Hypothesis of $M1$ Dominance in $\frac{3}{2}^+$-Isobar Production Reactions*

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The hypothesis that the $\rho N\Delta$ and $A_2N\Delta$ vertices are predominantly of the magnetic-dipole ($M1$) type is investigated in detail with reference to the $\frac{3}{2}^+$-isobar production reactions at high energies. The incorporation of this hypothesis for the Regge exchanges of ρ and A_2 mesons is greatly facilitated by our choice of Reggeizing the invariant amplitudes, a choice which avoids spurious parametrizations of the unknown Regge residues and which automatically satisfies all the constraint conditions at the normal thresholds and pseudothresholds. The simple model which assumes only $M1$ couplings and $SU(3)$ symmetry at the Regge vertices is in satisfactory agreement with the available high-energy data on the differential cross sections and density matrix elements of the reactions $\pi^+p \to \pi^0(\eta)\Delta^{++}$, $\pi^-p \to \pi^-\Delta^+$, and $K^+p \to K^0\Delta^{++}$. The presence of a small $E2$ coupling at the vector-meson vertex, in addition to the dominant $M1$ coupling, is shown to provide an improved fit with experiment near the forward direction.

1. INTRODUCTION

RECENTLY Jackson and Hite[1] have emphasized that certain kinematic singularities in the helicity amplitudes arising from an application of crossing symmetry cannot in fact be present in the formula for the differential cross section. For this reason, they suggest that the phenomenological fits based on the conventional Regge parametrization of the parity-conserving, regularized, t-channel helicity amplitudes together with the assumption that the residues are slowly varying functions of t have to be reexamined. The discussion of which amplitudes are free from kinematic singularities is perhaps easy in the framework of transverse amplitudes.[2] However, these amplitudes are not convenient for the purposes of incorporating the Regge asymptotic behavior. On the other hand, the use of invariant amplitudes facilitates the construction of Reggeizable amplitudes which automatically satisfy all the constraints imposed at the various pseudo- and normal thresholds. A minor disadvantage in this approach is, of course, that a linearly independent set of amplitudes has to be chosen each time a different spin configuration is considered. For the special class of isobar-production reactions we shall be considering in this paper, we follow the approach based on the invariant amplitudes.

A large amount of high-energy experimental data on a variety of reactions involving quasi-two-body final states has become available in the past few years. We will be especially interested in meson-baryon scattering reactions resulting in a pseudoscalar meson and a spin-$\frac{3}{2}^+$ isobar. Qualitatively, one might expect reactions in which the t-channel quantum numbers are identical but the $\frac{3}{2}^+$ isobar is replaced by a $\frac{1}{2}^+$ baryon, to exhibit similar structures in their t distributions. A list of the reactions that are of interest here is given in the second column of Table I. For these reactions, the t-channel quantum numbers isospin I (only the lowest possible value of I is noted), hypercharge Y, and G parity are noted in the third column. The possible Regge exchanges corresponding to each set of quantum numbers are given in column 4. The last column lists reactions in which a $\frac{1}{2}^+$ baryon is emitted instead of a $\frac{3}{2}^+$ baryon but otherwise have the same t-channel quantum numbers.

The existing data on the reactions (1) and (2) are suggestive of a turnover near $t\sim 0$ and a dip at $t\sim -0.6$ $(\mathrm{GeV}/c)^2$ followed by a bump very similar to the data on the corresponding prototype reaction. In the π^-p-charge-exchange reaction, the initial turnover and dip-bump structure can be explained by assuming the dominance of the helicity-flip amplitude over the nonflip amplitude. Because of the similarities in the t distributions, assuming spin to be an inessential complication, we expect that in the isobar reaction the helicity-flip amplitude(s) should predominate over the nonflip and double-flip helicity amplitudes. Thus one is naturally led to consider the Stodolsky-Sakurai hypothesis that $\rho N\Delta$ couplings are of pure magnetic dipole type. In general, spins do give rise to "essential complications." When the external particles have large spins we expect the t distributions to be flat, away from $t\sim 0$, because of the presence of various helicity-flip amplitudes which begin contributing away from the forward direction. The available data on the reactions (3) and (5) appear to show slightly flatter t distributions compared to their prototype reactions, while the data on reaction (4) at 5 GeV/c and those on K^-p charge exchange seem to be somewhat similar [as in the case of reactions (1) and (2)]. Thus in order to meaningfully discuss the differences or similarities in the various isobar-production reactions among themselves or in relation to their prototype reactions, it becomes necessary to undertake a detailed study of these reactions.

Moreover, all the isobar reactions in Table I can be related to each other by assuming $SU(3)$ symmetry (all the isobars considered belong to the well-known $\frac{3}{2}^+$-decuplet representation and thus there are no complica-

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[1] J. D. Jackson and G. E. Hite, Phys. Rev. **169**, 1248 (1968).
[2] A. Kotanski, Acta Phys. Polon. **29**, 699 (1966); **30**, 629 (1966); G. Cohen-Tannoudji, A. Morel, and H. Navelet, Ann. Phys. (N. Y.) **46**, 239 (1968).

TABLE I. Correspondence of $\frac{3}{2}^+$-isobar production reactions with baryon production reactions on the basis of t-channel quantum numbers.

Reaction number	Reaction $0^-+\frac{1}{2}^+\to 0^-+\frac{3}{2}^+$	t-channel quantum numbers I	$\lvert Y\rvert$	G	Possible t-channel exchanges	Prototype reaction $0^-+\frac{1}{2}^+\to 0^-+\frac{1}{2}^+$
(1)	$\pi^-p\to\pi^-\Delta^+$	1	0	+	ρ	$\pi^-p\to\pi^0n$
(2)	$\pi^+p\to\pi^0\Delta^{++}$	1	0	+	ρ	
(3)	$\pi^+p\to\eta^0\Delta^{++}$	1	0	−	A_2	$\pi^-p\to\eta^0n$
(4)	$K^+p\to K^0\Delta^{++}$	1	0	±	ρ,A_2	$K^-p\to\bar{K}^0n$
(5)	$K^-p\to\pi^-Y^{*+}$	$\frac{1}{2}$	1	⋯	K^*,K^{**}	$K^-p\to\pi^-\Sigma^+$

tions due to f/d ratios) so that such a study could also afford a testing ground for higher symmetries.

In Sec. 2, we will discuss the invariant-amplitude approach to be used in the analysis of the isobar reactions. Section 3 is devoted to constructing a model for reactions of the type $\pi(K)+p\to\pi(K)+\Delta$ and presenting fits to the available data. In Sec. 4, we summarize our approach to the problem and propose future modifications and refinements which can be made as more data become available. The connection between the helicity amplitudes and the invariant amplitudes is given in Appendix A and the expressions for multipoles in terms of invariant amplitudes are presented in Appendix B.

2. FORMALISM

We now proceed to a description of processes of the type $N+\Pi\to N^*+\Pi'$ in terms of four invariant amplitudes (Π and Π' are pseudoscalar mesons, while N and N^* represent a $\frac{1}{2}^+$ baryon and $\frac{3}{2}^+$ isobar, respectively). We write the s-channel amplitude T^s as (see Fig. 1)

$$T^s=\bar{u}^\mu(p_2)i\gamma^5M_\mu{}^su(p_1)\,, \tag{2.1}$$

where

$$M_\mu{}^s=AQ_\mu+B(-i\gamma^5)\epsilon_{\mu\nu\lambda\rho}P^\nu Q^\lambda K^\rho +CK_\mu+D\gamma\cdot KK_\mu. \tag{2.2}$$

Here $u^\mu(p_2)$ is a Rarita-Schwinger wave function describing a spin-$\frac{3}{2}$ particle of mass m_2;

$$\begin{aligned} Q&=k_1-k_2\,,\\ P&=p_1+p_2\,, \quad (s\text{ channel})\\ K&=k_1+k_2\,; \end{aligned} \tag{2.3}$$

and A, B, C, and D are a set of invariant amplitudes which are assumed to satisfy the Mandelstam representation. The notation and the conventions used for the metric and the γ matrices are explained in Appendix A.

