The Born amplitude corresponding to the potential $V(s,\mathbf{r})$ is given by

$$f_1(s,\Delta)=-\frac{1}{4\pi}\int d\mathbf{r}\, e^{i\Delta\cdot\mathbf{r}}V(s,\mathbf{r}). \tag{A1}$$

Inserting $V(s,\mathbf{r})$ from Eq. (1) and carrying out the integration over $d\mathbf{r}$, we obtain

$$f_1(s,\Delta)=-\frac{g(s)}{\Delta^2+\mu^2}\int \rho_c(\boldsymbol{\zeta})e^{-i\Delta\cdot\boldsymbol{\zeta}}d\boldsymbol{\zeta}\times\int \rho_c(\boldsymbol{\zeta}')e^{i\Delta\cdot\boldsymbol{\zeta}'}d\boldsymbol{\zeta}'. \tag{A2}$$

Let us next evaluate the integral

$$I\equiv\int \rho_c(\mathbf{r})e^{i\Delta\cdot\mathbf{r}}d\mathbf{r}. \tag{A3}$$

We consider a coordinate system in the barycentric frame where the incident momentum $\mathbf{k}$ is in the the z direction and the final momentum $\mathbf{k}'$ lies in the xz plane. If in this system the position vectors $\mathbf{r}=(x,y,z)$ and $\boldsymbol{\xi}=(\xi,\theta',\varphi')$ are related to each other in the following way: $x=\xi\sin\theta'\cos\varphi'$, $y=\xi\sin\theta'\sin\varphi'$, and $z=\xi\cos\theta'/\gamma$, then $\rho_c(\mathbf{r})=\gamma\rho(\xi)$; also,

$$I=\int \rho(\xi)e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\xi^2d\xi\sin\theta'd\theta'\,d\varphi'. \tag{A4}$$

Now,

$$(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}=k\xi\cos\theta'/\gamma-(k\cos\theta\,\xi\cos\theta'/\gamma+k\sin\theta\,\xi\sin\theta'\cos\varphi'). \tag{A5}$$

Inserting (A5) in (A4) and carrying out the $d\varphi'$ integration, we obtain

$$\begin{aligned}I&=2\pi\int \rho(\xi)e^{ik\xi\cos\theta'(1-\cos\theta)/\gamma}J_0(k\xi\sin\theta'\sin\theta)\times\xi^2d\xi\sin\theta'd\theta'\\&=4\pi\int\rho(\xi)\xi^2d\xi\int_0^1\cos[k\xi x(1-\cos\theta)/\gamma]\times J_0(k\xi(1-x^2)^{1/2}\sin\theta)dx\\&=4\pi\int\rho(\xi)\xi^2\frac{\sin\tau\xi}{\tau\xi}d\xi,\end{aligned} \tag{A6}$$

where

$$\tau=[k^2(1-\cos\theta)^2/\gamma^2+k^2\sin^2\theta]^{1/2}. \tag{A7}$$

If $F(\Delta^2)$ is the form factor corresponding to the spherically symmetric distribution $\rho(\xi)$, then

$$F(\Delta^2)=\int\rho(\xi)e^{i\Delta\cdot\boldsymbol{\xi}}d\boldsymbol{\xi}=4\pi\int\rho(\xi)\frac{\sin\Delta\xi}{\Delta\xi}\xi^2d\xi. \tag{A8}$$

Comparing (A6) with (A8), we find

$$I=F(\tau^2). \tag{A9}$$

The two integrals in (A2) are both equal to I because $\rho_c(-\mathbf{r})=\rho_c(\mathbf{r})$. From (A2) and (A9) we then get

$$f_1(s,\Delta)=-g(s)\frac{F^2(\tau^2)}{\Delta^2+\mu^2},$$

which is Eq. (2).

