In most applications of the current algebra, σ terms have been neglected. We found, for $K \to 3\pi$, that it was necessary to include such terms to obtain conformity with the low-energy constraints and reasonable agreement with experiment.

A model based on η-pole dominance was introduced to explain deviations from the $\Delta I = \frac{1}{2}$ rule in both $K \to 2\pi$ and $K \to 3\pi$ decays, and gave good agreement with experiment.

In studying $\eta \to 3\pi$ decays, we found that meson-pole models gave a good account of the energy spectrum, provided that the Weinberg amplitude was used for the low-energy $\pi\pi$ scattering, whereas the vector-exchange model gave results significantly different from experiment. Thus the $\eta \to 3\pi$ analysis lends weight to what our study of K decays suggests, namely, that the Weinberg scattering amplitude is preferable to the amplitude based on ρ exchange.

The most remarkable feature that our work demonstrates is the consistency of current algebra and phenomenological Lagrangian methods in describing strong, weak, and electromagnetic interactions, with strong-interaction vertices taken from phenomenological Lagrangians enforcing the low-energy theorems for weak decays, and giving good agreement with experiment.

ACKNOWLEDGMENT

I am grateful to Professor J. J. Sakurai for his patient guidance and helpful discussions, criticisms, and suggestions.

PHYSICAL REVIEW VOLUME 178, NUMBER 5 25 FEBRUARY 1969

Baryon Supermultiplets of $SU(6) \times O(3)$ in a Quark-Diquark Model*

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(Received 15 October 1968)

A model in which a baryon is a bound state of a quark and diquark is used to obtain a classification of baryon resonances according to a broken $SU(6) \times O(3)$ scheme. If the interaction between quark and diquark has an appreciable space-exchange contribution, a level ordering is obtained which is consistent with the present experimental information. The results are compared to those obtained with other models.

1. INTRODUCTION

It has been suggested that a quark-diquark model be used in calculating strong[1,2] and electromagnetic[3,4] properties of baryons. In this model, the diquark, although formed as a bound state of two quarks, is regarded as essentially elementary in its interaction with a quark to form a baryon. The practical advantage of using this model is that if a baryon is composed of two particles, calculations are simpler than if the baryon is composed of three particles, as in the usual quark model. A disadvantage is that some of the symmetry and conceptual simplicity of the quark model is lost.

In two previous papers,[2,4] properties of baryons have been calculated assuming that the relevant symmetry of the problem is that of the group $SU(6)$. The quark was assumed as usual to belong to a six-dimensional representation of $SU(6)$, while the diquark was assumed to belong to a 21-dimensional representation of the group. This implies that the diquark consists of an $SU(3)$ sextet of spin one and an $SU(3)$ triplet (belonging to the $\bar{3}$ representation) of spin zero. The groups $SU(6)$ and $SU(3)$ were broken in the model by mass splittings within the quark and diquark multiplets and by symmetry-breaking interactions.

Mesons have been treated only perfunctorily[1–4] because it is simpler to treat them as a quark-antiquark pair rather than as a diquark-antidiquark pair. If a meson multiplet belonging to a 27-dimensional representation of $SU(3)$ is found, such a multiplet can be interpreted as a bound state of a diquark and antidiquark. In this paper we shall not consider mesons.

The results of the previous papers on the quark-diquark model incorporating broken $SU(6)$ were confined to consideration of the ground-state 56-dimensional multiplet consisting of the baryon octet and decuplet of $SU(3)$. In this paper we consider the baryon excited states classified according to the group $SU(6) \times O(3)$. We compare the qualitative predictions of the model with those of a quark shell model with harmonic oscillator potential. This model was introduced by

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Greenberg[5] and amplified and summarized by Dalitz.[6] We also compare with the work of Mitra[7] who uses a quark model with separable s-wave potentials.

A number of properties of the diquark have been stated in Refs. 2 and 4. In particular, values of the quantum numbers and mass-splitting parameters have been given. Here, we shall define only those properties which are most necessary for present applications, and refer the reader to Refs. 2 and 4 for further details. Briefly, the additive quantum numbers of the diquark are those obtained from a two-quark system, but dynamical properties, such as mass splittings, are regarded as free parameters not to be calculated from the corresponding properties of the quark.

In the usual quark model, best agreement with experiment has been obtained under the assumption that quarks obey effective Bose statistics.[5–7] This symmetry could follow from the assumption that quarks are parafermions or order three as suggested by Greenberg,[5] or have internal degrees of freedom as suggested by a number of authors.

However, the assumptions of the existence of paraquarks or quarks with other degrees of freedom are not by themselves sufficient to require that three-quark systems have symmetric wave functions, since wave functions of mixed symmetry and antisymmetric wave functions can also occur within these schemes. Dalitz[6] simply postulates that three-quark systems which are antisymmetric or have mixed symmetry lie much higher in energy (e.g.$\sim$10 GeV) than the symmetric states.

