TABLE II. $\langle r^2\rangle_\rho$ under the assumption of a t-channel σ-exchange $\pi\pi$ interaction.

m_σ (MeV)	650	765	900
$\langle r^2\rangle_\rho$ (F^2)	-0.0008	0.0004	-0.0004

stituting our expression into Eq. (4) results in an integral which converges and can be evaluated exactly. (The coupling constant $g_{\sigma\pi\pi}$ is determined from the decay $\sigma\rightarrow\pi\pi$ under the assumption of a decay width of 100 MeV.)

The continuum contribution to the pion radius is tabulated in F^2 for three possible values of the σ mass m_σ to give an idea of the variation of the continuum contribution as a function of the σ mass. (See Table II). Continuum contributions to the pion radius are negligible in this case and not inconsistent with zero when one considers the accuracy of this calculation.

ACKNOWLEDGMENTS

The author wishes to thank Dr. M. A. B. Bég and Dr. Ivan Muzinich for their advice and guidance.

PHYSICAL REVIEW VOLUME 178, NUMBER 5 25 FEBRUARY 1969

Decay Rate of $\Sigma^+\rightarrow p+\gamma$ from Unsubtracted Dispersion Relations*

G. M. PAPAIOANNOU†

Department of Physics, University of Illinois, Urbana, Illinois 61801

(Received 4 October 1968)

Assuming unsubtracted dispersion relations, we set up integral representations for the weak photonic amplitudes F_1 and F_2. The integrals involve the magnetic and electric dipole amplitudes M_{1-} and E_{0+} of the photo-pion production and the pion-nucleon scattering phase shifts of the states P_{11}, P_{31}, S_{11}, and S_{31}. The integrals were carried out on a computer and the relative decay rate of $\Sigma^+\rightarrow p\gamma$ to its total rate was found to be 0.71×10^{-3}, which is comparable with the latest experimental value $(1.9\pm0.4)\times10^{-3}$.

I. INTRODUCTION

THE present investigation is based on the hypothesis of unsubtracted dispersion relations. This hypothesis was first thought of and applied by Goldberger and Treiman[1] in their investigation of the form factors a and b in β decay and μ capture. They assumed a once-subtracted dispersion relation for the form factor a and an unsubtracted one for the form factor b. Nishijima and co-workers[2] extended the no-subtraction hypothesis to all weak amplitudes involved in a weak process. By means of unsubtracted dispersion relations, Goldberger and Treiman, who were the first to apply the dispersion theory to weak interactions, calculated the π-μ decay and reproduced the experimental lifetime of the charged pion. Nishijima and co-workers succeeded in giving a dynamical derivation of the selection rule $|\Delta I|=\frac{1}{2}$ for the nonleptonic decays of strange particles. They also combined the assumption of unsubtracted dispersion relations with unitary symmetry to offer a unified interpretation of the Goldberger-Treiman relation, Gell-Mann–Okubo, and Coleman-Glashow mass formulas, and the Cabibbo theory of semileptonic interactions.

The investigation of the present problem was initiated by Kawaguchi and Nishijima[3] and by Iso and Kawaguchi.[4] They analyzed the photon mode of the hyperon decay by use of the unitarity of the S matrix. Behrends[5] also investigated the same problem. He constructed on invariance grounds the general matrix element of the photon mode of decay of the hyperon and wrote the lifetime and angular distributions of the decay products in terms of the three physical parameters of the problem. In addition, he gave an estimate of these parameters and of the branching ratios (photon mode to neutral pion mode) by use of perturbation theory. On the basis of these estimates he suggested the possibility of detecting the mode experimentally. Indeed, this turned out to be the case. More recently the same problem was investigated by Iwao.[6] He made use of the Suzuki-Sugawara Hamiltonian of the weak hadronic decay and obtained for the relative decay rate of $\Sigma^+\rightarrow p\gamma$ to its total rate 2.7×10^{-4} which was in good agreement with the old experimental value $(3.7\pm0.8)\times10^{-4}$.

* Supported by the National Science Foundation under Grant No. NSF GP-9273.

† On leave of absence from the Nuclear Research Center "Demokritos," Athens, Greece.

[1] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958); **111**, 354 (1958); Nuovo Cimento **9**, 451 (1958).

[2] E. R. McCliment and K. Nishijima, Phys. Rev. **128**, 1970 (1962); K. Nishijima, *ibid.* **133**, B1092 (1964); K. Nishijima and M. H. Saffouri, Progr. Theoret. Phys. (Kyoto) Suppl. (commemoration issue), 207 (1965); K. Nishijima and L. J. Swank, Phys. Rev. **146**, 1161 (1966); K. Nishijima, *ibid.* **157**, 1459 (1967).

[3] M. Kawaguchi and K. Nishijima, Progr. Theoret. Phys. (Kyoto) **15**, 182 (1956).

[4] G. Iso and M. Kawaguchi, Progr. Theoret. Phys. (Kyoto) **16**, 177 (1956).

[5] R. E. Behrends, Phys. Rev. **111**, 1691 (1958).

[6] S. Iwao, Helv. Phys. Acta **40**, 239 (1967).

The motive for reinvestigating the same problem arose from the difference in the value of the branching ratio $R((\Sigma^+ \to p\gamma)/\Sigma^+_{\text{tot}})$ between the old experiments of Carrara *et al.*[7] and of Burnstein *et al.*[8] and the new ones by Bazin *et al.*[9] and by Quareni *et al.*[10] The new value of the branching ratio is $(1.9\pm0.4)\times10^{-3}$, as given by Rosenfeld *et al.*[11]

Our method of investigating the problem is different from the methods used by the authors above. It is, of course, based on the unitarity of the S matrix which was used by the authors of Refs. 3 and 4, but the general method of resolving and analyzing the problem is different. We have arrived at our theoretical result, which is comparable with the experimental one, by using a model depending to a good extent upon experiment. The representations that we have derived relate the photonic weak decay amplitudes to integrals over known quantities; thus an accurate numerical evaluation of these integrals can be carried out. The physical processes upon which our model depends are the nonleptonic weak decay of Σ^+ and the pion photoproduction process. Both of them played a key role in our investigation and have been investigated experimentally. Finally, the general arguments valid for all invariant amplitudes treated by means of dispersion relations hold for ours as well. The arguments are that they vanish for very large values of energy and have the proper analytic properties, so that the dispersion relations they satisfy exist. A word about the order of presentation of our topics is given. In Sec. II we write down the dispersion relations for the photonic amplitudes F_1 and F_2. In Sec. III we evaluate the absorptive parts of F_1 and F_2 and set up their integral representations. In Sec. IV we write down the formula for the decay rate of Σ^+ and give its numerical evaluation. In the same section we also present some concluding remarks.