For the purposes of Reggeization we shall consider the related t-channel process,

$$\bar{N}^*+N\to\bar{\Pi}+\Pi'\,, \tag{2.4}$$

obtained by crossing the isobar and the initial meson: $p_2\to -p_2$ and $k_1\to -k_1$. The auxiliary momenta P, Q, and K become

$$\begin{aligned} Q&=-k_1-k_2\,,\\ P&=p_1-p_2\,, \quad (t\text{ channel})\\ K&=k_2-k_1\,. \end{aligned} \tag{2.5}$$

The t-channel amplitude is then given by

$$T=\bar{u}_c{}^\mu(p_2)i\gamma^5M_\mu u(p_1)\,, \tag{2.6}$$

where M_μ is obtained from $M_\mu{}^s$ by inserting into it the t-channel expressions given by Eqs. (2.5), and $u_c{}^\mu$ is the charge-conjugate spin-$\frac{3}{2}$ wave function. The helicity amplitudes in the t-channel barycentric frame are derived in Appendix A. Defining $T=-(8m_1m_2)^{1/2}f$ and neglecting an over-all phase factor, we have

$$\begin{aligned} f_{\frac12\frac12}=(\sqrt{3}m_2)^{-1}&[(M^2-t)^{1/2}(t-\Delta^2)]^{-1}\\ &\times[-A(t-\Delta^2)^2(t-M^2)-C(t-\Delta^2)V_1\\ &\qquad+D(4m_2\phi-V_1V_2)]\,, \end{aligned} \tag{2.7a}$$

$$\begin{aligned} f_{\frac12-\frac12}=\tfrac{2}{3}\sqrt{3}(\sqrt{\phi})&[(M^2-t)^{1/2}(t-\Delta^2)]^{-1}\\ &\times[B(t-M^2)(t-\Delta^2)+C(t-\Delta^2)\\ &\qquad+D(V_2+V_1/m_2)]\,, \end{aligned} \tag{2.7b}$$

$$\begin{aligned} f_{\frac32\frac12}=2(\sqrt{\phi})&[(M^2-t)^{1/2}(t-\Delta^2)]^{-1}\\ &\times[B(t-M^2)(t-\Delta^2)-C(t-\Delta^2)-DV_2]\,, \end{aligned} \tag{2.7c}$$

$$f_{\frac32-\frac12}=4\phi[(M^2-t)^{1/2}(t-\Delta^2)]^{-1}[D]. \tag{2.7d}$$

The normalization of the f's is such that the unpolarized differential cross section for the s-channel reaction $\Pi+N\to\Pi'+N^*$ is given by the formula

$$\frac{d\sigma}{dt}=\frac{|f_{\frac12\frac12}|^2+|f_{\frac12-\frac12}|^2+|f_{\frac32\frac12}|^2+|f_{\frac32-\frac12}|^2}{32\pi[s-(m_1+\mu_1)^2][s-(m_1-\mu_1)^2]}. \tag{2.8}$$

In Eqs. (2.7), the first and second subscripts of the f's refer to the helicities of the spin-$\frac{3}{2}$ particle and nucleon, respectively. The symbols M and Δ are the sum and

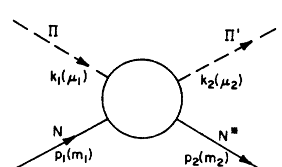

FIG. 1. Schematic representation of the s-channel scattering process $N+\Pi\to N^*+\Pi'$. The symbols in parentheses refer to the masses of the corresponding particles whose four-momenta are denoted by p and k.

difference of the baryon masses and the rest of the symbols are defined in Appendix A.

The manner in which the kinematic factors $(t-M^2)$ and $(t-\Delta^2)$ enter the helicity amplitudes is of considerable importance in the Regge-pole fits to the experimental data. In the case of $N^*(1238)$ production, $M^2=4.7$ (GeV)^2 and $\Delta^2=0.09$ (GeV)^2. The factor $(t-M^2)$ is then a slowly varying function of t, whereas the factor $(t-\Delta^2)$ is a rapidly varying function of t near the forward direction ($t\sim 0$). Consequently, if the expressions in the square brackets of Eqs. (2.7) are Reggeized with smooth residue functions,[3] a strong forward peak is predicted in the s-channel differential cross section, contrary to the experimental evidence. This point has been the subject of a detailed discussion by Jackson and Hite,[1] who emphasize that the helicity amplitudes must satisfy certain kinematical constraints at the t-channel normal $(t=M^2)$ and pseudo- $(t=\Delta^2)$ thresholds. These constraint conditions must be satisfied by any parametrization of the helicity amplitudes if spurious dependences on t, especially near the forward direction in Δ-isobar production, are not to be introduced. However, the constraint conditions (especially the derivative condition) are hard to incorporate, in practice, in phenomenological fits.

In our program of parametrizing the *invariant* amplitudes, however, all the Jackson-Hite conditions are automatically fulfilled. This is because *all* of the kinematical information for the given scattering process is contained in the factors that go into defining the covariant matrix element in terms of the invariant amplitudes. The method of parametrizing the invariant amplitudes, then, corresponds in some sense to the parametrization of dynamics and hence this method admits an unrestricted and meaningful parametrization.

As a basis of the parametrization of invariant amplitudes, we use the Regge-pole model. The general principle here will be to Reggeize a given invariant amplitude according to the helicity amplitude describing the greatest change in helicity, in which that particular invariant amplitude appears. More precisely, we treat the invariant amplitude D as a double helicity-flip amplitude ($f_{\frac{3}{2}-\frac{1}{2}}$), B and C as single helicity-flip amplitudes ($f_{\frac{1}{2}-\frac{1}{2}}$), and A as a nonflip amplitude ($f_{\frac{1}{2}\frac{1}{2}}$), for the purposes of Reggeization. Thus we introduce the following Reggeized invariant amplitudes:

$$A=A_0(t)[\tfrac{1}{2}(\alpha+1)\pm\tfrac{1}{2}(\alpha-1)] \times(\alpha+1)\xi_\pm[(s-u)/2s_0]^\alpha, \quad (2.9a)$$

$$B=B_0(t)\alpha(\alpha+1)\xi_\pm[(s-u)/2s_0]^{\alpha-1}, \quad (2.9b)$$

$$C=[C_0(t)/B_0(t)]B, \quad (2.9c)$$

$$D=D_0(t)\alpha(\alpha^2-1)\xi_\pm[(s-u)/2s_0]^{\alpha-2}, \quad (2.9d)$$

where

$$\xi_\pm=i+\begin{bmatrix}-\cot\frac{1}{2}\pi\alpha\\ +\tan\frac{1}{2}\pi\alpha\end{bmatrix}, \quad (2.10)$$

and the rest of the notation is conventional. The upper sign in the signature factor in Eq. (2.10) and in Eq. (2.9a) corresponds to an even-trajectory exchange (e.g., A_2) and the lower sign to an odd-trajectory exchange (e.g., ρ). The quantity in the square brackets in Eq. (2.9a) is the usual ghost-killing factor. We have chosen the Gell-Mann mechanism for A_2 exchange, according to which the helicity-flip amplitude is nonzero at $\alpha=0$. The functions $A_0(t)$, etc., may require an additional parametrization as will be discussed in Sec. 3.

3. ANALYSIS OF THE DATA

Differential cross-section data[4] exist on the following particular cases of the general reaction we wish to consider:

$$\pi^+p\to\pi^0\Delta^{++}(P_{\rm lab}=4,\ 8\ {\rm GeV}/c), \quad (3.1a)$$

$$\pi^-p\to\pi^-\Delta^{+}(P_{\rm lab}=8\ {\rm GeV}/c), \quad (3.1b)$$

$$K^+p\to K^0\Delta^{++}(P_{\rm lab}=3,\ 3.5,\ 5\ {\rm GeV}/c), \quad (3.1c)$$

$$\pi^+p\to\eta\Delta^{++}(P_{\rm lab}=3\text{–}4,\ 8\ {\rm GeV}/c). \quad (3.1d)$$

We shall not consider the available data on the above reactions at lower incident momenta inasmuch as we are not considering the effects due to direct-channel resonances in our model. It may be noted that since the experimental evidence for an enhancement in the K^+p system appears to be only near 1.9 GeV ($P_{\rm lab}\sim 1.2$ GeV/c), we can utilize the data at intermediate energies ($P_{\rm lab}\sim 2$–4 GeV/c) also for the reaction (3.1c). Data on the isobar-decay density matrix elements[5] are avail-

[3] L-L. C. Wang, Phys. Rev. **142**, 1187 (1965).

