commutator, $[V_{K^0}, A_{\pi^-}]=0$, taken, for example, between the $\langle n(\mathbf{q})|$ and $|\Sigma^+(\mathbf{q})\rangle$ states with $|\mathbf{q}|\to\infty$. Using the $SU(3)$ approximation for the V_{K^0} and pion PCAC for the A_{π^-}, we obtain a broken $SU(3)$ sum rule

$$\left(\frac{1}{\sqrt{2}}\right)g_{\Sigma^0\Sigma^+\pi^-}\left(\frac{1}{2m_\Sigma}\right)-\left(\sqrt{\frac{3}{2}}\right)g_{\Sigma^+\Lambda\pi^-}\left(\frac{1}{m_\Sigma+m_\Lambda}\right)+g_{pn\pi^-}\left(\frac{1}{2m_p}\right)=0, \quad (23)$$

whereas exact $SU(3)$ symmetry gives $(\sqrt{\frac{1}{2}})g_{\Sigma^0\Sigma^+\pi^-}-(\sqrt{\frac{3}{2}})g_{\Sigma^+\Lambda\pi^-}+g_{pn\pi^-}=0$. This sum rule (23) can also be derived from the sum rules (3) and (21). These observations show the internal consistency of our sum rules based on the $SU(3)$ approximation.

VI. CONCLUDING REMARKS

We have given an argument that the exact $SU(3)$ symmetry sum rules for the axial-vector semileptonic hyperon decay coupling constants remain valid to a good approximation even in the broken-symmetry world. Therefore, more precise determination of the parameters of semileptonic processes may be able to settle comfortably the question of whether we are dealing with one Cabibbo angle or not. However, one point has to be remembered. In this paper we have assumed that there are no other $\frac{1}{2}^+$ baryons (N') beside the usual $\frac{1}{2}^+$ octet (N). If this is not correct, we have to worry about the possible N-N' mixing. As the simplest example, suppose there exists an $I=Y=0$ $\frac{1}{2}^+$ baryon N'^0 which belongs to the $SU(3)$ singlet in the symmetry limit. Then our sum rule, for example, $g_{\Sigma\Lambda}=(\sqrt{\frac{3}{2}})g_{pn}+g_{p\Lambda}$ must be modified into $g_{\Sigma\Lambda}=(\sqrt{\frac{3}{2}})g_{pn}\cos\phi+g_{p\Lambda}$,[6] where ϕ denotes the Λ^0-N'^0 mixing angle.

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High-Energy pp Scattering and Hadronic Structure of the Proton

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Present high-energy large-angle pp angular distribution in the momentum range 8–21 GeV/c is investigated using an optical-model description. Observed differential scattering cross sections can be explained in terms of *two hadronic distributions* of the proton and *Lorentz contraction* of the distributions. It is inferred that these distributions interact by exchanging *vector mesons:* ω, and a new one, ω', having the same quantum numbers as ω.

SOME time ago we reported an analysis[1] of 90° fixed-angle pp scattering data of Akerlof *et al.*[2] Our analysis was based on two optical potentials with radial dependence of the form $\exp[-\mu(r^2+\beta^2)^{1/2}]/(r^2+\beta^2)^{1/2}$ and distorted-wave Born approximation of the corresponding amplitudes. The radial dependences of the potentials were related to two spherically symmetric hadronic (or nucleon-matter) distributions of the proton.[3] It is, however, intuitively clear that in the barycentric system at high energies the two protons would be highly Lorentz-contracted in the direction of their relative momentum. Similar ideas have been expressed by other authors.[4] To incorporate the effect of Lorentz contraction in a model calculation, some authors have assumed that the potential becomes

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[3] M. M. Islam, Nuovo Cimento 48, 251 (1967); in *Lectures In Theoretical Physics*, edited by A. O. Barut and W. E. Brittin (Gordon and Breach Science Publishers, Inc., 1968), Vol. 10B, p. 97.

[4] N. Byers and C. N. Yang, Phys. Rev. **142**, 976 (1966).

