Dispersion-Theoretic Approach to Hard-Pion Current-Algebra Results for ρ-π and A_1-π Scattering*

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Following recent attempts to explore possible connections between current algebra, dispersion relations, and Regge pole theory, we consider ρ-π and A_1-π scattering amplitudes, and calculate them both from the hard-pion current algebra for four-point functions and dispersion relations with saturation by single-particle intermediate states. The two are found to lead to mutually consistent results provided we introduce a subtraction in certain amplitudes. However, from Regge pole theory, many of these are found to require no subtractions. Thus, whereas the current algebra and the pole-dominated dispersion relations understandably give similar results, the Regge pole theory appears to give distinctly different results.

I. INTRODUCTION

SINCE the famous Adler-Weisberger[1] calculation of the g_A/g_V ratio, the algebra of vector and axial-vector currents has been widely used to investigate strong, weak, and electromagnetic interactions of elementary particles and resonances.[2] Gradually, the techniques employed have been perfected and many discrepancies arising in earlier current-algebra calculations have been removed in recent applications. For instance, we recall that in most of the earlier current-algebra applications, one worked with soft mesons (i.e., one worked in the limit of the four-momenta of the pseudoscalar meson involved going to zero). Thus, in these calculations (of decay and scattering processes), the "gradient coupling" term was missing. This approximation led to many wrong current-algebra predictions for a large number of processes.[3] Later, it was pointed out[4] that, within a current-algebra framework, it is sufficient to work with zero-mass pseudoscalar mesons (i.e., their four-momenta square going to zero) instead of having them soft as well. Thus, in a most straightforward manner, the "gradient coupling" term (term of first order in meson momenta) came to be incorporated in the current-algebra calculations. This scheme has led to much better current-algebra predictions.[5]

This modified current-algebra scheme has been developed following two different approaches. In the first approach,[6] attempts are made to combine soft-meson current algebra with dispersion relations, and in the second approach,[7] one writes generalized Ward-Takahashi identities assuming the lowest-order momentum dependence. However, both the approaches are essentially the same in that in both, in addition to the equal-time commutator terms, one evaluates and retains the pole contributions. Here, we would like to mention that recently, a number of chiral Lagrangian models[8] have also been proposed and these, too, lead to much improved and similar current-algebra results.

Our object in this paper is to study meson-meson scattering (in particular ρ-π and A_1-π scattering) in the context of hard-pion $SU(2)\times SU(2)$ chiral algebra and fixed-t or -s dispersion relations. Recently, we considered virtual photon-pion scattering in a similar framework.[9] There, we showed that the hard-pion current-algebra results for a four-point function are reproducible from dispersion relations (with one subtraction) and the current-algebra results for a three-point function. Now, we extend similar considerations to ρ-π and A_1-π scattering amplitudes. From a purely dispersion-theoretic viewpoint, one can write either unsubtracted or subtracted dispersion relations in the kinematical variables for the invariant amplitudes of the problem. So far as the dispersion integral is concerned, it may be determined by calculating the absorptive parts in a simple

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[1] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); Phys. Rev. **140**, B736 (1965); W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); Phys. Rev. **143**, 1302 (1966).

[2] See, for example, the works listed in B. Renner, *Current Algebra and Their Applications* (Pergamon Press, Inc., London, 1968); S. Adler and R. Dashen, *Current Algebras and Applications to Particle Physics* (W. A. Benjamin, Inc., New York, 1968).

[3] Here we quote the example of a soft-pion current-algebra calculation for the decay $A_1 \to \rho\pi$; such a calculation gives the A_1 width $\approx$800 MeV, whereas the experimental value is close to 100 MeV; see, e.g., B. Renner, Phys. Letters **21**, 143 (1966).

[4] S. Okubo, R. E. Marshak, and V. S. Mathur, Phys. Rev. Letters **19**, 407 (1967); see also S. Okubo, in *Proceedings of the International Conference on Particles and Fields, Rochester, 1967* (Wiley-Interscience Publishers, Inc., New York, 1967); S. G. Brown and G. B. West, Phys. Rev. Letters **19**, 812 (1967).

[5] See papers quoted in Refs. 4 and 6.

[6] See S. Okubo *et al.*, Ref. 4; T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 1067 (1967); H. T. Nieh, *ibid.* **21**, 116 (1968).

[7] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967); I. S. Gerstein and H. J. Schnitzer, *ibid.* **170**, 1638 (1968); see also K. C. Gupta and J. S. Vaishya, *ibid.* **170**, 1530 (1968); I. S. Gerstein, H. J. Schnitzer, and S. Weinberg, *ibid.* **175**, 1873 (1968).

[8] An account of the chiral Lagrangian models is given in K. Kawarabayashi, talk given at the Eleventh Boulder Conference, 1968 (unpublished); see also D. A. Geffen and S. Gasiorowicz, Argonne National Laboratory Report, 1968 (unpublished).

[9] K. C. Gupta and J. S. Vaishya (to be published); see also R. Chanda, R. N. Mohapatra, and S. Okubo, Phys. Rev. **170**, 1344 (1968).

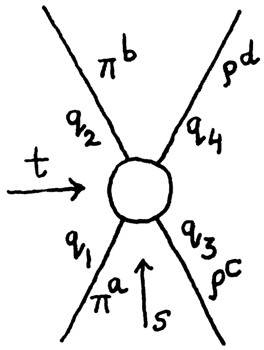

FIG. 1. Kinematics of ρ-π scattering.

saturation scheme of retaining only one-particle intermediate-state contributions. However, the main difficulty in such an approach has always been the question of subtractions: (a) whether to introduce a subtraction or not in a particular amplitude, and (b) how to know the subtraction constant. In recent years, Regge pole theory has been developed to provide an answer to (a) in that it gives the high-energy behavior of the invariant amplitude and thus, perhaps, serves the purpose of a reliable criterion for the convergence of the dispersion relation. According to Regge theory, the energy dependence of a strong-interaction scattering amplitude is essentially determined by its t-channel quantum numbers, i.e., by the Regge pole trajectory enchanged in the t channel. In such an approach, Regge cuts have not been accounted for so far and, further, one has to make plausible and perhaps justifiable assumptions[10] about the trajectories that represent the dynamical information of the pole model.

Recently, it has been shown[11] that, in the case of charged photon-pion scattering, the current algebra and Regge pole theory lead to contradictory results and in order to conciliate the two, one has to introduce fixed J-plane singularities. Further, the present authors have shown in an aforementioned paper[9] that the current-algebra results for the virtual photon-pion scattering can be reproduced from a dispersion-theoretic approach only if one introduces a subtraction in two of the four invariant amplitudes and determines the subtraction constant from current algebra. Further, it was pointed out that the Regge pole theory would lead to the requirement of no subtractions in these form factors. Thus, current algebra was, again, found to disagree with the Regge pole theory. It is important to point out that these conclusions are subject to the assumed validity of the saturation scheme (of including only 1^+, 1^-, and 0^- states) adopted in the previous and the present calculations. Next, we briefly outline our approach to the problem under consideration.

In order to obtain the hard-pion current algebra results for the ρ-π and A_1-π scattering amplitudes, we make use of the recently developed four-point functions of Gerstein and Schnitzer.[12] Also, we write dispersion relations (with and without subtractions) for the invariant amplitudes and saturate the dispersion integrals by A_1, π, and ρ meson states only as these belong to a representation of the underlying $SU(2)$ group. The consequences of including other possible 1^- states, e.g., ω and ϕ, in the theory are also discussed. Throughout our analysis, we ignore the existence of the scalar meson in the $\pi\pi$ system and, also, the so-called "σ terms."[13] In Sec. II, we fix our notations and definitions. In Sec. III, for the sake of completeness, we give some of the essential steps of the four-point functions analysis and derive the hard-pion current-algebra results for the invariant amplitudes occurring in ρ-π and A_1-π scattering. In Sec. IV, we derive the absorptive parts of the amplitudes to be used later on in the dispersion relations. In Sec. V, we carry out the main analysis and show the analogy between current algebra and once subtracted dispersion relations with the subtraction constants taken from current algebra. Finally, in Sec. VI, we discuss our results and the questions arising therein.