[4] The differential cross-section data are taken from the following sources: (a) $\pi^+p\to\pi^0\Delta^{++}$: 4-GeV/$c$ data from Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I. C.)-München Collaboration, Phys. Letters **10**, 229 (1964). The values from this reference are multiplied by 1.13 (see footnote 20 of Ref. 15); 8-GeV/c data from Aachen-Berlin-CERN Collaboration, Phys. Letters **19**, 608 (1965); (b) $\pi^-p\to\pi^-\Delta^+$: Preliminary data at 8-GeV/c from Brookhaven National Laboratory (BNL)–Carnegie-Mellon University (CMU) Collaboration, N. C. Hien, Bull. Am. Phys. Soc. **13**, 713 (1968); and R. M. Edelstein (private communication). The data as used do not include quantitative estimates of systematic errors; (c) $K^+p\to K^0\Delta^{++}$: 3-, 3.5-, and 5-GeV/c data from Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, A. Moisseev, F. Muller, J.-M. Perreau, A. Prokes, V. Yarba, W. De Baere, J. Debaisieux, P. Dufour, F. Grard, J. Heughebaert, L. Pape, P. Peeters, F. Verbeure, and R. Windmolders, Nuovo Cimento **46A**, 539 (1966); for the 5-GeV/c data we have taken those points which were based on improved statistics as given in Ref. 16; (d) $\pi^+p\to\eta\Delta^{++}$: 4-GeV/c data from D. Brown, G. Gidal, R. W. Birge, R. Bacastow, S. Y. Fung, W. Jackson, and R. Pu, Phys. Rev. Letters **19**, 664 (1967), and D. Brown, University of California Radiation Laboratory Report No. UCRL 18254, 1968 (unpublished); 8-GeV/c data from Aachen-Berlin-CERN Collaboration, Phys. Letters **19**, 608 (1965). We have taken the modified version of the data as given in Ref. 17.

[5] The data on the decay density matrix elements come from the following sources: (a) $\pi^+p\to\pi^0\Delta^{++}$: 3-4-GeV/$c$ points from G. Gidal, G. Borreani, D. Brown, F. Lott, S. Y. Fung, W. Jackson, and R. Pu, University of California Radiation Laboratory Report No. UCRL 18351, 1968 (unpublished); 4- and 8-GeV/c points from Ref. 15; (b) $K^+p\to K^0\Delta^{++}$: 3-, 3.5-, and 5-GeV/c points from Ref. 16; 5.5-GeV/c points from the work of A. Callahan and D. Gillespie, Johns Hopkins University, (private communication); (c) $\pi^+p\to\eta\Delta^{++}$: 3-4-GeV/c data from D. Brown, University of California Radiation Laboratory Report No. UCRL 18254, 1968 (unpublished).

able for the reactions (3.1a) and (3.1c), and for the reaction (3.1d) at the lower beam momentum.

There are several striking features of the experimental data which must be explained by any theoretical effort purporting to give a model of the reactions (3.1). In the reactions (3.1a) and (3.1b), the cross-section data suggest a "convexity" near $t\sim 0$ followed by a dip[6] near $t\sim -0.6$ (GeV/c)^2. On the other hand, for the reactions (3.1c) and (3.1d) the cross sections show no clear minimum near $t\sim -0.6$ (GeV/c)^2. Moreover, the latter (η production) reaction shows a somewhat flatter distribution near $t\sim 0$.

As opposed to the differential cross-section data, the density matrix elements show a remarkable similarity for the reactions (3.1a), (3.1c), and (3.1d), where they are known. On the basis of the available data the density matrix elements, independent of the reaction and energy and for $0\lesssim |t| \lesssim 0.3$ (GeV/c)^2, have the approximate values

$$\rho_{33}\simeq 0.2\text{–}0.4, \quad \mathrm{Re}\rho_{31}\simeq 0.0, \quad \mathrm{Re}\rho_{3-1}\simeq 0.2, \tag{3.2}$$

where the ρ's have the same meaning as those defined by Gottfried and Jackson.[7]

We will now review previous attempts to explain some or all of these features. One of the first models of Δ production in πN collisions was that of Stodolsky and Sakurai,[8] which was based on the exchange of an elementary ρ meson. The $\rho N\Delta$ vertex was given by assuming the validity of the $\rho-\gamma$ analogy and by assuming that the $\gamma N\Delta$ coupling was of a magnetic-dipole type, as had been indicated by experiments on the electroproduction of the Δ. This model was used to predict the density matrix elements describing the decay angular distribution of the Δ and gave the well-known results

$$\rho_{33}=\tfrac{3}{8}, \quad \mathrm{Re}\rho_{31}=0, \quad \mathrm{Re}\rho_{3-1}=\tfrac{1}{8}\sqrt{3}. \tag{3.3}$$

An extension of this model so that it might explain the differential cross sections of Δ production in πp and Kp collisions was made by Stodolsky[9] and by Jackson and Pilkuhn.[10] By retaining the essential magnetic-dipole character of the $\rho N\Delta$ vertex and introducing a form factor, these authors attempted to fit the then existing data on differential cross sections. A model based on the exchange of an elementary particle, however, cannot generally explain the observed energy dependence of the differential cross sections. For this reason, Roy[11] extended the Jackson-Pilkuhn version of the Sakurai-Stodolsky model by essentially replacing the ρ propagator by the Regge factor $(i+\tan\frac{1}{2}\pi\alpha_\rho)$ and including the energy dependence s^{α_ρ}. However, we know now that in addition to ρ exchange the important contribution of A_2 exchange should be considered in Kp collisions. Nevertheless, the successful feature of these models mentioned so far is that they all assume that the $\rho N\Delta$ coupling is of the magnetic-dipole type and consequently predict the relations (3.3) for the reaction $\pi p\rightarrow\pi\Delta$.

Attempts directed toward describing the differential cross sections in the Regge formalism, without any particular reference to elementary exchange, were made by Thews[12] and by Caprasse and Stremnitzer.[13] The latter authors included in their parametrization of helicity amplitudes the usual α factors capable of giving a dip in the differential cross sections at $t\sim -0.6$ (GeV/c)^2. These approaches to the problem may be criticized for their disregard of the kinematic singularities contained in the t-channel helicity amplitudes, which must be removed before performing the Sommerfeld-Watson transformation.[14] Recently, Krammer and Maor[15,16] have analyzed the available data by means of a detailed parametrization of kinematical singularity-free helicity amplitudes. Their fits and predictions of density matrix elements, and the prediction[17] of the results at 8 GeV/c in the reaction (3.1d), are good. Such parametrizations, however, are subject to the criticism of Jackson and Hite and therefore must be reexamined.

The two main features emerging from the models just reviewed are (a) the generality of parametrizing the four helicity amplitudes and (b) the dominance of the magnetic-dipole transition at the $\rho N\Delta$ vertex. With regard to the latter feature we wish to remark that the recent data[5] on density matrix elements (averaged in t) in the reaction $\pi^+p\rightarrow\eta\Delta^{++}$ indicate that the $A_2N\Delta$ vertex is also of the magnetic-dipole type:

$$\rho_{33}=0.46\pm0.05, \quad \mathrm{Re}\rho_{31}=0.01\pm0.05, \quad \mathrm{Re}\rho_{3-1}=0.22\pm0.07. \tag{3.4}$$

Our parametrization in what follows will be based on the Reggeization of the four invariant amplitudes with the assumption of the *dominance of the magnetic-dipole transitions at both the* $\rho N\Delta$ *and* $A_2N\Delta$ *vertices*. An additional assumption will of course be that of the validity of $SU(3)$ symmetry.

In order to isolate that invariant amplitude or combination of invariant amplitudes which controls the $M1$ transition, we must analyze the multipole structure of all the invariant amplitudes. This is done in Appendix B.