II. DISPERSION RELATIONS

The decay of the hyperon Σ^+ into a proton and a photon,

$$\Sigma^+ \to p+\gamma\,, \tag{1}$$

is assumed to be described by the amplitude[12]

$$\bar{u}_p J_{\Sigma^+}(s) u_{\Sigma^+} = (p_0/m)^{1/2}(2k)^{1/2}\langle pk\text{out}|\,\bar{j}_{\Sigma^+}(0)\,|0\rangle u_{\Sigma^+}\,, \tag{2}$$

where $s=-p_{\Sigma^+}{}^2$ is the square of the c.m. energy, p_0 is the final-state proton energy, k is the final-state photon energy, and m is the proton mass. $\bar{j}_{\Sigma^+}$ is the source of the Σ^+ field and is defined by

$$\bar{j}_{\Sigma^+}(x) = (-\gamma_\mu{}^T\partial/\partial x_\mu + M)\bar{\Psi}_{\Sigma^+}(x)\,. \tag{3}$$

Here M is the mass of Σ^+; u_p and u_{Σ^+} are the Dirac spinor functions describing the proton and hyperon and they are invariantly normalized according to $\bar{u}_p u_p = \bar{u}_{\Sigma^+}u_{\Sigma^+} = 1$. They satisfy the Dirac equations $(i\gamma\cdot p+m)u_p=0$ and $(i\gamma\cdot P+M)u_{\Sigma^+}=0$. We have chosen throughout for the Dirac γ matrices the Pauli representation. We write $\beta\alpha_k = i\gamma_k$ $(k=1,2,3)$, $\beta=\gamma_4$, and $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu}$. Finally, our metric is such that $p^2=\mathbf{p}^2-p_0{}^2$.

Now from the general invariance arguments, i.e., Lorentz and gauge invariances, J must have the form[5]

$$J_{\Sigma^+}(s) = [F_1(s)+i\gamma_5 F_2(s)]\gamma\cdot k\gamma\cdot\epsilon\,, \tag{4}$$

where we have assumed explicitly that J is considered to be a function of s for fixed $k^2=0$. Application of the Lehmann-Symanzik-Zimmermann (LSZ)[13] reduction formula to Eq. (2) yields, after neglecting an equal-time commutator,

$$\bar{u}_p J_{\Sigma^+}(s) u_{\Sigma^+} = i\left(\frac{p_0}{m}\right)^{1/2}\int d^4x\, e^{-ik\cdot x}\epsilon_\mu{}^{(\lambda)}(k) \times\langle p|\,T(j_\mu(x)\bar{j}_{\Sigma^+}(0))\,|0\rangle u_{\Sigma^+}\,, \tag{5}$$

where $T(\ )$ denotes the Wick product, $\epsilon_\mu{}^{(\lambda)}(k)$ is the polarization vector of the photon of helicity λ and energy-momentum k, and j_μ is the nucleon electromagnetic current. By making use of the expression

$$T(j_\mu(x)\bar{j}_{\Sigma^+}(0)) = [j_\mu(x),\bar{j}_{\Sigma^+}(0)]\theta(x) + \bar{j}_{\Sigma^+}(0)j_\mu(x) \tag{6}$$

and observing that the second term makes no contribution to the matrix element of the physical Σ^+ decay, we may write Eq. (5) in the form of the retarded commutator:

$$\bar{u}_p J_{\Sigma^+}(s) u_{\Sigma^+} = i\left(\frac{p_0}{m}\right)^{1/2}\int d^4x\, e^{-ik\cdot x}\theta(x) \times\langle p|\,[\epsilon_\mu j_\mu(x),\bar{j}_{\Sigma^+}(0)]\,|0\rangle u_{\Sigma^+}\,. \tag{7}$$

Here $\theta(x)$ is the step function (which vanishes for $x_0<0$ and is unity for $x_0>0$) and we have simply written ϵ_μ instead of $\epsilon_\mu{}^{(\lambda)}(k)$. The remainder of our discussion is based on Eq. (7). We shall use Eq. (7) to investigate the invariant amplitudes F_1 and F_2. We shall write down the unsubtracted dispersion relations for these invariant functions without proof.

$$F_{1,2}(s) = \frac{1}{\pi}\int_{(m+\mu)^2}^{\infty} ds'\frac{\text{Im}F_{1,2}(s')}{s'-s-i\epsilon}\,. \tag{8}$$

We should emphasize, however, that the properties of F_1 and F_2 which are required by Eq. (8) are that F_1

[7] R. Carrara, M. Cresti, A. Grigoletto, S. Limentani, L. Peruzzo, R. Santangelo, and R. B. Willman, Phys. Letters **12**, 72 (1964).

[8] R. A. Burnstein, T. B. Day, F. Martin, M. Sakitt, R. G. Glasser, N. Seeman, and A. Y. Herz, Phys. Rev. Letters **10**, 307 (1963).

[9] M. Bazin, H. Blumenfeld, V. Nauenberg, L. Seidlitz, and C. Y. Chang, Phys. Rev. Letters **14**, 154 (1965).

[10] G. Quareni, A. Quareni Vignudelli, A. M. Cartacci, M. G. Dagliana, M. Della Corte, G. Tomasini, A. Gainotti, C. Lamborizio, S. Mora, and I. Ortalli, Nuovo Cimento **40A**, 928 (1965).

[11] A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, Mattas Roos, Paul Söding, and C. G. Wohl, University of California Radiation Laboratory Report No. UCRL-8030, 1968 (unpublished).

[12] E. R. McCliment and K. Nishijima, Phys. Rev. **128**, 1970 (1962).