II. DEFINITIONS

The scattering amplitude for the process $\pi^a+\rho^c \to \pi^b+\rho^d$ is defined by

$$\langle\rho^d(q_4),\pi^b(q_2)|s|\rho^c(q_3),\pi^a(q_1)\rangle = 1+i(2\pi)^4(16q_{10}q_{20}q_{30}q_{40})^{-1/2} \times\delta^4(q_1+q_3-q_2-q_4)T\,, \quad (2.1)$$

where T is the reaction matrix and the various channels, isospin, and momenta are as shown in Fig. 1. From invariance arguments alone, we can write the following decomposition for T:

$$T=\sum_{i=1}^{4} A_i(s,t,u)I_{\mu\nu}{}^i\epsilon^\mu(q_3)\epsilon^\nu(q_4)\,. \quad (2.2)$$

Here, the ϵ's are the polarization vectors for the two-vector mesons and $A_i(s,t,u)$ are the invariant amplitudes, and are functions of the variables s, t, and u. These kinematical variables are defined below:

$$s=(q_1+q_3)^2\,,\quad u=(q_1-q_4)^2\,,\quad t=(q_1-q_2)^2\,. \quad (2.3)$$

[10] Generally, one takes $\alpha^{I=0}(t=0)=1$, $\alpha^{I=1}(t=0)<1$, and $\alpha^{I=2}(t=0)<0$. Of these, the last is certainly open to criticism. Also, more complications set in, in case Regge cuts are important, since these may lie above $\alpha^{I=2}(t=0)=0$. For the cuts see, e.g., R. J. N. Phillips, Phys. Letters **24B**, 342 (1967); I. J. Muzinich, Phys. Rev. Letters **18**, 381 (1967). For ρ-π scattering alone, see V. de Alfaro, S. Fubini, G. Furlan, and C. Rossetti, Phys. Letters **21**, 576 (1966), and also Ref. 23.

[11] J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, Phys. Rev. Letters **18**, 32 (1967); V. Singh, *ibid.* **18**, 36 (1967).

[12] For details see the paper by I. S. Gerstein and H. J. Schnitzer, Ref. 7. We follow the notation, etc., of this paper.

[13] The so-called scalar terms are assumed to be of little importance; the good results obtained in almost all of the current-algebra calculations justify this *a priori* assumption; also see Ref. 7.

Also,

$$s+t+u=2(m_\rho^2+m_\pi^2)\,,\qquad 4\nu=(s-u)\,. \tag{2.4}$$

The Lorentz covariants $I_{\mu\nu}{}^i$ are given below

$$I_{\mu\nu}{}^1=P_\mu P_\nu\,,\quad I_{\mu\nu}{}^2=\tfrac{1}{2}(P_\mu Q_\nu+Q_\mu P_\nu)\,,\quad I_{\mu\nu}{}^3=Q_\mu Q_\nu\,,\quad I_{\mu\nu}{}^4=g_{\mu\nu}\,, \tag{2.5}$$

where

$$P=\tfrac{1}{2}(q_1+q_2)\,,\quad Q=\tfrac{1}{2}(q_3+q_4)\,. \tag{2.6a}$$

Also, it is convenient to define another variable Δ,

$$\Delta=\tfrac{1}{2}(q_2-q_1)\,. \tag{2.6b}$$

Thus,

$$s=(P+Q)^2\,,\quad u=(P-Q)^2\,,\quad t=4\Delta^2\,,\quad \nu=P\cdot Q\,. \tag{2.6c}$$

Next, let us fix the isospin decomposition. From charge independence, we can write

$$A^{abcd}(s,t)=A^1\delta^{ab}\delta^{cd}+A^2\delta^{ac}\delta^{bd}+A^3\delta^{ad}\delta^{bc}\,. \tag{2.7}$$

The amplitudes for definite isotopic spins $I=0$, 1, 2 in the t channel are readily obtained:

$$\begin{aligned}A^{(0)}&\equiv A^{(I=0)}(s,t)=(3A^1+A^2+A^3)\,,\\ A^{(1)}&\equiv A^{(I=1)}(s,t)=(A^2-A^3)\,,\\ A^{(2)}&\equiv A^{(I=2)}(s,t)=(A^2+A^3)\,.\end{aligned} \tag{2.8}$$

Further, we notice that the s- to u-channel crossing symmetry leads to the following constraints:

$$\begin{aligned}A_i{}^I(s,u,t)&=(-)^I A_i{}^I(u,s,t)\,,\quad i=1,3,4\\ A_2{}^I(s,u,t)&=(-)^{I+1}A_2{}^I(u,s,t)\,.\end{aligned} \tag{2.9}$$

Now, the kinematics and notations for $\pi^a+A_1{}^c\to\pi^b+A_1{}^d$ are self-evident. For the sake of clarity, we shall use the letter B in place of A in the case of A_1-π scattering, i.e., now,

$$T=\sum_{i=1}^{4}B_i(s,t,u)I_{\mu\nu}{}^i\epsilon^\mu(q_3)\epsilon^\nu(q_4)\,, \tag{2.10}$$

the ϵ's being the polarization vectors for the two A_1 mesons. (Throughout the text, A meson or A_1 meson stands for the 1^+ meson of mass 1070 MeV.)

III. HARD-PION CURRENT-ALGEBRA CALCULATION OF ρ-π AND A_1-π SCATTERING

Recently, a number of authors[14] have investigated four-point functions in general, and π-π scattering in particular, using the chiral $SU(2)\times SU(2)$ algebra of currents and the hypothesis of partially conserved axial-vector current (PCAC). As a result, two elegant approaches have emerged: (i) the phenomenological chiral Lagrangian approach,[8] and (ii) the Ward-Takahashi identities[7] approach. The former has been developed as a powerful technique to calculate a large number of processes involving pions with a nonlinear realization of the pion field. The other approach leads to similar results though the techniques used are different. The two approaches are identical in that in both one sums up the tree diagrams only. In this section, we will calculate the invariant amplitudes earlier defined for both ρ-π and A_1-π scattering using the four-point functions developed by Gerstein and Schnitzer,[7] following the Ward-Takahashi identities approach. For the sake of completeness, we first mention the essential features of this paper. The main assumptions involved are

(1) local chiral $SU(2)\times SU(2)$ algebra of vector and axial-vector currents,

(2) PCAC hypothesis relating the pion field to the divergence of the axial-vector current,

(3) dominance of the 1^-, 1^+, and 0^- channels created by the currents from vacuum by the ρ, A_1, and π mesons only (thus the currents are used as interpolating fields),

(4) crossing symmetry requirements, and

(5) smooth momentum dependence of the primitive functions (this and saturation by one-particle states implies that one is summing up the tree diagrams only, to lowest order in perturbation theory).

The technique consists in setting up Ward identities for the proper functions (i.e., functions obtained after extracting the π, ρ, and A_1 pole structures) obeying the chiral algebra and the PCAC relation. These are then solved under the assumptions listed above and also under the further assumptions of (a) no $I=1$ part in the so-called Schwinger terms, and (b) nonexistence of 0^+ mesons. Since the calculations do not involve any limit for the pion four-momenta, the solutions contain the "gradient coupling" terms and so lead to hard-pion current-algebra results.

Now we consider the ρ-π case in details and then it is easy to repeat the steps for the A_1-π case. We define

$$T^{(2)}(q_1,-q_2,q_3)_{\lambda\sigma}{}^{abcd}=-i\int d^4xd^4yd^4z\,e^{-iq_1\cdot x+iq_2\cdot y-iq_3\cdot z}\times\langle T(\partial_\mu A_\mu{}^a(x),\partial_\nu A_\nu{}^b(y),V_\lambda{}^c(z),V_\sigma{}^d(0))\rangle_0 \tag{3.1}$$

and

$$A_{\lambda\sigma}=(m_\pi^2-q_1^2)(m_\pi^2-q_2^2)(m_\rho^2-q_3^2)(m_\rho^2-q_4^2)\times f_\pi^{-2}m_\pi^{-4}g_\rho^{-2}\epsilon_\lambda(q_3)\epsilon_\sigma(q_4)\,. \tag{3.2}$$

Thus, from these definitions and Eq. (2.1),

$$T=T^{(2)}(q_1,-q_2,q_3)_{\lambda\sigma}{}^{abcd}A_{\lambda\sigma}\,. \tag{3.3}$$

Here, we have used the PCAC relation[15]

$$\partial_\mu A_\mu{}^a(x)=f_\pi m_\pi^2\phi^a(x) \tag{3.4}$$

[14] For an interesting four-point current-algebra calculation of π-π scattering, see R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. Letters **20**, 475 (1968), and papers in Refs. 7 and 8.