Lorentz-contracted,[5,6] while others have put in phenomenologically Lorentz-contraction factors.[7] We have made a formulation of this Lorentz contraction given a spherically symmetric distribution at rest, and then have applied this to our previous spherically symmetric hadronic distributions. We find a number of interesting consequences to follow, such as for fixed s the large-angle elastic differential cross section falls off exponentially in $k_\perp$ (=transverse momentum), as in Orear's formula.[8,9] In view of recent extensive high-energy large-angle pp elastic scattering data of Allaby *et al.*,[10] we have reinvestigated pp scattering with Lorentz contraction of hadronic distributions taken into account. Results of our investigation together with a number of predictions bearing on the structure of the proton are reported here.

To investigate the Lorentz-contraction effect, let us suppose that each proton has a spherically symmetric hadronic matter distribution $\rho(r)$ at rest. When the proton is moving with relativistic energy in the barycentric system, the hadronic distribution becomes Lorentz-contracted in the direction of motion, say the z axis. Let us denote this distribution by $\rho_c(\mathbf{r})$, where $\mathbf{r}$ is the radius vector from the center of the nucleon. Conservation of "hadronic charge" then gives $\rho_c(\mathbf{r}) = \gamma\rho(\xi)$, where $\gamma=(1-v^2)^{-1/2}=(s/4m^2)^{1/2}$, and $x=\xi_1$, $y=\xi_2$, $z=\xi_3/\gamma$. As in I, we assume that during scattering of two protons their hadronic distributions interact through a complex Yukawa potential. The corresponding optical potential is then given by[3]

$$V(s,\mathbf{r})=g(s)\int\rho_c(\boldsymbol{\zeta})d\boldsymbol{\zeta}\int\rho_c(\boldsymbol{\zeta}')d\boldsymbol{\zeta}'e^{-\mu|\mathbf{r}+\boldsymbol{\zeta}-\boldsymbol{\zeta}'|}/|\mathbf{r}+\boldsymbol{\zeta}-\boldsymbol{\zeta}'|\,. \tag{1}$$

The Born amplitude that we obtain from this potential (see Appendix) using the relation $\rho_c(\mathbf{r})=\gamma\rho(\xi)$ is

$$f_1(s,\Delta)=-g(s)\frac{F^2(\tau^2)}{\Delta^2+\mu^2}, \tag{2}$$

where $\Delta^2=-t=2k^2(1-\cos\theta)$, $\tau=\Delta(1-\Delta^2/s)^{1/2}=k\sin\theta \times[1+(4m^2/s)\tan^2(\frac{1}{2}\theta)]^{1/2}$, and F is the form factor that corresponds to the spherically symmetric distribution $\rho(\xi)$:

$$F(\Delta^2)=\int\rho(\xi)e^{i\boldsymbol{\Delta}\cdot\boldsymbol{\xi}}d\boldsymbol{\xi}. \tag{3}$$

[5] G. A. Armoudian, D. Ito, and K. Mori, Nuovo Cimento **37**, 1739 (1965).
[6] H. Yabuki, Progr. Theoret. Phys. (Kyoto) **36**, 415 (1966).
[7] A. D. Krisch, in *Lectures In Theoretical Physics*, edited by W. E. Brittin and A. O. Barut (Gordon and Breach Science Publishers, Inc., 1968) Vol. 9B, p. 1.
[8] J. Orear, Phys. Letters **13**, 190 (1964).
[9] Exponential falloff in $k_\perp$ has also been obtained in an "incoherent droplet" model by K. Huang [Phys. Rev. **146**, 1075 (1966)].
[10] J. V. Allaby, G. Cocconi, A. N. Diddens, A. Klovning, G. Matthiae, E. J. Sacharidis, and A. M. Wetherell, Phys. Letters **25B**, 156 (1967).

Comparing (2) with the Born amplitude that we get for the spherically symmetric distribution,

$$f_1(s,\Delta)=-g(s)\frac{F^2(\Delta^2)}{\Delta^2+\mu^2}, \tag{4}$$

we notice that the effect of the Lorentz contraction has been to replace $F(\Delta^2)$ by $F(\tau^2)$. Physically, this is what we should expect since in going from one to the other only the source distribution is modified.

