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Low-Energy $\bar{K}N$ Scattering and Finite-Energy Sum Rules*

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Finite-energy sum rules are used to test the consistency of Kim's multichannel effective-range analysis of low-energy $\bar{K}N$ data with high-energy total cross sections.

FINITE-ENERGY sum rules (FESR) have been proposed by a number of authors[1–3] recently to relate low-energy data to the Regge-pole parameters describing high-energy cross-section data. These sum rules are statements of consistency that follow from dispersion relations and the assumed high-energy behavior of a given amplitude. In earlier applications[1] they were used to check whether the Regge parameters describing high-energy data were also consistent with low-energy data. On the other hand, if one has a fairly accurate knowledge of the high-energy region, one can use the FESR to check the consistency of a parametrization of low-energy data with high-energy data. We propose to use FESR to test the low-energy multichannel effective-range analysis of $\bar{K}N$ scattering reported by Kim[4] recently. One of the remarkable results of Kim's analysis is the calculation of $K\Lambda p$ and $K\Sigma p$ coupling constants, in excellent agreement with pure $SU(3)$ for the meson-baryon-baryon vertex. The contribution from the region below the $\bar{K}N$ threshold to the dispersion relation used to calculate these coupling constants is very important. Thus extrapolation of the low-energy data to this region is of crucial importance, as emphasised by Kim[4] and others.[5] In view of this situation, we consider it worthwhile to exploit FESR as further constraints on the data in question. We find that Kim's parametrization does not satisfy these constraints.

We consider the amplitude f_P defined by

$$f_P=f(K^-p)+f(K^+p)+f(K^-n)+f(K^+n)\,,$$

where f_P is normalized by the optical theorem,

$$\mathrm{Im}f_P\equiv(k/4\pi)S_P=(k/4\pi)[\sigma(K^-p)+\sigma(K^+p)+\sigma(K^-n)+\sigma(K^+n)]\,,\quad(1)$$

where $\sigma(KN)$ are the KN total cross sections and k is the laboratory momentum of the K meson.

At high energy, f_P is dominated by the P and P' Regge poles and is given by

$$f_P^{\mathrm{asy}}=\beta_P(E/\mu)^{\alpha}(i-\cot\tfrac{1}{2}\pi\alpha_P)+\beta_{P'}(E/\mu)^{\alpha_{P'}}(i-\cot\tfrac{1}{2}\pi\alpha_{P'})\,,\quad(2)$$

where β_i and α_i are the residue and trajectory functions of the P and P' trajectories and E and μ are lab energy and mass of the K. We take $\alpha_P=1$, as is usually assumed, and $\alpha_{P'}=0.64$ from our recent analysis[6] of $\pi^\pm p$ scattering using sum rules. We have two free parameters, β_P and $\beta_{P'}$, whose numerical values we shall need shortly in order to examine Kim's low-energy analysis. These two parameters can be determined directly by fitting the total cross-section data. A more accurate determination of β_P and $\beta_{P'}$ may be made, however, by making use of a superconvergence sum rule to constrain these parameters. The amplitude $f'=f_P-f_P^{\mathrm{asy}}$ satisfies the superconvergence relation

$$\int_0^N \mathrm{Im}(f_P-f_P^{\mathrm{asy}})EdE=0\,.$$

Substituting Eq. (2) and explicitly integrating Ef_P^{asy}, we obtain

$$N^2\left[\frac{\beta_P}{3}\left(\frac{N}{\mu}\right)+\frac{\beta_{P'}}{2+\alpha_{P'}}\left(\frac{N}{\mu}\right)^{\alpha_{P'}}\right]=2\pi\sum_{j=\Sigma,\Lambda}E_j(R_p{}^j+R_n{}^j)+\int_{E_{\Lambda\pi}}^N E\,\mathrm{Im}f_P(E)dE\,,\quad(3)$$

where N is an asymptotic energy such that (2) is valid and is taken to be 4 BeV in our calculations[7], and

$$\begin{aligned}&E_j=(M^2-M_j{}^2-\mu^2)/2M\,,\\&R_p{}^\Sigma=(g_{K\Sigma p}{}^2/4\pi)X(\Sigma)\,,\quad R_p{}^\Lambda=(g_{K\Lambda p}{}^2/4\pi)X(\Lambda)\\&R_n{}^\Sigma=2R_p{}^\Sigma\,,\quad R_n{}^\Lambda=0\,,\\&X(j)=[(M_j-M)^2-\mu^2]/8M^2\end{aligned}\quad(4)$$

and M is the nucleon mass. $E_{\Lambda\pi}$ denotes the $\Lambda\pi$ threshold.