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Hadronic and Electromagnetic Structure of the Proton

M. M. Islam and Kashyap V. Vasavada
Department of Physics, University of Connecticut, Storrs, Connecticut 06268
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Vector-meson–nucleon form factors obtained from high-energy, large-momentum-transfer pp scattering are used to determine the proton magnetic form factor G_{Mp}/μ_p. With essentially one adjustable parameter, all present large-momentum-transfer data can be fitted. The neutron magnetic form factor G_{Mn}/μ_n is predicted and is found to have a zero at $\Delta^2\approx2.0$ (GeV/c)^2.

PHYSICALLY it is of great interest to see what kind of structure of the proton is revealed by present high-energy large-momentum-transfer pp and ep elastic scattering. Although in pp scattering the structure seen will be hadronic, while in ep it will be electromagnetic, one intuitively expects that there is some connection between them. Wu and Yang[1] argued on the basis of the exponential fall of pp elastic scattering in $k_\perp$ that the electromagnetic form factor of the proton should be related to the fourth root of pp

[1] T. T. Wu and C. N. Yang, Phys. Rev. 137, B708 (1965).

differential scattering cross section ($k_\perp$=transverse momentum). Further, they suggested that the appropriate change of variable would be $k_\perp \to \Delta$ when we go from pp elastic scattering to electromagnetic form factor (Δ=momentum transfer). Schopper[2] attempted to relate large-momentum-transfer pp and ep scattering by assuming that the form factors indicated by optical-potential analysis of pp scattering[3] are also the form factors occurring in ep scattering. Recently, Abarbanel, Drell, and Gilman[4] have noted that present pp large-momentum-transfer data tend to approach the fourth power of the proton magnetic moment form factor with increasing energy, and have suggested that this may be due to the emergence of a current-current interaction in pp scattering with form factor proportional to the square of electromagnetic form factors. Chou and Yang[5] have proposed a model in which the Born amplitude for pp scattering at asymptotic energies is proportional to the square of the proton electromagnetic form factor.

It has been shown by Rosen and Islam[6] that present high-energy large-momentum-transfer pp data can be explained as due to two optical potentials whose radial dependence indicates two hadronic distributions of the proton. These distributions interact with corresponding distributions by exchanging heavy mesons. It has been argued in I that one of these should be identified with the ω meson. The other meson, called ω', has been conjectured to be another vector meson having the same quantum numbers as ω. The pp elastic scattering analysis gives us the ωNN and $\omega' NN$ form factors and this provides us a way of connecting large-momentum-transfer pp scattering with the proton electromagnetic structure. The purpose of this paper is to explore this connection quantitatively and compare with present form factor data.

The form factors which the optical potential analysis of pp scattering provides us are

$$F_i(\Delta^2)=\{\beta_i(\Delta^2+\mu_i^2)^{1/2}K_1[\beta_i(\Delta^2+\mu_i^2)^{1/2}]\}^{1/2}, \quad (i=1,\,2;\quad \Delta^2=-t). \tag{1}$$

With the identification that these are ωNN ($i=1$) and $\omega' NN$ ($i=2$) form factors, our model predicts that the isoscalar part of the proton electromagnetic form factor should be dominated by two terms given by

$$\sum_{i=1}^{2}\frac{\gamma_i F_i(\Delta^2)}{1+\Delta^2/\mu_i^2}. \tag{2}$$

At present, experimental data at large momentum transfers are only available for the proton magnetic form factor $G_{Mp}(\Delta^2)$.[7] This form factor has both isoscalar and isovector contributions. We thus have to take the isovector part into account in some way, since Eq. (2) can only give us the isoscalar part. To this end we assume, as is generally done, that the isovector contribution is essentially due to the ρ meson. We further assume that the ρNN form factor has the same functional dependence on Δ^2 as the ωNN and $\omega' NN$ form factors. With these assumptions, we then obtain

$$G_{Mp}(\Delta^2)/\mu_p=\sum_{i=0,1,2}\frac{\gamma_i F_i(\Delta^2)}{1+\Delta^2/\mu_i^2}, \tag{3}$$

where the $i=0$ term corresponds to ρ-meson contribution; $\mu_p=2.79$ is the proton magnetic moment. Written explicitly, the ρNN form factor is given by

$$F_0(\Delta^2)=\{\beta_0(\Delta^2+m_\rho^2)^{1/2}K_1[\beta_0(\Delta^2+m_\rho^2)^{1/2}]\}^{1/2}. \tag{4}$$