Following I, we write the scattering amplitude in the form[11]

$$f(s,\Delta)=ik\int_0^\infty bdb\,J_0(b\Delta)[1-e^{2i\delta_0(s,b)}] +\sum_{i=1}^{2}2k\int_0^\infty bdb\,J_0(b\Delta)e^{2i\delta_0(s,b)}\delta_i(s,b)\,; \tag{5}$$

$\delta_i(s,b)$ is now determined by a Born amplitude of the form (2), rather than (4), through the relation

$$\delta_i(s,b)=\frac{1}{2k}\int_0^\infty\Delta d\Delta\,J_0(b\Delta)f_{1,i}(s,\Delta)\,. \tag{6}$$

The spherically symmetric distributions used earlier corresponded to form factors

$$F_i(\Delta^2)=\{\beta_i(\Delta^2+\mu_i^2)^{1/2}K_1[\beta_i(\Delta^2+\mu_i^2)^{1/2}]\}^{1/2},\quad (i=1,2). \tag{7}$$

With the Lorentz contraction taken into account, we thus obtain from (2)

$$f_{1,i}(s,\Delta)=-\frac{g_i(s)}{\Delta^2+\mu_i^2}\beta_i(\tau^2+\mu_i^2)^{1/2}K_1[\beta_i(\tau^2+\mu_i^2)^{1/2}]. \tag{8}$$

These Born amplitudes together with Eq. (6) give us the phase-shift functions $\delta_i(s,b)$. We have parametrized the diffraction amplitude using the empirical formula

$$1-e^{2i\delta_0(s,b)}=(\sigma_T{}^d/\pi R^2)(1+i\epsilon)e^{-2b^2/R^2}, \tag{9}$$

where ϵ takes into account any real part arising from diffraction scattering at present energies. Inserting (6) and (9) into (5), we obtain

$$f(s,\Delta)=\frac{ik\sigma_T{}^d}{4\pi}(1+i\epsilon)e^{-R^2\Delta^2/8} +\sum_{i=1}^{2}\left\{f_{1,i}(s,\Delta)-\frac{\sigma_T{}^d}{4\pi}(1+i\epsilon) \times\int_0^\infty\Delta'd\Delta' f_{1,i}(s,\Delta')e^{-R^2(\Delta^2+\Delta'^2)/8}I_0(R^2\Delta\Delta'/4)\right\}. \tag{10}$$

[11] It is worthwhile to restate that the $i=1$, 2 terms correspond to distorted-wave Born approximation (DWBA) of the amplitudes due to potentials $V_1(s,r)$ and $V_2(s,r)$. Later on, we shall relate these potentials to ω and ω' exchanges. The validity of our using DWBA ultimately then rests on the following physical assumption: Single exchanges of vector mesons with complex coupling and form factors, Eq. (2), are adequate when absorption is taken into account.

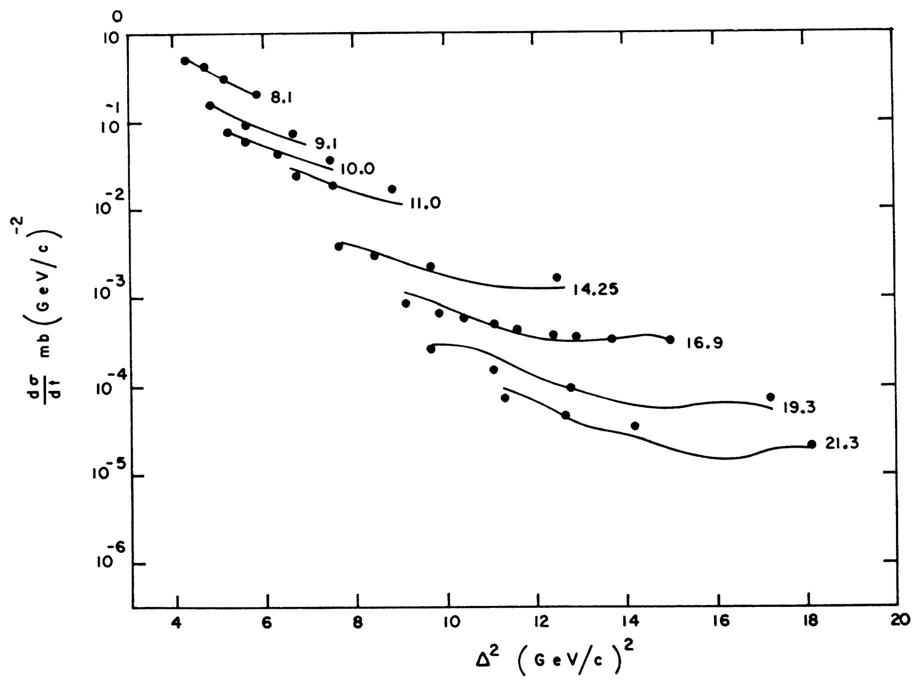

FIG. 1. Solid lines represent fits to the experimental data of Allaby *et al.* for lab momenta in the range 8–21 GeV/c.