[13] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **2**, 425 (1955).

and F_2 be analytic everywhere in the complex s plane with a branch cut along the positive real axis from $s=(m+\mu)^2$ to ∞. We also require that F_1 and F_2 vanish for large values of s. One might also expect that a reality condition $F_{1,2}(s)=[F_{1,2}(s^*)]^*$ would hold for these invariant functions which satisfy dispersion relations of the type given by Eq. (8).

In order to calculate the absorptive parts of F_1 and F_2 we go back to Eq. (7); we write in the usual way $\bar{u}_p J_{\Sigma^+} u_{\Sigma^+} \equiv M = D + iA$ and identify A, the absorptive part, with the contribution from the term $\frac{1}{2}$ in the expression of $\theta(x_0)=\frac{1}{2}+\frac{1}{2}\epsilon(x_0)$. We then obtain

$$A(s)=\frac{1}{2}\left(\frac{p_0}{m}\right)^{1/2}\int d^4x\, e^{-ik\cdot x}\langle p|[\epsilon_\mu j_\mu(x),\bar{j}_{\Sigma^+}(0)]|0\rangle u_{\Sigma^+}, \quad (9)$$

which we find from time-reversal invariance may be written

$$A(s)=\bar{u}_p[\mathrm{Im}F_1(s)+i\gamma_5\,\mathrm{Im}F_2(s)]\gamma\cdot k\gamma\cdot\epsilon u_{\Sigma^+}. \quad (10)$$

If we insert in Eq. (9) a complete set of intermediate states, use translational invariance, and carry out the space-time integrations, we obtain

$$A(s)=\frac{1}{2}\left(\frac{p_0}{m}\right)^{1/2}(2\pi)^4\sum_n\langle p|\epsilon_\mu j_\mu(0)|n\rangle \times\langle n|\bar{j}_{\Sigma^+}(0)|0\rangle u_{\Sigma^+}\delta^4(p_n-p_{\Sigma^+}), \quad (11)$$

where p_n is the energy momentum of the intermediate physical state $|n\rangle$.

Equation (11) shows that the imaginary parts of F_1 and F_2 appear as a sum of contributions from a complete set of states $|n\rangle$ which may be conveniently chosen as "in" or "out" states.

In addition, Eq. (11) is a relativistically invariant one holding in an arbitrary Lorentz frame, a fact which allows us to carry out our dynamical computations in any desired frame of reference. Later we shall state explicitly the Lorentz frame in which we evaluate the absorptive amplitude $A(s)$.

We also notice that in order to ensure reality of $A(s)$ in all stages of the approximation, use is made of the Goldberger-Treiman trick by writing the sum over the complete set of states $|n\rangle$ as half the sum over out plus in states.

We realize the difficulties which we are to be confronted with in an attempt to evaluate the contributions to the imaginary parts of F_1 and F_2 coming from all intermediate states allowed by a group of selection rules. We must make use of physical arguments to single out the important and physically tractable intermediate states. We need consider only states of positive unit charge and unit baryon number.

We shall therefore proceed on the assumption that only the intermediate state $|N\pi\rangle$ need be considered. We believe that the pion-nucleon intermediate state, at low energies at least, is the most important contributor to the amplitudes F_1 and F_2. In addition, retaining just this state provides us with a definite model which can be evaluated and compared with experiment. The intermediate state of a nucleon and a photon $|N\gamma\rangle$ could also be considered but its contribution's being smaller by a factor of $\sim 1/137$ (second order in the electromagnetic coupling) than that of the pion-nucleon intermediate state allows us to neglect it. We omitted single-baryon intermediate states since they do not contribute to F_i when the initial hyperon is on the mass shell.[12] We have also excluded intermediate states consisting of a nucleon and two pions and a strange particle and a pion.

Consequently, keeping just the pion-nucleon intermediate state we easily pass from Eq. (11) to

$$A(s)=\frac{1}{2}\left(\frac{p_0}{m}\right)^{1/2}(2\pi)^4\sum_{\alpha'}\int\frac{d^3p'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3} \times\langle p|\epsilon_\mu j_\mu(0)|N\pi(p',q';\alpha')\rangle \times\langle N\pi(p',q';\alpha')|\bar{j}_{\Sigma^+}(0)|0\rangle u_{\Sigma^+}\delta^4(p'+q'-P_{\Sigma^+}), \quad (12)$$

where $\sum_{\alpha'}$ refers to the sum over the discrete quantum numbers (spin and isotopic spin) of the physical $|N\pi\rangle$ system, whose corresponding four-momenta are p' and q'.

Observation of Eq. (12) reveals that we have succeeded in expressing the absorptive parts of F_1 and F_2 as a product of two experimentally known matrix elements.

One is the matrix element corresponding to the nonleptonic weak decay of the hyperon Σ^+ and the other is the matrix element of the inverse photoproduction process. We may therefore take them over directly from experiment.

The weak-process matrix element has, on general invariance grounds, the representation

$$\langle N\pi(p'q')|\bar{j}_{\Sigma^+}(0)|0\rangle u_{\Sigma^+} =(m/p_0'2q_0')^{1/2}\bar{u}_N(A+\gamma_5 B)u_{\Sigma^+}, \quad (13)$$

where A is the parity-violating S-wave amplitude and B is the parity-conserving P-wave amplitude. Both of them are energy- and isotopic-spin-dependent and satisfy unsubtracted dispersion relations of the form

$$A^{(I)}(s)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\frac{\mathrm{Im}A^{(I)}(s')}{s'-s-i\epsilon} \quad (14)$$

and

$$B^{(I)}(s)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\frac{\mathrm{Im}B^{(I)}(s')}{s'-s-i\epsilon}. \quad (15)$$

Here I denotes the total isotopic spin of the pion-nucleon system and takes the values $\frac{1}{2}$ and $\frac{3}{2}$. As will be analyzed below, the total isotopic spin eigenamplitudes $A^{(I)}$ and $B^{(I)}$ will be given in terms of the amplitudes $A^{(+,0)}$ and $B^{(+,0)}$, which for zero values of the energy s are known experimentally. The superscripts $(+)$ and (0) refer to the decay of Σ^+ into a neutron and a π^+ and into a proton and a π^0, respectively.