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[1] K. Igi, Phys. Rev. **130**, 820 (1963).
[2] L. Sertorio and M. Toller, Phys. Letters **18**, 191 (1965); A. A. Logunov *et al.*, *ibid.* **24B**, 181 (1967); K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967).
[3] M. S. K. Razmi and Y. Ueda, Phys. Rev. **162**, 1738 (1967); Nuovo Cimento **52B**, 948 (1967).
[4] J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967); **19**, 1079 (1967).
[5] M. Lusignoli *et al.*, Phys. Letters **21**, 229 (1966); N. Zovko, *ibid.* **23**, 143 (1966); H. P. C. Rood, Nuovo Cimento **50A**, 493 (1967).
[6] R. K. Logan and M. S. K. Razmi, Phys. Rev. **169**, 1167 (1968); Nuovo Cimento **55A**, 577 (1968). We have checked that our conclusions are independent of this particular choice for $\alpha_{P'}$.
[7] We have checked that our results are independent of the particular choice of N as long as N is chosen greater than 3 BeV.

TABLE I. Evaluation of the right-hand side of Eq. (5). $E_0=[(0.53\text{ BeV})^2+\mu^2]^{1/2}=0.73$ BeV.

Term	Contribution (mb BeV)	Method of calculation [a]
$-\frac{1}{2}\pi\,\mathrm{Re}f_P(\mu)$	6.594	
$\int_\mu^4 \frac{dE\,E}{E^2-\mu^2}[\mathrm{Im}f(K^+p)+\mathrm{Im}f(K^+n)]$	10.396	Numerical integration using the optical theorem and the experimental values of Refs. 8–12.
$\int_{E_0}^4 \frac{dE\,E}{E^2-\mu^2}[\mathrm{Im}f(K^-p)+\mathrm{Im}(K^-n)]$	15.351	Same as directly above.
$\int_{\Lambda\pi}^{E_0} \frac{dE\,E}{E^2-\mu^2}[\mathrm{Im}f(K^-p)+\mathrm{Im}f(K^-n)]$	-6.488	Numerical integration using the values[b] of Imf from Kim's multichannel analysis.[b]
$2\pi \sum_{j=\Sigma,\Lambda} \frac{E_j(R_p{}^j+R_n{}^j)}{E_j{}^2-\mu^2}$	0.096	From Kim's values for the coupling constants in Ref. 4.
	25.948	

[a] See Ref. 11. [b] See Ref. 4.

The right-hand side of (3) was evaluated using the experimental data[8–12] on total cross sections wherever possible. This includes K^+N in the region $\mu<E<N$ and K^-N in the region $E_0<E<N$, where $E_0=[(0.53\text{ BeV})^2+\mu^2]^{1/2}$. The remainder including the Born terms and the K^-N contribution in the region $E_{\Lambda\pi}<E<E_0$ comes from Kim's analysis.[4] Kim's terms contribute less than 0.1% of the total and hence do not appreciably effect the determination of the β_i.

The sum rule described above reduces the number of free parameters in Eq. (2) to one. This one parameter was determined by a fit to data of Galbraith *et al.*[13] shown in Fig. 1(a). The best fit was obtained with $\beta=2.65\pm0.06$ mb BeV and $\beta'=1.21\pm0.09$ mb BeV, yielding a χ^2 of 1.9.

Turning to our central problem, we now consider a particular FESR, namely, an Igi-type sum rule for KN scattering,[1] which is more sensitive to the low-energy data and, therefore, may be used to check the validity of Kim's analysis. The sum rule reads

$$\beta_P\left(\frac{N}{\mu}\right)+\frac{\beta_{P'}}{\alpha_{P'}}\left(\frac{N}{\mu}\right)^{\alpha_{P'}}=\tfrac{1}{2}\pi\left[-\mathrm{Re}f_P(\mu)+4\sum_{j=\Sigma,\Lambda}\frac{E_j(R_p{}^j+R_n{}^j)}{E_j{}^2-\mu^2}\right]+P\int_{E_{\Lambda\pi}}^{N}\frac{E\,\mathrm{Im}f_P(E)}{E^2-\mu^2}dE. \quad (5)$$

Inspection of (5) shows that this sum rule emphasizes the importance of the low-energy region much more than sum rule (3). That is to say, the contribution of the low-energy region in (5) is far from being negligible compared to the contribution of the medium-energy region. Therefore, we believe that sum rule (5) may be regarded as a sound check on any given parametrization of low-energy data. We evaluated the left-hand side of (5) using the values above, and it turns out to be 28.6

