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Beta Spectrum of W^{187}

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The β spectrum of W^{187} has been analyzed with a thoroughly tested Siegbahn-Slatis β-ray spectrometer equipped with a plastic well-type detector, whose counting efficiency is unity down to 50 keV. The highest-energy group ($\frac{3}{2}^- \to \frac{5}{2}^+$) is nonunique first-forbidden with a $\log ft$ value 8.0, and it deviates from the statistical shape as a result of the selection-rule effect. Its shape could be fitted with that of a modified B_{ij} approximation, namely, $C(W)=q^2+9L_1/L_0+D$ with the value of $D=23\pm3$. Though the calculations based on the Nilsson model and the j-j coupling shell model predict a deviation from the ξ approximation, only the former could give better agreement with experiment. A detailed analysis of the shape factor yields the end point of the outer group as 1314±2 keV. Subtraction of this outer group, corrected for its shape, revealed inner groups with maximum energies and intensities 625±5 (66%), 453±7 (3%), and 330±10 (5%) keV. Information is also obtained on a few conversion lines which are resolved by the spectrometer.

I. INTRODUCTION

THE decay of W^{187} has been studied by several authors,[1–11] but there still remain discrepancies in the levels and level spins over 511 keV and the β feeds to these levels. Funke *et al.*[9] have detected β groups with end points 710, 625, and 470 keV. They have estimated the intensities of these groups from γ-intensity balances. But Baskova *et al.*,[10] using both single and coincidence magnetic spectrometers, have identified only three β groups: 1335, 635, and 335 keV. We have attempted to detect these additional β groups by the usual subtraction analysis. Reliable information on the inner groups can be obtained by a Fermi-Kurie (FK) analysis, provided the outer groups are linearized with their true shapes. The FK plot of the highest energy group decaying to the ground level of Re^{187} was reported to be first-forbidden unique ($\Delta J=2$, yes) by Cork *et al.*[1] and Dubey *et al.*,[2] whereas Gallagher *et al.*[3] and Bisgaard *et al.*[5] report a linear FK plot. With a view to resolving this ambiguity and also obtaining detailed information about this transition, a precise analysis of the shape of this group is undertaken in the present study. A complete study of the conversion-line energies, intensities, and internal-conversion coefficients (ICC) was reported by Bisgaard *et al.*[5,8] Their evaluation of the conversion coefficients was based on normalization between the conversion-line intensities of their work and the photon intensities of Gallagher *et al.*,[3] assuming that the 686-keV transition is a pure $E1$ with $\alpha_k=0.00323$. In the present work, we have normalized our conversion-line intensities with the γ intensities of Gallagher *et al.*[3] through the intense 479-keV transition, which is known to be pure $E2$ with $\alpha_k=0.018$.

A measurement of the energy dependence of the shape of the once-forbidden nonunique transitions can contribute information about the nuclear matrix elements. The theoretical shape factor[12] can be expressed, in general, as $C(W)=k[1+aW+(b/W)+CW^2]$, where k, a, b, and c are functions of matrix-element parameters. The nuclear matrix elements which contribute to first-forbidden nonunique transitions ($\Delta J=\pm1$, yes) are $\eta x=-C_v\int \mathbf{r}$, $\eta\xi' y=-C_v\int i\boldsymbol{\alpha}$, $\eta u=C_A\int i\boldsymbol{\sigma}\times\mathbf{r}$, and $\eta z=C_A\int B_{ij}$. A factor $\xi'=(1/4R)$ is attached to the relativistic matrix elements to make them of order unity. In addition to the above, two more matrix elements, viz., $\eta w=C_A\int\boldsymbol{\sigma}\cdot\mathbf{r}$ and $\eta\xi' v=C_A\int i\gamma_5$ contribute to the $\Delta J=0$ (yes) transitions. The β-decay observables are expressed in terms of x, y, u, and z and their combinations $Y=\xi' y-\xi(u+x)$ and $V=\xi' v+\xi w$. In the ξ approximation, the constant term of order ξ^2 [$\xi=(\alpha z/2R)$] dominates, and the transitions consistent with ξ approximation exhibit statistical shape. When there is a suppression of the constant term due to cancellation among matrix-element combinations, the energy-dependent terms involving a, b, and c show up. The energy-dependent terms can also arise whenever a selection-rule effect such as j forbiddenness or K forbiddenness attenuates lower-rank matrix elements, except the B_{ij}, thereby leading to a domination of z, V, and Y. In both cases, the transition deviates from the ξ approximation and is characterized by large $\log ft$ values and shape deviation. If one assumes the B_{ij} matrix element to be dominant, then the transition can be analyzed by modified B_{ij} approximation[12] which requires

$$V\sim Y\sim z\gg x\sim u\sim y\sim w$$

and the shape can be described by $C(W)=q^2+9L_1/L_0+D$, where $q=W_0-W$ and $D=12Y^2$. W_0 and W are transition and electron energies. L_0 and L_1 are com-

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binations of electron radial wave functions defined in Ref. 13. The $C(W)$ of Sb^{124}, for which j selection is known to be operative, increases by 20% per m_0c^2, and the analysis with the modified B_{ij} approximation yields $z/Y=1.23$. For $W^{187}(\frac{3}{2}-)\rightarrow Re^{187}(\frac{5}{2}+)$ decay, $C(W)$ increases by 15% per m_0c^2, and $z/Y=0.72$ (Sec. V). The enhancement of B_{ij} matrix element is less pronounced in this case. For this case, the decaying neutron and proton belong to different major shells, and $\Delta j=\Delta K=1$. Hence j and K forbiddenness do not apply. But the Nilsson quantum numbers, characterizing the initial and final levels, introduce additional selection rules which have different effects on different matrix elements. Even though these selection rules are not as rigorous as j-selection rules, the Nilsson model predicts matrix-element ratios which satisfactorily explain the observed shape deviation.