The absorptive parts of $A^{(I)}$ and $B^{(I)}$ are again obtained by use of unitarity which allows these amplitudes to be again expressed in terms of the pion-nucleon scattering amplitudes and the nonleptonic weak amplitudes, both of which are known experimentally. Consequently, applying unitarity and considering again just the pion-nucleon intermediate state we obtain

$$\mathrm{Im}A^{(I)}(s)=qf_1^{(I)}(s)A^{(I)}(s)\,, \tag{16}$$

$$\mathrm{Im}B^{(I)}(s)=qf_2^{(I)}(s)B^{(I)}(s)\,. \tag{17}$$

Here I stands for the total isotopic spin and takes the values $\frac{1}{2}$ and $\frac{3}{2}$, and q is the c.m. momentum. Finally, the c.m. amplitudes f_1 and f_2 are given by

$$f_1=\sum_{l=0}^{\infty} f_{l+}P_{l+1}'(x)-\sum_{l=2}^{\infty} f_{l-}P_{l-1}'(x)\,, \tag{18}$$

$$f_2=\sum_{l=1}^{\infty} (f_{l-}f_{l+})P_l'(x)\,, \tag{19}$$

where $f_{l\pm}$ is the scattering amplitude in the state of parity $-(-1)^l$ and total angular momentum $j=l\pm\frac{1}{2}$. $P_l'(x)$ is the first derivative of the conventionally normalized Legendre polynomials and

$$f_{l\pm}=(e^{i\delta_{l\pm}}\sin\delta_{l\pm})/q\,, \tag{20}$$

where $\delta_{l\pm}$ is the phase shift in the state $l\pm$. Here $\delta_{l\pm}$ is a function of the c.m. variable s, and is slightly complex but will be taken to be real.

Since only $l=0$ and $l=1$ (S and P) amplitudes contribute, Eqs. (16) and (17) with the help of Eqs. (18)–(20) give

$$\mathrm{Im}A^{(2I)}=e^{i\delta_{2I,2J}}\sin\delta_{2I,2J}A^{(2I)}\,, \tag{21}$$

$$\mathrm{Im}B^{(2I)}=e^{i\delta_{2I,2J}}\sin\delta_{2I,2J}B^{(2I)}\,, \tag{22}$$

where J is the total angular momentum taking only the value $\frac{1}{2}$, and in addition we have slightly changed the notation.

The fact that we are considering just the pion-nucleon intermediate-state contribution allows us to write

$$[\mathrm{Im}A^{(2I)}]_{|N\pi>}=[\mathrm{Im}f_{2I,2J}\,\mathrm{Im}A^{(2I)}+\mathrm{Re}f_{2I,2J}\,\mathrm{Re}A^{(2I)}]\times\theta[s-(m+\mu)^2]\,, \tag{23}$$

with an identical equation for B.

In Eq. (23) the left-hand side denotes the pion-nucleon contribution to $\mathrm{Im}A$, whereas the right-hand side involves the true coefficient A. The step function is inserted to remind us that the pion-nucleon state contributes only for values of the energy s corresponding to physical scattering $s>(m+\mu)^2$.

Since we are considering just the pion-nucleon contribution we can write $[\mathrm{Im}A]_{|N\pi>}=\mathrm{Im}A$, and therefore Eq. (23) reduces to

$$\mathrm{Im}A^{(2I)}(s)=\tan\delta_{2I,2J}(s)\,\mathrm{Re}A^{(2I)}(s)\theta[s-(m+\mu)^2]\,, \tag{24}$$

where

$$\tan\delta_{2I,2J}(s)=\frac{\mathrm{Re}e^{i\delta_{2I,2J}}\sin\delta_{2I,2J}}{1-\mathrm{Im}e^{i\delta_{2I,2J}}\sin\delta_{2I,2J}}\,. \tag{25}$$

We can now substitute the pion-nucleon state contribution into the dispersion relations (14) and (15) and, treating these as integral equations, solve for A and B. The solutions are readily obtained if we use the Omnès[14] method. We obtain

$$A^{(2I)}(s)=A^{(2I)}(0)\exp\left(\frac{s}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\frac{\delta_{2I,2J}(s')}{s'(s'-s-i\epsilon)}\right)\,, \tag{26}$$

with an identical solution for B.

We now return to the other factor

$$\langle p|\epsilon_\mu j_\mu(0)|N\pi(p'q'\cdot\alpha')\rangle\,,$$

which is the matrix element for the inverse photoproduction process and which from invariance principles again may be written as[15]

$$\langle p|\epsilon_\mu j_\mu(0)|N\pi(p'q')\rangle=(m^2/p_0p_0'2q_0')^{1/2}\bar{u}_p(ps)\times[M_1A_1+M_2A_2+M_3A_3+M_4A_4]u_N(p's')\,, \tag{27}$$

where the fundamental forms of $M_1,\cdots,M_4$ are those of Chew-Goldberger-Low-Nambu (CGLN)[16]:

$$M_1=i\gamma_5\gamma\cdot\epsilon\gamma\cdot k\,, \tag{28}$$

$$M_2=i\gamma_5[(p+p')\cdot\epsilon q'\cdot k-(p+p')\cdot kq'\cdot\epsilon]\,, \tag{29}$$

$$M_3=\gamma_5[\gamma\cdot\epsilon q'\cdot k-\gamma\cdot kq'\cdot\epsilon]\,, \tag{30}$$

$$M_4=\gamma_5[\gamma\cdot\epsilon(p+p')\cdot k-\gamma\cdot k(p+p')\cdot\epsilon-im\gamma\cdot\epsilon\gamma\cdot k]\,. \tag{31}$$

The quantities A_1–A_4 are invariant functions of the energy and isotopic spin and the last dependence is made explicit by writing

$$A_i=A_i^{(+)}\delta_{\alpha 3}+A_i^{(-)}\tfrac{1}{2}[\tau_\alpha,\tau_3]+A_i^{(0)}\tau_\alpha,\quad (i=1,\cdots,4) \tag{32}$$

where the τ_α's are the Pauli spin matrices and α is the isotopic spin index of the pion.