I_0 is the modified Bessel function of the first kind. The integral in (10) can be easily evaluated on a computer because of the strong convergence factor $\exp(-R^2\Delta'^2/8)$ which provides a natural cutoff. Using Eq. (10), we have fitted the data of Allaby *et al.*[10] in the energy range 8–21 GeV/c. The fits are shown in Fig. 1 together with the experimental data. Values obtained for the energy-independent parameters are $R=1.2$ F ($1\text{F}\equiv10^{-13}$ cm), $\sigma_T{}^d=36.2$ mb, $\epsilon=0.249$, $\mu_1=0.801$ GeV, $\mu_2=2.163$ GeV, $\beta_1=0.615$ F, $\beta_2=0.338$ F. The two complex coupling constants $g_1(s)$ and $g_2(s)$ are energy-dependent parameters in our model and their values are given in Table I.[12] Our calculated values of $\text{Re}f(\theta=0)/\text{Im}f(\theta=0)$ and σ_{total} are in reasonable agreement with experimental results in the energy range 8–26 GeV/c.[13] Equation (5) shows that we can have a real part of the scattering amplitude as well as variation of the total cross section with energy; therefore, some of the criticisms of a pure diffraction model[14] do not apply to our model.

To see asymptotically what our model predicts, let us note that for Δ large the main contribution to the integrals in (5) come from small values of b, so that the absorption correction factor $\exp[2i\delta_0(s,b)]$ can be replaced by its value at $b=0$. Equation (5) then gives for large Δ,

$$f(s,\Delta)\approx\sum_{i=1}^{2}\left[1-\frac{\sigma_T{}^d}{\pi R^2}(1+i\epsilon)\right]f_{1,i}(s,\Delta)$$

$$=\left[-1+\frac{\sigma_T{}^d}{\pi R^2}(1+i\epsilon)\right]\left\{\sum_{i=1}^{2}\frac{g_i(s)}{\Delta^2+\mu_i{}^2}\beta_i(\tau^2+\mu_i{}^2)^{1/2}\times K_1[\beta_i(\tau^2+\mu_i{}^2)^{1/2}]\right\}. \quad (11)$$

The diffraction term which falls off exponentially in Δ^2 is negligible at present energies and has been dropped in (11). Using the asymptotic formula $K_1(z)=(\pi/2z)^{1/2}e^{-z}$ and $\beta_2<\beta_1$, we find from (11) that for high-energy large momentum transfer

$$f(s,\Delta)\approx\left[-1+\frac{\sigma_T{}^d}{\pi R^2}(1+i\epsilon)\right]\frac{g_2(s)}{\Delta^2}(\tfrac{1}{2}\pi\beta_2\tau)^{1/2}e^{-\beta_2\tau}. \quad (12)$$

Since $\tau=k\sin\theta=k_\perp$ for large s, (12) gives

$$\frac{d\sigma}{d\Omega}\approx\left|1-\frac{\sigma_T{}^d}{\pi R^2}(1+i\epsilon)\right|^2|g_2(s)|^2\frac{\pi\beta_2k_\perp}{2\Delta^4}e^{-2\beta_2k_\perp}. \quad (13)$$

The exponential falloff is precisely the same as in Orear's empirical formula[8]

$$d\sigma/d\Omega=(A/s)e^{-ak_\perp}. \quad (14)$$

Equation (13), however, predicts that $\log_{10}(s(d\sigma/d\Omega))$ plotted against $k_\perp$ is not a single straight line but a *number of approximately parallel lines.* The reason for obtaining a different line for each energy is that the coefficient of the exponential in (13) is strongly energy-

[12] The optical model does not provide us any knowledge on the s dependence of $g_1(s)$ and $g_2(s)$. To discuss the s dependence, we have to relate Eq. (10) with asymptotic theories, as considered later.