II. APPARATUS AND CONTROL EXPERIMENTS

A Siegbahn-Slatis β-ray spectrometer is used to investigate the β spectrum. The work of Paul[14,15,16] compares the scattering properties of the various types of spectrometers and assigns a negligible scattering effect to the Intermediate-image focusing spectrometer. We have confirmed this conclusion of Paul by direct measurement of scattering effects and also by a study of line-shape parameters.[17] Of the various effects which distort a β continuum, the instrumental distortion arises from the scattering of electrons within the instrument and, as such, is related to the constructional aspects of the instrument and hence cannot be eliminated by the experimenter. The reason for the anti-scattering property of this spectrometer is that it requires, by virtue of its strong field gradient, only two baffles (the entrance baffle and the central baffle), whereas the ordinary lens spectrometers and flat types require many baffles to define the electron trajectory.

The scintillation detection is preferred to a Geiger-Müller counter, since it provides a further energy selection at the detector and serves to discriminate the electrons of momentum selected by the spectrometer from the scattered electrons and the counter background. We have used an NE 102 plastic well of suitable geometry whose back-scattering and detector noise are negligible compared to an anthracene or a semiconductor detector. The plastic is coupled to a high-gain photomultiplier, namely, 6810 A, outside the pole piece by a short Lucite pipe. For the discrimination levels used in the present work, the back-scattering correction amounted to 0.2% at 80 keV and the counting efficiency was unity down to 50 keV. The LKB Siegbahn-Slatis β-ray spectrometer has the advantage that its annular slit can be closed from outside and reproduced at will. This enabled us to determine the source-dependent background at every measurement point. In most reported literature, the background spectrum is constructed from the count-rate at zero field and the spectrometer current-setting beyond the endpoint. This procedure is ambiguous, because the source-dependent background arising from the photons varies with the current-setting.

We found that the scatter of points in the shape-factor plot is more than could be warranted by mere counting statistics and is due to the limited precision of the current-measuring device. In the present work we employ a high-precision Leeds and Northrup K-type potentiometer whose linearity has been carefully checked.

An exact resolution correction[17] is applied without making any assumption concerning the line shape:

$$C_R=1-\left\{0.0001692\frac{p}{N}\frac{dN}{dp}+0.00002539\frac{p^2}{N}\frac{dN^2}{dp^2}\right\}.$$

The line-shape parameters in the above expression, as well as the transmission and resolution of the spectrometer, were found energy-independent, indicating the absence of any scattering in the spectrometer or source.

The source material was obtained from Bhabha Atomic Research Centre, Trombay, where it was prepared by neutron irradiation of enriched W^{186} metal in the CIRUS reactor. No foreign activities were found in the source, except very small traces of long-lived W^{181} which decays by 100% electron-capture. All data were taken early in the decay of each source. The sources were prepared by evaporation of sodium tungstate on thin Zapon films of 200 $\mu g/cm^2$ thickness. The sources were of 50 $\mu g/cm^2$ thickness and 1–2-mm diam. The source-centering was checked carefully.

III. EXPERIMENTAL PROCEDURE

As a check on the spectrometer response, we measured the shape of Y^{90}. A least-squares fitting of the experimental points (Fig. 1) showed the form factor of Y^{90} to be $(q^2+9L_1/L_0)[1-(0.001\pm0.003)W]$. This indicates that the spectrometer is free from any distorting effect to within 0.3%. Before we could ascribe any eventual limit on the spectral distortion, we eliminated all possible uncertainties in the method of analysis. Our investigation[18] of the 965-keV transition of Au^{198} shows that the same data analyzed with Fermi functions taken from National Bureau of Standards (NBS) tables[19] and with those from Dzhelepov and

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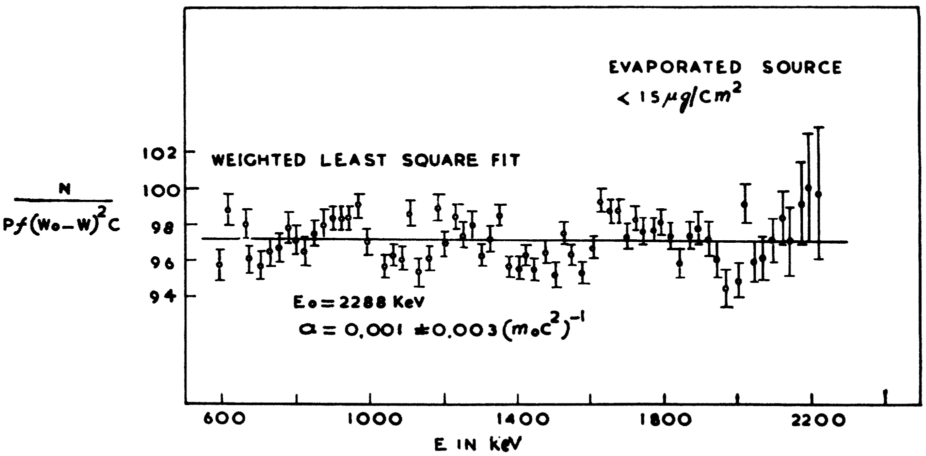

FIG. 1. Small-order shape factor of Y^{90}.