In the quark-diquark model, on the other hand, since the diquark is treated as an elementary particle quite distinct from the quark, it is most natural to assume that symmetry considerations play no role. If the quark-diquark wave functions have to be symmetrized under the interchange of a quark and either of the quark constituents of the diquark, then the calculational simplicity of the quark-diquark model is lost. In other words, either the diquark must be considered as elementary or any interference terms arising from any symmetrization of the wave functions must be negligible.

If a diquark is to be considered as elementary for the purposes of calculation, then any excited levels in it must lie quite high in energy. This means that the low-lying baryon excited states will arise from radial and orbital excitations of the two-particle quark-diquark system. On the other hand, in the usual three-quark model, there may be excitations between the first and second quarks, in addition to excitations between the third quark and the c.m. of the first and second. Therefore, in general many more low-lying levels occur in the three-quark model than in the quark-diquark model. This is in contrast to the results for the lowest 56-dimensional multiplet, where we found[2,4] that the results for the quark-diquark and three-quark models were very similar.

The differences in the excited states are particularly striking between the quark-diquark model and the three-quark model with harmonic-oscillator potential discussed by Greenberg[5] and Dalitz.[6] On the other hand, the model of Mitra[7] gives somewhat similar predictions to those of the quark-diquark model. This is because in Mitra's model, the potential between two quarks is effective only in s states, and thus, many of the excitations expected in the usual quark model are eliminated. It is not too surprising that the forces in a quark model can be chosen in such a way that the results are similar to those for a quark-diquark model. In fact, if this were not so, the quark-diquark model would not make sense as a dynamical approximation to a three-quark model.

Although, as we have remarked, our results differ quite strikingly from those of Dalitz,[6] the experimental evidence is still insufficient to distinguish between the two models. This is because at present only a few baryon supermultiplets of $SU(6)\times O(3)$ are reasonably well established.

2. ORDERING OF THE SUPERMULTIPLET ENERGY LEVELS

We assume that the largest term of the quark-diquark interaction is invariant under the group $SU(6)\times O(3)$. This means that there will exist baryonic supermultiplets which can be classified according to this group. We denote one of these supermultiplets by $(\mathbf{N},L\pi)$, where $\mathbf{N}$ is the multiplicity of the $SU(6)$ representation, L is the orbital angular momentum of the quark-diquark system, and π is the parity.

This same classification has been given by Greenberg[5] in the three-quark model, using a shell model with harmonic-oscillator potential, where L and π represent the total orbital angular momentum and parity of the three-quark system. Subsequently, Karl and Obryk[8] have classified the allowed harmonic-oscillator shell-model states. We use this latter classification for comparison with the predictions of the quark-diquark model.

Because the quark belongs to the six-dimensional representation of $SU(6)$ and the diquark to a **21**, we have the $SU(6)$ numerology

$$\mathbf{6}\times\mathbf{21}=\mathbf{56}+\mathbf{70}. \qquad (1)$$

Therefore, we immediately have the following rule:

Rule I. All low-lying baryon multiplets belong either to a 56- or 70-dimensional representation of $SU(6)$.

In order to obtain a baryon belonging to a **20** of $SU(6)$, a diquark would have to be excited from a **21**

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to a **15**. If this were easy to do, then treating the diquark as elementary would not be a good approximation.

Another feature of the quark-diquark model concerns the relation between L and π. If we define the relative parity of the quark and diquark as $+$, we immediately have the following prediction:

Rule II. The orbital angular momentum and parity of all low-lying levels are related by

$$\pi=(-1)^L. \qquad (2)$$

According to Dalitz, the supermultiplets which are best established experimentally are

$$(\mathbf{56}, 0+), (\mathbf{70}, 1-), (\mathbf{56}, 0+), (\mathbf{56}, 2+). \qquad (3)$$

It is seen that these representations agree with rules I and II of the quark-diquark model.

In the quark harmonic-oscillator model classification, the observed $(\mathbf{70}, 1-)$ level, (one quantum excitation) lies lower in energy than the unobserved $(\mathbf{70}, 0+)$ which corresponds to two excitations. The reason that the $(\mathbf{70}, 0+)$ does not lie below the $(\mathbf{70}, 1-)$ is the requirement that the quarks obey effective Bose statistics. Since in the quark-diquark model we do not have this symmetry requirement, we must obtain a similar result by a property of the quark-diquark interaction. The simplest interaction which gives the desired result is a space-exchange potential which is attractive in even-parity states for the **56** and attractive for odd-parity states in the **70**. (This space-exchange interaction could result from the exchange of a quark between the quark and diquark). We therefore write the quark-diquark interaction V as

