M. Adelson-Velsky, Phys. Letters 13, 185 (1964).

PHYSICAL REVIEW VOLUME 178, NUMBER 5 25 FEBRUARY 1969

S-Matrix Theory of Currents. I

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(Received 11 October 1968)

We consider the problem of the determination of currents and the off-mass-shell extension of scattering amplitudes within the context of analytic S-matrix theory, keeping up to three particles inside the unitary relations. The technique used is a straightforward application of the work by Mandelstam on the N/D formulation of the three-body problem. It is found that anomalous thresholds do not play any role in the problem and that two cases can be solved explicitly, namely, the extension off the mass shell of a scattering amplitude and the computation of a form factor which tends to zero asymptotically.

1. INTRODUCTION

The present theory of strong interactions is mainly built upon the analyticity properties of scattering amplitudes implemented with the properties of unitarity and crossing, together with some necessary assumptions which are not yet quite clear about asymptotic behavior.[1] On the other hand, electrodynamics and weak interactions are formulated in terms of currents, a notion which is quite simple within the framework of quantum field theory but more difficult to handle by S-matrix theory. However, much can be learned about the structure of currents and their form factors by using dispersion techniques, as was first observed in the work of Frazer and Fulco[2] on the isovector form factors of the nucleon.

Outstanding questions which arise in this domain are the following:

(1) Assuming the S-matrix for strong interactions to be known, is it possible to derive from it the matrix elements of some current operators?

(2) If this process is possible, will it lead to a unique type of current or to several different types, or, in other words, are the currents of electrodynamics and weak interactions uniquely defined?

(3) Is there a distinction between different quantum numbers such as, for instance, the quantum numbers ($I=0$ or 1, $I_3=0$, $S=0$, $J^P=1^-$) of the electric current,

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[1] See, e.g., R. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S Matrix* (Cambridge University Press, New York, 1964).

[2] W. Frazer and J. Fulco, Phys. Rev. Letters 2, 364 (1959); Phys. Rev. 117, 1609 (1960).