[6] The recent data of G. Gidal *et al.*, Ref. 5, on the reaction $\pi^+p\rightarrow\pi^0\Delta^{++}$ at the laboratory momenta 3–4 GeV/c clearly show the turnover near the forward direction and the dip at $t\sim -0.6$ (GeV)^2. We have, however, not used this energy-averaged data in our fits.

[7] K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964).

[8] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters **11**, 90 (1963).

[9] L. Stodolsky, Phys. Rev. **134**, B1099 (1964).

[10] J. D. Jackson and H. Pilkuhn, Nuovo Cimento **33**, 906 (1964); **34**, 1841 (1964).

[11] D. P. Roy, Nuovo Cimento **40**, 513 (1965).

[12] R. L. Thews, Phys. Rev. **155**, 1624 (1967).

[13] H. Caprasse and H. Stremnitzer, Nuovo Cimento **44A**, 1245 (1966).

[14] L. L. Wang, Phys. Rev. **153**, 1664 (1967).

[15] M. Krammer and U. Maor, Nuovo Cimento **50A**, 963 (1967).

[16] M. Krammer and U. Maor, Nuovo Cimento **52A**, 308 (1967).

[17] M. Krammer, Nuovo Cimento **52A**, 932 (1967).

In our problem each amplitude is a combination of three multipoles [longitudinal electric quadrupole ($L2$), transverse electric quadrupole ($E2$), and magnetic dipole ($M1$)]:

$$\begin{aligned}&A \text{ contributes only to } L2\,,\\ &B \text{ contributes only to } M1\,,\\ &C \text{ and } D \text{ each contribute to } L2,\ E2, \text{ and } M1.\end{aligned} \tag{3.5}$$

With this information (see also Appendix B) we can construct models described by amplitudes corresponding to any given multipolarity.

Model 1

We shall now consider a Regge-pole model which incorporates the Stodolsky-Sakurai hypothesis for the $\rho N\Delta$ vertex as well as the hypothesis of $M1$ coupling for the $A_2N\Delta$ vertex. This will be done by setting $A=C=D=0$ and using the expression (2.9b) for the amplitude B. The Regge residues corresponding to the exchange of vector (V) and tensor (T) mesons will be chosen to have the following simple forms:

$$B_{0V}(t)=C^VB_{0V}e^{b_Vt}, \tag{3.6}$$

$$B_{0T}(t)=C^TB_{0T}e^{b_Tt}. \tag{3.7}$$

The constant $C^V(C^T)$ is the product of the relevant $SU(3)$ Clebsch-Gordan coefficients for the coupling of the vector (tensor) meson to the pseudoscalar mesons and baryons. We shall choose the scale parameter s_0 in Eq. (2.9b) to be 1 (GeV)^2, and take the ρ and A_2 trajectories to be fairly well established[16,18] and given by

$$\alpha_\rho=0.56+1.0t+0.16t^2, \tag{3.8}$$

$$\alpha_{A_2}=0.41+0.93t, \tag{3.9}$$

where t is expressed in (GeV/c)^2. For the products of the $SU(3)$ Clebsch-Gordan coefficients appearing in Eqs. (3.6) and (3.7) we shall adopt the convention $C^V=1$ and $C^T=0$ for the reaction (3.1a), $C^V=0$ and $C^T=1$ for the reaction (3.1d). In a 4-parameter fit to the 69 data points[4] on the reactions (3.1a)–(3.1d) in this model we obtained a minimum χ^2 of 66.8. The values of the parameters are (we use natural units $\hbar=c=1$)

$$\begin{aligned}B_{0V}&=34.4\pm1.0\ (\mathrm{GeV})^{-3}, & b_V&=0.32\pm0.09\ (\mathrm{GeV})^{-2}\\ B_{0T}&=27.7\pm1.3\ (\mathrm{GeV})^{-3}, & b_T&=0.15\pm0.10\ (\mathrm{GeV})^{-2}.\end{aligned} \tag{3.10}$$

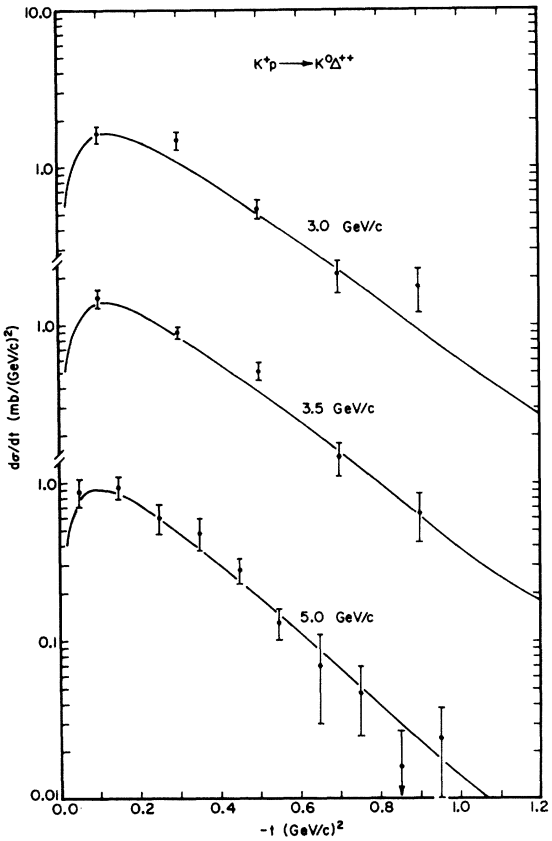

FIG. 3. Experimental data (Ref. 4) and fits of model 1 for the reaction $K^+p \to K^0\Delta^{++}$ for $P_{\mathrm{lab}}=3$, 3.5, and 5 GeV/c. The fits obtained from model 2 do not differ sufficiently from that of model 1 to be shown.

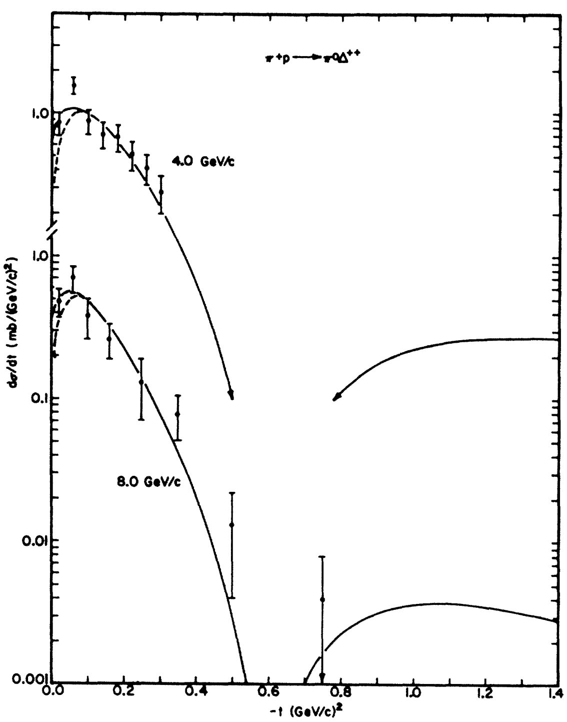

FIG. 2. Experimental data (Ref. 4) and fits of models 1 and 2 for the reaction $\pi^+p \to \pi^0\Delta^{++}$ for $P_{\mathrm{lab}}=4$ and 8 GeV/c. The solid curve is the fit obtained from model 2, while the dashed curve is the fit obtained from model 1, where it differs from model 2.

[18] D. D. Reeder and K. V. L. Sarma, Phys. Rev. **172**, 1566 (1968).