Zyrianova[20] (corrected for finite size) differ by 3% in the slope. The use of different Fermi functions and screening corrections extrapolates the FK plot to a slightly different W_0, which can cause discrepancies of the above magnitude. We have used the exact Fermi function $f=F_0L_0=\frac{1}{2}(f_{+1}^2+g_{-1}^2)$ (s.c.), where f_{+1} and g_{-1} are the ERWFs of Bhalla and Rose,[13] and s.c. is the screening correction of Bühring.[21] The factor $9L_1/L_0$ is again corrected for screening according to the tables of Bühring.[21] A FORTRAN program FERMKURI calculates the momentum and energy of each point, interpolates the functions f_1, g_{-1}, f_2, and g_{-2} of Bhalla and Rose tables, and applies decay correction, resolution correction, back-scattering correction, and efficiency correction (below 50 keV only). The program draws the FK plot and determines the endpoint energy.

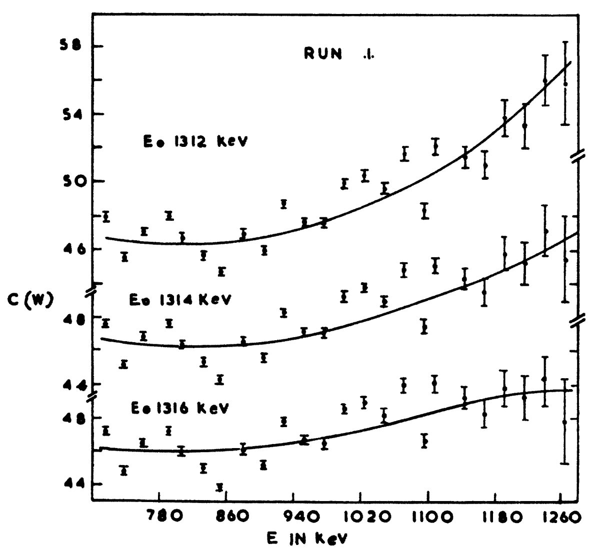

FIG. 2. Shape factor of the 1314-keV β component of W^{187}. The figure shows the changes in the slopes of the curve near the end point when it is varied around its true value.

[20] B. S. Dahelepov and L. N. Zyrianova, *The Influence of the Atomic Electric Field upon Beta Decay* (USSR Academy of Sciences, Moscow, 1956).

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Shape of the Highest-Energy β Group

We have subjected the energy region 700–1280 keV to the shape-factor analysis. Figure 2 shows the behavior of the shape factor in the neighborhood of W_0, where it blows up or down when the endpoint energy is changed from its genuine value. On account of the curvilinear nature of the shape, a precise determination of W_0 in this way is difficult. In a detailed analysis shown in Fig. 3 we have applied the criterion that the energy-independent factor $C(W)/(q^2+9L_1/L_0+D)$ should be linear near W_0 for correct values of W_0 and its