[15] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).

and the field-current identity[16]

$$\rho_\mu{}^a(x)=g_\rho{}^{-1}V_\mu{}^a(x)\,, \tag{3.5}$$

$\phi(x)$ and $\rho(x)$ being the pion and ρ-meson fields. Clearly, g_ρ is the coupling constant arising when the current $V_\mu(x)$ connects one ρ-meson state to vacuum. $V_\mu(x)$ and $A_\mu(x)$ are the vector and axial-vector currents, respectively, satisfying the usual equal-time current commutation relations and thus generating the $SU(2)\times SU(2)$ algebra of currents. Following Ref. 12, we can write

$$\begin{aligned}T^{(2)}(q_1,-q_2,q_3)_{\lambda\sigma}{}^{abcd}\\ =-i(m_\pi{}^2-q_1{}^2)^{-1}(m_\pi{}^2-q_2{}^2)^{-1}f_\pi{}^4m_\pi{}^8\\ \times\Delta_{\lambda\lambda'}{}^V(q_3)\Delta_{\sigma\sigma'}{}^V(q_4)M^{(2)}(q_1,-q_2,q_3)_{\lambda'\sigma'}{}^{abcd},\end{aligned} \tag{3.6}$$

where the Δ^V's are the vector propagators and $M_{\lambda'\sigma'}{}^{(2)abcd}$ is the reduced diagonalized four-point function with two vector currents (for details see Ref. 12). The result obtained for M is

$$\begin{aligned}&m_\pi{}^4f_\pi{}^4M^{(2)}(q_1,-q_2,q_3)_{\lambda\sigma}{}^{abcd}\\ &=c_A{}^2q_{1\mu}q_{2\nu}M_c{}^{(2)}(q_1,-q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abcd}-\tfrac{1}{2}i(\epsilon^{ade}\epsilon^{bce}+\epsilon^{bde}\epsilon^{ace})[\Delta^V(q_3)^{-1}{}_{\lambda\sigma}+\Delta^V(-q_4)^{-1}{}_{\lambda\sigma}]\\ &\quad+i\epsilon^{abe}\epsilon^{cde}\Gamma^{(3)}(q_3,q_1-q_2)_{\lambda\alpha\sigma}\Delta^V(q_1-q_2)_{\alpha\alpha'}[c_A{}^2q_{1\mu}q_{2\nu}\Gamma^{(1)}(q_1,-q_2)_{\mu\nu\alpha'}+\tfrac{1}{2}(q_1+q_2)_{\alpha'}]\\ &\quad+\{i\epsilon^{bde}\epsilon^{ace}[[-c_Aq_{2\nu}\Gamma^{(1)}(q_1+q_3,-q_2)_{\alpha\nu\sigma}+\Delta^V(-q_4)^{-1}{}_{\alpha\sigma}]\Delta^A(q_1+q_3)_{\alpha\alpha'}\\ &\quad\times[c_Aq_{1\mu}\Gamma^{(1)}(-q_2-q_4,q_1)_{\alpha'\mu\lambda}+\Delta^V(q_3)^{-1}{}_{\alpha'\lambda}]-[c_A{}^2q_{2\nu}(q_1+q_3)_\alpha\Gamma^{(1)}(-q_2,q_1+q_3)_{\nu\alpha\sigma}\\ &\quad+\tfrac{1}{2}(-q_1-q_2-q_3)_\sigma+(c_A-\tfrac{1}{2}c_V)(q_1+q_2+q_3)_\alpha\Delta^V(-q_4)^{-1}{}_{\alpha\sigma}]f_\pi{}^{-2}[m_\pi{}^2-(q_1+q_3)^2]^{-1}\\ &\quad\times[c_A{}^2q_{1\mu}(q_2+q_4)_\alpha\Gamma^{(1)}(q_1,-q_2-q_4)_{\mu\alpha\lambda}+\tfrac{1}{2}(q_1+q_2+q_4)_\lambda+(c_A-\tfrac{1}{2}c_V)(-q_1-q_2-q_4)_\alpha\Delta^V(q_3)^{-1}{}_{\alpha\lambda}]]\\ &\quad+(q_1\leftrightarrow-q_2,\ \mu\leftrightarrow\nu,\ a\leftrightarrow b)\}\,.\end{aligned} \tag{3.7}$$

In writing the above expression, we have dropped the scalar term assuming it to be small. Further, $\Gamma_{\mu\nu\lambda}{}^{(3)}(q_1,q_2)$, the proper three-point function is defined as

$$\begin{aligned}\int d^4xd^4y\,e^{-iq_1\cdot x-iq_2\cdot y}\langle T(V_\mu{}^a(x),V_\nu{}^b(y),V_\lambda{}^c(0))\rangle_0\\ =i\epsilon^{abc}\Delta_{\mu\mu'}{}^V(q_1)\Delta_{\nu\nu'}{}^V(q_2)\Delta_{\lambda\lambda'}{}^V(k)\Gamma_{\mu'\nu'\lambda'}{}^{(3)}(q_1,q_2)\end{aligned} \tag{3.8}$$

with $k=(q_1-q_2)$. $\Gamma_{\mu\nu\lambda}{}^{(1)}(q_1,q_2)$ is a similar three-point function with one vector and two axial-vector currents,

$$\begin{aligned}\int d^4xd^4y\,e^{-iq_1\cdot x-iq_2\cdot y}\langle T(A_\mu{}^a(x),\bar{A}_\nu{}^b(y),V_\lambda{}^c(0))\rangle_0\\ =i\epsilon^{abc}\Delta_{\mu\mu'}{}^A(q_1)\Delta_{\nu\nu'}{}^A(q_2)\Delta_{\lambda\lambda'}{}^V(k)\Gamma_{\mu'\nu'\lambda'}{}^{(1)}(q_1,q_2)\,,\end{aligned} \tag{3.9a}$$

where

$$\bar{A}_\mu{}^a(x)=A_\mu{}^a(x)+iq_\mu m_\pi{}^{-2}\partial_\nu A_\nu{}^a(x)\,. \tag{3.9b}$$

Further, the axial-vector and the vector propagators are

$$\Delta_{\mu\nu}{}^A(q)=\frac{c_Am_A{}^2}{(m_A{}^2-q^2)}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_A{}^2}\right),$$

$$\Delta_{\mu\nu}{}^V(q)=\frac{c_Vm_\rho{}^2}{(m_\rho{}^2-q^2)}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_\rho{}^2}\right) \tag{3.10}$$

with

$$c_A=-g_A{}^2/m_A{}^2\,,\quad c_V=-g_\rho{}^2/m_\rho{}^2\,. \tag{3.11}$$

By definition, $M_c{}^{(2)}(q_1,q_2,q_3)_{\lambda\sigma}{}^{abcd}$ is the proper four-point function and it does not contain any (π,ρ,A_1) poles. This is generally called the "contact term." The proper vertices, so far as the current algebra is concerned, do not have any poles and so can be so approximated as to be smooth functions of momenta. The results for these are

$$\begin{aligned}\Gamma^{(3)}(q_1,q_2)_{\nu\lambda\sigma}\\ =c_V{}^{-2}m_\rho{}^{-2}[g_{\nu\lambda}(q_2-q_1)_\sigma\\ -g_{\lambda\sigma}(q_1+2q_2)_\nu+g_{\sigma\nu}(2q_1+q_2)_\lambda]\,,\end{aligned} \tag{3.12}$$

$$\begin{aligned}\Gamma^{(1)}(q_1,q_2)_{\nu\lambda\sigma}\\ =c_V{}^{-1}c_A{}^{-1}m_A{}^{-2}[g_{\nu\lambda}(q_2-q_1)_\sigma\\ -g_{\lambda\sigma}(q_1+2q_2)_\nu+g_{\sigma\nu}(2q_1+q_2)_\lambda\\ +\delta(g_{\nu\sigma}(q_1+q_2)_\lambda-g_{\lambda\sigma}(q_1+q_2)_\nu)]\,,\end{aligned} \tag{3.13}$$

$$\begin{aligned}M_c{}^{(2)}(q_1,q_2,q_3)_{\mu\nu\lambda\sigma}{}^{abcd}\\ =ic_V{}^{-2}c_A{}^{-1}m_A{}^{-2}[-\epsilon^{abe}\epsilon^{cde}(\tfrac{3}{2}+\delta)(g_{\nu\lambda}g_{\mu\sigma}-g_{\lambda\mu}g_{\nu\sigma})\\ -(\epsilon^{ade}\epsilon^{bce}+\epsilon^{bde}\epsilon^{ace})\\ \times(\tfrac{1}{2}g_{\nu\lambda}g_{\mu\sigma}+\tfrac{1}{2}g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\nu}g_{\lambda\sigma})]\,.\end{aligned} \tag{3.14}$$

Here, δ is a parameter[17] related to the anomalous magnetic moment of the A_1 meson. Throughout our analysis, we will use the sum rules[18]

$$f_\pi{}^2=c_A-c_V\,, \tag{3.15}$$

$$m_{A_1}{}^2=2m_\rho{}^2\,, \tag{3.16}$$

and

$$g_A{}^2=g_\rho{}^2=2m_\rho{}^2f_\pi{}^2\,. \tag{3.17}$$

Thus,

$$c_A-\tfrac{1}{2}c_V=0\,. \tag{3.18}$$

[16] See, for example, N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967); J. J. Sakurai, Report No. EFINS 67-64 (unpublished).