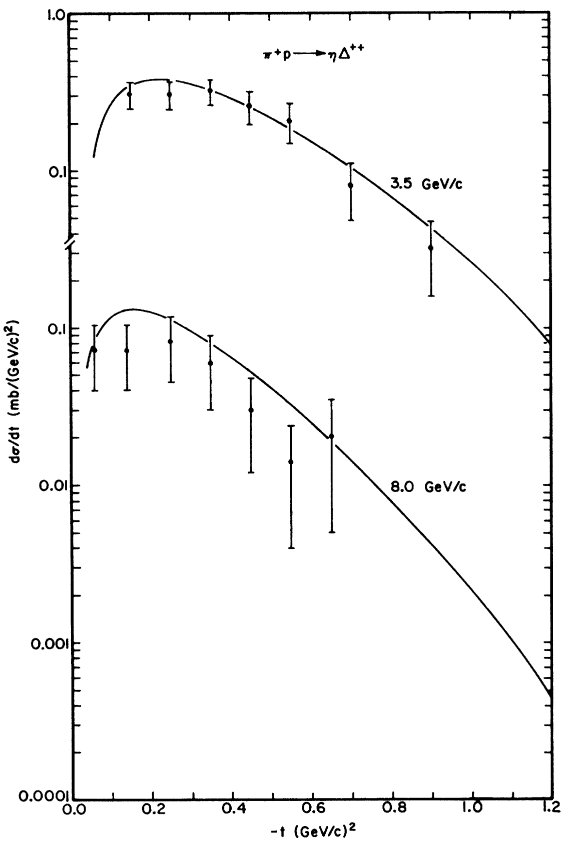

FIG. 4. Experimental data (Ref. 4) and fits of model 1 for the reaction $\pi^+p \to \eta^0\Delta^{++}$ for $P_{lab}=3.5$ and 8 GeV/c. The fits obtained from model 2 do not differ sufficiently from that of model 1 to be shown.

Using the $SU(3)$ Clebsch-Gordan coefficients for the reaction (3.1c) (according to the convention stated earlier $C^V=-1/\sqrt{2}$, $C^T=\sqrt{3}/\sqrt{2}$) we find that the relative sign between B_{0V} and B_{0T} is positive. In presenting Eqs. (3.10) we have chosen the B_{0V} to be positive.

The resulting fits to the differential cross-section data are given in Figs. 2–4 by dashed curves. The 14 data points on $\pi^-p \to \pi^-\Delta^+$ at 8 GeV/c have been used in obtaining the fits to the reactions (3.1), but are not presented here pending publication of the final data.[4] Because of $SU(2)$ invariance at the ρ vertices, however, the theoretical differential cross section of the reaction $\pi^-p \to \pi^-\Delta^+$ can be obtained from the theoretical cross section for the reaction $\pi^+p \to \pi^0\Delta^{++}$ at the corresponding laboratory momentum by multiplying the latter by the factor $\frac{2}{3}$. Predictions of this model of the decay density matrix elements evidently are those given in Eq. (3.3) which are denoted by dotted lines in Figs. 5 and 6. In general we observe that the fits are satisfactory in the framework of this "$M1$-dominance model." The dip at $t\sim -0.6$ (GeV/c)^2 in the reaction (3.1a) is a consequence of the factor $\alpha_\rho(t)$ in the amplitude. The absence of a similar dip in the reaction (3.1d) is due to the fact that we have introduced[16] only one factor of α_{A_2} in the residue corresponding to the exchange of the A_2. The small size of the parameters b_V and b_T is an indication that the functions $B_{0V}(t)$ and $B_{0T}(t)$ are almost independent of t. In fact, by slightly changing the slopes of the ρ and A_2 trajectories it is possible to make $B_{0V}(t)$ and $B_{0T}(t)$ independent of t. These changes are certainly within the accuracy with which the slopes are known.

Finally, from the fact that a simultaneous fit to the available data was possible, we make the following comments: (i) The available data[4] are consistent with each other; (ii) no free-scaling parameters are necessary to take into account the normalization errors; (iii) the data are inconsistent with the $SU(3)$ symmetric couplings as applied to the factorized Regge residues. It should be emphasized, however, that $SU(3)$ symmetry is invoked only at the meson vertex because, as in the case of the corresponding reactions involving a spin-$\frac{1}{2}$ final baryon (see last column of Table I), only the ratios of the couplings $g_{\rho\pi\pi}/g_{\rho KK}$ and $g_{A_2\pi\eta}/g_{A_2KK}$ are assumed from $SU(3)$ symmetry. Total cross sections [integrated over t from $t\sim 0$ up to -1.5 (GeV/c)^2] calculated on the basis of $SU(3)$ symmetry and the parameters given in Eqs. (3.8)–(3.10), are displayed in Fig. 7 as a function of the laboratory momentum of the incident meson for the reactions $\pi^-p \to \pi^-\Delta^+$, $K^+p \to K^0\Delta^{++}$, and $\pi^+p \to \eta\Delta^{++}$. A verification of the predictions in Fig. 7 would afford a test of the Regge asymptotic behavior of the scattering amplitude in the Δ-production reactions.

Model 2

From the present data on the isobar-production reactions it is difficult to conclude that the differential cross sections vanish in the forward direction. According to model 1, however, we expect to see a vanishing cross section in the forward direction for all reactions (3.1) and also at $t\sim -0.6$ (GeV/c)^2 for reactions of the type $\pi p \to \pi\Delta$. These features of the preceding model together with the predictions on the density matrix elements are certainly consistent with the present data. Nevertheless, it would be worthwhile investigating the effect of introducing one more amplitude in our analysis in addition to the pure $M1$ amplitude B. Such an analysis would be instructive at least in the following two respects: to see the stability of the solution already obtained in model 1 and to make a crude estimate of the differential cross section in the forward direction. However, the data containing measurements relatively close to the forward direction are at present available only on the first two reactions in (3.1). For this reason we shall introduce the extra amplitude only in connection with the vector-exchange amplitudes.

We choose the extra amplitude to be D and motivate this choice by the following observations: Only D enters all four t-channel helicity amplitudes and contributes to all the multipoles $E2$, $L2$, and $M1$ (see

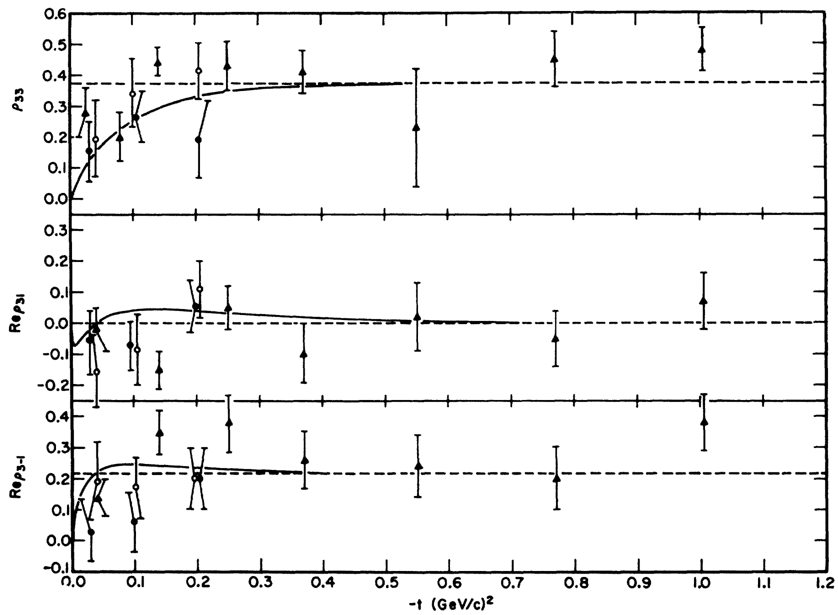

FIG. 5. Experimental data (Ref. 5) at 3–4 GeV/c (▲), 4 GeV/c (○), and 8 GeV/c (●) and predictions of models 1 and 2 for the density matrix elements in the reaction $\pi^+p \to \pi^0\Delta^{++}$. The dotted lines are Stodolsky-Sakurai values and the predictions of model 1. The solid curves are the nearly energy-independent predictions of model 2.