$$V=U+U'P_{qd}(P_{56}-P_{70})+v, \qquad (4)$$

where U is a term which acts equally in the **56** and **70** representations, U' is a term which acts differently in the two representations, and v contains all terms which break $SU(6)\times O(3)$. Here P_{qd} is a quark-diquark space-exchange operator and P_{56} and P_{70} are $SU(6)$ projection operators for the **56** and **70**, respectively. We assume that U, and U', and v satisfy the inequalities

$$|U|>|U'|>|v|.$$

We shall now neglect the symmetry-breaking term v and concentrate on the $SU(6)\times O(3)$ invariant terms U and U'. It can be seen that if $U'<0$, not only does $U'P_{qd}(P_{56}-P_{70})$ lower the energies of the super multiplets $(\mathbf{70}, 1-)$, $(\mathbf{70}, 3-)\cdots$ relative to $(\mathbf{70}, 0+)$, $(\mathbf{70}, 2+)\cdots$ but it raises the supermultiplets $(\mathbf{56}, 1-)$, $(\mathbf{56}, 3-)\cdots$ relative to $(\mathbf{56}, 0+)$, $(\mathbf{56}, 2+)\cdots$. From (3) we see that this scheme is consistent with the available experimental evidence. In fact, *for large enough exchange potentials* we have the following rule:

Rule III. All low-lying $SU(6)$ multiplets of even parity belong to the 56-dimensional representation, and all low-lying multiplets of odd parity belong to the **70**.

The problem exists as to what functional form to take for the interactions U and U'. One assumption is to take a harmonic-oscillator potential and to interpret the energy levels as squares of baryon masses. This leads to equal spacing of levels in mass squared, for which there is some evidence. However, we want to obtain features of the spectrum which are insensitive to the exact functional form of the potential. Therefore, we shall consider the problem for a square-well potential in addition to a harmonic oscillator, to see which features of the spectrum are common to both.

The energy spectra for harmonic oscillator and square-well potentials are, of course, well known. In the square-well case, in the limit of a large exchange potential, we have the usual square-well level ordering, with the additional requirement that rule III is satisfied. The level ordering is

$$(\mathbf{56}, 0+), (\mathbf{70}, 1-), (\mathbf{56}, 2+), (\mathbf{56}, 0+), (\mathbf{70}, 3-), (\mathbf{70}, 1-), (\mathbf{56}, 4+), \cdots. \qquad (5)$$

On the other hand, as we decrease the strength of the exchange potential, we intersperse even-parity **70** multiplets and odd-parity **56** multiplets within the level ordering given in Eq. (5). We give two examples, corresponding to different strengths of U'. One possible level ordering is

$$(\mathbf{56}, 0+), (\mathbf{70}, 1-), (\mathbf{56}, 2+), (\mathbf{56}, 0+), (\mathbf{70}, 3-), (\mathbf{70}, 0+), (\mathbf{70}, 1-), (\mathbf{56}, 1-), (\mathbf{56}, 4+), (\mathbf{70}, 2+), \cdots. \qquad (6)$$

For still weaker U' we obtain

$$(\mathbf{56}, 0+), (\mathbf{70}, 1-), (\mathbf{70}, 0+), (\mathbf{56}, 2+), (\mathbf{56}, 0+), (\mathbf{56}, 1-), (\mathbf{70}, 3-), (\mathbf{70}, 2+), \cdots. \qquad (7)$$

We cannot make the exchange potential appreciably weaker than that which gives the level ordering of Eq. (7), or the unobserved level $(\mathbf{70}, 0+)$ would lie below the observed $(\mathbf{70}, 1-)$ level.

In the harmonic-oscillator case, in the limit of a large exchange potential, we have the level ordering

$$(\mathbf{56}, 0+), (\mathbf{70}, 1-), \{(\mathbf{56}, 0+), (\mathbf{56}, 2+)\}, \{(\mathbf{70}, 1-), (\mathbf{70}, 3-)\}, \{(\mathbf{56}, 0+), (\mathbf{56}, 2+), (\mathbf{56}, 4+)\}, \cdots. \qquad (8)$$

where degenerate levels are enclosed in curly brackets This level ordering conforms to rule III.

For smaller exchange potentials, we again get levels violating rule III which become interspersed into the level ordering of Eq. (8). However, if we require that the $(\mathbf{70}, 1-)$ level lie below the $(\mathbf{70}, 0+)$ level, we find that there is only one low-lying level violating rule III, namely, the $(\mathbf{70}, 0+)$ level. One possible level ordering is

$$(\mathbf{56}, 0+), (\mathbf{70}, 1-), (\mathbf{70}, 0+), \{(\mathbf{56}, 0+), (\mathbf{56}, 2+)\}, \{(\mathbf{70}, 1-), (\mathbf{70}, 3-)\}, \{(\mathbf{56}, 0+), (\mathbf{56}, 2+), (\mathbf{56}, 4+)\}, (\mathbf{56}, 1-), \cdots. \qquad (9)$$

This is the level ordering with the smallest exchange potential consistent with the (**70**, 1−) lying lower than the (**70**, 0+). For a somewhat larger exchange potential [to raise the energy of the (**70**, 0+) level], the (**56**, 1−) level is raised considerably. A characteristic feature of both square-well and harmonic-oscillator potentials of the structure (4) is embodied in the following rule:

Rule IV. If rule III is broken, the lowest energy level to do so is a (**70**, 0+).