[13] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters **19**, 857 (1967).

[14] V. Barger, CERN Topical Conference on High Energy Collisions of Hadrons, 1968 (unpublished).

TABLE I. Values of the energy-dependent parameters g_1 and g_2.

p_L (GeV/c)	8.1	9.1	10.0	11.0	14.25	16.9	19.3	21.3
$\lvert g_1(s)\rvert$ (GeV)	29.44	26.87	27.31	29.49	29.07	29.41	30.59	30.92
arg $g_1(s)$ (rad)	-0.369	-0.390	-0.411	-0.457	-0.636	-0.818	-1.223	-2.010
$\lvert g_2(s)\rvert$ (GeV)	36.20	27.40	26.88	20.86	15.95	14.73	7.525	5.531
arg $g_2(s)$ (rad)	-1.314	-0.602	-0.024	-0.898	$+0.121$	$+1.078$	-0.513	-1.031

dependent. These predictions are in qualitative agreement with CERN data in the range 17–21 GeV/c. However, in a quantitative analysis one has to bear in mind that the $i=1$ and $i=2$ amplitudes have a large interference term. It is important to note here that $d\sigma/d\Omega$ given by Eq. (13) is in agreement with theoretical results on the asymptotic angular dependence[15] as well as on the fastest allowed decrease[16] with s of the differential scattering cross section derived from boundedness and analyticity conditions in the $\cos\theta$ plane. The exponential dependence on $k_\perp$ predicted by our model is also consistent with the statistical model.[17] Formula (13) suggests that instead of $(s(d\sigma/d\Omega))$, a better quantity will be $(\Delta^4(d\sigma/dt))$ to be plotted against $k_\perp$. For $\theta=90°$ this, of course, becomes the Orear plot.

To see what kind of pp cross section our model predicts for energies above 20 GeV, we have made a simple parametrization of the coupling constants $g_i(s)$:

$$g_i(s)=A_i(s/m^2)^{-\nu_i}e^{i\varphi_i(s)}, \quad \varphi_i(s)=\chi_i+c_i(s/m^2)^{-\eta_i}. \quad (15)$$

The reason for choosing this parametrization is discussed later. The parameters have been determined by using the higher-energy data of Allaby *et al.*[10] and of Cocconi *et al.*[18] and the values are $A_i=3.075\times10^1$, 7.053×10^5 GeV; $\nu_i=0.0$, 3.045; $\chi_i=-2.356$, -0.785, $c_i=2.443$, 0.0, $\eta_i=0.506$, 0.0 ($i=1$, 2).[19] We then have calculated the angular distributions at 30 and 70 GeV/c. The results are given in Fig. 2 together with the calculation of Chou and Yang.[20,21] Our results indicate that the Chou-Yang model is not asymptotic. In fact, from (15) we find that for $s\to\infty$ and Δ^2 fixed, the $i=1$ and $i=2$ terms in (10) behave as $s^{-\nu_1}$ and $s^{-\nu_2}$, respectively, so that asymptotically their contributions vanish compared to the diffraction term in (10). Hence, our model predicts that the elastic pp scattering is pure diffraction scattering asymptotically ($s\to\infty$, t fixed), which then should be the same for $p\bar{p}$ and pn, and should correspond to the exchange of vacuum quantum numbers.

What predictions can we make for other processes, such as $p\bar{p}$ scattering? If the mesons corresponding to the two optical potentials we have considered have the same G parity, say -1, their Born amplitudes change sign when we go from pp to $p\bar{p}$ scattering. Equation (11) then shows that for s large and fixed, the large-angle cross section for both processes should be the same. However, at present energies the diffraction parameters σ_T^d, R, and ϵ appear to be different for pp and $p\bar{p}$ scattering. Consequently, what we may see at finite energy is that the large-momentum-transfer angular distributions are similar for pp and $p\bar{p}$, but somewhat displaced from each other. What about pn scattering? If both the mesons have the same isospin (0 or 1), then again from (11) we conclude that, for Δ large, pp and pn differential cross sections should be the same. If, however, one of the mesons has isospin 0 and the other 1, then one of the terms in (11) will change sign when we go from pp to pn scattering. Correspondingly, the interference term between them will change sign. Since, in the 8–11 GeV/c lab momentum range we find the interference contribution in pp very large, this would imply that the pn cross section will be very different from pp. However, the limited experimental results available at present indicate that pp and pn scattering are the same.[22] We therefore conclude that both the mesons have the same isospin.