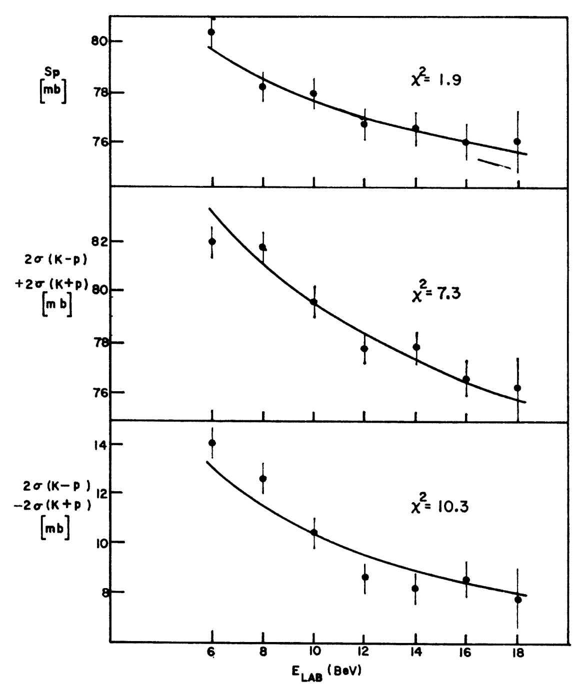

FIG. 1. (a). *Fit* to S_p. (b). *Fit* to $2\sigma(K^-p)+2\sigma(K^+p)$. (c). *Fit* to $2\sigma(K^-p)-2\sigma(K^+p)$.

[8] J. D. Davies *et al.*, Phys. Rev. Letters **18**, 62 (1967).
[9] R. L. Cool *et al.*, Phys. Rev. Letters **16**, 1228 (1966); **17**, 102 (1966).
[10] R. Good and N. Xuong, Phys. Rev. Letters **14**, 191 (1965).
[11] S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1962).
[12] V. J. Stenger *et al.*, Phys. Rev. **134**, B1111 (1964).
[13] W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965).

TABLE II. Comparison of experimental values of $\sigma(K^{\pm}n)$ with predictions of $K^{\pm}p$ sum rules.

$E_{\rm lab}$ (BeV)	$\sigma(K^+n)$ experimental (mb)	$\sigma(K^+n)$ from $K^{\pm}p$ sum rule (mb)	$\sigma(K^-n)$ experimental (mb)	$\sigma(K^-n)$ from $K^{\pm}p$ sum rule (mb)
6	17.5±0.4	26.6±2.7	21.9±0.4	25.4±2.7
8	17.6±0.4	25.1±2.6	19.7±0.4	24.0±2.6
10	17.5±0.4	24.0±2.5	20.6±0.4	23.0±2.5
12	17.6±0.4	23.3±2.4	20.2±0.4	22.3±2.4
14	17.5±0.4	22.7±2.3	20.1±0.4	21.8±2.3
16	17.4±0.4	22.2±2.3	20.3±0.6	21.3±2.3
18	17.6±0.4	21.8±2.2	20.3±1.1	21.0±2.2

±0.7 mb BeV. The right-hand side of (5) is evaluated, as before, by using the low-energy parametrization of Kim and the data of the authors in Refs. 8–12. The contributions of the various terms on the right-hand side of Eq. (5) are given in Table I. They give a total contribution of 25.9 mb BeV,[14] which represents a discrepancy with the left-hand side of 2.7 mb BeV. While this discrepancy of 2.7 mb BeV may not seem very dramatic,[15] it is none the less significant, especially when one considers that the over-all contribution from Kim's multichannel analysis is only −6.488 mb BeV.

If the discrepancy is due to an error from this contribution, then Kim's determination of the coupling constants is in difficulty. The reason for this is that Kim's determination[4] of the coupling constants from the dispersion relations differed from the previous determinations precisely because his contribution to the dispersion integrals differed from the others in this energy region ($E_{\Lambda\pi}<E<E_0=0.73$ BeV). The discrepancy, however, might have its source in the other terms. The scattering-length term, $-\frac{1}{2}\pi\,\mathrm{Re}f_P(\mu)$, has the value[11,12,16] 6.594 mb BeV with an error of only ±0.3 mb BeV. Estimating the error from the integrals over the total cross sections is difficult. The statistical error turns out to be very small because of the large number of terms contributing. The significant error is the systematic error which is not known exactly. The usual estimation for total cross section on proton targets is usually 1 to 2% which would represent an error from the cross-section terms of only 0.25 to 0.5 mb BeV. The major source of possible error, however, in our opinion is the determination of the $K^{\pm}n$ cross sections from the $K^{\pm}d$ data, using the Glauber correction.