The form factors $F_1(\Delta^2)$ and $F_2(\Delta^2)$ in (3) are completely known, since pp scattering furnishes us the parameters μ_1, β_1, μ_2, β_2.[6] There are thus four unknown parameters in (3), namely, γ_0, β_0, γ_1, and γ_2 (m_ρ is given by experiment). Of these four two are determined, say γ_0 and γ_1, by the requirement that at $\Delta^2=0$ the normalization of the isoscalar part is 0.158, and of the isovector part, 0.842. One more parameter, β_0, is determined by the condition that the observed mean square radius[8] $\langle r^2\rangle^{1/2}=0.813$ F be reproduced. We are thus left with one free parameter γ_2 to fit the experimental results. The fit obtained by us is shown in Fig. 1 together with the experimental data.[7] Also shown is the phenomenological dipole fit $G_{Mp}/\mu_p=(1+\Delta^2/0.71)^{-2}$. The agreement of the theoretical curve with the experimental results is gratifying in view of the fact that we have essentially one adjustable parameter to fit the whole large-momentum-transfer region. The values of all the parameters are given in Table I.

TABLE I. Parameters resulting from the fit to the experimental data.

	i	μ_i (GeV)	β_i (F)	γ_i
ρ	0	0.760	0.875	2.863
ω	1	0.801	0.615	0.268
ω'	2	2.163	0.338	0.168

Once all the parameters in (3) are known, this equation predicts what the magnetic form factor of the neutron will be. The theoretically predicted curve is shown in Fig. 2 and compared with experimental results.[9] The most important prediction is the zero in

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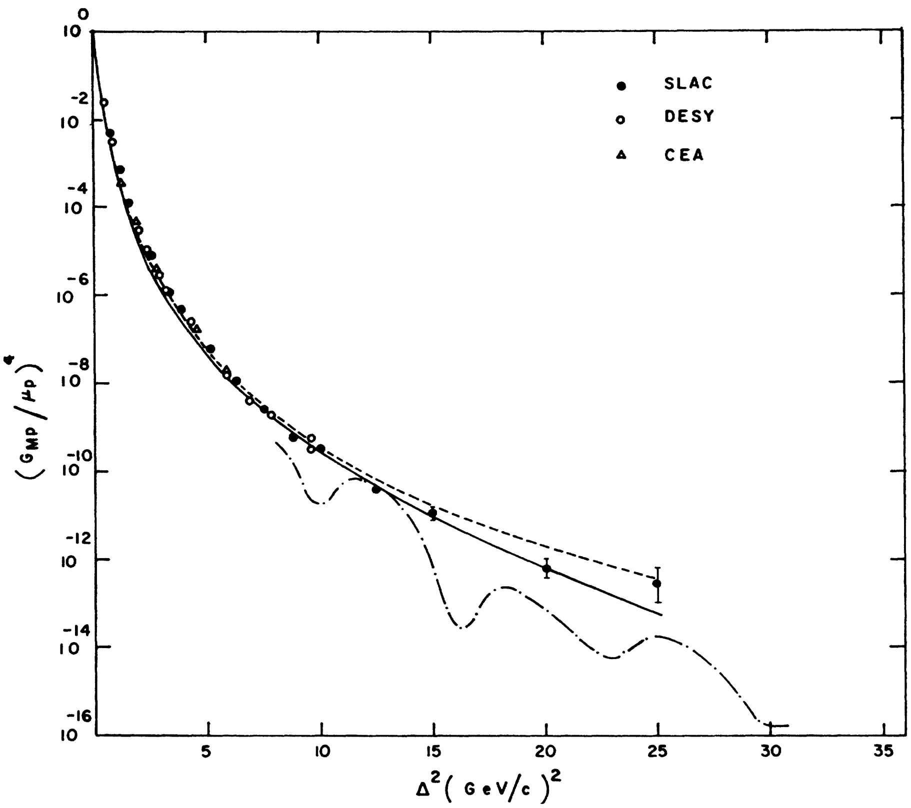