In Fig. 2 we have also plotted our calculated cross sections for $p_L=8.1$, 10.0, 14.25, and 16.9 GeV/c. Closer examination of the low-momentum-transfer angular distributions indicate that we do obtain a shrinkage of the diffraction peak. This shrinkage is due to the $i=1$ term on the right-hand side in Eq. (10), which interferes strongly and constructively with the diffraction amplitude. In the differential cross section $d\sigma/dt=\pi|f(s,\Delta)/k|^2$ the diffraction amplitude is energy-independent, but the $i=1$ amplitude decreases with energy thus producing the shrinkage. We identify this term as arising from the ω-meson exchange on the basis that (i) the real part of the corresponding potential is repulsive and that (ii) it is a large contribution in the forward direction which should be the same for pn scattering but differ in sign for $p\bar{p}$ scattering.[23] This identification at once leads to the conclusion that ω

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[19] The phases of the coupling constants $g_1(s)$ and $g_2(s)$ are not well determined in our analysis due to lack of sufficient data. Therefore, detailed shape of our theoretical curves should be taken qualitatively.

[20] T. T. Chou and C. N. Yang, Phys. Rev. Letters **20**, 1213 (1968).

[21] See, also, L. Durand, III, and R. Lipes, Phys. Rev. Letters **20**, 637 (1968).

[22] A. Bialas and O. Czyzewski, Nuovo Cimento **49A**, 273 (1967); J. Cox, M. L. Perl, M. N. Kreisler, and M. J. Longo, Phys. Rev. Letters **21**, 645 (1968).

[23] Present experimental observations indicate that forward pp and pn scattering are basically similar while pp and $p\bar{p}$ are quite different (see, for example, Ref. 14).