[17] The parametrization $\delta=-1$ leads to good results for the $\rho\to\pi\pi$, $A_1\to\rho\pi$, $K^*\to K\pi$, $Q\to\rho K$, $Q\to K^*\pi$, and $\phi\to K\bar{K}$ decay widths, see Ref. 7; also then $\xi=0$ [see Eq. (5.15)] and it agrees with the conclusions of Ref. 14.

[18] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); K. Kawarabayashi and M. Suzuki, *ibid.* **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966); J. J. Sakurai, Phys. Rev. Letters **19**, 803 (1967).

Using Eqs. (3.1)–(3.18), it is a straightforward though tedious algebra to calculate T and hence the invariant amplitudes A_i defined earlier. Our results are

$$A_1(s,t)=(4f_\pi^2)^{-1}\left\{\epsilon^{ace}\epsilon^{bde}\left[\frac{(3-\delta)^2m_\rho^2}{2(m_\pi^2-s)}+\frac{(1+\delta)^2m_\rho^2-2\delta(2+\delta)m_\rho^2+\delta^2t}{2(2m_\rho^2-s)}\right]+(a\leftrightarrow b,\, s\leftrightarrow u)\right\}, \tag{3.19}$$

$$A_2(s,t)=(4f_\pi^2)^{-1}\left\{\epsilon^{abe}\epsilon^{cde}\left[\frac{32m_\rho^2[1-(1+\delta)t(4m_\rho^2)^{-1}]}{(m_\rho^2-t)}-(8+8\delta+2\delta^2)\right]\right.$$
$$\left.+\left[\epsilon^{ace}\epsilon^{bde}\left(\frac{(3-\delta)^2m_\rho^2}{(m_\pi^2-s)}+\frac{(1+\delta)^2m_\rho^2-2\delta^2m_\pi^2+\delta^2t}{(2m_\rho^2-s)}\right)-(a\leftrightarrow b,\, s\leftrightarrow u)\right]\right\}, \tag{3.20}$$

$$A_3(s,t)=(4f_\pi^2)^{-1}\left\{\epsilon^{ace}\epsilon^{bde}\left[-2\delta(2+\delta)+\frac{(3-\delta)^2m_\rho^2}{2(m_\pi^2-s)}\right.\right.$$
$$\left.\left.+\frac{(1+\delta)^2m_\rho^2+2\delta(2+\delta)m_\rho^2-4\delta^2m_\pi^2+\delta^2t}{2(2m_\rho^2-s)}\right]+(a\leftrightarrow b,\, s\leftrightarrow u)\right\}, \tag{3.21}$$

$$A_4(s,t)=(4f_\pi^2)^{-1}\left\{\tfrac{1}{8}\epsilon^{abe}\epsilon^{cde}(u-s)\left[\frac{16m_\rho^2[1-(1+\delta)t(4m_\rho^2)^{-1}]}{(m_\rho^2-t)}-(2+\delta)^2\right]\right.$$
$$\left.+\left[\epsilon^{ace}\epsilon^{bde}\left((1+\delta)m_\rho^2-\tfrac{1}{8}\delta(4+\delta)t+\tfrac{1}{4}\delta^2(m_\rho^2-m_\pi^2)-\frac{[(2+\delta)m_\rho^2-\delta m_\pi^2]^2}{4(2m_\rho^2-s)}\right)+(a\leftrightarrow b,\, s\leftrightarrow u)\right]\right\}. \tag{3.22}$$

Also, using Eqs. (2.7), (2.8), and (3.19)–(3.22), it is easy to obtain the t-channel amplitudes with definite isotopic spins. Here, we write these in such a form so as to display clearly the structure in the variables ν and Δ^2;

$$A_i^{(2)}(\nu,\Delta^2)=-\tfrac{1}{2}A_i^{(0)}(\nu,\Delta^2),\quad i=1,2,3,4 \tag{3.23}$$

$$A_1^{(0)}(\nu,\Delta^2)=(4f_\pi^2)^{-1}\left\{\left[\frac{(3-\delta)^2m_\rho^2}{2(\nu_\pi-\nu)}+\frac{(1+\delta)^2m_\rho^2-2\delta(2+\delta)m_\rho^2+4\delta^2\Delta^2}{2(\nu_A-\nu)}\right]+(\nu\leftrightarrow-\nu)\right\}, \tag{3.24}$$

$$A_1^{(1)}(\nu,\Delta^2)=(4f_\pi^2)^{-1}\left\{\left[\frac{(3-\delta)^2m_\rho^2}{4(\nu_\pi-\nu)}+\frac{(1+\delta)^2m_\rho^2-2\delta(2+\delta)m_\rho^2+4\delta^2\Delta^2}{4(\nu_A-\nu)}\right]-(\nu\leftrightarrow-\nu)\right\}, \tag{3.25}$$

$$A_2^{(0)}(\nu,\Delta^2)=(4f_\pi^2)^{-1}\left\{\left[\frac{(3-\delta)^2m_\rho^2}{(\nu_\pi-\nu)}+\frac{(1+\delta)^2m_\rho^2-2\delta^2m_\pi^2+4\delta^2\Delta^2}{(\nu_A-\nu)}\right]-(\nu\leftrightarrow-\nu)\right\}, \tag{3.26}$$

$$A_2^{(1)}(\nu,\Delta^2)=(4f_\pi^2)^{-1}\left\{\left[-[6+4\delta+2(1+\delta)^2]+\frac{32m_\rho^2[1-(1+\delta)m_\rho^{-2}\Delta^2]}{(m_\rho^2-4\Delta^2)}+\frac{(3-\delta)^2m_\rho^2}{2(\nu_\pi-\nu)}\right.\right.$$
$$\left.\left.+\frac{(1+\delta)^2m_\rho^2-2\delta^2m_\pi^2+4\delta^2\Delta^2}{2(\nu_A-\nu)}\right]+(\nu\leftrightarrow-\nu)\right\}, \tag{3.27}$$

$$A_3^{(0)}(\nu,\Delta^2)=(4f_\pi^2)^{-1}\left\{\left[-4\delta(2+\delta)+\frac{(3-\delta)^2m_\rho^2}{2(\nu_\pi-\nu)}+\frac{(1+\delta)^2m_\rho^2+2\delta(2+\delta)m_\rho^2-4\delta^2m_\pi^2+4\delta^2\Delta^2}{2(\nu_A-\nu)}\right]+(\nu\leftrightarrow-\nu)\right\}, \tag{3.28}$$

$$A_3^{(1)}(\nu,\Delta^2)=(4f_\pi^2)^{-1}\left\{\left[\frac{(3-\delta)^2m_\rho^2}{4(\nu_\pi-\nu)}+\frac{(1+\delta)^2m_\rho^2+2\delta(2+\delta)m_\rho^2-4\delta^2m_\pi^2+4\delta^2\Delta^2}{4(\nu_A-\nu)}\right]-(\nu\leftrightarrow-\nu)\right\}, \tag{3.29}$$

$$A_4^{(0)}(\nu,\Delta^2)=(4f_\pi^2)^{-1}\left\{\left[2(1+\delta)m_\rho^2+\tfrac{1}{2}\delta^2(m_\rho^2-m_\pi^2)-\delta(4+\delta)\Delta^2-\frac{[(2+\delta)m_\rho^2-\delta m_\pi^2]^2}{4(\nu_A-\nu)}\right]+(\nu\leftrightarrow-\nu)\right\}, \tag{3.30}$$

$$A_4^{(1)}(\nu,\Delta^2)=(4f_\pi^2)^{-1}(-\nu)\left\{\frac{[(2+\delta)m_\rho^2-\delta m_\pi^2]^2}{4(\nu_A^2-\nu^2)}-(2+\delta)^2+\frac{16m_\rho^2[1-(1+\delta)m_\rho^{-2}\Delta^2]}{(m_\rho^2-4\Delta^2)}\right\}. \tag{3.31}$$

Here,

$$\nu_\pi = \tfrac{1}{2}(2\Delta^2 - m_\rho^2) \tag{3.32}$$

and

$$\nu_A = \tfrac{1}{2}(2\Delta^2 + m_\rho^2 - m_\pi^2). \tag{3.33}$$

We notice a singular feature in that the amplitude $A_4(1)$ has ν as an over-all multiplying factor and hence, its ν dependence is quite different from that of the other amplitudes.