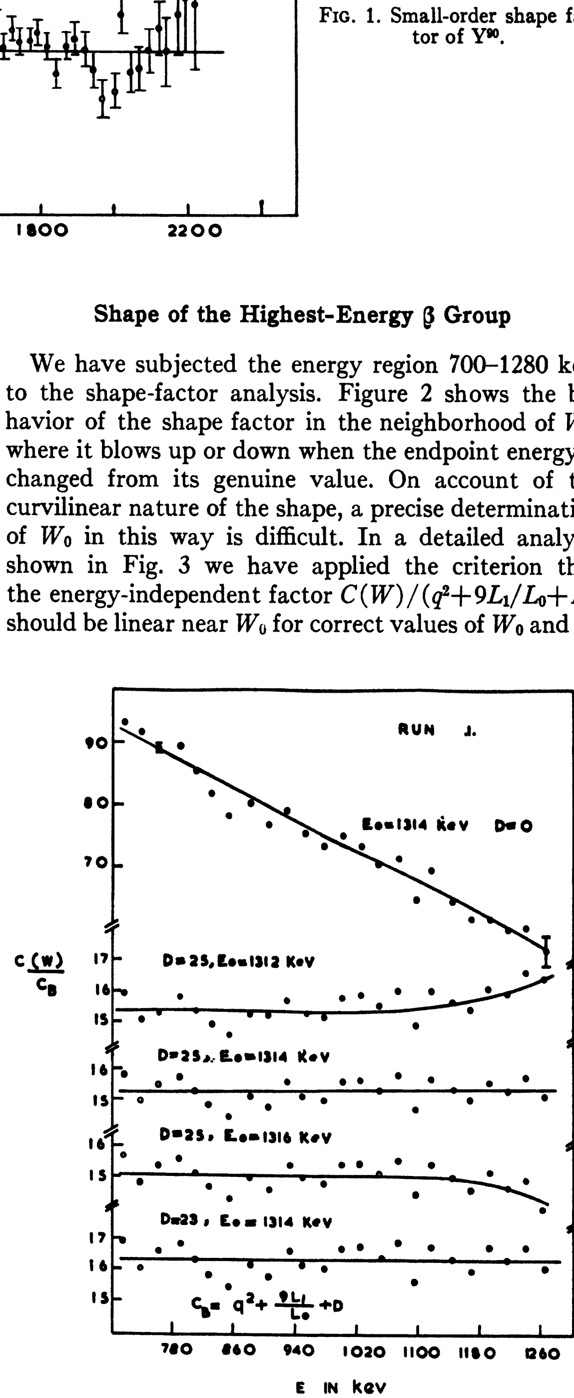

FIG. 3. Shape of the 1314-keV β component of W^{187} corrected for modified B_{ij} shape. The least-squares-fitted straight line to the corrected plot has small positive and negative slopes for $D=25$ and $D=21$, respectively. The slope vanishes for $D=23$.

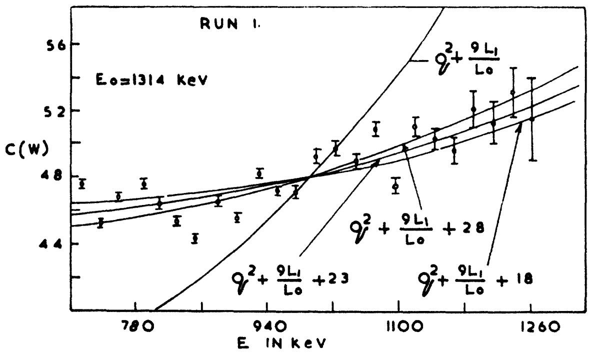

FIG. 4. Shape factor of the 1314-keV ($\frac{3}{2}^- \to \frac{5}{2}^+$) component of W^{187}.

slope should vanish for correct values of D. Figure 3 shows that this shape factor curves up or down at the high-energy side when W_0 is changed from its true value. On the other hand, a change of D shifts the least-squares-fitted line from the low-energy side, thereby changing its slope. A program called BETASHAP evaluated W_0 and D by a nonlinear weighted least-squares fitting. These results are in agreement with those obtained by the graphical method and are given in Table I.

Figure 4 shows the actual shape factor for the correct endpoint, along with the unique and "modified B_{ij} shapes" for different values of D. For a value of W_0 slightly smaller than the correct value, the experimental curve $C(W)=N/pf(W_0-W)^2$ shifts up, whereas q^2 in the modified B_{ij} shape shifts down. Hence the value of D required to fit the experimental points will be too large. On the other hand, the experimental value of D will be too small when W_0 is slightly larger than the genuine value.

TABLE I. W_0 and D as obtained from a computer least-squares fit to the data.

Parameters	Run 1	Run 2	Run 3
W_0 (keV)	1314±2	1316±2	1314±2
D	23±4	24±3	23±3

Analysis of β Branches and Conversion Lines

The FK plot corrected with the shape factor $C(W)=q^2+9L_1/L_0+23$ was used in order to subtract the high-energy group from the total spectrum (Fig. 5). The result of this subtraction yielded the spectrum for the next inner groups. The energy regions just above 625 and around 550 keV are obliterated by the existence of many conversion lines. Hence no information could be obtained for the 710- and 550-keV β groups. Successive subtractions yielded groups with endpoints 625 (66%), 453 (3%), and 330 (5%) keV. The 330-keV group will demand a level around 980 keV for Re^{187}, but this group has not been detected by Funke *et al.*[9] A thorough coincidence work with all the photons depopulating the higher-lying levels will help in establishing this group. Levels up to 947 keV have been postulated by Bashandy *et al.*,[11] but there is considerable ambiguity as to the position of most transitions in the decay scheme, so that the level structure is not well established. An intensity of 2% for the 710-keV group will be consistent with our present analysis. A subtraction of the 710-keV group with higher intensity changes radically the end point of the 625-keV group which is well established. Table II gives results of the present work as an average of three different runs along with intensities and log *ft* values beside the results of Funke *et al.*[9] and Baskova