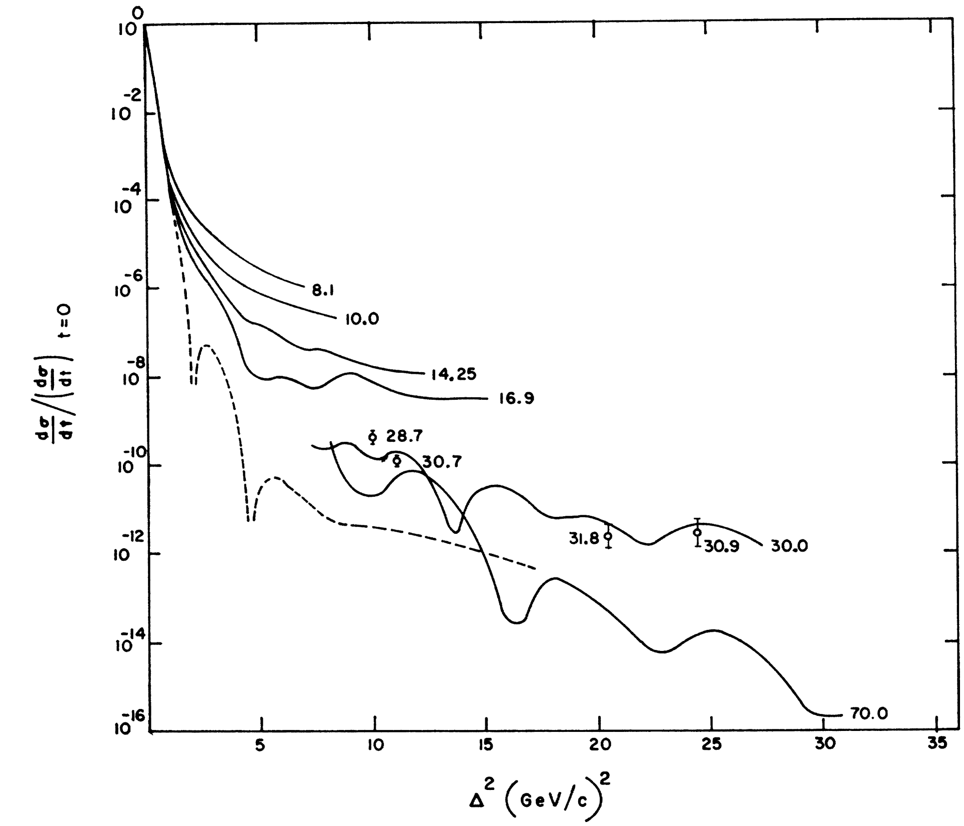

FIG. 2. Calculated pp differential scattering cross sections for various lab momenta. For $p_L=30$ and 70 GeV/c the parametrization given by Eq. (15) is used. The dashed curve represents the calculation of Chou and Yang (Ref. 20). Experimental points are from Cocconi *et al.* (Ref. 18).

plays a primary role in making the pp diffraction peak shrink, an observation which has been made earlier by one of us[24] and has been found qualitatively in agreement with the general behavior of diffraction peaks.

With the identification of the $i=1$ term in (10) as due to the ω meson, it is tempting to identify the $i=2$ term as due to another vector meson ω' having the same quantum numbers as ω, since from the argument above it should have the same isospin and also should have the same G parity, if large-angle pp and $p\bar{p}$ angular distributions approach each other at high energies.[25] This identification has strong implication on the electromagnetic structure of the proton which is investigated and compared with experimental results in the following paper.[26]

With the parametrization (15), the $i=1$ and $i=2$ terms in (10) behave as $\phi(t)/s^{\nu_i}$ for $s\rightarrow\infty$ and t fixed ($t<0$). This is precisely the form one is led to if the infinite sequence of Regge cuts associated with a trajectory are taken into account.[27] In fact, we can go one step further. If we consider the invariant amplitude $T(s,t)=Wf(s,\Delta)$, then our identification of the $i=1$ term as due to ω implies that $\alpha_\omega(0)=0.5-\nu_1=0.5$. This intercept of the ω trajectory is in good agreement with other results. Further, the ω' will now be the $J=1$ realization of the first daughter trajectory, so that one should expect the corresponding intercept to be $\alpha_{\omega'}(0)-2=-1.5$. Our value $\alpha_{\omega'}(0)=0.5-\nu_2=-2.5$ is in qualitative agreement with this reasoning.[28]

Finally, our results indicate that the proton has two hadronic distributions associated with ω and ω' exchange. The present high-energy large-momentum transfer scattering is dominated by the exchange of these two $I=0$ vector mesons. The root-mean-square radii of the corresponding distributions are respectively 0.44 and 0.20 F.

One of us (M.M.I.) would like to thank Professor N. P. Chang for a number of stimulating conversations.

APPENDIX

We present the various steps involved in the deduction of Eq. (2) from Eq. (1).

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[25] It is interesting to point out that to explain crossover phenomenon in Regge-pole model an extra ω-meson type contribution is needed [V. Barger and L. Durand, III, Phys. Rev. Letters **19**, 1295 (1967).

[26] M. M. Islam and K. V. Vasavada, following paper, Phys. Rev. **178**, 2140 (1969).

[27] J.-L. Gervais and F. J. Yndurain, Phys. Rev. Letters **20**, 27 (1968).

[28] Lack of quantitative agreement may be due to contributions from other daughter trajectories. Incidentally, if we assume a parallel daughter trajectory, then $m_{\omega'}=(\sqrt{5})m_\omega$, which is close to the value we have found.

The Born amplitude corresponding to the potential $V(s,\mathbf{r})$ is given by

$$f_1(s,\Delta)=-\frac{1}{4\pi}\int d\mathbf{r}\, e^{i\Delta\cdot\mathbf{r}}V(s,\mathbf{r}). \quad \text{(A1)}$$

Inserting $V(s,\mathbf{r})$ from Eq. (1) and carrying out the integration over $d\mathbf{r}$, we obtain

$$f_1(s,\Delta)=-\frac{g(s)}{\Delta^2+\mu^2}\int \rho_c(\boldsymbol{\zeta})e^{-i\Delta\cdot\boldsymbol{\zeta}}d\boldsymbol{\zeta}\times\int\rho_c(\boldsymbol{\zeta}')e^{i\Delta\cdot\boldsymbol{\zeta}'}d\boldsymbol{\zeta}'. \quad \text{(A2)}$$