Next, we give some of the steps for the A_1-π scattering case. The corresponding four-point function, now, is $M^{(0)}(q_1, -q_2, q_3)_{\lambda\sigma}{}^{abcd}$. This is given to be[12]

$$\begin{aligned} m_\pi^4 f_\pi^4 M^{(0)}(q_1, -q_2, q_3)_{\lambda\sigma}{}^{abcd} &= c_A^2 q_{1\mu} q_{2\nu} M_c{}^{(0)}(q_1, -q_2, q_3)_{\mu\nu\lambda\sigma}{}^{abcd} \\ &\quad + i\epsilon^{abe}\epsilon^{cde}\Gamma^{(1)}(q_3, -q_4)_{\lambda\sigma\alpha}\Delta^V(q_1-q_2)_{\alpha\alpha'}[-c_A^2 q_{1\mu}q_{2\nu}\Gamma^{(1)}(q_1, -q_2)_{\mu\nu\alpha'} - \tfrac{1}{2}(q_1+q_2)_{\alpha'}] \\ &\quad + \{ i\epsilon^{ade}\epsilon^{bce}([-c_A q_{1\mu}\Gamma^{(1)}(-q_4, q_1)_{\sigma\mu\alpha} + \Delta^A(q_4)^{-1}{}_{\sigma\alpha}]\Delta^V(q_3-q_2)_{\alpha\alpha'} \\ &\qquad \times [c_A q_{2\nu}\Gamma^{(1)}(q_3, -q_2)_{\lambda\nu\alpha'} + \Delta^A(q_3)^{-1}{}_{\lambda\alpha'}] + \tfrac{1}{2} i\Delta^A(q_3)^{-1}{}_{\sigma\lambda}) + (q_3 \leftrightarrow -q_4,\ \lambda\leftrightarrow\sigma,\ c\leftrightarrow d)\}. \end{aligned} \tag{3.34}$$

Again, $M_c{}^{(0)}$ is the primitive contact term and with the smoothness assumption for this, we have

$$\begin{aligned} M_c{}^{(0)}(q_1, -q_2, q_3)_{\mu\nu\lambda\sigma}{}^{abcd} &= i\xi c_A^{-3} m_A^{-2}[-\epsilon^{abe}\epsilon^{cde}\tfrac{3}{2}(g_{\nu\lambda}g_{\mu\sigma} - g_{\mu\lambda}g_{\nu\sigma}) \\ &\quad + (\epsilon^{ade}\epsilon^{bce} + \epsilon^{bde}\epsilon^{ace})(-\tfrac{1}{2}g_{\nu\lambda}g_{\mu\sigma} - \tfrac{1}{2}g_{\mu\lambda}g_{\nu\sigma} + g_{\mu\nu}g_{\lambda\sigma})]. \end{aligned} \tag{3.35}$$

The parameter ξ, in the above expression, appears as such because there is no vector constraint for the four-point function of axial-vector currents alone and so there is no condition to determine a scale for the primitive contact term and thus ξ is an unknown coupling constant. We will show in Sec. V that ξ is given in terms of the parameter δ, provided we have a proper combination of current algebra and dispersion relations for the scattering amplitudes. Now we define

$$\nu_\rho = \tfrac{1}{2}(2\Delta^2 - m_\rho^2 - m_\pi^2). \tag{3.36}$$

After a little algebra, the A_1-π scattering amplitudes are easily found

$$B_1{}^{(0)}(\nu,\Delta^2) = -(4f_\pi^2)^{-1}\left\{2[(1+\delta)^2 - 4\xi] + \frac{(1+\delta)^2 m_\rho^2 - 2m_\rho^2 - 2\delta^2\Delta^2}{(\nu_\rho - \nu)}\right\} + (\nu\leftrightarrow -\nu), \tag{3.37}$$

$$B_1{}^{(1)}(\nu,\Delta^2) = -(4f_\pi^2)^{-1}\left[\frac{(1+\delta)^2 m_\rho^2 - 2m_\rho^2 - 2\delta^2\Delta^2}{2(\nu_\rho - \nu)} - (\nu\leftrightarrow -\nu)\right], \tag{3.38}$$

$$B_2{}^{(0)}(\nu,\Delta^2) = -(2f_\pi^2)^{-1}\left[\frac{\delta^2 m_\pi^2 - m_\rho^2 - 2\delta^2\Delta^2}{(\nu_\rho - \nu)} - (\nu\leftrightarrow -\nu)\right], \tag{3.39}$$

$$B_2{}^{(1)}(\nu,\Delta^2) = -(2f_\pi^2)^{-1}\left\{\left[(12\xi - 1) - \frac{8(2+\delta)m_\rho^2}{(m_\rho^2 - 4\Delta^2)}[1 - (1+\delta)m_\rho^{-2}\Delta^2] + \frac{\delta^2 m_\pi^2 - m_\rho^2 - 2\delta^2\Delta^2}{2(\nu_\rho - \nu)}\right] + (\nu\leftrightarrow -\nu)\right\}, \tag{3.40}$$

$$B_3{}^{(0)}(\nu,\Delta^2) = -(4f_\pi^2)^{-1}\left\{\left[2[-(1+\delta)^2 + 2 - 4\xi] + \frac{2\delta^2 m_\pi^2 - (1+\delta)^2 m_\rho^2 - 2\delta^2\Delta^2}{(\nu_\rho - \nu)}\right] + (\nu\leftrightarrow -\nu)\right\}, \tag{3.41}$$

$$B_3{}^{(1)}(\nu,\Delta^2) = -(4f_\pi^2)^{-1}\left[\frac{2\delta^2 m_\pi^2 - (1+\delta)^2 m_\rho^2 - 2\delta^2\Delta^2}{2(\nu_\rho - \nu)} - (\nu\leftrightarrow -\nu)\right], \tag{3.42}$$

$$\begin{aligned} B_4{}^{(0)}(\nu,\Delta^2) = -(4f_\pi^2)^{-1}\Bigg\{\Bigg[& 8\xi(m_\pi^2 - 2\Delta^2) + (2+\delta)^2\nu_A + (2+\delta)[(2+\delta)m_\rho^2 - \delta m_\pi^2] \\ & + \frac{[(2+\delta)m_\rho^2 - \delta m_\pi^2]^2}{4(\nu_\rho - \nu)}\Bigg] + (\nu\leftrightarrow -\nu)\Bigg\}, \end{aligned} \tag{3.43}$$

$$B_4{}^{(1)}(\nu,\Delta^2) = -(4f_\pi^2)^{-1}(2\nu)\left\{\frac{8m_\rho^2[1 - (1+\delta)m_\rho^{-2}\Delta^2]}{(m_\rho^2 - 4\Delta^2)} - \tfrac{1}{2}(2+\delta)^2 + \frac{[(2+\delta)m_\rho^2 - \delta m_\pi^2]^2}{8(\nu_\rho^2 - \nu^2)}\right\}. \tag{3.44}$$

IV. DISPERSION INTEGRALS

We will study the fixed-t (or Δ^2) dispersion relations. Further, keeping in view that in the current-algebra calculation only ρ, A_1, and π states were included, we shall saturate the dispersion integrals by these states only. Implications of including other possible 1^-, 1^+, or 0^- states are pointed out at the end of this section and in the next section.

First, we calculate the absorptive parts for the ρ-π scattering amplitude that will contribute to fixed-t dispersion integrals. From Eqs. (3.1)–(3.3) we can write

$$\mathrm{Im}T=-(q_{10}q_{20})^{1/2}(2\pi)^4\sum_n [\langle\pi^b,q_2|V_\lambda{}^c(0)|n\rangle \times\langle n|V_\sigma{}^d(0)|\pi^a,q_1\rangle\delta^4(q_2-pn-q_3) -\langle\pi^b,q_2|V_\sigma{}^d(0)|n\rangle\langle n|V_\lambda{}^c(0)|\pi^a,q_1\rangle \times\delta^4(p_n-q_1-q_3)\bar{A}_{\lambda\sigma}], \quad (4.1)$$

where

$$\bar{A}_{\lambda\sigma}=(m_\rho{}^2-q_3{}^2)(m_\rho{}^2-q_4{}^2)\epsilon_\lambda(q_3)\epsilon_\sigma(q_4)g_\rho{}^{-2}. \quad (4.2)$$

It is obvious that only intermediate states with $G=-1$ can contribute to the right-hand side in Eq. (4.1). Such low-lying single-particle states are π, ω, ϕ, A_1, and A_2 poles. As mentioned earlier, to be consistent with the saturation scheme adopted in the $SU(2)\times SU(2)$ current-algebra approach, we include only π and A_1 states for the present. We summarize the results in the following.