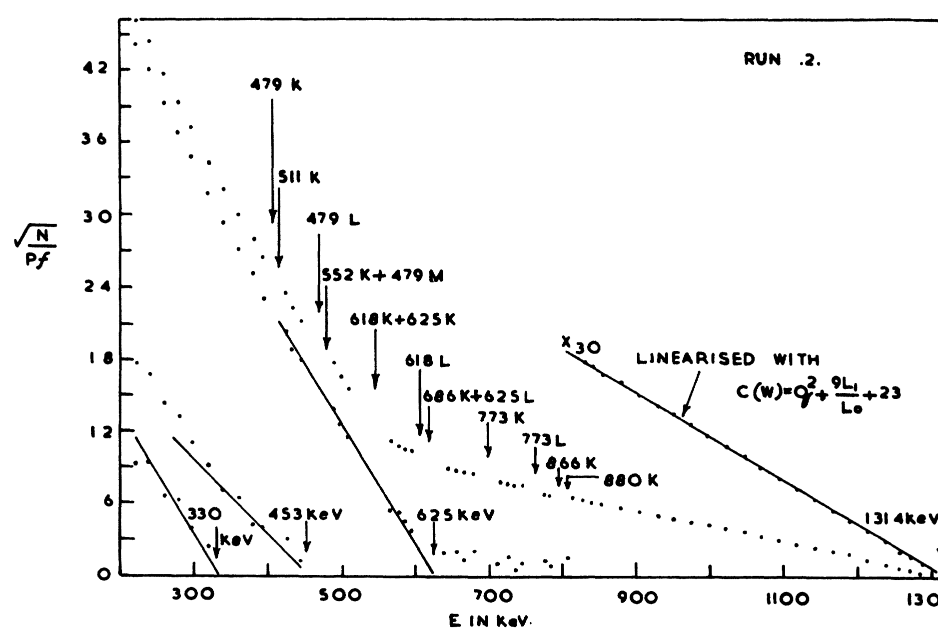

FIG. 5. Fermi-Kurie analysis of the total spectrum of W^{187}.

TABLE II. Data for the β decay of W^{187}.

β group	Funke *et al.*[a] Endpoint keV	Funke *et al.*[a] Intensity[d] %	logft	Baskava *et al.*[b] Endpoint keV	Baskava *et al.*[b] Intensity %	Bashandy *et al.*[c] Endpoint keV	Bashandy *et al.*[c] Intensity[d] %	Present work Endpoint keV	Present work Intensity %
	1320	18	8.0	1325±40	20	1330±5	18.3	1314±2	18
	800[e]	0.4	9.0	···	···	···	···	···	···
	710±40[f]	8.5	7.5	···	···	···	···	···	···
	650±30[f]	66	6.3	635±30[e]	70	629±6	73.8	625±5	66
	550±30[f]	5.7	7.3	···	···	···	···	···	···
	470±40[f]	1.28	7.8	···	···	···	···	453±7	3
	···	···	···	335	10	323±5	7.8	330±10 (logft=6.4)	5

[a] Reference 9.
[b] Reference 10.
[c] Reference 11.
[d] Obtained from gamma-intensity balances.
[e] Existence suggested from level scheme.
[f] Detected by beta-gamma coincidence work.

et al.[10] The resolution correction applied here is more realistic than that of Owen and Primakoff[22] and the endpoint energy is determined from the proper physical behavior of $C(W)$. In view of these considerations, the value of W_0 so determined, namely, 1314±2 keV, can be taken as the accurate Q^- value of W^{187}.

The spectrometer resolution being 1.3% could not resolve many of the lines. Table III gives both the line energies and their intensities expressed as a percentage of the total β disintegration. The transition energies calculated from the conversion-line positions agree well with the decay scheme (Fig. 6) and the recent measurements with Ge(Li) detectors.[23] The ICC of a few well-resolved lines were determined by normalizing our conversion-line intensities with the photon intensities of Gallagher *et al.*[3] through the K-conversion coefficient ($\alpha_k = 0.018$) of the 479-keV γ ray, which is a pure $E2$. Table III shows very good agreement between experimental and theoretical conversion coefficients for the expected multipoles. The α_k value of the 866-keV transition reported by Bashandy *et al.*[11] favors an $E2$ assignment to the 866-keV transition and this contradicts the $M1$ assignment in the present work as well as that of Gallagher *et al.*[3] and Birgaard *et al.*[8] The 511- and 625-keV transitions being pure $E2$, a spin of $\frac{1}{2}$ or $\frac{9}{2}$ is possible for both the levels. The assumption of $\frac{1}{2}^+$ or $\frac{9}{2}^+$ for the 511-keV level will lead to two mutually exclusive sets of spins ($\frac{3}{2}^+, \frac{1}{2}^+, \frac{3}{2}^+$) or ($\frac{7}{2}^+, \frac{9}{2}^+, \frac{9}{2}^+$) for the 618-, 625-, and 865-keV levels. The adoption of the set of high spins for the above levels by Bashandy *et al.*[11] is incompatible with the absence of any branching from the 511- and 625-keV levels to the 134-keV ($\frac{7}{2}^+$) level. Also the inelastic scattering of deuterons and alpha particles[24] and the results of Coulomb excitation experiments[25] on Re^{187} indicated that the 511-keV level is γ-vibrational, characterized by $K = K_0 - 2 = \frac{1}{2}$ with a large admixture of the $\frac{1}{2}+[400]$ state. The $K = K_0 + 2 = \frac{9}{2}^+$ built on the ground state of Re^{187} was observed at 840 keV. With the assumption of spin $\frac{1}{2}^+$ to the 511-keV level, the β branch to this level is first-forbidden with $\Delta J = 1$. The intensity of this β branch $\leq 0.4\%$, deduced from γ-intensity balances, leads to a logft value ≥ 9. This logft value is not inconsistent with the systematics of logft values of β feedings to vibrational levels, which normally involve strong configuration mixing.