Appendix B). Further, the singularity in the helicity amplitudes at the pseudothreshold is present only through the amplitude D.[19] Our model 2 then consists of using Eqs. (3.6) and (3.7) for $B_{0V}(t)$ and $B_{0T}(t)$, and assuming

$$D_{0V}(t)=C^V D_{0V} e^{d_V t}, \tag{3.11}$$

where $D_{0V}(t)$ is defined in Eq. (2.9d). The rest of the amplitudes are set equal to zero;

$$A_{0V}=A_{0T}=C_{0V}=C_{0T}=D_{0T}=0. \tag{3.12}$$

The trajectories and the scale parameters used are exactly those of model 1. Our best 6-parameter fit had a $\chi^2=56$ (for 69 data points) with the following values for the parameters:

$$\begin{aligned} B_{0V}&=33.0\pm1.3\ (\mathrm{GeV})^{-3}, & b_V&=0.25\pm0.08\ (\mathrm{GeV})^{-2},\\ B_{0T}&=27.8\pm1.3\ (\mathrm{GeV})^{-3}, & b_T&=0.16\pm0.10\ (\mathrm{GeV})^{-2},\\ D_{0V}&=-28\pm5\ (\mathrm{GeV})^{-2}, & d_V&=5.6\pm4.2\ (\mathrm{GeV})^{-2}. \end{aligned} \tag{3.13}$$

The corresponding fits to the data are also indicated in Figs. 2–4. Predictions of the density matrix elements are given in Figs. 5 and 6. As expected, the parameters D_{0V} and d_V are poorly determined due to the paucity of data near the forward direction. Comparing the values (3.13) with the corresponding values of model 1 it is gratifying to find that the same values for the parameters B_{0V} and b_V occur in both the models. The fits to the reactions (3.1a) and (3.1b) according to model 2 are much better than the corresponding ones in model 1; the χ^2 in model 2 being 25 as compared to 36 in model 1 for 34 data points.

Other models[20] similar to model 2 may easily be constructed by retaining the dominant amplitude B (for both vector and tensor exchanges) and introducing any one or a combination of A, C, and D. The construction of such models, however, awaits better data[21] on the isobar-production reactions.

4. DISCUSSION

We have presented a model which successfully describes the available data on the reactions $\pi p \to \pi\Delta$, $Kp \to K\Delta$, and $\pi p \to \eta\Delta$. The two dynamical principles on which the model is founded are: the Stodolsky-Sakurai hypothesis that the $\rho N\Delta$ coupling is dominantly of the magnetic-dipole type and the hypothesis that the $A_2N\Delta$ coupling is also dominantly of the magnetic-dipole type.[22] Both these hypotheses are indeed motivated chiefly by the present data on the density matrix elements. Other assumptions of the model are the dominance of a peripheral Regge exchange and the

[19] Note that the "dynamical exception" in the terminology of Jackson and Hite corresponds to the situation in which $D=0$ independent of the other three amplitudes.

[20] The data near the forward direction may easily be fitted by considering the amplitude A which contributes only to the helicity nonflip amplitude. Because of the presence of the factor $(t-\Delta^2) \simeq -\Delta^2(1-11t)$ before A in Eq. (2.7a), it is expected that the residue associated with A should have a large exponential parameter $a_V \simeq 10$–15 (GeV/c)^2.

[21] After this work was completed, we became aware of the experimental results of M. Aderholz *et al.*, Aachen-Berlin-CERN-London (I. C.)-Vienna Collaboration [Nucl. Phys. **B5**, 606 (1968)] on $K^-p \to K^-\Delta^+$ at 10.1-GeV/c incident momentum. Our predicted values (based on Model 2) of the cross section are in excellent agreement with these data except for the first two points near the forward direction where the theoretical points lie low by about a factor 2 to 3. This discrepancy near the forward direction could be, however, in large measure due to the uncertainties involved in identifying the background events. We thank Dr. M. Deutschmann for mentioning this point to us.

[22] Arguments based on the static model suggest that both the vector and tensor couplings to the baryons are dominantly of the $M1$ type; see R. Dashen and S. Frautschi, Phys. Rev. **152**, 1450 (1966).

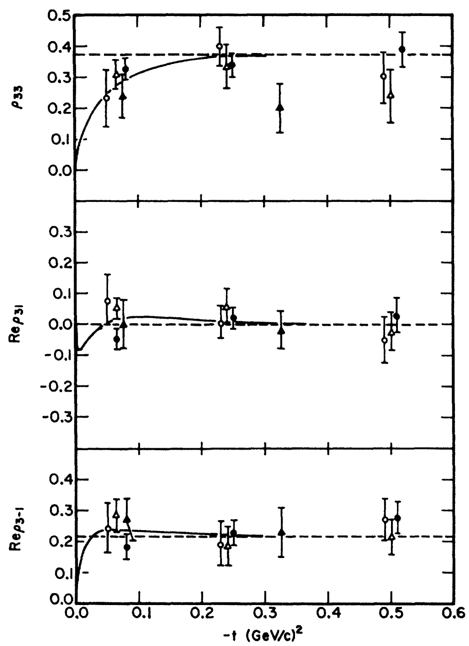

FIG. 6. Experimental data (Ref. 5) at 3 GeV/c (○), 3.5 GeV/c (△), 5 GeV/c (●), and 5.5 GeV/c (▲) and predictions of models 1 and 2 for the density matrix elements in the reaction $K^+p \to K^0\Delta^{++}$. The dashed lines are the predictions of model 1. The solid curves are the nearly energy-independent predictions of model 2

validity of $SU(3)$ symmetry for the factorized Regge residues. Our procedure of Reggeization is consistent with all the Jackson-Hite constraints (including the

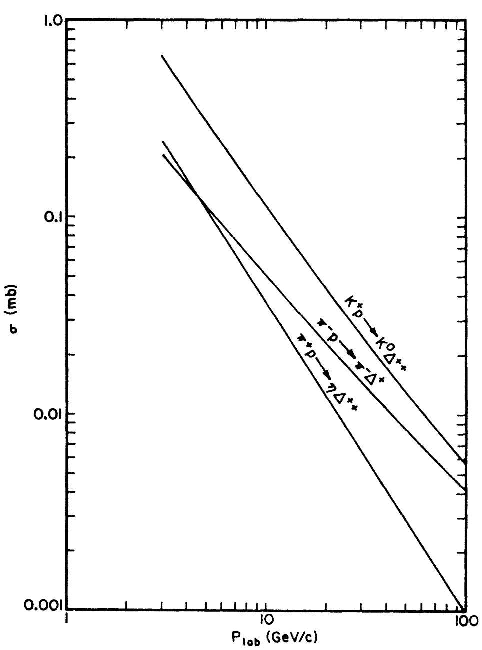

FIG. 7. Predictions of model 1 of the energy dependence of the integrated differential cross sections for the reactions $\pi^-p \to \pi^-\Delta^+$, $K^+p \to K^0\Delta^{++}$, and $\pi^+p \to \eta\Delta^{++}$. The predictions of model 2 differ from those of model 1 only by very small amounts and are therefore not shown.

differential-constraint conditions) at the normal and pseudothresholds, since we have chosen to Reggeize the invariant amplitudes. The assumption of $M1$ dominance in both the vector and tensor exchanges is implemented conveniently by retaining the invariant amplitude which is known to contribute only to a magnetic-dipole transition at the baryon vertex (see Ref. 9 and Appendix B).

The above hypothesis on the similarity of the $\rho N\Delta$ and $A_2N\Delta$ couplings may in fact be related to the backward-scattering experiments on the reactions $\pi^-p \to p\rho^-$ and $\pi^-p \to pA_2^-$. Assuming the exchange of Δ^{++} trajectory to be the dominant mechanism and neglecting the possible off-mass-shell effects [involved in going from the Δ-production reactions to the reactions $\pi^-p \to p\rho^-(A_2^-)$], we can expect the shapes of $d\sigma/du$ for the ρ^- and A_2^- production to be similar. It is very interesting that the results of Anderson *et al.*[23] at 16 GeV/c are in qualitative agreement with these expectations.

Obviously we have not utilized the full generality of the parametrization in terms of the four invariant amplitudes. Such a general analysis, however, can meaningfully be attempted only when we have more extensive and better data than are presently available. An attempt to analyze the data on the production of $Y^*(1385)$ in terms of K^* and K^{**} exchanges can be undertaken when more data become available. It would be interesting to examine the possible corrections to the conventional Regge amplitude arising from diffraction scattering in the initial and final states, as is recently being done for some of the prototype reactions[24] of Table I. Finally, on the basis of our investigation we feel that the Reggeization of the invariant amplitudes provides a simple and probably meaningful parametrization and therefore such an approach could be used to advantage in the analysis of the other reactions as well.