FIG. 1. Solid curve represents our theoretical fit. Dashed curve represents the dipole fit. Dot-dashed curve is the projected pp angular distribution at $p_L=70$ GeV/c given in Ref. 6. Experimental data are taken from Ref. 7.

the neutron magnetic moment G_{Mn}/μ_n. The deviation for small Δ^2 of the theoretical curve from the experimental results may be for a number of reasons, such as those discussed below. However, recalling that this is a predicted curve based on large-momentum-transfer theory, there is good reason to consider it very encouraging.

In Eq. (3) we have completely ignored the ϕ-meson contribution. The reason for this is primarily to avoid introducing unknown parameters, since no knowledge of the ϕNN form factor is provided by the high-energy pp scattering.[10] Our assumption that both the electric and the magnetic vector-meson–nucleon form factors have the same Δ^2 dependence is also prompted by the necessity of keeping the number of adjustable parameters down to a minimum. This assumption may be more critical for the ρ contribution, since it is known that the ρNN magnetic coupling is very large while that of the ωNN is negligible. That vector-meson–nucleon form factors must play an important role in understanding present form factor data has been noted by other authors.[11] In the vector-dominance theory of Kroll, Lee, and Zumino[12] vector-meson–nucleon form factors together with propagators naturally arise. The usual phenomenological forms taken for the VNN form factors[11,13] indicate that they fall off as the inverse of Δ^2 for $\Delta^2 \to \infty$. On the other hand, it has often been conjectured that the form factors fall off exponentially in Δ, corresponding to the fastest rate of decrease.[14,15] Our form factors indeed fall off exponentially, $F_i(\Delta^2) \sim \Delta^{1/4} \exp(-\frac{1}{2}\beta_i\Delta)$. Further, each of the form factors given by (1) is analytic in the t plane ($t=-\Delta^2$) with a right-hand cut from $t=\mu_i^2$ to $t=+\infty$, so that each has the right analyticity property.[16]

We now want to make a number of comments on the way we have connected pp scattering with the proton electromagnetic structure. In our model the connection arises from the observation that from pp scattering we can obtain the form factors of the vector-meson–nucleon vertex. In the case of pp scattering, however, the form factors which are operative corresponds to Lorentz-contracted hadronic distributions and thus asymptotically fall off in $k_\perp$. In ep scattering, on the other hand, the form factors can only be functions of Δ^2 and cor-

[10] In fact all indications from high-energy scattering are that ϕNN coupling is negligible.

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[14] A. Martin, Nuovo Cimento **37**, 671 (1965).

[15] A. M. Jaffe, Phys. Rev. Letters **17**, 661 (1966).

[16] The dipole fit $G_{Mp}/\mu_p=(1+\Delta^2/0.71)^{-2}$, which agrees better than any previous theoretical fits with present data, does not have the right analyticity behavior. In this regard, for a critical discussion on the failures of previous theoretical models, see M. Goitein, J. R. Dunning, Jr., and R. Wilson, Phys. Rev. Letters **18**, 1018 (1967).