IV. INTERPRETATION OF THE SHAPE OF THE 1314-keV β COMPONENT ON THE BASIS OF THE NILSSON MODEL

The idea of using the Nilsson model to interpret the β decay of W^{187} stems from the fact that the level scheme of Re^{187} has been explained on this basis, except the 511- and 686-keV levels which are γ vibrational. For strongly deformed odd-A nuclei, the energy levels

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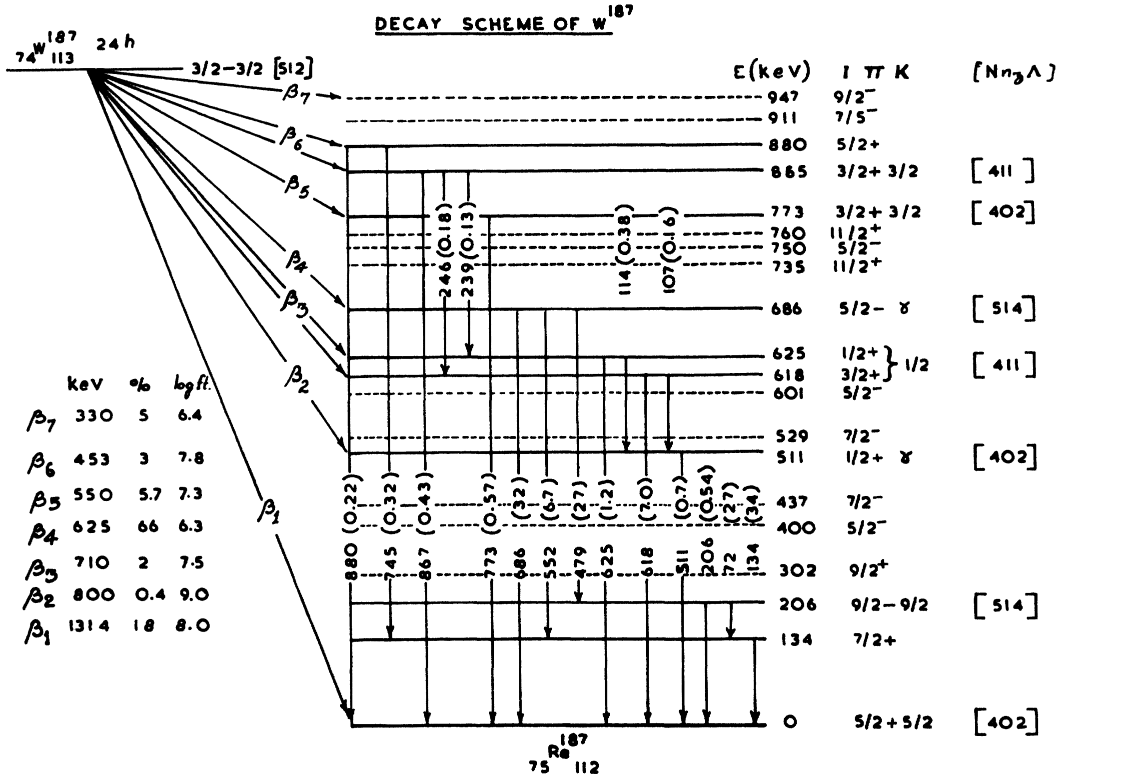

FIG. 6. Decay scheme of W^{187}. The decay scheme is due to Funke *et al.* (Ref. 9) and Bashandy *et al.* (Ref. 11) The β endpoints and intensities obtained from the present work are combined with the information deduced from the level scheme. The Nilsson-state identification in terms of asymptotic quantum numbers are given in square brackets.

TABLE III. Data on conversion lines of W^{187}. Binding energies for Re^{187} were taken to be $E_K = 71.7$ keV and $E_L = 12.5$ keV.

Conversion-line energy keV	Transition energy keV	Conversion-line intensity (% of total β activity)	K/L	γ intensities of Gallagher *et al.*[a,b]	Experimental ICC	Theoretical ICC[c]
407±2	478.7K	0.48	4	26.67		($E2$) = 0.018
439±2	511K	0.014	···	0.69	0.02±0.003	($E2$) = 0.016
466±3	478.7L	0.12	···	···	···	···
480±4	552K+479M	0.065	···	···	···	···
546±3	618K+625K	0.23	···	···	···	···
605±3	618L	0.033	···	6.93	0.0048±0.003	($M1$) = 0.0045
614±5	686K+625L	0.12	···	···	···	···
701±3	773K	0.066	6.47	4.32	0.0153±0.003	($M1$) = 0.017
760±4	773L	0.0102			0.0024±0.005	($M1$) = 0.0025
794±3	866K	0.0058		0.411	0.0141±0.006	($M1$) = 0.013
807±3	880K	···	···	···	···	···

[a] γ intensities of Gallagher *et al.* are normalized to the present conversion-line intensities through α_k of the 479-keV transition.
[b] Reference 3.
[c] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Co., Amsterdam, 1958).