The isovector transition amplitudes $A_i^{(+,-)}$ may be expressed in terms of the amplitudes $A_i^{(1,3)}$ corresponding to the values $\frac{1}{2}$ and $\frac{3}{2}$ of the total isotopic spin I in the final state.

$$A_i^{(1)}=A_i^{(+)}+2A_i^{(-)}\,, \tag{33}$$

$$A_i^{(3)}=A_i^{(+)}-A_i^{(-)}\,. \tag{34}$$

---

[14] R. Omnès, Nuovo Cimento 8, 316 (1958).

[15] Here the matrix element corresponding to the inverse photo-pion-production is the time-reversed matrix element of the direct process. We shall not deal explicitly with the time-reversal operation. Its over-all effect is, when we evaluate the real and imaginary parts of the amplitudes F_1 and F_2 via the representations (59)–(62) and Eq. (24), to change i into $-i$ and that is all. This is so because we consider the time-reversal operation to be equivalent to the complex conjugation.

[16] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

We should also keep in mind that the isoscalar transition amplitudes $A_i^{(0)}$ always lead to final states with isotopic spin $\frac{1}{2}$.

III. EVALUATION OF ABSORPTIVE AMPLITUDE

Before substituting Eqs. (13) and (27) into (12) in order to effect its evaluation, it is appropriate to discuss the sum over the spins and isospins of the pion-nucleon system. The spin summation is easily carried out, giving

$$\sum_{s'} u_N(p's')\bar{u}_N(p's')=\frac{m-i\gamma\cdot p'}{2m}. \qquad (35)$$

The isotopic spin summation is also carried out easily if we take into consideration the fact that the initial and final states have positive unit charge; consequently, from charge conservation the intermediate physical states entering are the $|p\pi^0\rangle$ and $|n\pi^+\rangle$ states. In what follows we shall not specify the charges, but instead shall proceed with our calculations by making use of the isotopic spin variables in our sum over states. In other words, what we shall do is to express, by using the Clebsch-Gordan coefficients, the nonleptonic weak amplitudes A and B in terms of eigenamplitudes of total isotopic spin through the relations

$$A^{(+)}(\Sigma^+\to n+\pi^+)=\tfrac{1}{3}(A^{(3)}+2A^{(1)}), \qquad (36)$$

$$A^{(0)}(\Sigma^+\to p+\pi^0)=\tfrac{1}{3}\sqrt{2}(-A^{(3)}+A^{(1)}), \qquad (37)$$

with identical relations for B.

We can do the same for the invariant amplitudes corresponding to these particular photoproduction processes to obtain

$$A_1(p+\gamma\to n+\pi^+)=\sqrt{2}(A_1^{(0)}+A_1^{(-)}) \qquad (38)$$

and

$$A_1(p+\gamma\to p+\pi^0)=(A_1^{(0)}+A_1^{(+)}), \qquad (39)$$

with similar relations for the rest of the amplitudes A_2, A_3, and A_4.

Since we wish ultimately to evaluate the right-hand side of Eq. (12) in the c.m. system of the photoproduction process which is also the rest frame of Σ^+, it is necessary that we reduce Eq. (12) to such a form which explicitly exhibits the dependence of the photoproduction process on the c.m. parameters, such as the c.m. amplitudes and the c.m. angle specifying the direction of the intermediate meson. In addition, in order to effect this reduction we must pass from the Dirac matrices and spinors to the Pauli matrices and two-component spinors defined by

$$u(ps)=\frac{1}{[2m(E+m)]^{1/2}}\begin{pmatrix}E+m\\ \boldsymbol{\sigma}\cdot\mathbf{p}\end{pmatrix}\chi(s),$$

where $\chi(s)$ represents the Pauli spinors which have the z axis as the quantization axis.

Consequently, in order to use the c.m. dependence of the photoproduction process, we introduce a quantity $\mathfrak{F}$ by means of the relation

$$\bar{u}(ps)\sum_{i=1}^{4} A_iM_iu(p's')=4\pi\frac{W}{m}\chi^\dagger(s)\mathfrak{F}\chi(s'), \qquad (40)$$

where W is the c.m. energy and the matrix $\mathfrak{F}$ is defined from Eq. (40) to be

$$\mathfrak{F}=i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathfrak{F}_1+\frac{\boldsymbol{\sigma}\cdot q'\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})}{q'k}\mathfrak{F}_2+\frac{i\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}'\cdot\boldsymbol{\varepsilon}}{kq'}\mathfrak{F}_3+\frac{i\boldsymbol{\sigma}\cdot\mathbf{q}'\mathbf{q}'\cdot\boldsymbol{\varepsilon}}{q'^2}\mathfrak{F}_4, \qquad (41)$$

where $\mathfrak{F}_1,\cdots,\mathfrak{F}_4$ are functions of energy and angle in the c.m. system and q' and k are the pion and photon three-dimensional momenta.

The angular dependence of $\mathfrak{F}_1,\cdots\mathfrak{F}_4$ may be made explicit by means of an expansion involving derivatives of Legendre polynomials.

$$\mathfrak{F}_1=\sum_{l=0}^{\infty}[lM_{l+}+E_{l+}]P_{l+1}'(x)+[(l+1)M_{l-}+E_{l-}]P_{l-1}'(x), \qquad (42)$$

$$\mathfrak{F}_2=\sum_{l=1}^{\infty}[(l+1)M_{l+}+lM_{l-}]P_l'(x), \qquad (43)$$

$$\mathfrak{F}_3=\sum_{l=1}^{\infty}[E_{l+}-M_{l+}]P_{l+1}''(x)+[E_{l-}+M_{l-}]P_{l-1}''(x), \qquad (44)$$

$$\mathfrak{F}_4=\sum_{l=1}^{\infty}[M_{l+}-E_{l+}-M_{l-}-E_{l-}]P_l''(x). \qquad (45)$$

Here x is the cosine of the polarization angle which, together with the aximuthal, specifies completely the direction of the three-momentum of the intermediate pion, given by

$$\mathbf{q}'=q'(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta).$$

The energy-dependent amplitudes $M_{l\pm}$ and $E_{l\pm}$ refer to transitions initiated by magnetic and electric radiation, respectively, leading to final states of orbital angular momentum l and total angular momentum $l\pm\frac{1}{2}$. Superscripts ($\pm$, 0) may be added to each quantity in formulas (41)–(45) in order to designate the isotopic spin character of the transition.