ACKNOWLEDGMENTS

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[23] E. W. Anderson, E. J. Bleser, H. R. Blieden, G. B. Collins, D. Garelick, J. Menes, F. Turkot, D. Birnbaum, R. M. Edelstein, N. C. Hien, T. J. McMahon, J. Mucci, and J. Russ, in Contribution to the Fourteenth International Conference on High Energy Physics, Vienna, 1968 (unpublished).

[24] See, e.g., F. Henyey, G. L. Kane, Jon Pumplin, and Marc Ross, Phys. Rev. Letters 21, 946 (1968), and related references therein.

APPENDIX A: DERIVATION OF t-CHANNEL HELICITY AMPLITUDES

We use the real metric for which $p^2=m^2$. The γ matrices are defined so that γ^0 is Hermitian and γ^i, $i=1,2,3$, are anti-Hermitian. The matrix $\gamma^5=+i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\epsilon^{0123}=+1$.

In the t-channel c.m. frame

$$p_1=(E_1,\,-p\hat{z}),\quad p_2=(E_2,\,+p\hat{z}),$$
$$k_1=(w_1,\,k\cos\theta\,\hat{z}+k\sin\theta\,\hat{x}),$$

and

$$k_2=(w_2,\,-k\cos\theta\,\hat{z}-k\sin\theta\,\hat{x}).$$

Here p_2 refers to the 4-momentum of the anti-isobar whose 3-momentum is chosen to be along the positive z direction (see Fig. 8) and θ is the t-channel scattering angle. The charge-conjugate spinor for the isobar will be denoted by u_c^{μ}, which is defined as

$$u_c^{\mu}=i\gamma^2u^{\mu *}. \tag{A1}$$

Since the momentum of the proton is along the negative z direction, in order to compute the helicity amplitudes we make a 180° rotation about the y axis

$$u_{\lambda_2}(-p\hat{z})=(\exp\tfrac{1}{2}i\sigma_2\pi)u_{\lambda_2}(p\hat{z}), \tag{A2}$$

where

$$u_{\lambda_2}(p\hat{z})=N_1\begin{pmatrix}\xi_{\lambda_2}\\ \sigma_3\rho_1\xi_{\lambda_2}\end{pmatrix}, \tag{A3}$$

$$\rho_1=p/(E_1+m_1), \tag{A4}$$

$$N_1=[(E_1+m_1)/2m_1]^{1/2}. \tag{A5}$$

Our procedure is to first compute the helicity amplitudes in the *rest frame* of the $\bar{N}^*$ and then to make a Lorentz transformation to get the corresponding amplitudes in the t-channel c.m. system.

The Rarita-Schwinger wave function in the rest frame of the spin-$\frac{3}{2}$ particle is

$$u_{\lambda}{}^0=0,\quad \mathbf{u}_{\lambda}=\begin{pmatrix}\boldsymbol{\xi}_{\lambda}\\ 0\end{pmatrix},$$

where $\boldsymbol{\xi}_{\lambda}$ is a combination of the Pauli spinors ξ's, and unit vectors e's, in the 3-dimensional spherical basis, describing the helicity state λ of the isobar

$$\boldsymbol{\xi}_{3/2}=\xi_{1/2}e_{+1},$$
$$\boldsymbol{\xi}_{1/2}=(\xi_{-1/2}e_{+1}+\sqrt{2}\xi_{1/2}e_0)/\sqrt{3}.$$

The remaining two helicity states (for $\lambda=-\frac{1}{2},-\frac{3}{2}$) are not needed for our purpose. The vectors e's are defined as

$$e_0=\hat{z},\quad e_{+1}=-(\hat{x}+i\hat{y})/\sqrt{2}.$$

Attaching a bar to distinguish the relevant quantities in the rest frame of $\bar{N}^*$, the helicity amplitudes[25] are

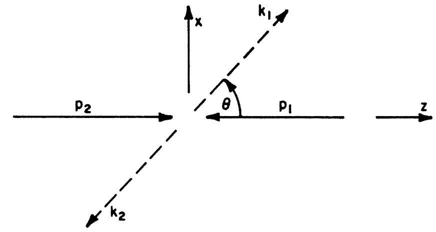

FIG. 8. Conventions for the t-channel scattering corresponding to the reaction $\bar{N}^*(p_2)+N(p_1)\to\bar{\Pi}(k_1)+\Pi'(k_2)$.

given by

$$\bar{T}_{\lambda_1\lambda_2}\equiv\bar{T}_{00,\lambda_1\lambda_2}$$
$$=-(-)^{s_2-\lambda_2}\bar{N}_1(0\;\;\xi_{\lambda_1}{}^{i\dagger}\sigma_2)i\gamma^5\bar{M}_i\begin{pmatrix}\sigma_2\xi_{\lambda_2}\\ +i\sigma_1\bar{\rho}_1\xi_{\lambda_2}\end{pmatrix}, \tag{A6}$$

where

$$\bar{M}_i=A\bar{Q}_i+B(-i\gamma^5)\epsilon_{i\nu\lambda\rho}\bar{P}^{\nu}\bar{Q}^{\lambda}\bar{K}^{\rho}+C\bar{K}_i+D\gamma\cdot\bar{K}\bar{K}_i, \tag{A7}$$
$$\bar{K}=\bar{k}_2-\bar{k}_1,$$
$$\bar{Q}=(-m_2-\bar{E}_1,\,\bar{Q}\hat{z}),$$
$$\bar{P}=(\bar{E}_1-m_2,\,-\bar{Q}\hat{z}).$$

A calculation of (A6) for the various helicity states yields

$$\bar{T}_{\frac{1}{2}\frac{1}{2}}=-i\sqrt{\tfrac{2}{3}}\bar{N}_1\{A\bar{Q}+C\bar{K}_z+\tfrac{1}{2}D\times[-\bar{K}_x{}^2\bar{\rho}_1+2\bar{K}_z(\bar{K}_0+\bar{K}_z\bar{\rho}_1)]\}, \tag{A8}$$

$$\bar{T}_{\frac{1}{2}-\frac{1}{2}}=-i(\sqrt{\tfrac{1}{6}})\bar{N}_1\bar{K}_x[+B(2\bar{Q}m_2\bar{\rho}_1)+C+D(3\bar{K}_z\bar{\rho}_1+\bar{K}_0)], \tag{A9}$$

$$\bar{T}_{\frac{3}{2}\frac{1}{2}}=-i(\sqrt{\tfrac{1}{2}})\bar{N}_1\bar{K}_x[+B(2\bar{Q}m_2\rho_1)-C-D(\bar{K}_z\rho_1+\bar{K}_0)], \tag{A10}$$

$$\bar{T}_{\frac{3}{2}-\frac{1}{2}}=-i(\sqrt{\tfrac{1}{2}})\bar{N}_1\bar{K}_x{}^2[-D\bar{\rho}_1]. \tag{A11}$$

To obtain the corresponding amplitudes in the t-channel c.m. frame, we perform a Lorentz boost along the positive z direction so that if $\bar{q}$ is any 4-vector in the isobar rest frame it is related to the corresponding 4-vector q in the c.m. frame by

$$\bar{q}_0=\gamma(q_0-\beta q_z),\quad \bar{q}_z=\gamma(q_z-\beta q_0),$$
$$\bar{q}_x=q_x,\quad \bar{q}_y=q_y, \tag{A12}$$

where $\gamma=E_2/m_2$, $\beta=p/E_2$. Using the abbreviations

$$M=m_1+m_2,\quad \Delta=m_2-m_1,$$
$$\mu=\mu_1+\mu_2,\quad \delta=u_2-\mu_1, \tag{A13}$$

and (A12) we have the following relations:

$$\bar{Q}=p(\sqrt{t})/m_2,\quad \bar{N}_1=[(t-\Delta^2)/4m_1m_2]^{1/2},$$
$$\bar{K}_x=-2k\sin\theta,\quad \bar{K}_z=\gamma(-2k\cos\theta-\beta\mu\delta/\sqrt{t}),$$
$$\bar{K}_0=\gamma(\mu\delta/\sqrt{t}+2\beta k\cos\theta),\quad 4kp\cos\theta=u-s-M\Delta\mu\delta/t,$$
$$4kp\sin\theta=(4\phi/t)^{1/2},$$
$$\phi=st(m_1^2+m_2^2+\mu_1^2+\mu_2^2-s-t)-t(\mu_1^2-m_1^2)(\mu_2^2-m_2^2)-sM\Delta\mu\delta-(\mu_1^2m_2^2-\mu_2^2m_1^2)(M\Delta-\mu\delta). \tag{A14}$$

[25] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).