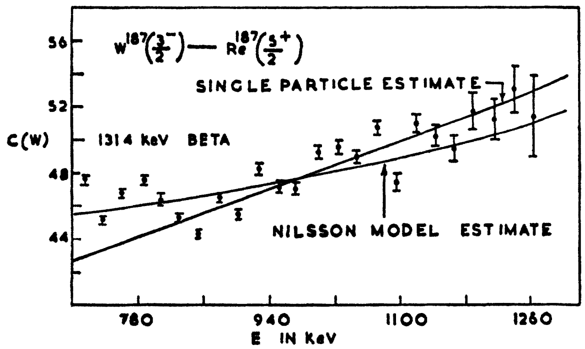

FIG. 7. Shape factor of the 1314-keV β component of W^{187}. The theoretical shape factors for single-particle as well as Nilsson model predictions are also shown. At lower energies, the single-particle shape factor distinctly deviates from experimental shape, whereas Nilsson-model estimates are in good agreement.

have been successfully described as single-particle states in an axially symmetric potential, with rotational states based on them[26,27] but the isobars W^{187} and Re^{187} lie on the outskirts of the deformed region $140 \leq A \leq 190$, and the deformation $\delta \approx 0.19$ is small and, therefore, the assumption of strong coupling may be uncertain. A study of the β decay of W^{187} will be useful for testing the predictions of the Nilsson model near the limits of the region where the strong coupling theory is known to be applicable.

Theoretical expressions for both relativistic and nonrelativistic nuclear matrix elements for β transitions of arbitrary forbiddenness using Nilsson wave functions have been developed by Bodgan[28,29] and Tuong *et al.*[30] for one- and two-particle configurations, respectively, in a deformed potential. We use the expressions of Bogdan for the Kotani nuclear matrix-element parameters.[12] The matrix elements which contribute to the present case of $\Delta J = 1$ and x, u, z, and Y. By setting $z = 1$, the standard matrix element is given by $\eta = C_A \int B_{ij}$ and $f_c t = \pi^3 \ln 2 / |\eta|^2$. W^{187} with 113 neutrons and an intrinsic state of $p_{3/2}$ can be assigned from the Nilsson diagram[26] the Nilsson state $\frac{3}{2}-[512]$. The 75th proton of Re^{187} is in a $d_{5/2}$ state and is characterized by the Nilsson level $\frac{5}{2}+[402]$. The states are labeled by the quantum numbers in the asymptotic limit of Nilsson orbitals and the wave function in terms of Nilsson amplitudes $a_{l\Lambda}$ is written as

$$X_\Omega = K\pi[N, n_z, \Lambda] = \sum_{l,\Lambda} a_{l\Lambda} \, | N l \Lambda \Sigma \rangle \tag{1}$$

with the normalization

$$\sum_{l,\Lambda} a_{l\Lambda}{}^2 = 1.$$

The intitial and final wave functions of the present case are

$$X_{\Omega=3/2} = a_{51} \, | 551+ \rangle + a_{31} \, | 531+ \rangle + a_{11} \, | 511+ \rangle + a_{52} \, | 552- \rangle + a_{32} \, | 532- \rangle$$

and

$$X_{\Omega=5/2} = a_{42} \, | 442+ \rangle + a_{22} \, | 422+ \rangle + a_{43} \, | 443- \rangle.$$

From the expressions of Bogdan,[28,29] it is seen that the Clebsch-Gordan coefficients appearing in the reduced matrix elements of $\int \mathbf{r}$ and $\int i\boldsymbol{\alpha}$ impose certain conditions to the quantum numbers Λ_i, Λ_f, $k = K_f - K_i$ and $q = -K_f - K_i$:

$$k = \Lambda_i - \Lambda_f \quad \text{if} \quad \Sigma_i = \Sigma_f$$
$$q = \Lambda_i + \Lambda_f \quad \text{if} \quad \Sigma_i = -\Sigma_f. \tag{2}$$

As no pair of quantum numbers Λ_i and Λ_f occurring in (1) satisfies at least one of the conditions (2), the matrix elements $\int \mathbf{r}$ and $\int i\boldsymbol{\alpha}$ vanish. The two remaining matrix elements are contributed by only those pairs of initial and final Nilsson amplitudes for which $\Sigma_f - \Sigma_i = 1$ and $l_i + 1 \geq l_f \geq l_i - 1$, and the ratio z/u comes out independent of the Nilsson amplitudes in the present case. The matrix-element ratios are therefore given by