Finally, from conservation of the total angular momentum and the selection rule $|\Delta I|=\frac{1}{2}$, the states we expect to be allowed for the pion-nucleon system are the ones characterized by $I=\frac{1}{2}$, $J=\frac{1}{2}$ and $I=\frac{3}{2}$, $J=\frac{1}{2}$.

In other words, the states in question are the P_{11}, P_{31} and S_{11}, S_{31} states. Having thus outlined the general procedure, we come back to Eq. (12), which, after we substitute into it Eqs. (13) and (27) and carry out the isotopic spin summation of the pion-nucleon system, becomes

$$A(s)=\frac{\sqrt{2}m}{3(2\pi)^2}\sum_{s'}\int d^4p'd^4q'\theta(p_0')\theta(q_0')\delta(p'^2+m^2)\delta(q'^2+\mu^2)\delta^4(p'+q'-P_{\Sigma^+})$$
$$\times\{\bar{u}(ps)[(A_1{}^{(+)}M_1+A_2{}^{(+)}M_2+A_3{}^{(+)}M_3+A_4{}^{(+)}M_4)+(A_1{}^{(0)}M_1+A_2{}^{(0)}M_2+A_3{}^{(0)}M_3+A_4{}^{(0)}M_4)]$$
$$\times u(p's')\bar{u}(p's')[(A^{(1)}+\gamma_5B^{(1)})-(A^{(3)}+\gamma_5B^{(3)})]u_{\Sigma^+}$$
$$+\bar{u}(ps)[(A_1{}^{(-)}M_1+A_2{}^{(-)}M_2+A_3{}^{(-)}M_3+A_4{}^{(-)}M_4)+(A_1{}^{(0)}M_1+A_2{}^{(0)}M_2+A_3{}^{(0)}M_3+A_4{}^{(0)}M_4)]$$
$$\times u(p's')\bar{u}(p's')[2(A^{(1)}+\gamma_5B^{(1)})+(A^{(3)}+\gamma_5B^{(3)})]u_{\Sigma^+}\}\,, \quad (46)$$

where we have added two one-dimensional mass-shell δ functions in order to pass from the three-dimensional integrations to four dimensions, and

$$\theta(p_0)=1 \quad (p_0>0)$$
$$=0 \quad (p_0<0).$$

Using Eq. (40) and effecting the spin summation and the δ-function integrations by going to the rest frame of the Σ^+ particle, we obtain

$$A(s)=\frac{q'}{12\pi[m(E+m)]^{1/2}}\int d\Omega(q')\chi^\dagger(s)[(E+m)(\mathfrak{F}^{(+)}+2\mathfrak{F}^{(-)}+3\mathfrak{F}^{(0)})A^{(1)}-(\mathfrak{F}^{(+)}+2\mathfrak{F}^{(-)}+3\mathfrak{F}^{(0)})B^{(1)}\boldsymbol{\sigma}\cdot\mathbf{q}'$$
$$-(E+m)(\mathfrak{F}^{(+)}-\mathfrak{F}^{(-)})A^{(3)}+(\mathfrak{F}^{(+)}-\mathfrak{F}^{(-)})B^{(3)}\boldsymbol{\sigma}\cdot\mathbf{q}']\chi(s_{\Sigma^+}). \quad (47)$$

Since only $l=0$ and $l=1$ (S and P) amplitudes contribute, Eq. (47) can be further reduced, if use is made of Eqs. (41)–(45), to

$$A(s)=\{(E+m)q'/3[m(E+m)]^{1/2}\}[(E_{0+}{}^{(+)}+2E_{0+}{}^{(-)}+3E_{0+}{}^{(0)})A^{(1)}\chi^\dagger(s)i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\chi(s_{\Sigma^+})$$
$$-(E_{0+}{}^{(+)}-E_{0+}{}^{(-)})A^{(3)}\chi^\dagger(s)i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\chi(s_{\Sigma^+})]-\{q'^2/3k[m(E+m)]^{1/2}\}[(M_{1-}{}^{(+)}+2M_{1-}{}^{(-)}+3M_{1-}{}^{(0)})B^{(1)}]$$
$$\times\chi^\dagger(s)i\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\chi(s_{\Sigma^+})-(M_{1-}{}^{(+)}-M_{1-}{}^{(-)})B^{(3)}\chi^\dagger(s)i\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\chi(s_{\Sigma^+}). \quad (48)$$

In order now to obtain explicit representations for the imaginary parts of the photonic decay amplitudes F_1 and F_2, we come back to Eq. (10), reduce it to a representation involving the Pauli matrices and spinors, carry out the isotopic spin analysis, and then compare it with Eq. (48). We obtain

$$\mathrm{Im}F_1{}^{(1)}(s)=-[q^2/k(m+s^{1/2})]B^{(1)}(s)(M_{1-}{}^{(1)}(s)+3M_{1-}{}^{(0)})\theta[s-(m+\mu)^2]\,, \quad (49)$$

$$\mathrm{Im}F_1{}^{(3)}(s)=-[q^2/k(m+s^{1/2})]B^{(3)}(s)M_{1-}{}^{(3)}(s)\theta[s-(m+\mu)^2]\,, \quad (50)$$

$$\mathrm{Im}F_2{}^{(1)}(s)=-[(E+m)q/k(m+s^{1/2})]A^{(1)}(s)(E_{0+}{}^{(1)}(s)+3E_{0+}{}^{(0)}(s))\theta[s-(m+\mu)^2]\,, \quad (51)$$

$$\mathrm{Im}F_2{}^{(3)}(s)=-[(E+m)q/k(m+s^{1/2})]A^{(3)}(s)E_{0+}{}^{(3)}(s)\theta[s-(m+\mu)^2]\,, \quad (52)$$

where E is the total energy of the decay proton and we have written q instead of q'. In obtaining Eqs. (49)–(52), use has been made of the relations

$$M_{l\pm}{}^{(1)}=M_{l\pm}{}^{(+)}+2M_{l\pm}{}^{(-)}\,, \quad (53)$$

$$M_{l\pm}{}^{(3)}=M_{l\pm}{}^{(+)}-M_{l\pm}{}^{(-)}\,, \quad (54)$$

with identical ones for the $E_{l\pm}{}^{(1)}$ and $E_{l\pm}{}^{(3)}$ amplitudes.