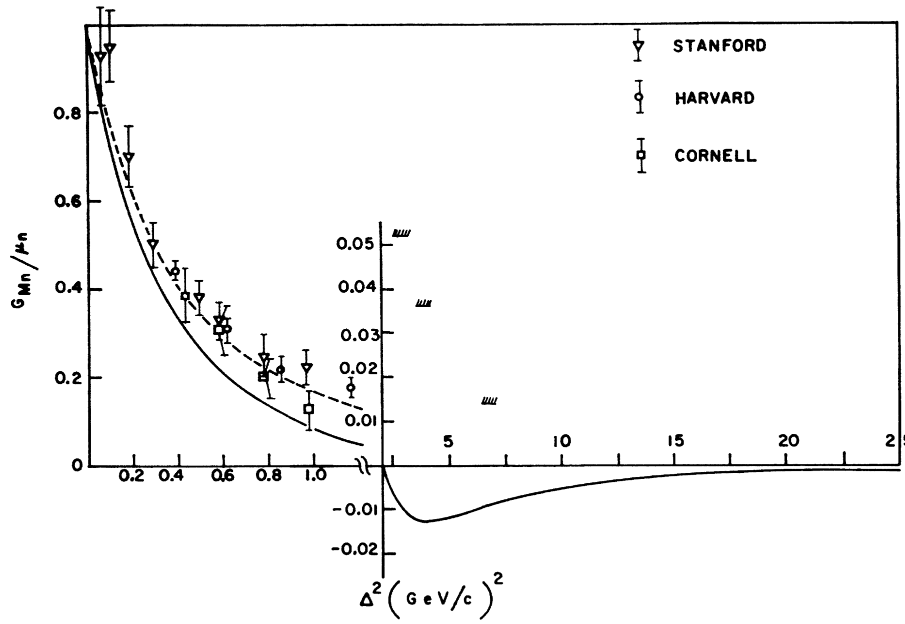

FIG. 2. Solid curve represents the predicted neutron magnetic form factor $G_{Mn}(\Delta^2)/\mu_n$. Experimental data are from Ref. 9. The dashed curve is the theoretical fit of $G_{Mp}(\Delta^2)/\mu_p$ given for comparison.

respondingly they fall off exponentially in momentum transfer Δ. This at once provides us with the reason why the appropriate change of variable is $k_\perp \to \Delta$ when we go from pp to ep scattering. Asymptotically our pp differential scattering cross section falls off as $d\sigma/d\Omega \sim C(s,\theta)\exp(-2\beta_2 k_\perp)$. From Eq. (3), on the other hand, we obtain $G_{Mp}/\mu_p \sim e^{-\beta_2\Delta/2}$ when $\Delta \to \infty$ ($\beta_2 < \beta_0, \beta_1$). We thus see that as far as the dominant exponential falloff is concerned $d\sigma/d\Omega$ is indeed proportional to the fourth power of $G_{Mp}(\Delta^2)/\mu_p$ when the change in variable $k_\perp \to \Delta$ is made. Thus, our model asymptotically reproduces the Wu-Yang conjecture. The main difference between our approach and Schopper's can be seen from the following remarks. Schopper attempts to connect pp scattering with the proton electromagnetic structure by assuming a formula of the type

$$(G_{Mp}/\mu_p)^2 = a_1 F_1(\Delta^2)^2 + a_2 F_2(\Delta^2)^2 \qquad (5)$$

and by neglecting μ_1^2 and μ_2^2 compared with Δ^2. In contrast, our identification of $F_1(\Delta^2)$ as ωNN and $F_2(\Delta^2)$ as $\omega' NN$ form factors leads us unambiguously to the last two terms in Eq. (3), which include vector-meson propagators. Further, Schopper assumes $(d\sigma/d\Omega)_{pp}$ to be proportional to $G_{Mp}{}^4$ and the change of variable to be $\Delta \to k_\perp$. On the other hand, these assumptions are not made in our model. As noted in I, our pp scattering asymptotically ($s \to \infty$, Δ^2 fixed) is pure diffraction scattering and, therefore, negligible outside the diffraction peak compared to the differential scattering cross sections predicted by the Abarbanel-Drell-Gilman model and by the Chou-Yang model. To get a quantitative feeling, we have plotted in Fig. 1 the projected pp angular distribution given in I for $p_L = 70$ GeV/c. As can be seen, the calculated cross sections are much lower at a number of momentum transfers than the fourth power of G_{Mp}/μ_p and have a great deal more structure.

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