$$x/z = 0, \qquad y/z = 0, \qquad u/z = -0.76$$

and

$$Y = 11.76z. \tag{3}$$

All β-decay observable quantities except ft values depend only on the ratios of matrix elements. The dependence of nuclear matrix elements on the model parameters (η, μ) occurs only through Nilsson base vectors of the initial and final states. η is defined[26] as $\eta = (\delta/\chi)[1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3]^{-6}$; μ determines the relative strengths of the spin-orbit and l^2 terms in the single-particle Hamiltonian. Nilsson has assigned values of μ for each N shell to both proton and neutron levels in order to reproduce the proper sequence of shell-model levels for zero deformation. For W^{187}, the transforming nucleon is initially in a neutron level from the shell $N_i = 5$ and, finally, in a proton level from the shell $N_f = 4$. For these, Nilsson[26] has assigned the values $\mu_i = 0.45$, $\chi_i = 0.05$, $\mu_f = 0.55$, and $\chi_f = 0.0613$, and has calculated the corresponding expansion coefficients. These coefficients are used to calculate the ft value of the transition under consideration. Berthier and Lipnik use $\mu = 0.7$ for the initial neutron level for $N_i = 5$ for Tm^{170}. However, this does not appear to be appropriate, since Nilsson and Mottleson[27] specify that the calculation with $\mu = 0.7$ is valid only for proton levels. However, Tuong *et al.*[30] have calculated matrix elements of Tm^{170} and Re^{186} decays with $\mu_i = 0.45$ $(N_i = 5)$ and $\mu_f = 0.55$ $(N_f = 4)$ and their results are in good agreement with experiment. One should normally vary both η and μ of the initial and final states in order to obtain a fit with experimental data. But this is not indispensable in the

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present case, as there exists only one experimental quantity, namely, the ft value, which is dependent on (η, μ), and a fit can be obtained by changing η_i or η_f or both.

From a comparison of theoretical and experimental values of magnetic and quadrupole moments, a deformation $\delta \approx 0.19$ has been assigned for the ground state of Re^{187}. Using Nilsson's[27] value of $\chi_f = 0.0613$ for the $N_f = 4$ shell, we obtain $\eta_f \approx 3$. Keeping $\eta_f = 3$, the calculations were performed for $\eta_i = 2$, 3, and 4. The best fit with experiment could be obtained for $\eta_i = 3$. $\eta_i = 3$ does not necessarily mean a modification of the deformation parameter in view of the relation for η. The relation $\eta = C_A \int B_{ij}$ is used in computing the theoretical ft value:

$$(f_c t)_{\text{Nilsson}} = \pi^3 \ln 2 \Big/ \left| C_A \int B_{ij} \right|^2$$

$$= 10^{8.2} \text{ sec.}$$

This is to be compared with $(ft)_{\text{exp}} = 10^{8.0}$ sec for the transition under study. In order to correct the experimental ft value for the energy dependence of the shape factor, Moszkowski's[31] definition of the "Fermi integral f" is modified to

$$f_c = \int_1^{W_0} F_0(Z, W) C(W) p W (W_0 - W)^2 dW.$$

The integral f_c is numerically evaluated using the experimental shape factor and $F_0(Z, W)$ of Bhalla and Rose.[13] This gives $(f_c t)_{\text{exp}} = 10^{8.17}$ sec which is in excellent agreement with the Nilsson prediction.

From the above results, we first draw the conclusion that in the case of W^{187}, the Nilsson model yields matrix elements which are not in agreement with the ξ approximation. This conclusion also supports the relatively large experimental value of $\log ft = 8.0$. In the second place one notices that the value of u is not small as expected on the basis of "modified B_{ij} approximation" and the vector-type matrix elements x and y vanish due to the additional selection rules.

In Fig. 7 the theoretical shape factor, calculated from Nilsson matrix elements using Simms's expressions,[32] is compared with the experimental shape factor. The experimental shape factor increases by 15% in the energy region 700–1280 keV, while the shape due to the Nilsson scheme increases by 11% only. The matrix elements of this transition are also calculated on the basis of the j-j coupling shell model, employing the general calculus of Rose and Osborne.[33] These are

$$x = 0.389, \qquad u = -0.463, \qquad z = 1$$

and

$$Y = 15.61.$$

The shape factor due to these matrix elements increases by 24% and is also shown in Fig. 7.

V. CONCLUSION

The 1314-keV transition of W^{187} exhibits a "selection-rule" effect and an analysis under the "modified B_{ij}" shape

$$C(W) = (z^2/12)(q^2 + 9L_1/L_0) + Y^2 \propto q^2 + 9L_1/L_0 + D$$

is also made. In Fig. 4 a fit is made with experimental shape for various values of D. We get $D = 23 \pm 3$, which gives $Y/z = 1.33 \pm 0.04$. But the experimental shape factor can be simulated by any combination of matrix-element parameters. Hence an individual determination of matrix elements through more experimental observables will be helpful. From the above considerations it follows that besides the "K-selection rule" mentioned by Kotani,[11] in the expression for reduced matrix elements corresponding to the first-forbidden transitions in the Nilsson one-particle model, different restrictions regarding the quantum numbers l_i, l_f, Λ_i, Λ_f, Σ_i, and Σ_f appear, which may have sometimes important effects as in the decay of W^{187}. To obtain conclusive evidence as to whether or not x and y are attenuated, as predicted by the Nilsson model, more experimental information is necessary. Since this is a ground-state transition, correlation experiments are not possible. Measurement of longitudinal polarization will not serve any useful purpose either, since longitudinal polarization is $-p/W$ for both the ξ approximation and the modified B_{ij} approximation. Even if the Nilsson predictions were strictly true for W^{187}, the longitudinal polarization will be only $1.03(-p/W)$, which is impossible to verify. But the electron distribution from oriented nuclei can throw much light on the problem.

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