(i) Pion-pole contribution: We define the coupling as

$$\langle\pi^a,k|V_\mu{}^c(0)|\pi^b,p\rangle=\frac{i\epsilon^{abc}}{(4k_0p_0)^{1/2}}(p+k)_\mu\frac{F_\pi(q^2)}{(m_\rho{}^2-q^2)}, \quad (4.3)$$

where $q_\mu=(p-k)_\mu$. A simple calculation gives,

$$\mathrm{Im}A_1{}^{(\pi)}(s)=4\pi[F_\pi{}^2(m_\rho{}^2)/g_\rho{}^2] \times[\epsilon^{ace}\epsilon^{bde}\delta(s-m_\pi{}^2)-(a\leftrightarrow b,s\leftrightarrow u)], \quad (4.4)$$

$$\mathrm{Im}A_2{}^{(\pi)}(s)=8\pi[F_\pi{}^2(m_\rho{}^2)/g_\rho{}^2] \times[\epsilon^{ace}\epsilon^{bde}\delta(s-m_\pi{}^2)+(a\leftrightarrow b,s\leftrightarrow u)], \quad (4.5)$$

$$\mathrm{Im}A_3{}^{(\pi)}(s)=\mathrm{Im}A_1{}^{(\pi)}(s), \quad (4.6)$$

$$\mathrm{Im}A_4{}^{(\pi)}(s)=0. \quad (4.7)$$

(ii) A_1-pole contribution: We write the most general invariant coupling

$$\langle A_1{}^b,p|V_\mu{}^c(0)|\pi^a,k\rangle=\frac{\epsilon^{abc}}{(4k_0p_0)^{1/2}}\frac{1}{(m_\rho{}^2-q^2)}\{[g_{\mu\alpha}(p^2-k^2)+(p+k)_\mu k_\alpha]C(q^2)+(g_{\mu\alpha}q^2+q_\mu k_\alpha)D(q^2)\}\epsilon_\alpha{}^{A_1}(p) \quad (4.8)$$

with $q=(k-p)$. The absorptive parts obtained are

$$\mathrm{Im}A_1{}^{(A_1)}(s,t)=\pi g_\rho{}^{-2}\{\epsilon^{ace}\epsilon^{bde}[\tfrac{1}{2}m_\rho{}^2(3C+D)^2-2(4m_\rho{}^2-t)C^2-4m_\rho{}^2CD]\delta(s-2m_\rho{}^2)-(a\leftrightarrow b,\ s\leftrightarrow u)\}, \quad (4.9)$$

$$\mathrm{Im}A_2{}^{(A_1)}(s,t)=2\pi g_\rho{}^{-2}\{\epsilon^{ace}\epsilon^{bde}[\tfrac{1}{2}m_\rho{}^2(3C+D)^2-2(2m_\pi{}^2-t)C^2]\delta(s-2m_\rho{}^2)+(a\leftrightarrow b,\ s\leftrightarrow u)\}, \quad (4.10)$$

$$\mathrm{Im}A_3{}^{(A_1)}(s,t)=\pi g_\rho{}^{-2}\{\epsilon^{ace}\epsilon^{bde}[\tfrac{1}{2}m_\rho{}^2(3C+D)^2+2C^2(4m_\rho{}^2+t-4m_\pi{}^2)+4m_\rho{}^2CD]\delta(s-2m_\rho{}^2)-(a\leftrightarrow b,\ s\leftrightarrow u)\}, \quad (4.11)$$

$$\mathrm{Im}A_4{}^{(A_1)}(s,t)=-\pi g_\rho{}^{-2}\{\epsilon^{ace}\epsilon^{bde}[(2m_\rho{}^2-m_\pi{}^2)C+m_\rho{}^2D]^2\delta(s-2m_\rho{}^2)-(a\leftrightarrow b,\ s\leftrightarrow u)\}. \quad (4.12)$$

In the above, we have used $m_{A_1}{}^2=2m_\rho{}^2$ and, also, C and D are at the point $q^2=m_\rho{}^2$. Similarly, it is straightforward to calculate the contributions of ω and ϕ pole contributions. Below, we give expressions for the ω pole. To obtain the ϕ contribution, one has to simply replace the symbol ω by ϕ. The coupling defined is

$$\langle\pi^a,k|V_\mu{}^b(0)|\omega^0,p\rangle=\frac{i\delta^{ab}}{(4p_0k_0)^{1/2}}\frac{G_{\omega\pi\rho}(q^2)}{(m_\rho{}^2-q^2)}\epsilon_{\mu\alpha\beta\gamma}k_\alpha p_\beta\epsilon_\gamma{}^\omega(p) \quad (4.13)$$

and the absorptive parts obtained are

$$\mathrm{Im}A_1{}^{(\omega)}(s,t)=\pi g_\rho{}^{-2}G_{\omega\rho\pi}{}^2[\delta^{ac}\delta^{bd}(\tfrac{1}{2}t-m_\rho{}^2)\delta(s-m_\omega{}^2)-(a\leftrightarrow b,\ s\leftrightarrow u)], \quad (4.14)$$

$$\mathrm{Im}A_2{}^{(\omega)}(s,t)=-2\pi g_\rho{}^{-2}G_{\omega\rho\pi}{}^2[\delta^{ac}\delta^{bd}(m_\pi{}^2-m_\omega{}^2-\tfrac{3}{4}t)\delta(s-m_\omega{}^2)+(a\leftrightarrow b,\ s\leftrightarrow u)], \quad (4.15)$$

$$\mathrm{Im}A_3{}^{(\omega)}(s,t)=-\pi g_\rho{}^{-2}G_{\omega\rho\pi}{}^2[\delta^{ac}\delta^{bd}(m_\rho{}^2-2m_\pi{}^2-2m_\omega{}^2+\tfrac{1}{2}t)\delta(s-m_\omega{}^2)-(a\leftrightarrow b,\ s\leftrightarrow u)], \quad (4.16)$$

$$\mathrm{Im}A_4{}^{(\omega)}(s,t)=-\pi g_\rho{}^{-2}G_{\omega\rho\pi}{}^2\{\delta^{ac}\delta^{bd}[m_\rho{}^2m_\pi{}^2-\tfrac{1}{4}t(m_\omega{}^2+m_\rho{}^2+m_\pi{}^2) +\tfrac{1}{16}t^2-\tfrac{1}{4}(m_\pi{}^2+m_\rho{}^2-m_\omega{}^2-\tfrac{1}{2}t)^2]\delta(s-m_\omega{}^2)-(a\leftrightarrow b,\ s\leftrightarrow u)\}. \quad (4.17)$$

It is easy to calculate the expressions corresponding to a definite isospin in a particular channel.

In an identical manner, it is straightforward to calculate the absorptive parts in the case of A_1-π scattering. Now, the only single-particle intermediate state possible is that with $G=+1$, namely, the ρ-meson state. On general invariance grounds, we can write

$$\langle\pi^a,k|A_\nu{}^b(0)|\rho^c,p\rangle=\frac{\epsilon^{abc}}{(4k_0p_0)^{1/2}}\frac{1}{(m_A{}^2-q^2)}[K_1(q^2)\epsilon_\nu(p)+K_2(q^2)(p-k)_\nu\epsilon(p)\cdot k+K_3(q^2)(p+k)_\nu\epsilon(p)\cdot k], \quad (4.18)$$

where $q=p-k$, and the K_i's are the form factors. We see that K_2 does not contribute since it gets multiplied by $q_\nu\epsilon_\nu^{A_1}(q)$ and the various contributions are

$$\mathrm{Im}B_1(\nu,\Delta^2)=\pi(4m_\rho^2f_\pi^2)^{-1}\epsilon^{ace}\epsilon^{bde}\{m_\rho^{-2}[K_1+\tfrac{1}{2}(m_\pi^2-m_\rho^2)K_3]^2 -K_3^2(m_\pi^2-2\Delta^2)-2K_1K_3\}\delta(\nu-\nu_\rho)-(\nu\leftrightarrow-\nu,\ a\leftrightarrow b)\,, \quad (4.19)$$

$$\mathrm{Im}B_2(\nu,\Delta^2)=\pi(2m_\rho^2f_\pi^2)^{-1}\epsilon^{ace}\epsilon^{bde}\{m_\rho^{-2}[K_1+\tfrac{1}{2}(m_\pi^2-m_\rho^2)K_3]^2-K_3^2(m_\pi^2-2\Delta^2)\}\delta(\nu-\nu_\rho)+(\nu\leftrightarrow-\nu,\ a\leftrightarrow b)\,, \quad (4.20)$$

$$\mathrm{Im}B_3(\nu,\Delta^2)=\pi(4m_\rho^2f_\pi^2)^{-1}\epsilon^{ace}\epsilon^{bde}\{m_\rho^{-2}[K_1+\tfrac{1}{2}(m_\pi^2-m_\rho^2)K_3]^2 -K_3^2(m_\pi^2-2\Delta^2)+2K_1K_3\}\delta(\nu-\nu_\rho)-(\nu\leftrightarrow-\nu,\ a\leftrightarrow b)\,, \quad (4.21)$$

$$\mathrm{Im}B_4(\nu,\Delta^2)=\pi(4m_\rho^2f_\pi^2)^{-1}\epsilon^{ace}\epsilon^{bde}[-K_1^2\delta(\nu-\nu_\rho)]-(\nu\leftrightarrow-\nu,\ a\leftrightarrow b)\,. \quad (4.22)$$