[14] It is hard to estimate the error in evaluating the right-hand side because one knows only the statistical error, which is very much smaller than the systematic error. According to most estimates, the systematic error in measuring the total cross sections is 1%, which would give the right-hand side an error of ±0.3.

[15] A more dramatic presentation of this discrepancy is obtained by pursuing the following line of thought. Sum rules (3) and (5) are used to determine β_P and $\beta_{P'}$ with the result that $\beta_{P'}$ is negative. This is in clear contradiction with the data on S_P, which show S_P approaching its asymptotic limit from above, not below.

[16] J. K. Kim, Phys. Rev. Letters 14, 29 (1965); Columbia University Report Nevis 149, 1966 (unpublished).

In order to test Kim's results without considering this possible source of error, we shall now consider a set of sum rules which only involve $K^{\pm}p$ data. We define $f^{\pm}=2[f(K^-p)\pm f(K^+p)]$. At high energies f^+ is dominated by the P, P', and R Regge poles and f^- by ω and ρ Regge poles. The asymptotic forms of $f^{\pm}$ are given by

$$f_A^+(E)=\beta_P(E/\mu)+\beta_{P'}(E/\mu)^{\alpha_{P'}}+\beta_R(E/\mu)^{\alpha_R} \quad (6)$$

and

$$f_A^-(E)=\beta_\omega(E/\mu)\alpha_\omega+\beta_\rho(E/\mu)\alpha_\rho. \quad (7)$$

As before, we take $\alpha_{P'}=0.64$. The remaining intercepts α_R, α_ω, and α_ρ are fairly well determined from previous analyses of other data. We take[17] $\alpha_R=0.32$, $\alpha_\omega=0.52$, and $\alpha_\rho=0.58$. Thus, we are left with three free parameters for f^+ and two for f^-. The three parameters of f^+ reduce to only one free parameter when we make use of the superconvergent sum rule and the Igi-type sum rule for f^+. The two sum rules are readily obtained from (3) and (5) by formally setting the $K^{\pm}n$ contributions to zero, multiplying the $K^{\pm}p$ contributions by a factor of 2 on the right-hand side, and adding the contribution of the R trajectory on the left-hand side. The single remaining parameter is determined by a least-squares fit to $2[\sigma(K^-p)+\sigma(K^+p)]$ with relation (6), which is shown in Fig. 1(b). We obtain

$$\beta_P=2.15\pm0.04\text{ mb BeV},\quad \beta_{P'}=3.74\pm0.10\text{ mb BeV},$$
$$\beta_R=-2.21\pm0.05\text{ mb BeV}.$$

In the case of f^-, the two parameters are reduced to one when we make use of the superconvergent relation[3] for f^-, namely,

$$N\sum_{i=\omega,\rho}\frac{\beta_i}{1+\alpha_i}\left(\frac{N}{\mu}\right)^{\alpha_i}=4\pi\sum_{j=\Sigma,\Lambda}R_p{}^j+\int_{E_{\Lambda\pi}}^{N}\mathrm{Im}f^-(E)dE. \quad (8)$$

The one free parameter is now determined by a least-squares fit to $2[\sigma(K^-p)-\sigma(K^+p)]$ using (7). This fit is shown in Fig. 1(c). We obtain

$$\beta_\omega=0.70\pm0.3\text{ mb BeV},\quad \beta_\rho=0.87\pm0.3\text{ mb BeV}.$$

Notice that the β_P and $\beta_{P'}$, which are normalized as before, have quite different values than the ones obtained with f_P. Furthermore, the R-pole residue is negative.

With all the parameters determined, we are now in a position to predict $K^{\pm}n$ high-energy cross sections. These predictions are shown in Table II where the

[17] Our results are independent of this particular choice of these intercepts.

experimental values are also listed for comparison. The disagreement between the predicted and experimental values is so large that we cannot possibly attribute the discrepancy to inaccuracies of the Glauber corrections. We feel that the disagreement stems from incompatibility of Kim's results with FESR.

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We wish to thank Dr. J. K. Kim for making available to us tabulated forms of the K^-N scattering amplitude. We also wish to thank Dr. T. Kycia for making available to us his results on KN total cross sections in the energy range 0.5 to 3.5 BeV prior to publication.