The representations (49)–(52) are consistent with regard to the superscript (0) since the so-called isoscalar amplitudes $M_{l\pm}{}^{(0)}$ and $E_{l\pm}{}^{(0)}$ always lead to final states with isotopic spin $\frac{1}{2}$.

The explicit representations (49)–(52) allow us to write the dispersion relations (8) as follows:

$$F_1{}^{(1)}(s)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\frac{\Lambda_1(s')B^{(1)}(s')[M_{1-}{}^{(1)}(s')+3M_{1-}{}^{(0)}(s')]\theta[s'-(m+\mu)^2]}{s'-s-i\epsilon}\,, \quad (55)$$

$$F_1{}^{(s)}(s)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\frac{\Lambda_1(s')B^{(3)}(s')M_{1-}{}^{(3)}(s')\theta[s'-(m+\mu)^2]}{s'-s-i\epsilon}\,, \quad (56)$$

$$F_2{}^{(1)}(s)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\frac{\Lambda_2(s')A^{(1)}(s')[E_{0+}{}^{(1)}(s')+3E_{0+}{}^{(0)}(s')]\theta[s'-(m+\mu)^2]}{s'-s-i\epsilon}\,, \quad (57)$$

$$F_2{}^{(s)}(s)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\frac{\Lambda_2(s')A^{(3)}(s')E_{0+}{}^{(3)}\theta[s'-(m+\mu)^2]}{s'-s-i\epsilon}\,. \quad (58)$$

Here we have set

$$\Lambda_1(s)=-\frac{q^2}{k(m+s^{1/2})},\quad \Lambda_2(s)=-\frac{(E+m)q}{k(m+s^{1/2})}.$$

We recall once again that the representations (55)–(58) are uncoupled in the total isotopic spin and the total angular momentum, and that the A and B nonleptonic weak decay amplitudes are the S- and P-wave amplitudes, respectively. Both of them, as mentioned previously, satisfy dispersion relations with no subtraction and are given through unitarity in terms of the pion-nucleon scattering amplitudes. The form of their solution was given in Sec. II.

Consequently, making use of the solutions of A and B, Eq. (26), we can cast the representations (55)–(58) into the following ones:

$$F_1^{(1)}(s)=\frac{B^{(1)}(0)}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\Lambda_1(s')\exp\left(\frac{s'}{\pi}\int_{(m+\mu)^2}^{\infty}ds''\frac{\delta_{11}(s'')}{s''(s''-s'-i\epsilon)}\right)\frac{[M_{1-}^{(1)}(s')+3M_{1-}^{(0)}(s')]}{s'-s-i\epsilon},\tag{59}$$

$$F_1^{(3)}(s)=\frac{B^{(3)}(0)}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\Lambda_1(s')\exp\left(\frac{s'}{\pi}\int_{(m+\mu)^2}^{\infty}ds''\frac{\delta_{31}(s'')}{s''(s''-s'-i\epsilon)}\right)\frac{M_{1-}^{(3)}(s')}{s'-s-i\epsilon},\tag{60}$$

$$F_2^{(1)}(s)=\frac{A^{(1)}(0)}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\Lambda_2(s')\exp\left(\frac{s'}{\pi}\int_{(m+\mu)^2}^{\infty}ds''\frac{\delta_{11}(s'')}{s''(s''-s'-i\epsilon)}\right)\frac{[E_{0+}^{(1)}(s')+3E_{0+}^{(0)}(s')]}{s'-s-i\epsilon},\tag{61}$$

$$F_2^{(3)}(s)=\frac{A^{(3)}(0)}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\Lambda_2(s')\exp\left(\frac{s'}{\pi}\int_{(m+\mu)^2}^{\infty}ds''\frac{\delta_{31}(s'')}{s''(s''-s'-i\epsilon)}\right)\frac{E_{0+}^{(3)}(s')}{s'-s-i\epsilon}.\tag{62}$$

These are the basic representations of the photonic decay amplitudes; they will be evaluated at $s=M_{\Sigma^+}{}^2$, which is relevant to the decay of Σ^+.

The phase shifts δ_{11} and δ_{31} in the representations (59) and (60) are those of the P_{11} and P_{31} states and the rest are those of the S_{11} and S_{31} states. They are the pion-nucleon phase shifts, and will be taken over from experiment. The representations (59)–(62) relate the photonic decay amplitudes to integrals over known quantities; consequently, we can make an accurate numerical evaluation of the right-hand sides of (59)–(62).

The numerical evaluation was done on a computer and the results are shown in Table I.

The values of the amplitudes $B^{(1)}(0)$, $B^{(3)}(0)$, $A^{(1)}(0)$, and $A^{(3)}(0)$ which appear in front of the integrals were taken over from experiment.[17] This reference gives the values of $A^{(+)}(0)$, $B^{(+)}(0)$, $A^{(0)}(0)$, and $B^{(0)}(0)$ corresponding to the nonleptonic decays of Σ^+ into a neutron and a π^+ and into a proton and a π^0, respectively. In passing from the $A^{(+)}$, $B^{(+)}$, $A^{(0)}$, and $B^{(0)}$ amplitudes to the total isotopic spin eigenamplitudes $A^{(3)}$, $B^{(3)}$, $A^{(1)}$, and $B^{(1)}$, use has been made of the relations (36) and (37). The experimental values for the $A^{(+)}$, $B^{(+)}$, $A^{(0)}$, and $B^{(0)}$ amplitudes are listed in Table II. For the pion-nucleon scattering S- and P-wave phase shifts and the electric and magnetic dipole amplitudes E_{0+} and M_{1-}, we used the results of Berends, Donnachie, and Weaver.[18] They have listed the real part of the electric and magnetic multipole transition amplitudes and their phases; consequently, one easily determines the imaginary part by means of Eq. (24). We should also recall that the phase of the amplitudes $E_{l\pm}$ and $M_{l\pm}$ leading to a final pion-nucleon state is, by Watson's theorem, equal to the scattering phase shift of that pion-nucleon state. The authors of Ref. 18 have given the real part of the multipole transition amplitudes up to a photon laboratory energy of 500 MeV. Consequently, in evaluating the integrals (59)–(62) the cutoff which was introduced as upper limit was 1.824 BeV^2. This cutoff value corresponds to a photon laboratory energy of 0.5 BeV and was chosen at this value in order to utilize the results of Ref. 18.