It is worthwhile to point out that the form factors C and D defined earlier and K_1, K_3 defined above all characterize the A_1-ρ-π vertex and hence are related in a trivial manner,

$$K_1=(2m_\rho^2-m_\pi^2)C+m_\rho^2D\,,\qquad K_3=2C\,, \quad (4.23)$$

all being at the point $q^2=m_\rho^2$.

V. SUBTRACTED DISPERSION RELATIONS AND CURRENT ALGEBRA

Now, we will analyze whether to write unsubtracted dispersion relations (USDR) or once-subtracted dispersion relations (OSDR) so as to reproduce the current-algebra results. First, let us assume USDR for the amplitude $A_1^{(I)}(\nu,\Delta^2)$ with Δ^2 fixed

$$A_1^{(I)}(\nu,\Delta^2)=\frac{1}{\pi}\int_{-\infty}^{+\infty}d\nu'\frac{\mathrm{Im}A_1^{(I)}(\nu',\Delta^2)}{(\nu'-\nu)} =\frac{1}{\pi}\int_0^{\infty}d\nu'\frac{\mathrm{Im}A_1^{(I)}(\nu',\Delta^2)}{(\nu'-\nu)}+\frac{1}{\pi}\int_{-\infty}^{0}d\nu'\frac{\mathrm{Im}A_1^{(I)}(\nu',\Delta^2)}{(\nu'-\nu)}\,, \quad (5.1)$$

the first integral being over the positive valence of ν' and the second one over the negative values. Also, using the crossing symmetry property, we can write

$$A_1^I(\nu,\Delta^2)=\frac{1}{\pi}\int_0^{\infty}d\nu'\,\mathrm{Im}A_1^{(I)}(\nu',\Delta^2)\left[\frac{1}{(\nu'-\nu)}+(-)^I\frac{1}{(\nu'+\nu)}\right]. \quad (5.2)$$

Feeding in the π and A_1 pole contributions obtained in Sec. IV, we obtain

$$A_1^{(0)}(\nu,\Delta^2)=\left[\frac{2F_\pi^2}{f_\pi^2m_\rho^2(\nu_\pi-\nu)}+\frac{m_\rho^2(3C+D)^2-16(m_\rho^2-\Delta^2)C^2-8m_\rho^2CD}{4f_\pi^2m_\rho^2(\nu_A-\nu)}\right]+(\nu\leftrightarrow-\nu)\,, \quad (5.3)$$

$$A_1^{(1)}(\nu,\Delta^2)=\left[\frac{F_\pi^2}{f_\pi^2m_\rho^2(\nu_\pi-\nu)}+\frac{m_\rho^2(3C+D)^2-16(m_\rho^2-\Delta^2)C^2-8m_\rho^2CD}{8f_\pi^2m_\rho^2(\nu_A-\nu)}\right]-(\nu\leftrightarrow-\nu)\,. \quad (5.4)$$

Comparing these with the current-algebra results obtained in Eqs. (3.24) and (3.25), we obtain

$$F_\pi(m_\rho^2)=\pm\tfrac{1}{4}(3-\delta)m_\rho^2\,,\qquad C(m_\rho^2)=\pm(\delta/2\sqrt{2})m_\rho\,. \quad (5.5)$$

We choose the positive sign for C and then a quadratic equation for D gives two solutions

$$D(m_\rho^2)=[(2-\delta)/2\sqrt{2}]m_\rho \quad (5.6)$$

and

$$D(m_\rho^2)=[(3\delta-2)/2\sqrt{2}]m_\rho\,. \quad (5.7\mathrm{a})$$

In the case where we choose negative sign for C, the solutions for D get an over-all negative sign, too. We notice that the solutions (5.5) and (5.6) are the usual three-point function results[19] and lead to correct predictions for $\rho\to\pi\pi$ and $A_1\to\rho\pi$ decay widths. Similarly, we find that in the case where we write USDR for the amplitudes $A_2^{(0)}(\nu,\Delta^2)$ and $A_3^{(1)}(\nu,\Delta^2)$, we obtain identical solutions as given by Eqs. (5.5) and (5.6), except that the second solution for $D(m_\rho^2)$ turns out to be different in different cases, e.g., in the case of $A_2^{(0)}(\nu,\Delta^2)$, in addition to the sum rules (5.5) and (5.6), now we get

$$D(m_\rho^2)=[(5\delta+2)/2\sqrt{2}]m_\rho\,. \quad (5.7\mathrm{b})$$

Thus, for the sake of consistency, we discard the other set of solutions for $D(m_\rho^2)$ and take it to be given by Eq. (5.6) alone. As mentioned earlier, the set

$$F_\pi(m_\rho^2)=\pm\tfrac{1}{4}(3-\delta)m_\rho^2\,,\qquad C(m_\rho^2)=\pm(\delta/2\sqrt{2})m_\rho\,,\qquad D(m_\rho^2)=\pm[(2-\delta)/2\sqrt{2}]m_\rho\,, \quad (5.8)$$

[19] See H. J. Schnitzer and S. Weinberg, Ref. 7.

leads to correct predictions[19] for the pion form factor, $\rho \rightarrow \pi\pi$ decay width and $A_1 \rightarrow \rho\pi$ decay width.

For the other amplitudes, we notice that in the current-algebra expressions (see Sec. III), we have, in addition to the s and u channels, t-channel contributions. On the other hand, since we are writing fixed-t dispersion relations, the t-channel pole is missing in the dispersion-theoretic analysis and so, accordingly, it has to be subtracted from the current-algebra expression to make a comparison. Thus, for a more meaningful comparison, it is perhaps desirable to write double dispersion relations in both the ν and Δ^2 variables simultaneously, and then to include only the pole contributions, thereby neglecting the boxlike diagrams arising from many-particle intermediate states.[20] For the present, we assume OSDR for the amplitudes $A_2^{(1)}$, $A_3^{(0)}$, and $A_4^{(0)}$. We give the details for one case, say for $A_2^{(1)}$. Writing OSDR for this amplitude,

$$A_2^{(1)}(\nu,\Delta^2)-A_2^{(1)}(\nu_0,\Delta^2) =\frac{(\nu-\nu_0)}{\pi}\int_0^\infty d\nu' \,\mathrm{Im}A_2^{(1)}(\nu',\Delta^2) \times\left[\frac{1}{(\nu'-\nu)(\nu'-\nu_0)}+\frac{1}{(\nu'+\nu)(\nu'+\nu_0)}\right]. \tag{5.9}$$

Substituting in the right-hand side of Eq. (5.9) from Eqs. (4.3)–(4.12),

$$A_2^{(1)}(\nu,\Delta^2)-A_2^{(1)}(\nu_0,\Delta^2) =(\nu-\nu_0)\left\{\frac{2F_\pi^2}{f_\pi^2m_\rho^2(\nu_\pi-\nu)(\nu_\pi-\nu_0)} +\frac{[m_\rho^2(3C+D)^2-8C^2(m_\pi^2-2\Delta^2)]}{4f_\pi^2m_\rho^2(\nu_A+\nu)(\nu_A+\nu_0)}\right\}. \tag{5.10}$$

Now we adopt the viewpoint that the subtraction constant is known from the current algebra.[6] Thus, the left-hand side in the above equation is known from Eq. (3.27), and a comparison of the two sides again gives the sum rules of Eqs. (5.8). Hence, the whole scheme appears consistent. A similar analysis holds good for the amplitudes $A_2^{(1)}$, $A_3^{(0)}$, and $A_4^{(0)}$.