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Broad-Area Subtraction Dispersion Relation for Forward pp Scattering*

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Dispersion relations in which the real part of an amplitude appears in a finite range of the dispersion integral are studied, particularly for forward pp scattering. Lindenbaum's measurement of $\alpha \equiv \mathrm{Re}C_{pp}(\nu)/\mathrm{Im}C_{pp}(\nu)$ is tested.

KNOWING the analytic property and the high-energy behavior of a scattering amplitude, several extensions of ordinary dispersion relations have been possible. One is the finite-energy sum rule,[1] in which the high-energy behavior is assumed to be exactly given by the Regge-pole model. The other is a new superconvergent dispersion relation,[2] in which both the real and the imaginary parts appear in the same dispersion integral. The distinction between a finite-energy sum rule and a superconvergent dispersion sum rule is that, whereas the former will break down if the Regge behavior is invalid at the cutoff energy, the latter uses the asymptotic Regge behavior only as a guide (other bounds are also possible), but, will not, however, be able to determine the Regge parameters in the ordinary sense, because the high-energy tail of the dispersion integral is too small to render such information.

In this paper a third kind of extension is studied.[3] The real part of the amplitude appears only in a finite range of the dispersion integral, which is more plausible to deal with experimentally. This bears the name broad-area subtraction method.[4] The sum rule is not superconvergent, but relies on the Pomeranchuk theorem.[5] The unknown parameters, e.g., subtraction constants, are also minimal.

We consider forward proton-proton elastic scattering as a specific example. Other cases like K^+p scattering can be done in a similar way.

Following the notation of Goldberger, Nambu, and Oehme,[6] the forward, unpolarized, pp scattering amplitude is given by

$$C_{pp}(\nu)=G_1(\nu)-2\nu G_2(\nu)+\nu^2 G_3(\nu)\,,$$

with $\mathrm{Im}C_{pp}(\nu)=(1/2m)(\nu^2-m^2)^{1/2}\sigma_{pp}(\nu)$. In this expression ν is the laboratory energy of the incident proton, and m is the proton rest mass. The amplitude $C_{pp}(\nu)$ has a pion pole at $\nu_\pi=-\nu_\mu=m-\mu^2/2m$ (μ = pion mass), a right-hand cut starting from m to ∞, a crossing cut from $-m$ to $-\infty$, and an unphysical cut from $\nu_{2\pi}=-\nu(2\mu)=m-2\mu^2/m$ to $-\infty$. The crossing relation is $C_{pp}(-\nu)=C_{\bar{p}p}^*(\nu)$. The ordinary (unsubtracted) dispersion relation[7] is of the form

$$\mathrm{Re}C_{pp}(\nu)=\frac{g_\pi^2}{2m}\frac{\mu^2}{8m^2}\frac{1}{\nu-\nu_\pi}+\frac{1}{\pi}\int_{-\nu_{2\pi}}^{m}\frac{\mathrm{Im}C_{\bar{p}p}(\nu')}{\nu'+\nu}d\nu' +\frac{1}{\pi}\int_m^\infty\left[\frac{\mathrm{Im}C_{pp}(\nu')}{\nu'-\nu}+\frac{\mathrm{Im}C_{\bar{p}p}(\nu')}{\nu'+\nu}\right]d\nu', \quad (1)$$

where $g_\pi^2/4\pi=14.6$.

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[1] A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967); K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967); R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).

[2] Yu-Chien Liu and S. Okubo, Phys. Rev. Letters **19**, 190 (1967).

[3] In the course of a more general survey (to appear elsewhere), the author came across the following papers in which the real part also appears only in a finite region of the dispersion integral: S. L. Adler, Phys. Rev. **137**, B1022 (1965); A. D. Martin and F. Poole, Phys. Letters **25B**, 343 (1967); C. H. Chan and F. T. Meiere, Phys. Rev. Letters **20**, 568 (1968). The derivation of the sum rule itself is very simple. It is how to extract physics from the sum rule that is important, e.g., the particular combinations of Eq. (2).

[4] S. L. Adler, Ref. 3. The author wishes to thank N. Zovko for bringing his attention to this paper and to the name "broad-area subtraction dispersion relation."

[5] I. Ya. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. **34**, 725 (1958) [English transl.: Soviet Phys.—JETP **7**, 499 (1958)].

[6] M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. (N. Y.) **2**, 226 (1957).

[7] The presence of the unphysical region has obscured the detection of either a wrong expression or a wrong factor in the dispersion relation for NN (KN) scattering in much of the previous work. The unsubtracted dispersion relation Eq. (1) helps to clarify these ambiguities. (Once one knows the unsubtracted dispersion relation, one can "derive" everything.)