Substituting the above relations in the amplitudes (A8)–(A11) we obtain the following helicity amplitudes in the t-channel c.m. frame:

$$
\begin{aligned}
T_{\frac{1}{2}\frac{1}{2}} &= g(\sqrt{\tfrac{1}{3}})m_2^{-1}[-A(t-\Delta^2)^2(t-M^2) \\
&\qquad -C(t-\Delta^2)V_1+D(4m_2\phi-V_1V_2)], \\
T_{\frac{1}{2}-\frac{1}{2}} &= g(\sqrt{\tfrac{4}{3}})(\sqrt{\phi})[B(t-M^2)(t-\Delta^2)+C(t-\Delta^2) \\
&\qquad +D(V_2+V_1/m_2)], \qquad \text{(A15)} \\
T_{\frac{3}{2}\frac{1}{2}} &= g\times 2(\sqrt{\phi})[B(t-M^2)(t-\Delta^2)-C(t-\Delta^2)-DV_2], \\
T_{\frac{3}{2}-\frac{1}{2}} &= g\times 4\phi[D],
\end{aligned}
$$

where

$$
\begin{aligned}
g &= i/\{[8m_1m_2(t-M^2)]^{1/2}(t-\Delta^2)\}, \\
V_1 &= (t+M\Delta)(s-u)+\mu\delta(\Delta^2+M\Delta-t+M^2), \\
V_2 &= \Delta(s-u)+\mu\delta M.
\end{aligned}
$$

APPENDIX B: MULTIPOLE NATURE OF THE INVARIANT AMPLITUDES

We shall determine the multipole nature of the invariant amplitudes using a method based on that developed by Durand, DeCelles, and Marr[26] (hereinafter referred to as DDM) in their investigation of vertex functions. The frame of reference best suited to this analysis is the s-channel brick-wall frame of the baryons. In this frame, we choose the spatial momentum of the final baryon (the isobar) to lie along the positive z direction and the scattering plane to be the xz plane (see Fig. 9).

The matrix element of the scattering amplitude (in the brick-wall frame) taken between helicity states is

$$
\begin{aligned}
T_{\lambda_2\lambda_1}{}^{b-w} &\equiv T_{\lambda_2 0,\lambda_1 0}{}^{b-w} \\
&= \langle p0s_2\lambda_2;\mathbf{k}_2|T|R(p0s_1\lambda_1);\mathbf{k}_1\rangle, \qquad \text{(B1)}
\end{aligned}
$$

where $R(p0s_1\lambda_1)$ denotes the rotated helicity state

$$e^{i\pi J_2}|p0s_1\lambda_1\rangle.$$

Such a matrix element can now be decomposed into amplitudes having a definite spin exchanged between the baryons and the mesons. In a notation similar to that of DDM, we write

$$T_{\lambda_2\lambda_1}{}^{b-w} = \sum_{J,M}\begin{pmatrix} s_2 & J & s_1 \\ -\lambda_2 & M & -\lambda_1\end{pmatrix} T_M{}^{(J)}. \qquad \text{(B2)}$$

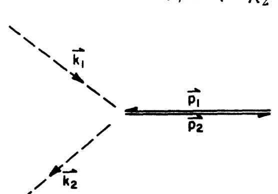

FIG. 9. The s-channel brick-wall frame.

[26] L. Durand, III, P. C. De Celles, and R. B. Marr, Phys. Rev. **126**, 1882 (1962).

In this particular frame, the angular-momentum dependence due to the baryons is taken care of by the $3-j$ symbol in (B2). The quantity $T_M{}^{(J)}$ contains in it the angular-momentum dependence induced by the meson vertex. Just as an analysis of the vertex functions made in this way yields information about the multipole structure of the vertices, so a similar analysis of the scattering amplitudes will yield such information in our problem. Because the baryons have positive parity, the parity of the exchanged state will be positive. In the language of electromagnetism, then, we are dealing with a magnetic-dipole $M1$ ($J^P=1^+$) and an electric-quadrupole $E2$ ($J^P=2^+$) transition between the nucleon and the isobar. Thus we can write

$$
\begin{aligned}
T_{\frac{3}{2}\frac{1}{2}}{}^{b-w} &= (1/\sqrt{5})T_2{}^{(2)}, \\
T_{\frac{3}{2}-\frac{1}{2}} &= -1/(2\sqrt{5})T_1{}^{(2)}+\tfrac{1}{2}T_1{}^{(1)}, \qquad \text{(B3)} \\
T_{\frac{1}{2}\frac{1}{2}}{}^{b-w} &= -\sqrt{3}/(2\sqrt{5})T_1{}^{(2)}-1/(2\sqrt{3})T_1{}^{(1)}, \\
T_{\frac{1}{2}-\frac{1}{2}}{}^{b-w} &= [1/\sqrt{(10)}]T_0{}^{(2)}.
\end{aligned}
$$

The amplitude $T_0{}^{(1)}$ vanishes identically by parity conservation. Only the single-helicity-flip amplitudes contain both the $M1$ and $E2$ transitions. The latter can be separated to yield

$$
\begin{aligned}
T_1{}^{(1)} &= (\tfrac{1}{2}\sqrt{3})[\sqrt{3}T_{\frac{3}{2}-\frac{1}{2}}{}^{b-w}-T_{\frac{1}{2}\frac{1}{2}}{}^{b-w}], \\
T_1{}^{(2)} &= -(\tfrac{1}{2}\sqrt{5})[T_{\frac{3}{2}-\frac{1}{2}}{}^{b-w}+\sqrt{3}T_{\frac{1}{2}\frac{1}{2}}{}^{b-w}]. \qquad \text{(B4)}
\end{aligned}
$$

In a manner similar to that of Appendix A, we can express the brick-wall helicity amplitudes in terms of the invariant amplitudes. In the case of equal meson masses we find

$$
\begin{aligned}
T_{\frac{1}{2}-\frac{1}{2}}{}^{b-w} &= (g'/\sqrt{3}m_2)[A(\Delta^2-t)^2(M^2-t)+C(\Delta^2-t)V_1 \\
&\qquad +D(4m_2\phi-V_1V_2)], \\
T_{\frac{1}{2}\frac{1}{2}}{}^{b-w} &= [2g'(\sqrt{\phi})/\sqrt{3}][B(\Delta^2-t)(M^2-t) \\
&\qquad +C(\Delta^2-t)-D(V_2+V_1/m_2)], \qquad \text{(B5)} \\
T_{\frac{3}{2}-\frac{1}{2}}{}^{b-w} &= +2g'(\sqrt{\phi})[-B(\Delta^2-t)(M^2-t) \\
&\qquad +C(\Delta^2-t)-DV_2], \\
T_{\frac{3}{2}\frac{1}{2}}{}^{b-w} &= -4g'\phi[D],
\end{aligned}
$$

where

$$
\begin{aligned}
g' &= i/\{[8m_1m_2(M^2-t)]^{1/2}(\Delta^2-t)\}, \\
V_1 &= (t+M\Delta)(s-u), \\
V_2 &= \Delta(s-u).
\end{aligned}
$$

Inserting the relevant amplitudes of (B5) into (B4) we have

$$
\begin{aligned}
T_1{}^{(1)} &= -2g'(\Delta^2-t)(\sqrt{\phi})\{2(M^2-t)B-C \\
&\qquad +[(s-u)/2m_2]D\}, \\
T_1{}^{(2)} &= -2(\sqrt{5})g'(\sqrt{\phi})\{C(\Delta^2-t) \\
&\qquad -[V_2+V_1/(2m_2)]D\}. \qquad \text{(B6)}
\end{aligned}
$$