[17] N. P. Samios, in Proceedings of the Argonne International Conference on Weak Interactions, [Argonne National Laboratory Report No. ANL-7130 (unpublished)].

[18] F. A. Berends, A. Donnachie, and D. Weaver, Nucl. Phys. **B4**, 1 (1967); **B4**, 55 (1967).

IV. NUMERICAL ESTIMATE

From the matrix element $M(s)=\bar{u}_pJ_{\Sigma^+}(a)u_{\Sigma^+}$, where J_{Σ^+} is given by Eq. (4), it is easy to write down the expression for the Σ^+ decay rate Γ. Making use of the standard techniques, one obtains

$$\Gamma(\Sigma^+\to p+\gamma)=\frac{(M_{\Sigma^+}{}^2-m^2)^2}{8\pi M_{\Sigma^+}{}^3}(|F_1|^2+|F_2|^2),\tag{63}$$

where, from the selection rule $|\Delta I|=\frac{1}{2}$, we may write

$$F_{1,2}=\tfrac{1}{3}\sqrt{2}(-F_{1,2}{}^{(3)}+F_{1,2}{}^{(1)}).\tag{64}$$

Hence, using the results of Table I, we find that the relative decay rate of $\Sigma^+\to p\gamma$ to its total rate to be 0.71×10^{-3}, which is comparable with the latest experimental[11] value $(1.9\pm0.4)\times10^{-3}$. We observe that the results obtained by means of this physical dynamical

TABLE I. Numerical values of the real and imaginary parts of F_1 and F_2 in $\mathrm{BeV}^{-3/2}\ \mathrm{sec}^{-1/2}$.

$\mathrm{Re}F_1^{(1)}$	$\mathrm{Im}F_1^{(1)}$	$\mathrm{Re}F_1^{(3)}$	$\mathrm{Im}F_1^{(3)}$	$\mathrm{Re}F_2^{(1)}$	$\mathrm{Im}F_2^{(1)}$	$\mathrm{Re}F_2^{(3)}$	$\mathrm{Im}F_2^{(3)}$
-9.2×10^2	$+1.06\times10^2$	$+8.55\times10^4$	$+2.68\times10^4$	-4×10^4	$+1.64\times10^4$	-1.98×10^4	-1.36×10^4

model are acceptable. The possible sources of the discrepancy might be found in the noninclusion of intermediate states consisting of two pions and a nucleon, a strange particle and a pion, etc.; we neglected, as mentioned previously, intermediate states involving a hyeron and a pion in the following two cases. The first case was the one referring to the nonleptonic decay of Σ^+ and the other to its photon mode of decay. Under the assumption of keeping just the pion-nucleon state, the nonleptonic decay of Σ^+ gave uncoupled integral equations in the total isotopic spin which, when solved using the Omnès method, led to Eq. (26). The same assumption applied to the photon mode of decay of Σ^+ led to the final integral representations (59)–(62). We can judge, however, by the results that the assumption of keeping only the pion-nucleon intermediate state as the most representative of the low-mass states led to rather acceptable results. The dispersion integrals (59)–(62) do receive their greatest contribution from this state and the choice of the cutoff shows, in addition, that this contribution comes from the region which is not very far from the energy threshold corresponding to a transition into the virtual intermediate pion-nucleon state. Finally, we should emphasize that there are several sources of uncertainty within the context of the dynamical model that we have used. For example, the decay constants $A^{(+)},\cdots,B^{(0)}$ are uncertain, as we see in Table II. For example, $A^{(3)}$ is uncertain by about 42.5%. The effect, however, of this high uncertainty does not affect the numerical evaluation significantly. This is due to the fact that the main contribution comes from the real and imaginary parts of the amplitudes $F_1^{(3)}$ and $F_2^{(1)}$, as we see in Table I. Moreover, the amplitudes $B^{(3)}$ and $A^{(1)}$ entering in the expressions of $F_1^{(3)}$ and $F_2^{(1)}$ are uncertain by about 5.2 and 4.4%, respectively. The amplitude $B^{(1)}$ is uncertain by about 20% but the amplitude $F_1^{(1)}$ hardly contributes anything, as we see again in Table I. Also, there must be some uncertainty in the determination of the photopion amplitudes and some uncertainty which depends upon the rate of convergence of the integrals. We have nothing to report on these two sources of uncertainty. The reason is that the authors of Ref. 18 give no uncertainty associated with the photopion amplitudes. Also, we have no data beyond the cutoff value which would allow us to see how the change of the cutoff affects the results. We did, however, calculate the decay rate of Σ^+ by taking into consideration the uncertainty in the decay constants $A^{(+)},\cdots,B^{(0)}$; the result differed by an insignificant amount from the one quoted in this paper.

TABLE II. Nonleptonic S- and P-wave decay amplitudes. The values listed are to be multiplied by $3.16\times10^6\ \mathrm{BeV}^{1/2}\ \mathrm{sec}^{1/2}$.

$A^{(+)}(\Sigma^+\to n+\pi^+)$	$A^{(0)}(\Sigma^+\to p+\pi^0)$	$B^{(+)}(\Sigma^+\to n+\pi^+)$	$B^{(0)}(\Sigma^+\to p+\pi^0)$
$+0.158\pm0.004$	$+0.144\pm0.011$	$+1.632\pm0.042$	-1.443 ± 0.114

ACKNOWLEDGMENTS

The author expresses his deep gratitude to Professor K. Nishijima for proposing the investigation and for his help and interest in this work. The author wishes also to express his thanks to Professor John Stack for many helpful and interesting discussions and critical comments on the manuscript. Finally, the author expresses his thanks to his colleague, S. Pieper, for his helpful suggestions on the programming.