In the case of the harmonic-oscillator potential, the (**70**, 0+) is the only possible low-lying level to violate rule III [there must be at least six levels lower than the (**56**, 1−) if the (**70**, 1−) lies below the (**70**, 0+)]. However, in the square-well case, there could exist other low-lying levels, for example (**56**, 1−) and (**70**, 2+).

It should be noted that the validity of rules III and IV depends on the nature of the assumed interaction between quark and diquark, and their violation would not be as serious for the model as the violation of rules I and II.

We now briefly remark on the symmetry-breaking potential v. Some of the properties of v have already been deduced[2] from the observed splitting of the (**56**, 0+) supermultiplet. It is difficult to obtain further properties of v for the following reasons. First, none of the other $SU(6)\times O(3)$ multiplets is complete from the experimental point of view. The existence of these higher supermultiplets has been deduced[6] chiefly from a knowledge of pion-nucleon resonances whose energies may be in error by ~50 MeV. Also, there is no general agreement about how to assign some of the resonances within supermultiplets. Despite these difficulties, we can say that at the very least v must contain a spin-orbit term differing in the doublet and quartet states of the quark-diquark system, and which also depends on $SU(3)$ multiplicity.

3. DISCUSSION

We have obtained the following principal results using the quark-diquark model: First, the only low-lying baryon states should be those that belong to the **56** and **70** representations of $SU(6)$. Second, the orbital angular momentum and parity of an $SU(6)\times O(3)$ supermultiplet should be related by $\pi=(-1)^L$. Third, for the most part, low-lying states belonging to the **56** should have even parity and those belonging to the **70** should have odd parity. If there are any exceptions, the lowest one should be a (**70**, 0+) supermultiplet. None of these predictions is contradicted by the available experimental data, but the data are insufficient to provide a severe test of the model.

These results disagree with those summarized by Dalitz[6] for the quark model with harmonic-oscillator potential. For example, in the Greenberg-Dalitz model, there is a (**20**, 1+) supermultiplet corresponding to a two-quanta excitation, and (**20**, 3−) and (**20**, 1−) supermultiplets in the three-quanta excitations. All such supermultiplets belonging to the **20** are forbidden in the quark-diquark model. The (**20**, 1+) quark harmonic-oscillator level is the only one with $\pi=(-1)^{L+1}$ for excitations of up to three-quanta, so that the $L-\pi$ relation does not provide a convenient independent way to discriminate between the two models.

It is interesting that in the quark harmonic-oscillator model, the unobserved levels (**20**, 1+), (**70**, 2+), and (**70**, 0+) are predicted to be degenerate with the observed levels (**56**, 2+) and (**56**, 0+). More experimental information will be needed to find out whether these supermultiplets are present in the baryon spectrum. A crucial test which distinguishes between the Greenberg-Dalitz shell model and the quark-diquark model is whether there exists a low-lying (**20**, 1+) supermultiplet. If it is found, the quark-diquark model will have to be drastically altered, for example, by including a negative-parity diquark belonging to the 15-dimensional representation of $SU(6)$. If such a diquark is needed, the model will become more unwieldy and much of its attractiveness will be lost.

On the other hand, the predictions of Mitra[7] are much closer to those of the quark-diquark model. In fact, Mitra's quark model with only s-wave quark-quark forces, gives just the results embodied in rules I, II, and III. The principal difference is that in the quark-diquark model, some low-lying states which violate rule III, such as (**70**, 0+), (**56**, 1−), and (**70**, 2+), are allowed, although they are by no means required. It may be that a quark model with dynamics similar to that assumed by Mitra can lead in a natural way to a quark-diquark model as a good approximation. In any such scheme, one will have to decide whether or not to include effective Bose symmetry for the quarks. Mitra's model contains this symmetry, while the quark-diquark model as presently formulated does not, but instead contains exchange forces. Since the two models give similar predictions, the question of symmetry may be a difficult one to answer.

ACKNOWLEDGMENTS

I should like to thank Professor R. H. Dalitz and Professor A. N. Mitra for helpful discussions. I am very grateful for the hospitality of Professor A. Salam and Professor P. Budini at the International Center for Theoretical Physics at Trieste where this work was begun, and for the hospitality of Professor R. H. Dalitz and Dr. R. J. N. Phillips at the Rutherford High Energy Laboratory where the work was completed.