Finally, we have to consider the amplitude $A_4^{(1)}(\nu,\Delta^2)$. As mentioned earlier, this amplitude is unique in that it has ν as an over-all factor. In order to remove this singular dependence, we define a new function $A_4^{(1)}(\nu,\Delta^2)=\nu a_4^{(1)}(\nu,\Delta^2)$ and write OSDR for $a_4^{(1)}(\nu,\Delta^2)$, with fixed Δ^2; the result obtained is

$$a_4^{(1)}(\nu,\Delta^2)-a_4^{(1)}(\nu_0,\Delta^2)=-(2m_\rho^2f_\pi^2)^{-1}\left\{\frac{[(2m_\rho^2-m_\pi^2)C+m_\rho^2D]^2}{(\nu_A^2-\nu^2)}-(\nu\rightarrow\nu_0)\right\}. \tag{5.11}$$

Again, comparing with the current-algebra result from Eq. (3.31), we obtain the set of solutions given in Eqs. (5.8). To sum up, we see that for ρ-π scattering, current algebra and dispersion relations lead to mutually consistent results, provided we write USDR *for the amplitudes* $A_1^{(0,1,2)}(\nu,\Delta^2)$, $A_2^{(0,2)}(\nu,\Delta^2)$, and $A_3^{(1)}(\nu,\Delta^2)$ and OSDR *for the amplitudes* $A_2^{(1)}(\nu,\Delta^2)$, $A_3^{(0,2)}(\nu,\Delta^2)$, $A_4^{(0,2)}(\nu,\Delta^2)$, and $(\nu)^{-1}A_4^{(1)}(\nu,\Delta^2)$.

We can repeat the whole procedure for the A_1-π case. Here, the energy behavior of the amplitudes is the same as in the case of ρ-π scattering. Once again, we see that current algebra and dispersion relations lead to mutually consistent results, provided we write USDR for the amplitudes $B_1^{(0,1,2)}$, $B_2^{(0,2)}$, and $B_3^{(1)}$, and OSDR for the amplitudes $B_2^{(1)}$, $B_3^{(0,2)}$, $B_4^{(0,2)}$, and $(\nu)^{-1}B_4^{(1)}$. To illustrate, we consider the amplitude $B_2^{(0)}(\nu,\Delta^2)$. A USDR gives

$$B_2^{(0)}(\nu,\Delta^2)=(m_\rho^2f_\pi^2)^{-1}\left\{\frac{\{m_\rho^{-2}[K_1+\frac{1}{2}(m_\pi^2-m_\rho^2)K_3]^2-K_3^2(m_\pi^2-2\Delta^2)\}}{(\nu_\rho-\nu)}-(\nu\leftrightarrow-\nu)\right\}. \tag{5.12}$$

Comparing with the current-algebra result, namely, Eq. (3.39), we obtain

$$K_3=\pm\delta m_\rho/\sqrt{2}, \tag{5.13}$$

$$K_1=\pm(m_\rho/\sqrt{2})[m_\rho^2-\tfrac{1}{2}(m_\pi^2-m_\rho^2)\delta]. \tag{5.14}$$

Similar solutions are obtained from USDR for the amplitudes $B_1^{(1)}$ and $B_3^{(1)}$. However, a USDR for $B_1^{(0)}(\nu,\Delta^2)$ gives in addition to Eqs. (5.13) and (5.14) another sum rule, i.e.,

$$(1+\delta)^2-4\xi=0$$

or

$$\xi=\tfrac{1}{4}(1+\delta)^2. \tag{5.15}$$

Thus, for $-1\leq\delta\leq0$,

$$0\leqslant\xi\leqslant0.25. \tag{5.16}$$

This is a very interesting sum rule, since it expresses the parameter ξ in terms of δ and hence, in a way, determines ξ.[21] Again, the whole analysis is carried out with OSDR for $B_2^{(1)}$, $B_3^{(0)}$, $B_4^{(0)}$, and $\nu^{-1}B_4^{(1)}$, leading to the aforementioned sum rules. Thus, we have clearly demonstrated that we can obtain mutually consistent results from dispersion relations and the algebra of currents, provided we introduce a subtraction in some

[20] Recently we have attempted to combine double dispersion relations with current algebra to obtain momentum-dependent physical axial-vector form factors for the K_{l4} decays, S. N. Biswas, R. Dutt, and K. G. Gupta, Ann. Phys. (N. Y.) (to be published).

[21] See also I. S. Gerstein and H. J. Schnitzer in Ref. 7.

of the invariant amplitudes of the problem. Finally, we see that in the above analysis we have not included the ω and ϕ contributions, chiefly because no such contributions are present in the current-algebra part. Obviously, if one includes these contributions in the dispersion integrals, one gets, for the sake of consistency, $g_{\omega\rho\pi}^2 \approx g_{\phi\rho\pi}^2 \approx 0$. However, it is not possible to deduce anything from this unless one incorporates the ω and ϕ states in the chiral-algebra scheme.[22]

VI. CONCLUDING REMARKS

We have calculated ρ-π and A_1-π scattering amplitudes both from the algebra of currents and dispersion relations with a subtraction. We have shown that in case the subtraction constant is taken from the current algebra, both the calculations lead to similar results. In addition, we have also determined the parameter ξ in terms of another inherent parameter δ (Eq. 5.15). This is an interesting sum rule and perhaps reflects the closure property of the A_1-ρ-π system.

Now, we would like to comment on our results in the light of Regge pole theory.[23] From Regge pole theory (without fixed poles), one expects that for fixed t and $\nu \to \infty$,

$$A_1{}^I(\nu,t) \sim \sum_i \gamma_{1i}(t)\nu^{\alpha_i{}^I(t)-2},$$

$$A_2{}^I(\nu,t) \sim \sum_i \gamma_{2i}(t)\nu^{\alpha_i{}^I(t)-1},$$

and

$$A_{3,4}{}^I(\nu,t) \sim \sum_i \gamma_{3,4i}(t)\nu^{\alpha_i{}^I(t)},$$

where $\gamma_{ij}(t)$ are the Regge residue functions and $\alpha_i{}^I(t)$ are the Regge trajectory parameters (see Ref. 10). Clearly, in case we take $0 \leq \alpha_i{}^I(0) \leq 1$, we find contradiction, in most cases, with the conclusions reached in the paragraph following Eq. (5.11) about the subtractions needed in the dispersion relations for the amplitudes $A_i{}^I(\nu,t)$. For an example, let us consider the amplitudes $A_i{}^1(\nu,t)$. From the high-energy behavior given by Regge pole theory, we expect that $\nu A_1{}^1(\nu,t)$ and $A_2{}^1(\nu,t)$ should satisfy USDR (thus $A_1{}^1$ can satisfy a superconvergent dispersion relation). If we do assume such a ν behavior and saturate the dispersion integrals by π and A_1 states only, the results obtained are much different from those given by the hard-pion current algebra. Now, an obvious question arises, namely: Is the saturation complete? We know that the ω contribution to ρ-π scattering is substantial and there may be other higher-lying states too. So far, our viewpoint has been that since in the current-algebra scheme only π, A_1, and ρ states have been included, we should do likewise in the dispersion-relations calculation. However, we notice that in the current-algebra calculation, because of the nonlinearity of the commutation relations, a contribution comes in which, in effect, plays the role of a subtraction constant. Also, we know that a subtraction constant is, in general, presumed to take care of the contributions coming from higher states. Thus, perhaps the observed anomaly between the current-algebra results and the Regge high-energy behavior for the invariant amplitudes can be resolved by including more states in the dispersion integrals, thereby effectively absorbing the subtraction constant in the dispersion integrals. So far as the A_1-π scattering amplitudes $B_i{}^I(\nu,t)$ are concerned, their high-energy behavior is similar to that of $A_i{}^I(\nu,t)$ and the above arguments hold good for these also.[24]

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[22] Actually $g_{\omega\rho\pi}$ is fairly large and one does not expect it to result from some kind of chiral-symmetry breaking and so this aspect is a bit puzzling; for further comments see D. A. Geffen and S. Gasiorowicz, Ref. 8.

[23] So far as the Regge pole theory is concerned, ρ-π and A_1-π problems are identical and a comprehensive account can be found in F. J. Gilman and H. Harari, Phys. Rev. **165**, 1803 (1968)

[24] After the completion of this manuscript, we noticed a relevant paper of ρ-π scattering; S. G. Brown and G. B. West, Phys. Rev. **174**, 1786 (1968).