Let us next evaluate the integral

$$I\equiv\int\rho_c(\mathbf{r})e^{i\Delta\cdot\mathbf{r}}d\mathbf{r}. \quad \text{(A3)}$$

We consider a coordinate system in the barycentric frame where the incident momentum $\mathbf{k}$ is in the the z direction and the final momentum $\mathbf{k}'$ lies in the xz plane. If in this system the position vectors $\mathbf{r}=(x,y,z)$ and $\boldsymbol{\xi}=(\xi,\theta',\varphi')$ are related to each other in the following way: $x=\xi\sin\theta'\cos\varphi'$, $y=\xi\sin\theta'\sin\varphi'$, and $z=\xi\cos\theta'/\gamma$, then $\rho_c(\mathbf{r})=\gamma\rho(\xi)$; also,

$$I=\int\rho(\xi)e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\xi^2d\xi\sin\theta'd\theta'\,d\varphi'. \quad \text{(A4)}$$

Now,

$$(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}=k\xi\cos\theta'/\gamma-(k\cos\theta\,\xi\cos\theta'/\gamma+k\sin\theta\,\xi\sin\theta'\cos\varphi'). \quad \text{(A5)}$$

Inserting (A5) in (A4) and carrying out the $d\varphi'$ integration, we obtain

$$\begin{aligned}I&=2\pi\int\rho(\xi)e^{ik\xi\cos\theta'(1-\cos\theta)/\gamma}J_0(k\xi\sin\theta'\sin\theta)\times\xi^2d\xi\sin\theta'd\theta'\\&=4\pi\int\rho(\xi)\xi^2d\xi\int_0^1\cos[k\xi x(1-\cos\theta)/\gamma]\times J_0(k\xi(1-x^2)^{1/2}\sin\theta)dx\\&=4\pi\int\rho(\xi)\xi^2\frac{\sin\tau\xi}{\tau\xi}d\xi,\end{aligned} \quad \text{(A6)}$$

where

$$\tau=[k^2(1-\cos\theta)^2/\gamma^2+k^2\sin^2\theta]^{1/2}. \quad \text{(A7)}$$

If $F(\Delta^2)$ is the form factor corresponding to the spherically symmetric distribution $\rho(\xi)$, then

$$\begin{aligned}F(\Delta^2)&=\int\rho(\xi)e^{i\Delta\cdot\boldsymbol{\xi}}d\boldsymbol{\xi}\\&=4\pi\int\rho(\xi)\frac{\sin\Delta\xi}{\Delta\xi}\xi^2d\xi.\end{aligned} \quad \text{(A8)}$$

Comparing (A6) with (A8), we find

$$I=F(\tau^2). \quad \text{(A9)}$$

The two integrals in (A2) are both equal to I because $\rho_c(-\mathbf{r})=\rho_c(\mathbf{r})$. From (A2) and (A9) we then get

$$f_1(s,\Delta)=-g(s)\frac{F^2(\tau^2)}{\Delta^2+\mu^2},$$

which is Eq. (2).

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Hadronic and Electromagnetic Structure of the Proton

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Vector-meson–nucleon form factors obtained from high-energy, large-momentum-transfer pp scattering are used to determine the proton magnetic form factor G_{Mp}/μ_p. With essentially one adjustable parameter, all present large-momentum-transfer data can be fitted. The neutron magnetic form factor G_{Mn}/μ_n is predicted and is found to have a zero at $\Delta^2\approx 2.0$ (GeV/c)^2.

PHYSICALLY it is of great interest to see what kind of structure of the proton is revealed by present high-energy large-momentum-transfer pp and ep elastic scattering. Although in pp scattering the structure seen will be hadronic, while in ep it will be electromagnetic, one intuitively expects that there is some connection between them. Wu and Yang[1] argued on the basis of the exponential fall of pp elastic scattering in $k_\perp$ that the electromagnetic form factor of the proton should be related to the fourth root of pp

[1] T. T. Wu and C. N. Yang, Phys. Rev. 137, B708 (1965).